

# Universes as Bigdata:

Superstrings, Calabi-Yau Manifolds & Machine-Learning

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1984:  $10 = 4 + 3 \times 2$

- Heterotic string [Gross-Harvey-Martinec-Rohm]:  $E_8 \times E_8$  or  $SO(32)$ , 1984 - 6
- String Phenomenology [Candelas-Horowitz-Strominger-Witten]: 1985
  - $E_8$  accommodates SM

$$SU(3) \times SU(2) \times U(1) \subset SU(5) \subset SO(10) \subset E_6 \subset E_8$$

- Standard Solution:  $\mathbb{R}^{3,1} \times X$ ,  $X$  Calabi-Yau (Ricci-flat Kähler)
- *mathematicians were independently thinking of the same problem:*
  - Riemann Uniformization Theorem in  $\dim_{\mathbb{C}} = 1$ : Trichotomy  $R < 0, = 0, > 0$
  - Euler, Gauss, Riemann  $\Sigma$ :  $\dim_{\mathbb{R}} = 2$ , i.e.,  $\dim_{\mathbb{C}} = 1$  (in fact Kähler)
  - $\chi(\Sigma) = 2 - 2g(\Sigma) = [c_1(\Sigma)] \cdot [\Sigma] = \frac{1}{2\pi} \int_{\Sigma} R = \sum_{i=0}^2 (-1)^i h^i(\Sigma)$






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# Generalizing a Classic Problem in Mathematics

					...
$g(\Sigma) = 0$	$g(\Sigma) = 1$	$g(\Sigma) > 1$			
$\chi(\Sigma) = 2$	$\chi(\Sigma) = 0$	$\chi(\Sigma) < 0$			
Spherical	Ricci-Flat	Hyperbolic			
+ curvature	0 curvature	- curvature			
Fano	Calabi-Yau	General Type			

- **CONJECTURE** [E. Calabi, 1954, 1957] / Thm [ST. Yau, 1977-8]  $M$  compact Kähler manifold  $(g, \omega)$  and  $([R] = [c_1(M)])_{H^{1,1}(M)}$ . Then  $\exists! (\tilde{g}, \tilde{\omega})$  such that  $([\omega] = [\tilde{\omega}])_{H^2(M; \mathbb{R})}$  and  $Ricci(\tilde{\omega}) = R$ .
- Strominger & Yau were neighbours at IAS in 1985

# Calabi-Yau Spaces as Algebraic Varieties

- $\dim_{\mathbb{C}} = 2$  ( $T^2$  as cubic elliptic curve in  $\mathbb{P}^2$ );  $\dim_{\mathbb{C}} = 2$ ? K3, quartic in  $\mathbb{P}^3$ ;
- **TMH: Homogeneous Eq in  $\mathbb{P}^n$ , degree =  $n+1$**  is Calabi-Yau of  $\dim_{\mathbb{C}} = n-1$
- We get 5 CY3 immediately (**Cyclic Manifolds**)
  - Degree 5 in  $\mathbb{P}^4$  (The Quintic Q)
  - Two degree 3 in  $\mathbb{P}^5$
  - One degree 2 and one degree 4 in  $\mathbb{P}^5$
  - Two degree 2 and one degree 3 in  $\mathbb{P}^6$
  - Four degree 2 in  $\mathbb{P}^7$
- Examples of **Complete Intersection CY3**  
 $\dim(\text{Ambient space}) - \#(\text{defining Eq.}) = 3$  (complete intersection)

# An Early Physical Challenge to Algebraic Geometry

- CY3  $X$ , tangent bundle  $SU(3) \Rightarrow$

①  $E_6$  GUT: commutant  $E_8 \rightarrow SU(3) \times E_6$ , then

② Wilson-line/discrete symmetry to break  $E_6$ -GUT to some SUSY version of Standard Model (generalize later)

③ Particle Spectrum:

Generation	$n_{27} = h^1(X, TX) = h_{\partial}^{2,1}(X)$
Anti-Generation	$n_{\overline{27}} = h^1(X, TX^*) = h_{\partial}^{1,1}(X)$

- Net-generation:  $\chi = 2(h^{1,1} - h^{2,1}) = \text{Euler Number}$

• 1986 Question: Are there Calabi-Yau threefolds with Euler number  $\pm 6$ ?

• None of our 5 obvious ones 😊

e.g., Quintic  $Q$  in  $\mathbb{P}^4$  is CY3  $Q_{\chi}^{h^{1,1}, h^{2,1}} = Q_{-200}^{1,101}$  so too many generations (even with quotient  $-200 \notin 3\mathbb{Z}$ )

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# The First Data-sets in Mathematical Physics/Geometry

- [Candelas-A. He-Hübsch-Lutken-Schimmrigk-Berglund] (1986-1990)
  - CICYs (complete intersection CYs) multi-deg polys in products of  $\mathbb{CP}^{n_i}$  CICYs
  - Problem: *classify all configuration matrices*; employed the best computers at the time (**CERN supercomputer**); q.v. magnetic tape and dot-matrix printout in Philip's office
  - 7890 matrices, 266 Hodge pairs  $(h^{1,1}, h^{2,1})$ , 70 Euler  $\chi \in [-200, 0]$
- [Candelas-Lynker-Schimmrigk, 1990]
  - Hypersurfaces in Weighted P4
  - 7555 inequivalent 5-vectors  $w_i$ , 2780 Hodge pairs,  $\chi \in [-960, 960]$
- [Kreuzer-Skarke, mid-1990s - 2000] Reflexive Polytopes
  - Hypersurfaces in (Reflexive, Gorenstein Fano) Toric 4-folds
  - 6-month running time on dual Pentium SGI machine
  - at least 473,800,776, with 30,108 distinct Hodge pairs,  $\chi \in [-960, 960]$

# Technically, Moses



**was the first person  
with a tablet  
downloading data  
from the cloud**

The age of data science in mathematical physics/string theory not as recent as you might think

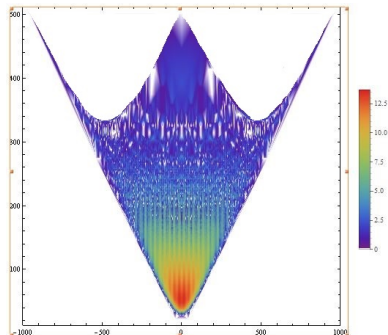
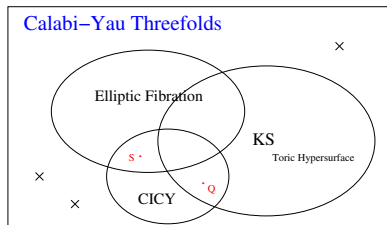
of course, experimental physics had been decades ahead in data-science/machine-learning

After 40 years of research by mathematicians and physicists  
.....

# The Compact CY3 Landscape

cf. YHH, *The Calabi-Yau Landscape: from Geometry, to Physics, to Machine-Learning*, 1812.02893, Springer, to appear, 2019/20

- $\sim 10^{10}$  data-points (and growing, still mined by many international collabs: London/Oxford, Vienna, Northeastern, Jo'burg, Munich, ...)
- a Georgia O'Keefe Plot for Kreuzer-Skarke



# The Geometric Origin of our Universe

- Each CY3 (+ bundles, discrete symmetries)  $X$  gives a 4-D universe
  - The geometry (algebraic geometry, topology, differential geometry etc.) of  $X$  determines the physical properties of the 4-D world
  - particles and interactions  $\sim$  cohomology theory; masses  $\sim$  metric; Yukawa  $\sim$  Triple intersections/integral of forms over  $X$



Ubi materia, ibi geometria

– Johannes Kepler (1571-1630)

- Our Universe:  $\left\{ \begin{array}{l} (1) \text{ probabilistic/anthropic?} \\ (2) \text{ Sui generis/selection rule?} \\ (3) \text{ one of multi-verse ?} \end{array} \right.$

cf. *Exo-planet/Habitable Zone search*

# Triadophilia

## Exact (MS)SM Particle Content from String Compactification

- [Braun-YHH-Ovrut-Pantev, Bouchard-Cvetic-Donagi 2005] first exact MSSM
- [Anderson-Gray-YHH-Lukas, 2007-] use alg./comp. algebraic geo & sift
- Anderson-Gray-Lukas-Ovrut-Palti  $\sim 200$  in  $10^{10}$  MSSM Stable Sum of Line Bundles over CICYs (Oxford-Penn-Virginia 2012-)

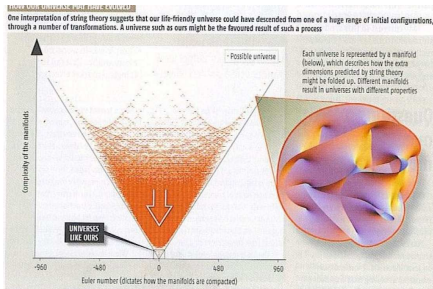
Constantin-YHH-Lukas '19:  $10^{23}$  exact MSSMs (by extrapolation on above set)?

A Special Corner

[New Scientist, Jan, 5, 2008 feature]

P. Candelas, X. de la Ossa, YHH,  
and B. Szendroi

“Triadophilia: A Special Corner of the  
Landscape” ATMP, 2008

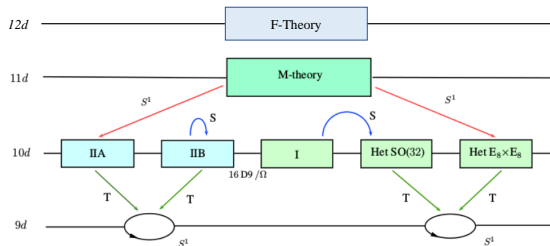


# The Landscape Explosion

meanwhile ... LANDSCAPE grew rapidly with

- D-branes Polchinski 1995
- M-Theory/ $G_2$  Witten, 1995
- F-Theory/4-folds Katz-Morrison-Vafa, 1996
- AdS/CFT Maldacena 1998 Alg Geo of AdS/CFT
- Flux-compactification Kachru-Kallosch-Linde-Trivedi, 2003, ...

# The Vacuum Degeneracy Problem



More  
solutions  
related by  
dualities

Fig. modified  
from

[https://www.  
physics.uu.se/](https://www.physics.uu.se/)

- String theory trades one hard-problem [quantization of gravity] by another [looking for the right compactification] (in many ways a richer and more interesting problem)
- KKLT 2003, Douglas, Denef 2005 - 6 at least  $10^{500}$  possibilities

# SUMMARY: Algorithms and Datasets in String Theory

- Growing databases and algorithms (many motivated by string theory): e.g., Singular, Macaulay2, GAP ( $10^7$  finite groups), SAGE, Bertini, grdb, etc; “Periodic table of shapes Project” classify Fanos, KS  $\sim 10^9$  CYs, Cremona’s  $\sim 10^7$  elliptic curves, ...
- Archetypical Problems
  - Classify configurations (typically integer matrices: polytope, adjacency, ...)
  - Compute geometrical quantity algorithmically
    - toric  $\leadsto$  combinatorics;
    - quotient singularities  $\leadsto$  rep. finite groups;
    - generically  $\leadsto$  ideals in polynomial rings;
    - Numerical geometry (homotopy continuation);
    - Cohomolgy (spectral sequences, Adjunction, Euler sequences)

# Where we stand ...

**The Good** Last 10-15 years: several international groups have bitten the bullet  
Oxford, London, Vienna, Blacksburg, Boston, Johannesburg, Munich, ...

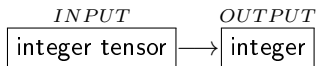
computed many geometrical/physical quantities and **compiled them into various databases Landscape Data** ( $10^9 \sim 10^{10}$  entries typically)

**The Bad** Generic computation **HARD**: dual cone algorithm (exponential),  
triangulation (exponential), Gröbner basis (double-exponential)  
... e.g., how to construct stable bundles over the  $\gg$  473 million KS  
CY3? Sifting through for SM computationally impossible ...

**The ??? Borrow new techniques from “Big Data” revolution**

# A Wild Question

- Typical Problem in String Theory/Algebraic Geometry:



- Q: Can (classes of problems in computational) Algebraic Geometry be “learned” by AI ? , i.e., can we “machine-learn the landscape?”
- [YHH 1706.02714] Deep-Learning the Landscape, PLB 774, 2017:  
Experimentally, it seems to be the case for many situations
- 2017

YHH (1706.02714), Seong-Kreft (1706.03346), Ruehle (1706.07024),  
Carifio-Halverson-Krioukov-Nelson (1707.00655)

Progress in String Theory

# A Prototypical Question

- Hand-writing Recognition, e.g., my 0 to 9 is different from yours:

1 2 3 4 5 6 7 8 9 0

- How to set up a bijection that takes these to  $\{1, 2, \dots, 9, 0\}$ ? Find a clever Morse function? Compute persistent homology? Find topological invariants?  
ALL are inefficient and too sensitive to variation.
- What does your iPhone/tablet do? What does Google do? **Machine-Learn**
  - Take large sample, take a few hundred thousand (e.g. NIST database)  
6 → 6, 8 → 8, 2 → 2, 4 → 4, 8 → 8, 7 → 7, 8 → 8,  
0 → 0, 4 → 4, 2 → 2, 5 → 5, 6 → 6, 3 → 3, 2 → 2,  
9 → 9, 0 → 0, 3 → 3, 8 → 8, 8 → 8, 1 → 1, 0 → 0, ...

Supervised ML in 1 min

# NN Doesn't Care/Know about Algebraic Geometry

- Hodge Number of a Complete Intersection CY is the association rule, e.g.

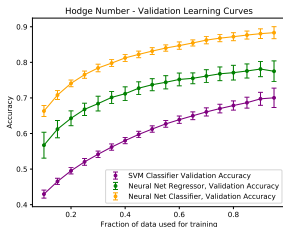
$$X = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}, \quad h^{1,1}(X) = 8 \quad \leadsto \quad \begin{array}{c} \text{Image of } X \end{array} \rightarrow 8$$

CICY is  $12 \times 15$  integer matrix with entries  $\in [0, 5]$  is simply represented as a  $12 \times 15$  pixel image of 6 colours Proper Way

- **Cross-Validation:**  $\left\{ \begin{array}{l} - \text{Take samples of } X \rightarrow h^{1,1} \\ - \text{train a NN, or SVM} \\ - \text{Validation on } \textit{unseen} X \rightarrow h^{1,1} \end{array} \right.$

# Deep-Learning Algebraic Geometry

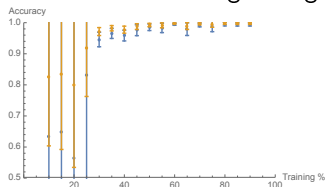
- YHH '17 Bull-YHH-Jejjala-Mishra '18:



Learning Hodge Number

$h^{1,1} \in [0, 19]$  so can set up 20-channel NN classifier, regressor, as well as SVM, bypass exact sequences

- YHH-SJ Lee'19: Distinguishing Elliptic Fibrations in CY3



bypass Oguiso-Kollar-Wilson

Theorem/Conjecture

learning curves for precision and Matthews  $\phi$

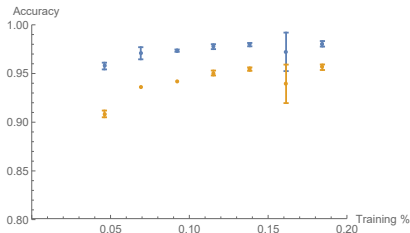
# More Success Stories in Algebraic Geometry

- Ruehle '17: genetic algorithm for bundle cohomology
- Brodie-Constantin-Lukas '19: EXACT formulae for line-bundle coho / complex surfaces Interpolation vs Extrapolation  $\leadsto$  Conjecture Formulation
- Ashmore-YHH-Ovrut '19: ML Calabi-Yau metric:
  - No known explicit Ricci-Flat Kähler metric ( except  $T^n$ ) (Yau's '86 proof non-constructive); Donaldson ['01-05] relatively fast method of *numerical* (balanced) such metrics
  - ML improves it to 10-100 times faster with equal/better accuracy (NB. *checking* Ricci-flat is easy)
- RMK: Alg Geo /  $\mathbb{C}$  amenable to ML: core computations (Grobner bases, syzygies, long exact sequences, etc)  $\sim$  integer (co-)kernel of matrices.

# Why stop at string/geometry?

[YHH-MH. Kim 1905.02263] Learning Algebraic Structures

- When is a Latin Square (Sudoku) the Cayley (multiplication) table of a finite group? (rmk: there is a known quadrangle-thm to test this) NN/SVM find to 94.9% ( $\phi = 0.90$ ) at 25-75 cross-validation.
- Can one look at the Cayley table and recognize a **finite simple group**?
  - bypass Sylow and Noether Thm



- rmk: can do it via character-table  $T$ , but getting  $T$  not trivial
- **SVM**: space of finite-groups (point-cloud of Cayley tables),  $\exists$  hypersurface separating simple/non-simple?

# Further Explorations


- Alessandretti, Baronchelli, YHH 1911.02008 ML/TDA@Birch-Swinnerton-Dyer  
BSD:  $L$ -function  $L(s, \mathcal{E})$  of elliptic curve  $\mathcal{E}$  has  $L(s \rightarrow 1, \mathcal{E})$  given in terms of precise quantities: rank  $r$ , torsion  $T$ , period  $\Omega$ , Tate-Shafarevich group  $\text{III}$ , conductor  $N$ , regulator  $R$ , Tamagawa number  $c$   
 $\text{III}$  and  $\Omega$  good with regression and boosted decision trees:  $\text{RMS} < 0.1$
- a **Reprobate**: predicting primes  
YHH 1706.02714: tried supervised ML on  $2 \rightarrow 3$ ,  $2, 3 \rightarrow 5$ ,  $2, 3, 5 \rightarrow 7$   
tried fixed window of  $(\text{yes/no})_{1,2,\dots,k}$  to  $(\text{yes/no})_{k+1}$ , no breaking banks yet.

# Why stop at the mathematics/physics?

[YHH-Jejjala-Nelson] “hep-th” 1807.00735

- **Word2Vec**: [Mikolov et al., '13] NN which maps words in sentences to a vector space **by context** (much better than word-frequency, quickly adopted by Google); maximize (partition function) over all words with sliding window ( $W_{1,2}$  weights of 2 layers,  $C_\alpha$  window size,  $D$  # windows)

$$Z(W_1, W_2) := \frac{1}{|D|} \sum_{\alpha=1}^{|D|} \log \prod_{c=1}^{C_\alpha} \frac{\exp([\vec{x}_c]^T \cdot W_1 \cdot W_2)}{\sum_{j=1}^V \exp([\vec{x}_c]^T \cdot W_1 \cdot W_2)}$$

- We downloaded all  $\sim 10^6$  titles of hep-th, hep-ph, gr-qc, math-ph, hep-lat from ArXiv since the beginning (1989) till end of 2017   
(rmk: Ginzparg has been doing a version of linguistic ML on ArXiv)  
(rmk: abs and full texts in future)

# Subfields on ArXiv has own linguistic particulars

- Linear Syntactical Identities

*bosonic + string-theory = open-string*

*holography + quantum + string + ads = extremal-black-hole*

*string-theory + calabi-yau = m-theory + g2*

*space + black-hole = geometry + gravity ...*

- binary **classification** (Word2Vec + SVM) of formal (hep-th, math-ph, gr-qc) vs phenomenological (hep-ph, hep-lat) : 87.1% accuracy (5-fold classification 65.1% accuracy). [ArXiv classifications](#)

- Cf. **Tshitoyan et al.**, “Unsupervised word embeddings capture latent knowledge from materials science literature”, **Nature** July, 2019: 3.3. million materials-science abstracts; uncovers structure of periodic table, predicts discoveries of new thermoelectric materials years in advance, and suggests as-yet unknown materials

# Summary and Outlook

- PHYSICS
- Use AI (Neural Networks, SVMs, Regressor ...) as
    1. Classifier deep-learn and categorize landscape data
    2. Predictor estimate results beyond computational power

- MATHS
- Not solving NP-hard problems, but stochastically bypassing the expensive steps of long sequence-chasing, Gröbner bases, dual cones/combinatorics **how is AI doing maths more efficiently without knowing any maths?**
  - Hierarchy of Difficulty ML struggles with:  
**numerical < algebraic geometry over  $\mathbb{C}$  < combinatorics/algebra < number theory**

- [Boris Zilber](#) [Merton Professor of Logic, Oxford]: “you’ve managed syntax without semantics...”

Alpha Go	→	Alpha Zero
ML	→	Voevodsky’s Dream; Automated Thm Pf

- cf. [Renner et al.](#), PRL/Nature News, Nov, 2019: ML (*SciNet*, *autoencoder*) finds heliocentrism from Mars positions alone.



**Sophia** (Hanson Robotics, HK)

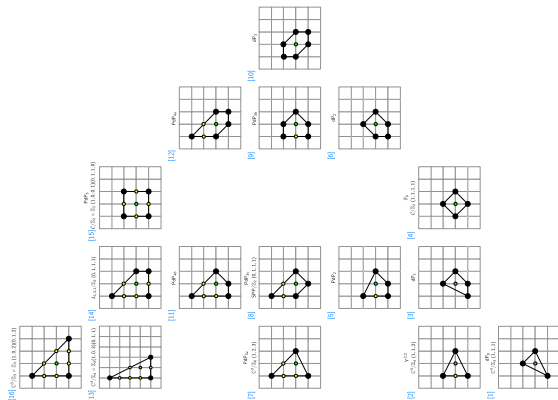
1st non-human citizen (2017, Saudi Arabia)

1st non-human with UN title (2017)

1st String Data Conference (2017)

# 16 Reflexive Polygons

Back to Reflexives



classify convex lattice polytopes with single interior point and all faces are distance 1 therefrom (up to  $SL(n; \mathbb{Z})$ )

**Kreuzer-Skarke:** 4319 reflexive polyhedra, 473,800,776 reflexive 4-polytopes,  
 Skarke: next number is at least 185,269,499,015.

## Major International Annual Conference Series

1986- First “Strings” Conference

2002- First “StringPheno” Conference

2006 - 2010 String Vacuum Project (NSF)

2011- First “String-Math” Conference

2014- First String/Theoretical Physics Session in SIAM Conference

2017- First “String-Data” Conference

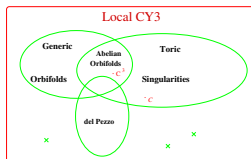
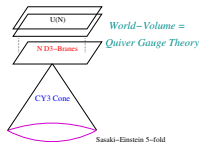
$$M = \left[ \begin{array}{c|cccc} n_1 & q_1^1 & q_1^2 & \cdots & q_1^K \\ n_2 & q_2^1 & q_2^2 & \cdots & q_2^K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n_m & q_m^1 & q_m^2 & \cdots & q_m^K \end{array} \right]_{m \times K}$$

- Complete Intersection Calabi-Yau (CICY) 3-folds
- $K$  eqns of multi-degree  $q_j^i \in \mathbb{Z}_{\geq 0}$   
embedded in  $\mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_m}$
- $c_1(X) = 0 \leadsto \sum_{j=1}^K q_r^j = n_r + 1$
- $M^T$  also CICY

- The Quintic  $Q = [4|5]_{-200}^{1,101}$  (or simply  $[5]$ );
- CICYs Central to string pheno in the 1st decade [Distler, Greene, Ross, et al.]  
 $E_6$  GUTS unfavoured; Many exotics: e.g. 6 entire anti-generations

# AdS/CFT as a Quiver Rep/Moduli Variety Corr.

a 20-year prog. joint with **A. Hanany**, S. Franco, B. Feng, et al.



D-Brane Gauge Theory  
(SCFT encoded as quiver)



Vacuum Space as affine Variety

- $(\mathcal{N} = 4 \text{ SYM}) \left( \text{quiver with } x, y, z \text{ nodes}, W = \text{Tr}([x, y], z) \right) \longleftrightarrow \mathbb{C}^3 = \text{Cone}(S^5)$  [Maldacena]

- THM [(P) Feng, Franco, Hanany, YHH, Kennaway, Martelli, Mekareeya, Seong, Sparks, Vafa, Vegh, Yamazaki, Zaffaroni ...]

(M) R. Bockland, N. Broomhead, A. Craw, A. King, G. Musiker, K. Ueda ...] (coherent component of)  
representation variety of a quiver is toric CY3 iff quiver + superpotential  
graph dual to a bipartite graph on  $T^2$

[Back to Landscape](#)

# A Single Neuron: The Perceptron

- began in 1957 (!! ) in early AI experiments (using CdS photo-cells)
- DEF: Imitates a **neuron**: activates upon certain inputs, so define
  - Activation Function  $f(z_i)$  for input tensor  $z_i$  for some multi-index  $i$ ;
  - consider:  $f(w_i z_i + b)$  with  $w_i$  weights and  $b$  bias/off-set;
  - typically,  $f(z)$  is sigmoid, Tanh, etc.
- Given **training data**:  $D = \{(x_i^{(j)}, d^{(j)})\}$  with input  $x_i$  and **known output**  $d^{(j)}$ , minimize

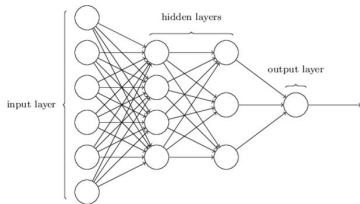
$$SD = \sum_j \left( f\left(\sum_i w_i x_i^{(j)} + b\right) - d^{(j)} \right)^2$$

to find optimal  $w_i$  and  $b \rightsquigarrow$  “learning”, then check against **Validation Data**

- Essentially (non-linear) regression

# The Neural Network: network of neurons $\leadsto$ the “brain”

- DEF: a **connected graph**, each node is a perceptron (*Implemented on Mathematica 11.1 + / TensorFlow-Keras on Python*)
  - 1 adjustable weights/bias;
  - 2 distinguished nodes: 1 set for input and 1 for output;
  - 3 iterated training rounds.



Simple case: forward directed only,  
called **multilayer perceptron**

- others: e.g., decision trees, support-vector machines (SVM), etc
- Essentially how brain learns complex tasks; **apply to our Landscape Data**

# Computing Hodge Numbers: Sketch

- Recall Hodge decomposition  $H^{p,q}(X) \simeq H^q(X, \wedge^p T^*X) \simeq$

$$H^{1,1}(X) = H^1(X, T_X^*), \quad H^{2,1}(X) \simeq H^{1,2} = H^2(X, T_X^*) \simeq H^1(X, T_X)$$

- Euler Sequence** for subvariety  $X \subset A$  is short exact:

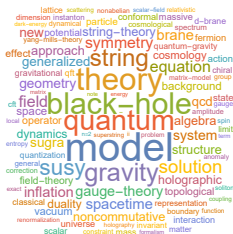
$$0 \rightarrow T_X \rightarrow T_M|_X \rightarrow N_X \rightarrow 0$$

- Induces **long exact sequence in cohomology**:

$$\begin{array}{ccccccc} 0 & \rightarrow & H^0(X, T_X) & \xrightarrow{\quad} & H^0(X, T_A|_X) & \rightarrow & H^0(X, N_X) \rightarrow \\ & & \searrow & & & & \\ & \rightarrow & \boxed{H^1(X, T_X)} & \xrightarrow{d} & H^1(X, T_A|_X) & \rightarrow & H^1(X, N_X) \rightarrow \\ & \rightarrow & H^2(X, T_X) & \rightarrow & \dots & & \end{array}$$

- Need to compute  $\text{Rk}(d)$ , cohomology and  $H^i(X, T_A|_X)$  (Cf. Hübsch)

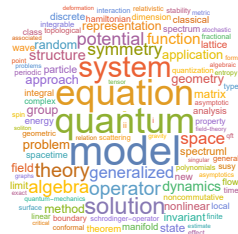
# ArXiv Word-Clouds



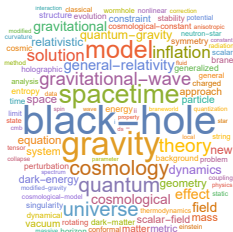
hep-th



hep-ph



math-ph



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# Classifying Titles

Compare, + non-physics sections, non-science (Times), pseudo-science (viXra)

Word2Vec + SVM		1	2	3	4	5	$\left\{ \begin{array}{ll} 1 & : \text{ hep-th} \\ 2 & : \text{ hep-ph} \\ 3 & : \text{ hep-lat} \\ 4 & : \text{ gr-qc} \\ 5 & : \text{ math-ph} \end{array} \right.$	
Actual								
1		40.2	6.5	8.7	24.0	20.6		
2		7.8	65.8	12.9	9.1	4.4		
3		7.5	11.3	72.4	1.5	7.4		
4		12.4	4.4	1.0	72.1	10.2		
5		10.9	2.2	4.0	7.8	75.1		

NN		1	2	3	4	5	6	7	8	9	10
Actual											
viXra-hep		11.5	47.4	6.8	13.	11.	4.5	0.2	0.3	2.2	3.1
viXra-qgst		13.3	14.5	1.5	54.	8.4	1.8	0.1	1.1	2.8	3.

6: cond-mat, 7: q-fin, 8: stat, 9: q-bio, 10: Times of India

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