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Cyclified Cohomotopy Flux Quantization of IIA Sugra via superspace dim. reduction of Hypothesis H

joint work w Hisham Sati, Urs Schreiber

G., Sati, Schreiber 24' [2403.16456] JHEP ⇝ 11D Sugra on superspace

G. 25' [2507.00140] LMP ⇝ Cou. hse der.

G., Sati 26' [260....] (soon) ⇝ 10D IIA via 11D Sugra.

Flux Quantization via Rat. Hom. Theory

- Equivalence [Quillen, Sullivan ...]

$$\mathrm{Ho}\,\mathrm{Rat\,Top}^{\text{f.c. un.}}_{\mathbb{Q}\to\mathbb{R}} \xrightarrow[L_{\mathbb{R}}]{\sim} \mathrm{Ho}\,\mathrm{hoo\text{-}Alg}^{\geq 0,\ \text{f.d. un.}}_{\mathbb{R}} \xrightarrow[CE]{\sim} \mathrm{Ho}\,\mathrm{dgcA}\mathrm{lg}^{\infty\ \geq 0}_{\mathbb{R}}$$

- Fluxes $\{F^i\}_{i\in I_{\text{finite}}} \subset \Omega^{\bullet}_{dR}(M)$ of a field theory

satisfy Bianchi identities

$$d_{dR} F^i = P^i(\underline{F})$$

- $\Longrightarrow$ identifies dg c.a.lg.

$$\mathbb{R}_{\mathrm{d}}[\![\varepsilon f^i]\!] \Big/ _{\mathrm{d}f^i = P^i(f)} =: CE(\mathfrak{a})$$

- so that

$$\vec{F}: \quad TM \xrightarrow{\text{loo-algebraic map}} \mathfrak{a}$$

$$\downarrow \qquad\qquad \downarrow$$

$$F^i \longleftarrow f^i$$

i.e. dually $\quad \Omega^{\bullet}_{\mathrm{dR}}(M) \longleftarrow CE(\mathfrak{a})$

Equivalently

- $\vec{F} : M \longrightarrow \Omega^1_{dR}(-; \mathfrak{a})_{cl}$ — classifying space $\in Fun(Corr, Set)$

- $\exists$ "cohomology" (via concordances)

$$\Omega^1_{dR}(-, \mathfrak{a}) \twoheadrightarrow H^1_{dR}(-; \mathfrak{a})$$

"non-abelian" [Fiorenza, Sati, Schreiber 23']

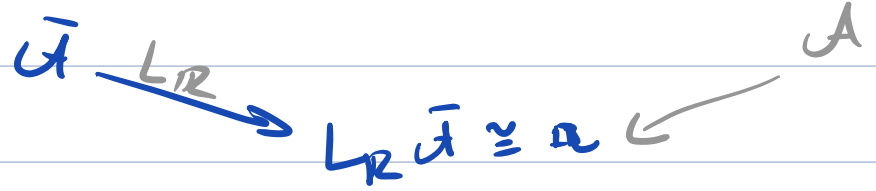
- And diff character

$$\mathcal{A} \longrightarrow \int \Omega^1_{dR}(-, \mathfrak{a})_{cl}$$

(w $Lie(G) \cong \mathfrak{a}$)

∞-groupoid of deformations

- ∃ infinitely many rationally equivalent spaces $\vec{\mathcal{A}} \in \mathrm{Top}$ w



Def

A "flux quantization law" for

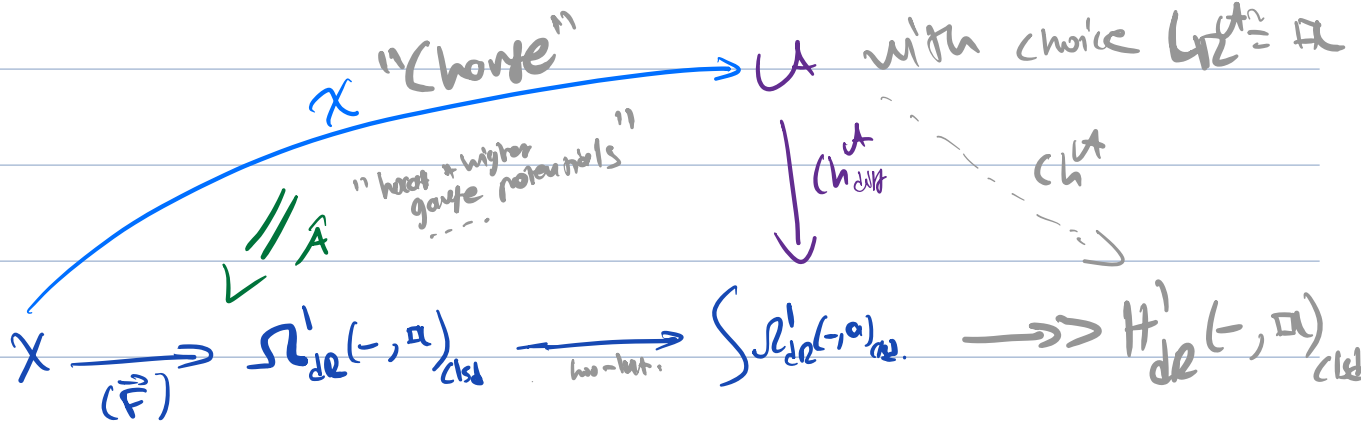
$$\vec{F} : M \longrightarrow \Omega^1_{dR}(-;\mathbb{R})_{cld}$$

is a <u>choice</u>

$\mathcal{A} \in \mathrm{Top}$ w

Def.

A global Flux/Charge quantized field conf. is $(\chi, \vec{F}, \hat{A})$



i.e.,

$$\mathcal{A}\text{-Quant.Fluxes} := \Omega^1_{dR}(-,\mathbb{R})_{cl} \times^{h}_{\Omega^2_{dR}(-,\mathbb{R})_{cl}} \mathcal{A}$$

Classifying space for

diff. form data

top. quantization data

Hypothesis H

- 11D Supergravity solutions $(e, \psi, \omega, G_4, G_7)$ (vielbein, gravitino, spin(1,10) gauge field.) on $X^{11} \in \mathrm{Mon}$ satisfy

$$dG_4 = 0$$

$$dG_7 = \tfrac{1}{2} G_4 G_4 + O(\psi)$$

(*) ω $G_7 = *G_4$

- $\mathbb{R}_d[q_4, q_7] \big/ \substack{dq_4 = 0 \\ dq_7 = \frac{1}{2} q_4 q_4} \cong CE(L_{\mathbb{R}} S^4)$

where $L_{\mathbb{R}} S^4 \simeq L S^4$ "rational 4-sphere"

Hypothesis H [Sati 13', Fiorenza-Sati-Schreiber 15'...]

The "C-field" is flux quantized in $\mathcal{A} = S^4 \in \mathrm{Top}$.

+ adjectives

Problem $G_4 = *_e G_7$? In dynamical e

Note: Also ignored in proposed twisted-M4 quantization for IIA/B, i.e. $F_{10-2k} = *_e F_{2k}$

- Moreover, interplay w/ other EOMs ?
  - Einstein sourcing
  - Fermionic corrections ?

- I.e. what is the full Flux-quantized phase space of 11D-Sugra ?

Flux quantization on 11D superspace

- Consider the super-Minkowski Lie alg.

$$\mathbb{R}^{1,10|32} \underset{\text{Sdm}}{\simeq} \mathrm{ISO}(\mathbb{R}^{1,10|32}) \Big/ \mathrm{Spin}(1,10)$$

dual to

$$CE(\mathbb{R}^{1,10|32}) := \mathbb{R}_{\mathrm{d}}[e^a, \psi^\beta] \Big/ \left(de^a = (\bar{\psi}\Gamma^a\psi) \right)$$

$$\underset{\mathrm{dR}}{\Omega^\bullet}(\mathbb{R}^{1,10|32}) \text{ c.i.}$$

[Fiorenza, Sati, Schreiber]

- ∃ (unique) $\mathbb{S}^4$-cocycle

$$\begin{array}{rcl} \mathbb{R}^{1,10\,|\,\mathbf{32}} & \xrightarrow{(G_4^{\bullet}, G_7^{\bullet})} & \mathbb{S}^4 \\ \frac{1}{2}\big(\overline{\psi}\,\Gamma_{a_1 a_2}\,\psi\big) e^{a_1} e^{a_2} =: G_4^{\bullet} & \longmapsto & g_4 \\ \frac{1}{5!}\big(\overline{\psi}\,\Gamma_{a_1 \cdots a_5}\,\psi\big) e^{a_1} \cdots e^{a_5} =: G_7^{\bullet} & \longmapsto & g_7 \end{array}$$

- Turns out that: (e, ψ, w, b_4, G_4)

(Full) 2 10 sugra sols on $X^{10,1}$ ∈ Man

⟹

special ∈ Global extensions of $\mathbb{S}^4$-cocycle on $X^{11|32}$ ∈ SMan

Def.

A **(11|32)-superspacetime** is a tuple

$$(X,\ (e, \psi, \omega))$$

where

(i) X is locally modelled on $\mathbb{R}^{11|32}$

(ii) (e, ψ, ω) is a **Cartan connection** (super-coframe, spin-connection (gauge field))

w.r.t. $\mathrm{Spin}(1,10) \longrightarrow \mathrm{ISO}(\mathbb{R}^{1,10|32})$

(nif) That is bosonic Torsion-free

$$de + \omega e = (\bar{\psi} \Gamma \psi)$$

Extends that model condition

$$de = (\bar{\psi} \Gamma \psi) \in CE(\mathbb{R}^{d+1,2})$$

Theorem [G., Sati, Schreiber 21, following Castellani, D'Auria, Fré 91 ...]

A $(11|32)$-superspacetime $(X, (e, \psi, \omega))$ carries a (G_4^s, G_7^s)-flux of the form

$$G_4^s := \frac{1}{4!}(G_4)_{a_1 \cdots a_4} e^{a_1} \cdots e^{a_4} + \frac{1}{2}\left(\overline{\psi}\Gamma_{a_1 a_2}\psi\right) e^{a_1} e^{a_2}$$

$$G_7^s := \frac{1}{7!}(G_7)_{a_1 \cdots a_7} e^{a_1} \cdots e^{a_7} + \frac{1}{5!}\left(\overline{\psi}\Gamma_{a_1 \cdots a_5}\psi\right) e^{a_1} \cdots e^{a_5}$$

bosonic curved contribution — flat cocycles

which is an $L\mathbb{S}^4$-cocycle $TX \longrightarrow L\mathbb{S}^4$

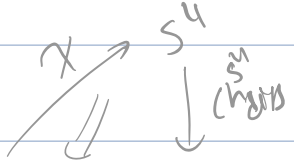
if and only if.

i) The restriction $i^*(e, \psi, \omega, G^S_4, G^S_7)$ to $\vec{X} \hookrightarrow X$ is a 7D sol.

ii) This uniquely extends to X.
"Rheonomically" [!] due existence of super-normal coords

Corollary 1 Geometrically natural (alternative) def. of (on-shell) 11D Sugra

Corollary 2 Applying S^4-flux quantization on $X^{11|32} \to \mathbb{R}^{1,2}(-15^4)$ shows:



i) Hodge duality not extra constraint to be imposed.

ii) Flux quant. compatible w • Einstein sourcing
• Fermionic sources

Dim. reduction to IIA superspace

- For $g \in \mathrm{hoo}\,\mathrm{Alg}$ (f.g. ≥ 0) w generators $\{e^i\}$ for $CE(g)$, $\mathrm{cyc}\,g \in \mathrm{hoo}\,\mathrm{Alg}$ is defined by

$$CE(\mathrm{cyc}\,g) = \mathbb{R}_d[e^i, \vec{s}e^i, w_2] \Big/ \begin{matrix} dw_2 = 0 \\ d\vec{s}e^i = -\vec{s}(d_g e^i) \\ d e^i = d_g e^i + w_2 \cdot \vec{s}e^i \end{matrix}$$

– Rationalization of $wX/\!/_{S^1}$ for $\mathbb{L}_{\mathbb{R}}X \simeq g$.

- $\exists$ adjunction $\mathrm{hoo}\,\mathrm{Alg} \underset{\mathrm{ext}}{\overset{\mathrm{cyc}}{\rightleftarrows}} \mathrm{hoo}\,\mathrm{Alg}_{/\mathbb{B}U(1)}$

– Rationalization of $\mathrm{Top} \to \mathrm{Top}_{/\mathbb{B}U(1)}$ $\quad \mathrm{Hom}_{\mathrm{Top}}(X \times \mathbb{B}U(1), Y) \simeq \mathrm{Hom}_{\mathrm{Top}_{/\mathbb{B}U(1)}}(X, Y)$

- Explicitly $\mathrm{Ham}_{\mathrm{loc}}(\hat{\mathfrak{h}}, g) \cong \mathrm{Ham}_{\mathrm{loc}/\mathrm{bun}}(\mathfrak{h}, \mathrm{cyc}(g))$

$(\mathrm{CE}(\hat{\mathfrak{h}}) := \mathrm{LE}(\mathfrak{h}) \cdot e / de = d_2)$

$f_2 \searrow \quad \swarrow \omega_2$

$\mathbb{b}\mathbb{u}(1)$

Example

$\mathrm{CE}(\mathbb{R}^{1,10|32}) \cong \mathrm{CE}(\mathbb{R}^{1,9|16\oplus\overline{16}}) \cdot e^{10} / de^{10} = (\bar{\psi}\Gamma^{10}\psi)$

$\mathbb{R}^{1,10\,|\,\mathbf{32}} \xrightarrow{(G_4, G_7)} \mathbb{L}S^4$

$\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2} \longmapsto g_4$

$\frac{1}{5!}(\bar{\psi}\Gamma_{a_1\cdots a_5}\psi)e^{a_1}\cdots e^{a_5} \longmapsto g_7$

$\mathbb{R}^{1,9\,|\,\mathbf{16}\oplus\overline{\mathbf{16}}} \xrightarrow{\mathrm{rdc}_{c_4^M}(G_4,G_7) = (F_2,F_4,F_6,H_3^A,H_7^A)} \mathrm{cyc}(\mathbb{L}S^4)$

$(\bar{\psi}\Gamma_{10}\psi) =: \mathrm{F}_2 \longmapsto \omega_2$

$\frac{1}{2}(\bar{\psi}\Gamma_{a_1a_2}\psi)e^{a_1}e^{a_2} =: \mathrm{F}_4 \longmapsto g_4$

$(\bar{\psi}\Gamma_a\Gamma_{10}\psi)e^a =: H_3^A \longmapsto \mathrm{s}g_4$

$\frac{1}{5!}(\bar{\psi}\Gamma_{a_1\cdots a_5}\psi)e^{a_1}\cdots e^{a_5} =: H_7^A \longmapsto g_7$

$-\frac{1}{4!}(\bar{\psi}\Gamma_{a_1\cdots a_4}\Gamma_{10}\psi)e^{a_1}\cdots e^{a_4} =: +F_6 \longmapsto \mathrm{s}g_7$

(rational)

not twisted K-theory

Prop "KK-reduction via cyclification" [G. Sati 26', following Fiorenza, Sati, Schreiber]

Let $Y \to X$ be an S^1 super-principal bundle w/ connection $e' \in \mathrm{Conn}_{U(1)}(Y)$.

Then bijection

(i) *S^1-invariant maps of super-L_∞-algebroids out of the tangent Lie algebroid TY into $\mathfrak{h}$,*
(ii) *maps of super-L_∞-algebroids out of $TX \cong T(Y/S^1)$ into $\mathrm{cyc}(\mathfrak{h})$ that preserve the curvature 2-form F_2,*

$$\left\{ TY \xrightarrow{F} \mathfrak{h} \right\}^{S^1\text{-inv}} \underset{\text{oxidation } \mathrm{oxd}_{F_2}}{\overset{\text{reduction } \mathrm{rdc}_{F_2}}{\underset{\sim}{\rightleftarrows}}} \left\{ TX \xrightarrow{\widetilde{F}} \mathrm{cyc}(\mathfrak{h}) \;\text{ with }\; F_2 \to b\mathbb{R} \leftarrow \omega_2 \right\} \tag{3}$$

given by

$$\begin{array}{ccc} TY & \xrightarrow{F} & \mathfrak{h} \\ F^i = F^i_{\mathrm{nw}} + e' \int_{S^1} F^i & \longmapsto & e^i \end{array} \quad \leftrightsquigarrow \quad \begin{array}{ccc} TX & \xrightarrow{\widetilde{F}} & \mathrm{cyc}(\mathfrak{h}) \\ F^i_{\mathrm{nw}} & \longmapsto & e^i \\ -\int_{S^1} F^i & \longmapsto & \mathrm{s}e^i \\ F_2 & \longmapsto & \omega_2 \,. \end{array} \tag{4}$$

(*) *This extends to a (strict) isomorphism of the corresponding ∞-groupoids of fluxes with their higher (globally defined)[3] concordances*

$$\int \Omega^1_{\mathrm{dR}}\big(Y;\mathfrak{h}\big)_{\mathrm{clsd,inv.}} \cong_{e'} \int \Omega^1_{\mathrm{dR}}\big(X;\mathrm{cyc}(\mathfrak{h})\big)_{\mathrm{clsd},F_2} \hookrightarrow \int \Omega^1_{\mathrm{dR}}\big(X;\mathrm{cyc}(\mathfrak{h})\big)_{\mathrm{clsd}} \in \mathrm{SimpSet}_{\mathrm{Kan}}\,.$$

Theorem [G., Sati 26']

A $(10|16\overline{16})$-superspacetime $(X, (e,\psi,\omega))$

carries a $(F_2^s, H_3^s, F_4^s, F_6^s, H_7^s)$-flux

bosonic curved flux — winding 10D term — (scaled) flat cocycles → on $\mathbb{R}^{10|16\overline{16}}$

$$
\begin{aligned}
F_2^s &:= \tfrac{1}{2}(F_2)_{a_1a_2}\, e^{a_1}e^{a_2} + \tfrac{2}{\Phi^2}(\overline{\psi}_X\Gamma_a\lambda)\, e^a + \tfrac{1}{\Phi}(\overline{\psi}\Gamma_{10}\psi)\\
H_3^s &:= +\tfrac{1}{3!}(H_3)_{a_1a_2a_3}\, e^{a_1}e^{a_2}e^{a_3} + (\overline{\psi}\Gamma_{a_1a_2}\lambda)\, e^{a_1}e^{a_2} - \Phi(\overline{\psi}\Gamma_{10a}\psi)\, e^a\\
F_4^s &:= \tfrac{1}{4!}(F_4)_{a_1\cdots a_4}\, e^{a_1}\cdots e^{a_4} + \tfrac{1}{2}(\overline{\psi}\Gamma_{a_1a_2}\psi)\, e^{a_1}e^{a_2}\\
F_6^s &:= +\tfrac{1}{6!}(F_6)_{a_1\cdots a_6}\, e^{a_1}\cdots e^{a_6} + \tfrac{2}{5!}(\overline{\psi}\Gamma_{a_1\cdots a_5}\lambda)\, e^{a_1}\cdots e^{a_5} - \tfrac{\Phi}{4!}(\overline{\psi}\Gamma_{10\,a_1\cdots a_4}\psi)\, e^{a_1}\cdots e^{a_4}\\
H_7^s &:= \tfrac{1}{7!}(H_7)_{a_1\cdots a_7}\, e^{a_1}\cdots e^{a_7} + \tfrac{1}{5!}(\overline{\psi}\Gamma_{a_1\cdots a_5}\psi)\, e^{a_1}\cdots e^{a_5}\,,
\end{aligned}
$$

where $\Phi \neq 0 \in C^\infty_{\mathrm{even}}(X)$ *and* $\lambda \in C^\infty_{\mathrm{odd}}\big(X, (\mathbf{16}\oplus\overline{\mathbf{16}})_{\mathrm{Spin}(1,9)}\big)$ *with*

$$
\mathrm{d}\Phi = \partial_a\Phi\, e^a_X + 2\big(\overline{\psi}_X\,\Gamma^{10}\,\lambda\big)\,,
$$

which is a cyclic 4-cocycle $TX \longrightarrow$ cyclic 4

if and only if.

i) The restriction to $\tilde{X} \hookrightarrow X$ is a IIA 100 Supra sol.

ii) This uniquely extends to X.

"Rheonomically"

Proof.

via Spin-Covariant Hit

sections of v.b. and locally def. gauge field.

a) Formulate S^1-invariance for (e_Y, ψ_Y, ω_Y) on $Y \in \mathrm{SMan}$

via Yosmean Hit derivative

$$\overset{\kappa}{\nabla} e_Y := \omega_3 e_Y + \underset{3}{B}(e) \triangleright e_Y$$

Each separately defined only locally

b) Partially gauge fix $\mathrm{Spin}(1,9) \hookrightarrow \mathrm{Spin}(1,10)$ [exists Remma 6.26']

s.t. $e_Y = (e_X, \phi \cdot e^1)$ $\quad \psi_Y = \psi_X + \lambda \cdot e^1$

for $e^1 \in \Gamma_{U(1)}(Y)$, (e_X, ψ_Y) super coframe, (ϕ, λ) superised dilaton/inio } on X

with $(h_3^k e_4, h_3^k \psi_4, h_3^2 \omega) \equiv (h_3 e_4, h_3 \psi_4, h_3 \omega_4)$

Kaluza-Klein basis — Naive basis

2)

$\left(\begin{array}{c} S^1\text{-sym.} \quad (G_4^S, G_7^S) \\ \mathbb{S}^4\text{-cocycle on } Y \end{array}\right) \xleftrightarrow[\sim]{\text{Prop.}} \left(\begin{array}{c} (F_2^S, H_3^S, F_4^S, F_6^S, H_7^S) \\ CYC(\mathbb{S}^4)\text{-cocycle} \\ \text{on } X \end{array}\right)$

$Y \downarrow X$

Thm 1 S

slice $\begin{array}{ccc} \tilde{Y} & \to & Y \\ \downarrow & & \downarrow \\ \tilde{X} & \to & X \end{array}$

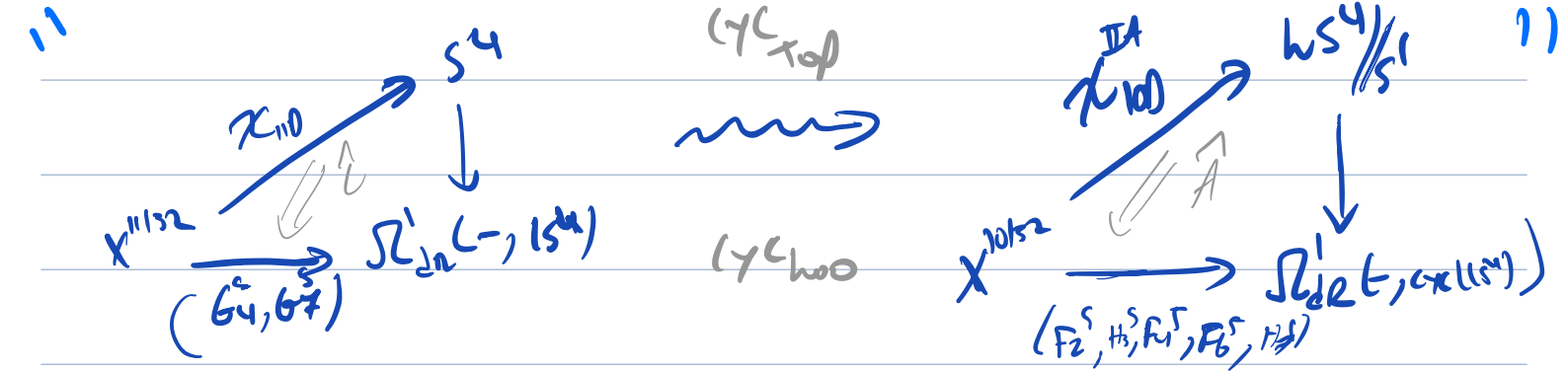
$\left(\begin{array}{c} S^1\text{-Sym 11d Sol.} \\ \text{on } \tilde{Y} \to Y \end{array}\right) \xleftrightarrow[\sim]{\text{Original derivation of IIA}} \left(\begin{array}{c} \text{10d IIA} \\ \text{Sugra Sol.} \\ \text{on } \tilde{X} \to X \end{array}\right)$

Original derivation of IIA
Townsend, Pernici 84'
Campbell, West 84'
Howe, Nawazic 85'

Corollary 1) Any flux-quant law $\mathcal{K}$, $\iota\mathcal{K} \cong \mathrm{cyc}(S^4)$ fully compatible w/ IIA EOMs.

2) Manifestly compatible w/ Hypothesis H → or any alternative $\tilde{H}$

via



Outlook

- Easy systematic extension to all toroidal / cyc^4 IID sugra reductions.

- $\exists!$ automorphism $cyc\,ls^4 \xrightarrow{\sim} cyc\,ls^4$

  $f_4 + f_2 f_2 \longleftarrow 1 \; f_4$

  $f_6 - f_2 f_4 - \frac{1}{2} f_2^3 \longleftarrow f_6$

$\Rightarrow$ new "dual" IIA sugra formulation ?

- T-duality and IIB flux quant. ?