# NAMBU POISSON M5-BRANE

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Apr. 2011
Cross Strait Meeting on
Particle Physics and Cosmology

# M5 IN C-FIELD

- M theory:
- M5-brane in C-field (3-form) potential background
- M5-brane worldvolume low energy effective theory
- M2-brane ending on M5-brane = self-dual “string” on M5
- self-dual “string” → self-dual 2-form potential
- volume-preserving diffeomorphism on M5 defined by C

# D-BRANE IN RR FIELD

- Dp-brane in RR (p-1)-form potential background
- D(p-1)-brane ending on Dp-brane coupled to RR (p-1)-form
- Ending of D(p-1)-brane = (p-2)-brane ⟶ worldvolume (p-2)-form potential B
- B = gauge potential for (p-1)-dim.-volume-preserving-diffeo.
- But are they new physical degrees of freedom in addition to U(1) gauge potential A ?

[Nambu 73] [Takhtajan 94]

# NAMBU POISSON BRACKET

- Nambu Poisson bracket

$$\{f, g, h\} = P^{\dot{\mu}\dot{\nu}\dot{\lambda}} \left(\partial_{\dot{\mu}} f\right) \left(\partial_{\dot{\nu}} g\right) \left(\partial_{\dot{\lambda}} h\right)$$

$$\dot{\mu}, \dot{\nu} = \dot{1}, \dot{2}, \dot{3}$$

- Skew-symmetry

$$\{f, g, h\} = -\{g, f, h\} = -\{h, g, f\}$$

- Leibniz rule

$$\{fg, h_1, h_2\} = f\{g, h_1, h_2\} + g\{f, h_1, h_2\}$$

- Jacobi identity

$$\{f_1, f_2, \{g_1, g_2, g_3\}\} =$$

$$\{\{f_1, f_2, g_1\}, g_2, g_3\} + \{g_1, \{f_1, f_2, g_2\}, g_3\} + \{g_1, g_2, \{f_1, f_2, g_3\}\}$$

# VOLUME-PRESERVING-DIFFEOMORPHISM (VPD)

For a 3D space, the coordinate transformation $\delta y^{\dot{\mu}} = \kappa^{\dot{\mu}}$

preserves the volume-form $dy^{\dot{1}} dy^{\dot{2}} dy^{\dot{3}}$

if $\partial_{\dot{\mu}} \kappa^{\dot{\mu}} = 0$

Transformation on functions can be expressed via NP bracket as

$$\delta\Phi = \kappa^{\dot{\mu}} \partial_{\dot{\mu}} \Phi = \sum_a \{f_a, g_a, \Phi\}$$

This can be generalized to higher/lower dimensions.

[Ho, Matsuo 08] [Ho, Imamura, Matuo, Shiba 08]

# M5 IN LARGE C-FIELD

- Worldvolume coordinates are divided into two groups by the C-field background

$$C = \frac{1}{6} C_{\dot{1}\dot{2}\dot{3}} \epsilon_{\dot{\mu}\dot{\nu}\dot{\lambda}} dy^{\dot{\mu}} dy^{\dot{\nu}} dy^{\dot{\lambda}}$$

$$x^{\mu} \quad (\mu = 1, 2, 3), \qquad y^{\dot{\mu}} \quad (\dot{\mu} = \dot{1}, \dot{2}, \dot{3})$$

- Gauge transformation for an Abelian 2-form potential:

$$\delta b_{\dot{\mu}\dot{\nu}} = \partial_{\dot{\mu}} \Lambda_{\dot{\nu}} - \partial_{\dot{\nu}} \Lambda_{\dot{\mu}}$$
$$\delta b_{\mu\dot{\mu}} = \partial_{\mu} \Lambda_{\dot{\mu}} - \partial_{\dot{\mu}} \Lambda_{\mu}$$

$$\kappa^{\dot{\lambda}} \equiv \epsilon^{\dot{\lambda}\dot{\mu}\dot{\nu}} \partial_{\dot{\mu}} \Lambda_{\dot{\nu}}(x, y)$$

- Field strengths are

$$H_{\lambda\dot{\mu}\dot{\nu}} = \partial_{\lambda} b_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b_{\lambda\dot{\nu}} + \partial_{\dot{\nu}} b_{\lambda\dot{\mu}}$$
$$H_{\dot{\lambda}\dot{\mu}\dot{\nu}} = \partial_{\dot{\lambda}} b_{\dot{\mu}\dot{\nu}} + \partial_{\dot{\mu}} b_{\dot{\nu}\dot{\lambda}} + \partial_{\dot{\nu}} b_{\dot{\lambda}\dot{\mu}}$$

# SELF-DUAL 2-FORM GAUGE FIELD THEORY

- Field content

$$X^i(x,y), \qquad \Psi(x,y)$$

$$b_{\dot{\mu}\dot{\nu}}, \qquad b_{\mu\dot{\mu}}$$

$$b_{\mu\nu}(x,y) \to \text{arise from solutions}$$

- Small C-field background → $\int CH$
- Large C-field background → VPD gauge symm.
- Self-Duality condition satisfied only after solving the equation of motion and change variables.

cf. [Pasti, Samsonov, Sorokin, Tonin 09][Furuuchi 10]

# GAUGE SYMMETRY IN LARGE C-FIELD

- gauge transformation (VPD)

$$
\begin{aligned}
\delta_\Lambda \Phi &= g\kappa^{\dot\rho}\partial_{\dot\rho}\Phi \qquad (\Phi = X^i, \Psi) \\
\delta_\Lambda b^{\dot\mu} &= \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu}, \\
\delta_\Lambda B_\mu{}^{\dot\mu} &= \partial_\mu \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} B_\mu{}^{\dot\mu} - g(\partial_{\dot\nu}\kappa^{\dot\mu}) B_\mu{}^{\dot\nu}
\end{aligned}
$$

- Field strengths

$$
\begin{aligned}
\mathcal{H}_{\lambda\dot\mu\dot\nu} &= \epsilon_{\dot\mu\dot\nu\dot\lambda}\mathcal{D}_\lambda X^{\dot\lambda} \\
&= H_{\lambda\dot\mu\dot\nu} - g\epsilon^{\dot\sigma\dot\tau\dot\rho}(\partial_{\dot\sigma} b_{\lambda\dot\tau})\partial_{\dot\rho} b_{\dot\mu\dot\nu}, \\
\mathcal{H}_{\dot1\dot2\dot3} &= g^2\{X^{\dot1}, X^{\dot2}, X^{\dot3}\} - \frac{1}{g} \\
&= H_{\dot1\dot2\dot3} + \frac{g}{2}(\partial_{\dot\mu} b^{\dot\mu}\partial_{\dot\nu} b^{\dot\nu} - \partial_{\dot\mu} b^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu}) + g^2\{b^{\dot1}, b^{\dot2}, b^{\dot3}\}
\end{aligned}
$$

# THE ACTION

$$S = S_X + S_\Psi + S_{gauge} \qquad S_{gauge} = S_{\mathcal{H}^2} + S_{CS}$$

$$\begin{aligned}
S_X &= \int d^3x d^3y \left[ -\frac{1}{2}(\mathcal{D}_\mu X^i)^2 - \frac{1}{2}(\mathcal{D}_{\dot\lambda} X^i)^2 \right. \\
&\left. -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot\mu}, X^i, X^j\}^2 - \frac{g^4}{12}\{X^i, X^j, X^k\}^2 \right] \\
S_\Psi &= \int d^3x d^3y \left[ \frac{i}{2}\overline{\Psi}\Gamma^\mu \mathcal{D}_\mu \Psi + \frac{i}{2}\overline{\Psi}\Gamma^{\dot\rho}\mathcal{D}_{\dot\rho}\Psi \right. \\
&\left. + \frac{ig^2}{2}\overline{\Psi}\Gamma_{\dot\mu i}\{X^{\dot\mu}, X^i, \Psi\} - \frac{ig^2}{4}\overline{\Psi}\Gamma_{ij}\Gamma_{\dot1\dot2\dot3}\{X^i, X^j, \Psi\} \right] \\
S_{\mathcal{H}^2} &= \int d^3x d^3y \left[ -\frac{1}{12}\mathcal{H}^2_{\dot\mu\dot\nu\dot\rho} - \frac{1}{4}\mathcal{H}^2_{\lambda\dot\mu\dot\nu} \right] \\
S_{CS} &= \int d^3x d^3y \, \epsilon^{\mu\nu\lambda}\epsilon^{\dot\mu\dot\nu\dot\lambda} \left[ -\frac{1}{2}\partial_{\dot\mu} b_{\mu\dot\nu}\partial_\nu b_{\lambda\dot\lambda} + \frac{g}{6}\partial_{\dot\mu} b_{\nu\dot\nu}\epsilon^{\dot\rho\dot\sigma\dot\tau}\partial_{\dot\sigma} b_{\lambda\dot\rho}(\partial_{\dot\lambda} b_{\mu\dot\tau} - \partial_{\dot\tau} b_{\mu\dot\lambda}) \right]
\end{aligned}$$

- covariant derivatives

$$\begin{aligned}
\mathcal{D}_\mu \Phi &= \partial_\mu \Phi - g B_\mu{}^{\dot\mu}\partial_{\dot\mu}\Phi. \\
\mathcal{D}_{\dot\mu}\Phi &= \frac{g^2}{2}\epsilon_{\dot\mu\dot\nu\dot\rho}\{X^{\dot\nu}, X^{\dot\rho}, \Phi\}
\end{aligned}$$

$$X^{\dot\mu}(y) \equiv \frac{y^{\dot\mu}}{g} + \frac{1}{2}\epsilon^{\dot\mu\dot\kappa\dot\lambda} b_{\dot\kappa\dot\lambda}(y)$$

# SUPERSYMMETRY

$$
\begin{aligned}
\delta_\epsilon X^i &= i\bar{\epsilon}\Gamma^i\Psi \\
\delta_\epsilon \Psi &= \mathcal{D}_\mu X^i \Gamma^\mu \Gamma^i \epsilon + \mathcal{D}_{\dot{\mu}} X^i \Gamma^{\dot{\mu}} \Gamma^i \epsilon \\
&\quad -\frac{1}{2}\mathcal{H}_{\mu\dot{\nu}\dot{\rho}}\Gamma^\mu\Gamma^{\dot{\nu}\dot{\rho}}\epsilon - \mathcal{H}_{\dot{1}\dot{2}\dot{3}}\Gamma_{\dot{1}\dot{2}\dot{3}}\epsilon \\
&\quad -\frac{g^2}{2}\{X^{\dot{\mu}}, X^i, X^j\}\Gamma^{\dot{\mu}}\Gamma^{ij}\epsilon + \frac{g^2}{6}\{X^i, X^j, X^k\}\Gamma^{ijk}\Gamma^{\dot{1}\dot{2}\dot{3}}\epsilon \\
\delta_\epsilon b_{\dot{\mu}\dot{\nu}} &= -i(\bar{\epsilon}\Gamma_{\dot{\mu}\dot{\nu}}\Psi) \\
\delta_\epsilon b_{\mu\dot{\nu}} &= -i\left(1 + g\mathcal{H}_{\dot{1}\dot{2}\dot{3}}\right)\bar{\epsilon}\Gamma_\mu\Gamma_{\dot{\nu}}\Psi + ig(\bar{\epsilon}\Gamma_\mu\Gamma_i\Gamma_{\dot{1}\dot{2}\dot{3}}\Psi)\partial_{\dot{\nu}}X^i
\end{aligned}
$$

# NP M5 TO NC D4

[Ho, Imamura, Matsuo, Shiba 08]

- Double Dimensional Reduction

$$y^{\dot{3}} \sim y^{\dot{3}} + 2\pi R$$

- VPD becomes Area-Preserving-Diff.

$$b_{\dot{1}\dot{2}} = b^{\dot{3}} = 0$$

$$b_{\dot{\alpha}\dot{3}} = a_{\dot{\alpha}}$$

$$b_{\mu\dot{3}} = a_{\mu}$$

$$b_{\mu\dot{\alpha}} \quad \text{integrated out}$$

$$\kappa^{\dot{\mu}} = \epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}} \partial_{\dot{\nu}} \Lambda_{\dot{\lambda}}$$

$$\Lambda_{\dot{1}}, \Lambda_{\dot{2}}$$

$$\Lambda_{\dot{3}} = \lambda$$

- Seiberg-Witten limit for NC D4 reinterpreted for NP M5 $\epsilon \to 0$ [Chen, Furuuchi, Ho, Takimi 10]

$$\ell_P \sim \epsilon^{1/3}, \qquad g_{\mu\nu} \sim 1, \qquad g_{\dot{\mu}\dot{\nu}} \sim \epsilon, \qquad C_{\dot{\mu}\dot{\nu}\dot{\lambda}} \sim 1$$

- Analogous to D-brane in B field background, where open strings coupled to B induce interactions through Moyal bracket, open membranes coupled to C induce Nambu-Poisson bracket.

- Same story generalized to Dp in constant RR (p-1)-form background

[Ho, Yeh 11]

# NP M5 TO NP D4

- Double Dimensional Reduction

$$x^2 \sim x^2 + 2\pi R$$

- VPD survives, also get U(1)

$$b^{\dot{\mu}}$$

$$b_{2\dot{\mu}} = a_{\dot{\mu}}$$

$$b_{\alpha\dot{\mu}} \quad \xrightarrow{dual} \quad a_\alpha$$

We will focus on the gauge fields.

- Low energy limit for NP M5 reinterpreted for NP D4

$$\ell_s \sim \epsilon^{1/2}, \qquad g_s \sim \epsilon^{-1/2}, \qquad g_{\alpha\beta} \sim 1, \qquad g_{\dot\mu\dot\nu} \sim \epsilon, \qquad C_{\dot\mu\dot\nu\dot\lambda} \sim 1$$

$$g_s \ell_s \ll 1$$ radius much smaller then 1/E

Background fields:

$$C_{012} = \frac{1}{(2\pi)^2 \ell_P^3} C_{\dot1\dot2\dot3}$$ No NP structure in (012) directions.

$$2\pi\alpha' B_{01} = 2\pi\ell_s^2 R C_{012} = \frac{C_{\dot1\dot2\dot3}}{2\pi}$$ finite B-field background (no NC)

# GAUGE SYMMETRY

- gauge transformation

$$
\begin{aligned}
\delta b^{\dot\mu} &= \kappa^{\dot\mu} + g\kappa^{\dot\nu}\partial_{\dot\nu} b^{\dot\mu} \\
\delta a_A &= \partial_A \lambda + g(\kappa^{\dot\nu}\partial_{\dot\nu} a_A + a_{\dot\nu}\partial_A \kappa^{\dot\nu})
\end{aligned}
$$

$A = (\mu \;\; \text{or} \;\; \dot\mu) = 0, 1, \dot{1}, \dot{2}, \dot{3}$ $\qquad$ $\lambda \equiv \Lambda_2$

- field strengths

$$
\begin{aligned}
\mathcal{H}_{\dot1\dot2\dot3} &= \partial_{\dot\mu} b^{\dot\mu} + \frac{1}{2} g(\partial_{\dot\nu} b^{\dot\nu}\partial_{\dot\rho} b^{\dot\rho} - \partial_{\dot\nu} b^{\dot\rho}\partial_{\dot\rho} b^{\dot\nu}) + g^2\{b^{\dot1}, b^{\dot2}, b^{\dot3}\} \\
\mathcal{F}_{\dot\mu\dot\nu} &\equiv \mathcal{H}_{\dot\mu\dot\nu 2} = F_{\dot\mu\dot\nu} + g[\partial_{\dot\sigma} b^{\dot\sigma} F_{\dot\mu\dot\nu} - \partial_{\dot\mu} b^{\dot\sigma} F_{\dot\sigma\dot\nu} - \partial_{\dot\nu} b^{\dot\sigma} F_{\dot\mu\dot\sigma}] \\
\mathcal{F}_{\alpha\dot\mu} &= {V^{-1}}_{\dot\mu}{}^{\dot\nu}(F_{\alpha\dot\nu} + gF_{\dot\nu\dot\delta}\hat{B}_\alpha{}^{\dot\delta}) \\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot\mu}\hat{B}_\beta{}^{\dot\mu} - F_{\dot\mu\beta}\hat{B}_\alpha{}^{\dot\mu} + gF_{\dot\mu\dot\nu}\hat{B}_\alpha{}^{\dot\mu}\hat{B}_\beta{}^{\dot\nu}]
\end{aligned}
$$

$$V_{\dot{\nu}}{}^{\dot{\mu}} \equiv \delta_{\dot{\nu}}^{\dot{\mu}} + g\partial_{\dot{\nu}} b^{\dot{\mu}}$$

$$M_{\dot{\mu}\dot{\nu}}^{\alpha\beta} \equiv V_{\dot{\mu}\dot{\rho}} V_{\dot{\nu}}{}^{\dot{\rho}} \delta^{\alpha\beta} - g\epsilon^{\alpha\beta} F_{\dot{\mu}\dot{\nu}}$$

$$(M^{-1})_{\alpha\gamma}^{\dot{\mu}\dot{\lambda}} M_{\dot{\lambda}\dot{\nu}}^{\gamma\beta} = \delta_{\dot{\nu}}^{\dot{\mu}} \delta_{\alpha}^{\beta}$$

$$\hat{B}_{\alpha}{}^{\dot{\mu}} \equiv (M^{-1})_{\alpha\beta}^{\dot{\mu}\dot{\nu}} (V_{\dot{\nu}}{}^{\dot{\lambda}} \partial^{\beta} b_{\dot{\lambda}} + \epsilon^{\beta\gamma} F_{\gamma\dot{\nu}})$$

# ACTION

$$S_{gauge} = \int d^2x d^3y \left\{ -\frac{1}{2}\mathcal{H}_{\dot{1}\dot{2}\dot{3}}\mathcal{H}^{\dot{1}\dot{2}\dot{3}} - \frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}\mathcal{F}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}\mathcal{F}_{\beta\dot{\mu}}\mathcal{F}^{\beta\dot{\mu}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta} \right\}$$

- To the lowest order

$$S_{gauge} \simeq \int d^2x d^3y \left\{ -\frac{1}{2}(H_{\dot{1}\dot{2}\dot{3}} + F_{01})^2 - \frac{1}{4}F_{AB}F^{AB} \right\}$$

# GENERALIZATION TO DP

- Nambu-Poisson bracket with (p-1) slots

$$\{f_1, f_2, \cdots, f_{p-1}\} \equiv \epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}}\partial_{\dot{\mu}_1} f_1 \partial_{\dot{\mu}_2} f_2 \cdots \partial_{\dot{\mu}_{p-1}} f_{p-1}$$

- Gauge fields

$$b^{\dot{\mu}_1} = \frac{1}{(p-2)!}\epsilon^{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}} b_{\dot{\mu}_2\cdots\dot{\mu}_{p-2}} \qquad X^{\dot{\mu}} = \frac{y^{\dot{\mu}}}{g} + b^{\dot{\mu}}$$

$$\delta a_A = [D_A, \lambda] + g(\kappa^{\dot{\mu}}\partial_{\dot{\mu}} a_A + a_{\dot{\mu}}\partial_A \kappa^{\dot{\mu}}) \qquad A = 0, 1, 2, \cdots, p$$

$$\begin{aligned}
\mathcal{F}_{\dot{\mu}\dot{\nu}} &= F_{\dot{\mu}\dot{\nu}} + g[\partial_{\dot{\sigma}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\nu}} - \partial_{\dot{\mu}} b^{\dot{\sigma}} F_{\dot{\sigma}\dot{\nu}} - \partial_{\dot{\nu}} b^{\dot{\sigma}} F_{\dot{\mu}\dot{\sigma}}] \\
\mathcal{F}_{\alpha\dot{\mu}} &= {V^{-1}}_{\dot{\mu}}^{\ \dot{\nu}}(F_{\alpha\dot{\nu}} + g F_{\dot{\nu}\dot{\delta}} \hat{B}_\alpha^{\ \dot{\delta}}) \\
\mathcal{F}_{\alpha\beta} &= F_{\alpha\beta} + g[-F_{\alpha\dot{\mu}}\hat{B}_\beta^{\ \dot{\mu}} - F_{\dot{\mu}\beta}\hat{B}_\alpha^{\ \dot{\mu}} + g F_{\dot{\mu}\dot{\nu}}\hat{B}_\alpha^{\ \dot{\mu}}\hat{B}_\beta^{\ \dot{\nu}}]
\end{aligned}$$

$$\mathcal{H}_{\dot{\mu}_1\dot{\mu}_2\cdots\dot{\mu}_{p-1}} \equiv g^{p-2}\{X^{\dot{\mu}_1}, X^{\dot{\mu}_1}, \cdots, X^{\dot{\mu}_{p-1}}\} - \frac{1}{g} = \partial_{\dot{\mu}} b^{\dot{\mu}} + \mathcal{O}(g)$$

$$\delta\mathcal{F}_{AB} = [\mathcal{F}_{AB}, \lambda - g\kappa^{\dot{\mu}}\partial_{\dot{\mu}}]$$

- symmetry algebra $[\delta_1, \delta_2] = \delta_3$

$$\begin{aligned}\lambda_3 &= [\lambda_1, \lambda_2] + g(\kappa_2^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_1 - \kappa_1^{\dot{\mu}}\partial_{\dot{\mu}}\lambda_2) \\ \kappa_3^{\dot{\mu}} &= g(\kappa_2^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_1^{\dot{\mu}} - \kappa_1^{\dot{\nu}}\partial_{\dot{\nu}}\kappa_2^{\dot{\mu}})\end{aligned}$$

- action $a_A = a_A^{U(1)} + a_A^{SU(N)}$

$$\begin{aligned}S_{gauge}^{Dp} = &\int d^2x d^{p-1}y \left\{ -\frac{1}{2}\frac{1}{(p-1)!}\mathcal{H}_{\dot{\mu}_1\cdots\dot{\mu}_{p-1}}\mathcal{H}^{\dot{\mu}_1\cdots\dot{\mu}_{p-1}} + \frac{1}{2g}\epsilon^{\alpha\beta}\mathcal{F}_{\alpha\beta}^{U(1)} \right. \\ &\left. -\frac{1}{4}\mathcal{F}_{\dot{\nu}\dot{\rho}}^{U(1)}\mathcal{F}_{U(1)}^{\dot{\nu}\dot{\rho}} + \frac{1}{2}\mathcal{F}_{\beta\dot{\mu}}^{U(1)}\mathcal{F}_{U(1)}^{\beta\dot{\mu}} - \frac{1}{4}\mathrm{tr}\left(\mathcal{F}_{AB}^{SU(N)}\mathcal{F}_{SU(N)}^{AB}\right)\right\}\end{aligned}$$

# CONCLUSION

- T-duality:
  - Dp in RR (p+1)-form background
  - (p+1)-form = (p-form)x(1-form)
  - VPD gauge field shares physical d.o.f. with momentum of Dp.
- Other branes in other backgrounds?
- Applications?