Charges and Duality in Gravity

Magnetic charges and exotic gravitons

Abu Dhabi, January 2023

Gravity and Magnetic Charges

- Circle reduction from D to D-1 dimensions $g_{mn} \to (g_{\mu\nu}, A_\mu, \phi)$
- Momentum $P^m \to (P^\mu, q)$
- q electric charge for A_μ.
- Magnetic charge $p_{\mu_1\mu_2\ldots\mu_{D-5}}$, D-5 form in D-1 dimensions
- D-dimensional origin: gravitational magnetic charge $K_{m_1m_2\ldots m_{D-5}}$, D-5 form in D dims
- $K_{m_1m_2\ldots m_{D-5}} \to (p_{\mu_1\mu_2\ldots\mu_{D-5}}, K_{\mu_1\mu_2\ldots\mu_{D-4}})$
- "K-charge" carried by KK monopoles, a "magnetic" gravitational charge

Superalgebra

- D-5 form K-charge appears in susy algebra for $D \geq 5$.
- e.g. for D=11:

$$\{Q, Q\} \sim P^m \Gamma_m + Z_{m_1 \ldots m_5} \Gamma^{m_1 \ldots m_5} + \ldots$$

- $m = (0, i)$
- $Z_{i_1 \ldots i_5}$: M5-brane charge
- $Z_{0 i_1 \ldots i_4}$: dual of $K_{i_1 \ldots i_6}$

K-Charge

- K-charge appears in algebra of conserved charges and so is a conserved charge
- "Magnetic" charge for gravity in $D \geq 5$
- Appears in BPS bounds
- Can be calculated from the super-algebra [CH '97]
- Expression gives correct BPS charge carried by KK monopoles

ADM Momentum

- Metric $g_{\mu\nu}$ asymptotic to Minkowski space $\bar{g}_{\mu\nu}$
- Frames $e^a_\mu, \bar{e}^a_\mu$, 1-forms $e^a, \bar{e}^a$ Spin-connections $\omega^a{}_b(e), \bar{\omega}^a{}_b(\bar{e})$. $e^a \to \bar{e}^a$ as $r \to \infty$

$$\Gamma^a{}_b = \omega^a{}_b - \bar{\omega}^a{}_b$$

- An asymptotic Killing vector k^μ of $g_{\mu\nu}$ is constant KV of $\bar{g}_{\mu\nu}$
- ADM $Q[k] \sim P \cdot k$, S is D-2 sphere at spatial infinity

$$Q[k] = \frac{1}{4} \int_S \Gamma_{ab} \wedge * (k \wedge \bar{e}^a \wedge \bar{e}^b)$$

(My rewriting of Nester's expression for ADM momentum)

K-charge

- "Asymptotic Killing form": Constant D-5 form ρ in background Minkowski space
- Corresponding K-charge $Q[\rho] \sim K \cdot \rho$

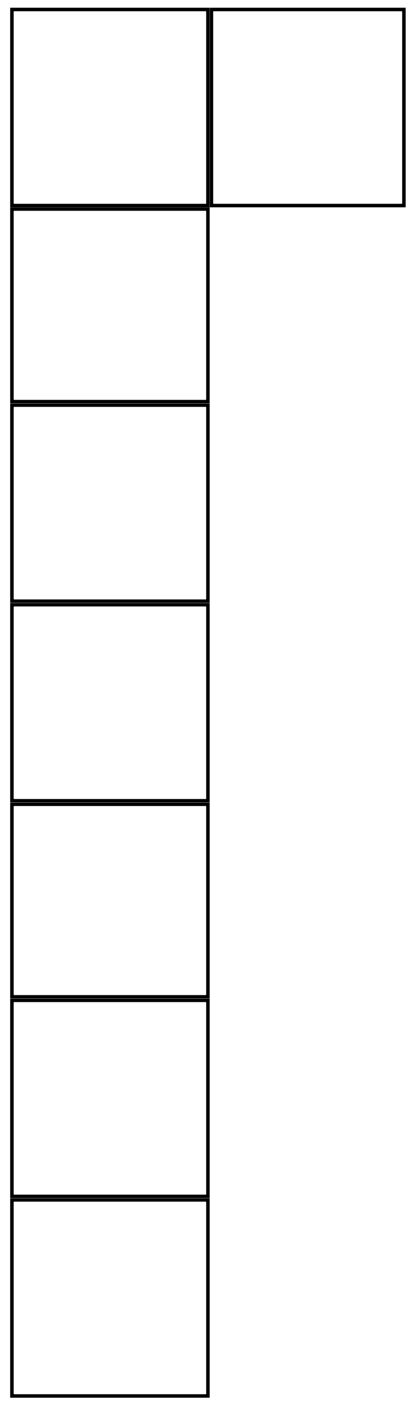
$$Q[\rho] = \frac{1}{4} \int_S \rho \wedge \Gamma_{ab} \wedge \bar{e}^a \wedge \bar{e}^b$$

- compare with ADM:

$$Q[k] = \frac{1}{4} \int_S \Gamma_{ab} \wedge * (k \wedge \bar{e}^a \wedge \bar{e}^b)$$

Dual Gravity Charges

- What is meaning of the charge K?
- Dual graviton charge?
- What is spectrum of states with charge K?
- Are there other magnetic charges?
- Dual "gravity" charges with hook Young tableaux suggested in various approaches
- Are there such charges too?



M-theory beyond the Planck Scale

- What does M-theory look like at energies much higher than the Planck scale?
- New highly symmetric phase?
- Conjecture [CH, 2000]: M-theory on T^6 has trans-Planckian phase which is a 6D theory with (4,0) supersymmetry
- Highly symmetric
- CONFORMAL theory of gravity without higher derivatives
- Exotic theory of gravity without g_{mn}

5D Superalgebra

- N supercharges Q^a, central charges $Z^{ab} = -Z^{ba}, X$

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab} P^\mu (C\Gamma_\mu)_{\alpha\beta} + C_{\alpha\beta}(Z^{ab} + X\Omega^{ab})$$

- $a, b = 1,\ldots,N$ are $USp(N) = Sp(N/2)$ R-symmetry indices
- Z Electric charges for Maxwell fields
- N=4 SYM: X carried by "instantonic solitons": YM instanton in $\mathbb{R}^4$ lifts to 0-brane in $\mathbb{R}^{4,1}$
- These are 0-branes of form **(YM instanton)x(time)**
- N=8 SUGRA: X is K-charge

5D SYM at Strong Coupling

$$\{Q^a_\alpha, Q^b_\beta\} = \Omega^{ab} P^\mu (C\Gamma_\mu)_{\alpha\beta} + C_{\alpha\beta}(Z^{ab} + X\Omega^{ab})$$

- Z^{ab} Electric charges carried by W-bosons
- YM instanton in $\mathbb{R}^4$ lifts to 0-brane in $\mathbb{R}^{4,1}$
- X proportional to instanton number n
- BPS states in massive tensor multiplets $dB = \pm M \star B$

$$M = |X| \propto n/g^2_{YM}$$

- Massless as $g_{YM} \to \infty$. KK tower for 6'th dimension $R = g^2_{YM}$
- Decompactifies to 6D (2,0) theory [Witten]
- 5D SYM non-renormalizable, so embed in UV complete theory

5D N=8 Supergravity at Strong Coupling

[CH, 2000]

- Embed in UV complete theory: M-theory on T^6
- BPS states with $M = |K|$ fit into multiplets with massive $C_{\mu\nu\rho\sigma}$
- **<u>IF</u>** such BPS states have spectrum

$$M = \frac{n}{l_{Planck}}$$

- Become light as $l_{Planck} \to \infty$
- Decompactification with K-states as KK tower?
- D=5 SUGRA → massless D=6 multiplet with $C_{\mu\nu\rho\sigma}$
- (4,0) supersymmetry, conformal multiplet!

(2,0) and (4,0)

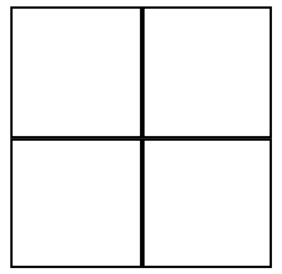
- Free theories of (2,0) and (4,0) 6D supermultiplets exist. (4,0) “square” of (2,0)
- Free (4,0): C_{MNPQ}, $8\ \psi^{\alpha\, a}_{MN}$, $27\ B^{ab}_{MN}$, $48\ \lambda^{abc}_{\alpha}$, $42\ \phi^{abcd}$. Sp(4) R-symmetry $a, b = 1,\ldots,8$
- Reduce on S^1 to 5D SYM and SUGRA
- A_μ, $g_{\mu\nu}$ from higher tensors B_{MN}, C_{MNPQ} with self-dual field strengths
- No conventional field theory interactions possible for (2,0),(4,0)
- Interacting (2,0) theory exists: non-lagrangian CFT, reduces to interacting 5D SYM
- Does an interacting (4,0) theory exist, reducing to 5D supergravity?
- Would be a conformal theory of gravity in 6D based on exotic tensor instead of metric!

Free (4,0) Theory

Light-cone gauge spectrum: Strathdee 1987; Covariant formulation CH 2000

Gauge field: symmetries of Riemann tensor

$$C_{MNPQ} = -C_{NMPQ} = -C_{MNQP} = C_{PQMN}$$

C 

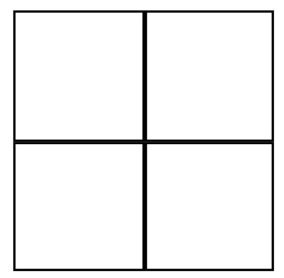
Free (4,0) Theory

Light-cone gauge spectrum: Strathdee 1987; Covariant formulation CH 2000

Gauge field: symmetries of Riemann tensor

$$C_{MNPQ} = -C_{NMPQ} = -C_{MNQP} = C_{PQMN}$$

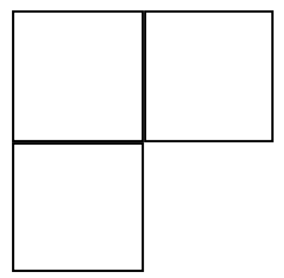
C



Gauge transformations

$$\delta C_{MNPQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

χ



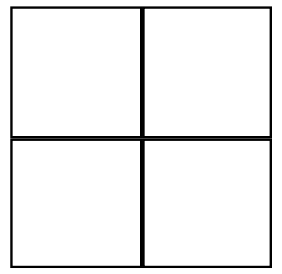
Free (4,0) Theory

Light-cone gauge spectrum: Strathdee 1987; Covariant formulation CH 2000

Gauge field: symmetries of Riemann tensor

$$C_{MNPQ} = -C_{NMPQ} = -C_{MNQP} = C_{PQMN}$$

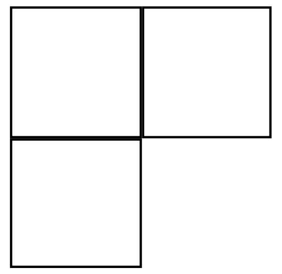
C



Gauge transformations

$$\delta C_{MNPQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

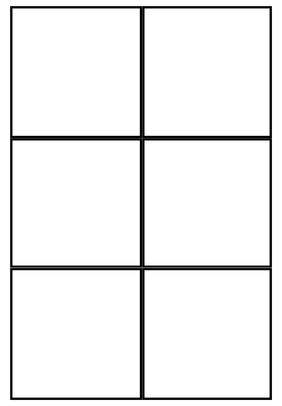
χ



Field strength

$$G_{MNPQRS} = \frac{1}{36}(\partial_M \partial_S C_{NPRS} + \ldots) = \partial_{[M}C_{NP][QR,S]}$$

G

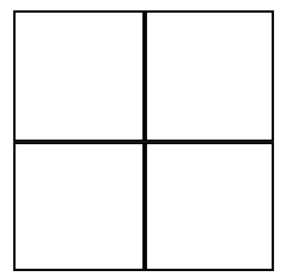


Free (4,0) Theory

Light-cone gauge spectrum: Strathdee 1987; Covariant formulation CH 2000

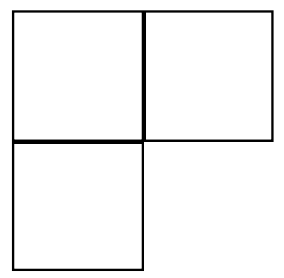
Gauge field: symmetries of Riemann tensor

$$C_{MNPQ} = -C_{NMPQ} = -C_{MNQP} = C_{PQMN}$$

C 

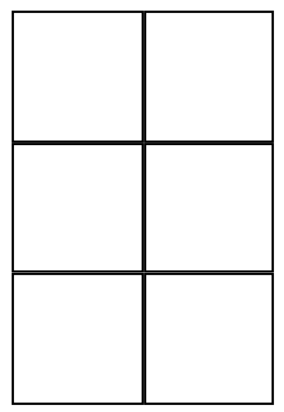
Gauge transformations

$$\delta C_{MNPQ} = \partial_{[M}\chi_{N]PQ} + \partial_{[P}\chi_{Q]MN} - 2\partial_{[M}\chi_{NPQ]}$$

χ 

Field strength

$$G_{MNPQRS} = \frac{1}{36}(\partial_M\partial_S C_{NPRS} + \dots) = \partial_{[M}C_{NP][QR,S]}$$

G 

Self-dual

$$G_{MNPQRS} = \frac{1}{6}\epsilon_{MNPTUV}G^{TUV}{}_{QRS}$$

$G = *G = G*$

Self-dual B-field →E-M duality

Reduce from 6D to 5D on S^1

Reduction of general B-field

$$B_{MN} \to (b_{\mu\nu}, A_\mu) \qquad b_{\mu\nu} = B_{\mu\nu}, A_\mu = B_{\mu 5}$$

$$H = dB \to (h, F) \qquad h = db, F = dA$$

Self-dual B-field → E-M duality

Reduce from 6D to 5D on S^1

Reduction of general B-field

$$B_{MN} \to (b_{\mu\nu}, A_\mu) \qquad b_{\mu\nu} = B_{\mu\nu}, A_\mu = B_{\mu 5}$$

$$H = dB \to (h, F) \qquad h = db, F = dA$$

If H self dual $\quad H = *H \quad \longrightarrow \quad h = *F$

Fields A, b are electromagnetic duals

Alternate formulations of same degrees of freedom for Free Theory

Theory can be written in terms of A or b, dual formulations

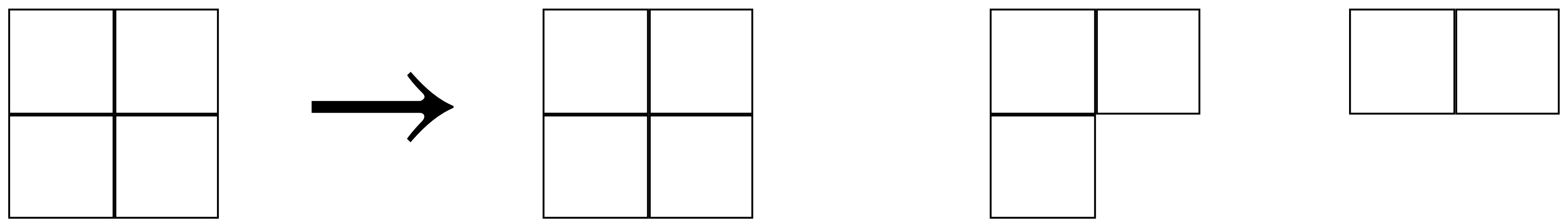
Self-dual C-field → Gravitational duality

Reduce from 6D to 5D on S^1

CH 2000

Reduction of general C-field

$$C_{MNPQ} \rightarrow (c_{\mu\nu\rho\sigma}, d_{\mu\nu\rho}, h_{\mu\nu})$$



$$c_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma}, d_{\mu\nu\rho} = C_{\mu\nu\rho 5}, h_{\mu\nu} = C_{\mu 5\nu 5}$$

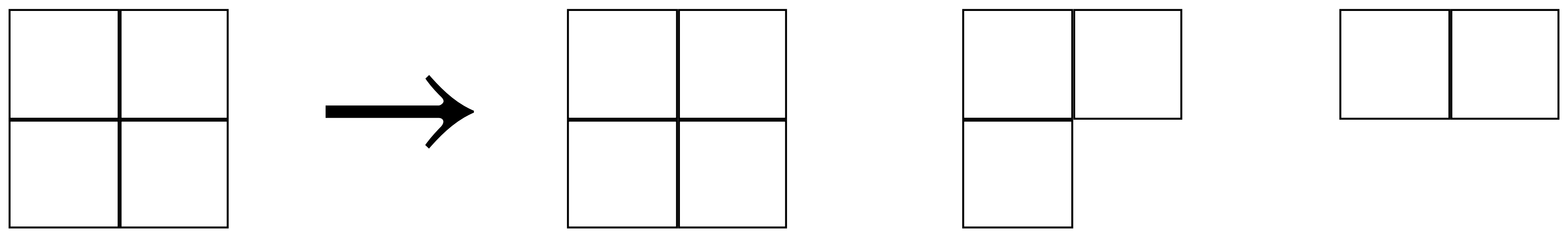
Self-dual C-field → Gravitational duality

Reduce from 6D to 5D on S^1

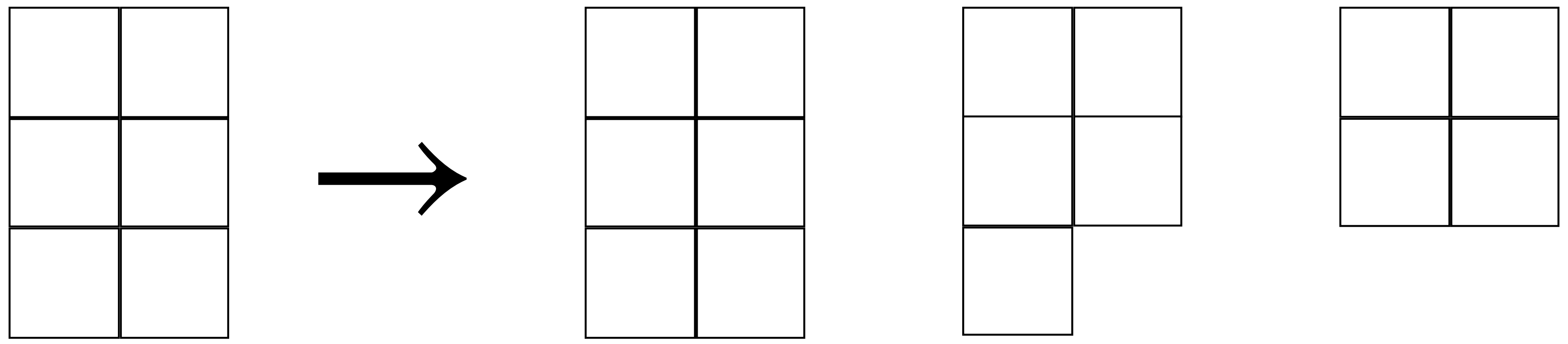
CH 2000

Reduction of general C-field

$$C_{MNPQ} \to (c_{\mu\nu\rho\sigma}, d_{\mu\nu\rho}, h_{\mu\nu})$$



$$G \sim \partial\partial C \to (G, S, R)$$



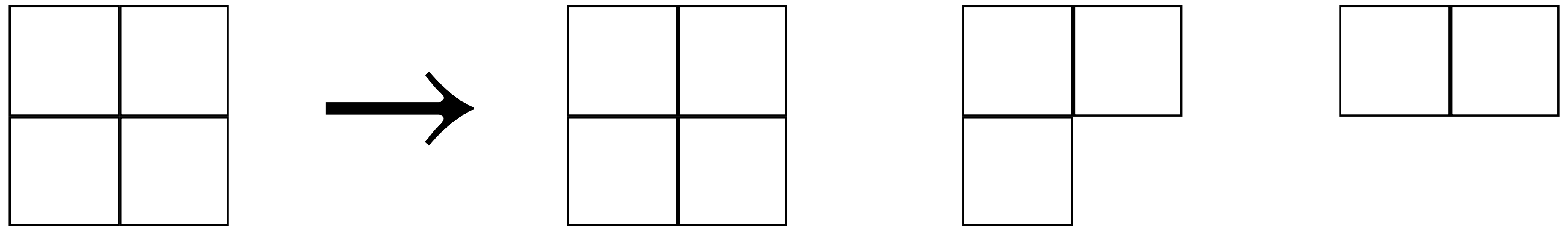
Self-dual C-field → Gravitational duality

Reduce from 6D to 5D on S^1

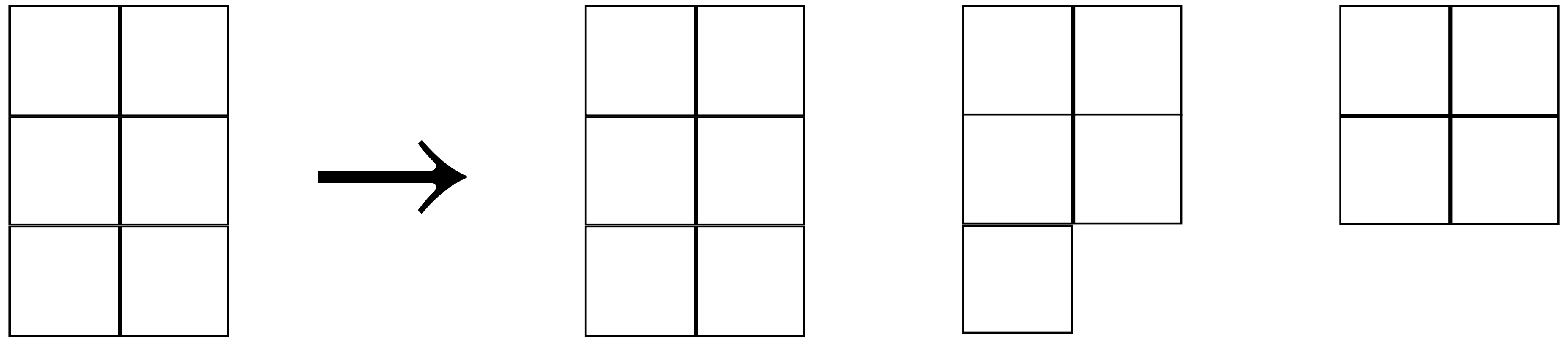
CH 2000

Reduction of general C-field

$$C_{MNPQ} \to (c_{\mu\nu\rho\sigma}, d_{\mu\nu\rho}, h_{\mu\nu})$$



$$G \sim \partial\partial C \to (G, S, R)$$



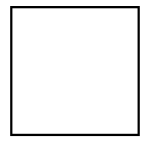
If G self dual $\quad G = *G = G* \quad \longrightarrow \quad S = *R, G = *R*$

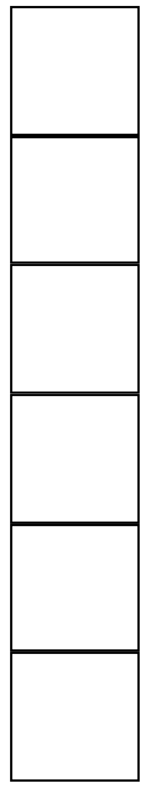
$h_{\mu\nu}$ is graviton, $d_{\mu\nu\rho}$ is *dual graviton*, $c_{\mu\nu\rho\sigma}$ is *double dual graviton*

Alternate formulations: can write free theory in terms of h,d or c

Duality of Free Fields in D dimensions

CH 2000

Photon A_μ 

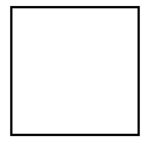
Dual Photon n-form $\tilde{A}_{\mu_1 \ldots \mu_n}$ 

$n = D - 3$

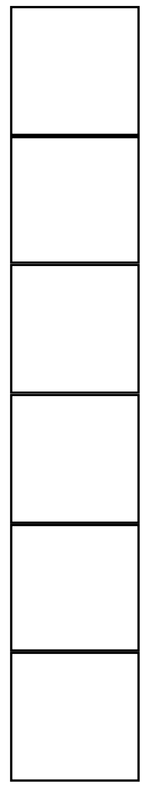
Duality of Free Fields in D dimensions

CH 2000

Photon A_μ



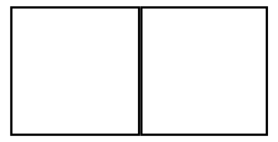
Dual Photon n-form $\tilde{A}_{\mu_1 \ldots \mu_n}$



$n = D - 3$

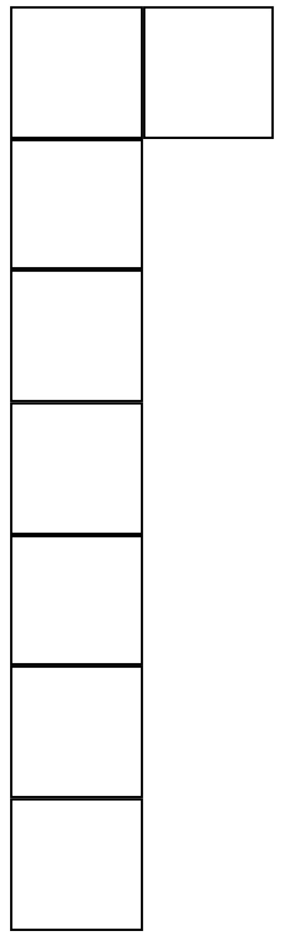
Graviton

$h_{\mu\nu}$ [1,1]



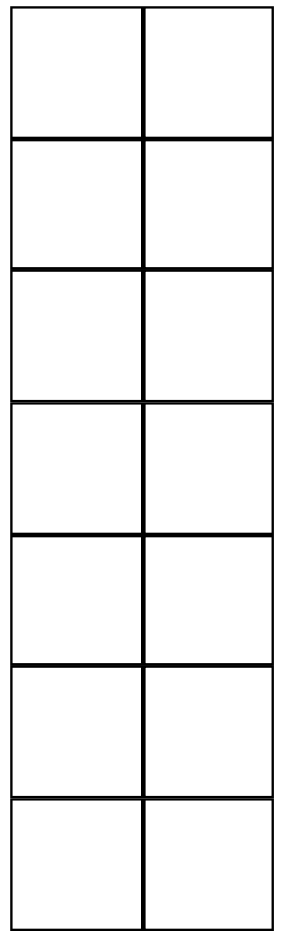
Dual Graviton

$D_{\mu_1 \mu_2 \ldots \mu_n \mid \nu}$ [n,1]



Double Dual Graviton

$C_{\mu_1 \mu_2 \ldots \mu_n \mid \nu_1 \nu_2 \ldots \nu_n}$ [n,n]



Gravitational duality interchanges field equations and Bianchi identities

If (4,0) limit exists…

- l_{Planck} NOT dimensionless. Limit is one to energies $E >> 1/l_{Planck}$
- Highly symmetric superconformal phase emerging at transplanckian energies
- 32 Supersymmetries + 32 conformal supersymmetries, $OSp(8^*/8) \supset SO(6,2) \times Sp(4)$
- Graviton arising from tensor C_{MNPQ}
- Exotic theory of gravity
- (4,0) Phase of M-theory?
- Test: Does M-theory on T^6 have states with K? Do they fit in KK tower?

Circle Reduction from 6D

- Momentum modes on S^1 $\rightarrow$ 0-branes in 5D. 5D theory shd have 0-brane solutions
- Interacting (2,0) $\rightarrow$ 5D SYM:
- KK modes identified with instantonic solitons of 5D SYM
- Free (2,0): can do reduction explicitly, get free SYM
- Instantons $\rightarrow$zero size singular instantons in Maxwell theory
- Free (4,0) theory: can do reduction explicitly, get linearised SUGRA
- KK modes in free theory: zero size singular gravitational instantons used to make instantonic 0-branes?

0-branes: gravitational instantons?

- Does non-linear 5D supergravity have suitable 0-branes?
- KK modes have multiplet structure of (self-dual 4D space) x (time)
- Gibbons Hawking form of gravitational instanton:

$$ds^2_{GH} = V^{-1}(dy + A_i dx^i)^2 + V dx^i dx^i \qquad i = 1,2,3 \qquad V = V(x^i)$$

$$\nabla^2 V = 0 \qquad F_{ij} = \epsilon_{ijk} \nabla^k V \qquad F_{ij} \equiv \partial_i A_j - \partial_j A_i$$

- KK monopole

$$ds^2 = -dt^2 + ds^2_{GH}$$

- Single centre:

$$V = 1 + U, \qquad U = \frac{N}{|x - x_o|}$$

Self-dual Euclidean

Taub-NUT

$$ds^2 = -dt^2 + ds^2_{GH} \qquad ds^2_{GH} = V^{-1}(dy + A_i dx^i)^2 + V dx^i dx^i \qquad V = 1 + \frac{N}{|x - x_o|}$$

F=*dV

Is this a suitable 0-brane solution for 5D supergravity?

Require to be asymptotic to 5D Minkowski space?

$$ds^2 = -dt^2 + ds^2_{GH} \qquad ds^2_{GH} = V^{-1}(dy + A_i dx^i)^2 + V dx^i dx^i \qquad V = 1 + \frac{N}{|x - x_o|}$$

F=*dV

Problems: 1)Singular at $x = x_0$ 2) Dirac string singularity e.g. $A_\pm = -\frac{N}{2}(\cos\theta \pm 1)d\phi$

Dirac string can be moved by diffeomorphisms. Curvature depends only on F=dA=*dV

$$ds^2 = -dt^2 + ds^2_{GH} \qquad ds^2_{GH} = V^{-1}(dy + A_i dx^i)^2 + V dx^i dx^i \qquad V = 1 + \frac{N}{|x - x_o|}$$

F=*dV

Problems: 1)Singular at $x = x_0$ 2) Dirac string singularity e.g. $A_\pm = -\frac{N}{2}(\cos\theta \pm 1)d\phi$

Dirac string can be moved by diffeomorphisms. Curvature depends only on F=dA=*dV

Usual approach:

If $N \in \mathbb{Z}$ and y periodic $y \sim y + 4\pi N$, construct manifold with two patches, one with A_+, one with $A\_$. Each patch has no Dirac string.

If $N = 1$, no singularity at $x = x_0$. For other N, orbifold singularity

Smooth complete manifold, S^1 fibration over $\mathbb{R}^3$.

Asymptotic to Hopf fibration over S^2 at infinity giving a (squashed) S^3

$$ds^2 = -dt^2 + ds^2_{GH} \qquad ds^2_{GH} = V^{-1}(dy + A_i dx^i)^2 + V dx^i dx^i \qquad V = 1 + \frac{N}{|x - x_o|}$$

F=*dV

Is this a suitable 0-brane solution for 5D supergravity?

Require to be asymptotic to 5D Minkowski space?

But for asymptotically flat, require non-periodic y. Do Dirac strings matter?

Can we think of this solution without compactifying y?

Gravitational Charges

Gauge symmetry: diffeomorphisms

Isometries: subroup preserving a given configuration

Generated by Killing vectors k^μ

$$\nabla_{(\mu} k_{\nu)} = 0$$

E.g. Minkowski space: Isometry is Poincare group, translations and Lorentz

Isometries: Global symmetry group with associated Noether charges

$$J_\mu = T_{\mu\nu} k^\nu \qquad \nabla_\mu J^\mu = 0$$

Conserved charge $$Q[k] = \int_\Sigma * J$$ Σ is spatial hypersurface

Use Einstein's equations to rewrite as integral over $S = \partial\Sigma$, sphere at spatial infinity

Gravitational Charges 2

General configurations have no isometries so no Noether charges

If spacetime is asymptotic to some $(\bar{M}, \bar{g})$ which has isometries

generated by KV $\bar{k}^\mu$, then (M, g) can have asymptotic Killing vectors

ADM construction: Generalised Noether charges $Q[\bar{k}]$

Can write as integrals over (D-2)-sphere at spatial infinity

Requires suitable boundary conditions

e.g. if $(\bar{M}, \bar{g})$ is Minkowski space: ADM momentum P^μ, angular momentum $J^{\mu\nu}$

Gravitational Charges 3

Linearised gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Linearise about Minkowski space

$$G^L_{\mu\nu}(h) = T_{\mu\nu}$$

$G^L_{\mu\nu}$: terms in Einstein tensor linear in $h_{\mu\nu}$

Gauge symmetry $\quad \delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$

Killing vector $\quad \partial_{(\mu}k_{\nu)} = 0$ $\quad$ Generate Poincare group

All configurations have isometry generated by such KV, so there are Noether charges

$$Q[k]$$

Linearisation of ADM charges. ADM from “integrating up” linearised construction

Charges in General Gauge Theories

Similar structure to Gravity

Isometries/invariances

For a given configuration, the gauge transformations preserving that configuration

give an "isometry group", typically finite dimensional.

Killing gauge parameters, Noether charges

General configurations have no isometries. If asymptotic to configuration with isometries t
can be asymptotic Killing gauge parameters and ADM-type charges

Linearising gauge symmetry about that configuration, these become Noether charges

ADM-type charges from "integrating up" the linearised construction

Antisymmetric Tensor Gauge Theories

D dimensions, general spacetime

$$dF = *\,\tilde{J}, \qquad d*F = *\,J$$

(n + 1)-form field strength F

n-form electric current J, (D - n - 2)-form magnetic current $\tilde{J}$

Conserved $\qquad d*\tilde{J}, = 0 \qquad d*J = 0$

If $\tilde{J} = 0$, locally there is n-form potential A $\qquad F = dA$

Gauge symmetry $\qquad \delta A = d\lambda$

Reducible: exact $\lambda = d\alpha$ don't act

Isometries/invariances

Killing gauge parameters: closed n-1 forms λ $\quad d\lambda = 0$

Modulo exact, so isometries correspond to cohomology classes

1-form currents $\quad *j = (\lambda \wedge *J)$ $\quad j_\mu = \frac{1}{(n-1)!} J_{\mu\nu_1\ldots\nu_{n-1}} \lambda^{\nu_1\ldots\nu_{n-1}}$

Conserved $\quad d*j[\lambda] = 0$

Conserved charge for each cohomology class $\quad Q[\lambda] = \int_\Sigma *j[\lambda]$

$$Q[\lambda + d\alpha] = Q[\lambda]$$

$$*j[\lambda] = \lambda \wedge *J = \lambda \wedge d(*F) = d(\lambda \wedge *F)$$

$$Q[\lambda] = \int_{\partial\Sigma} \lambda \wedge *F$$

If $J = 0$, locally there is (D-n-3)—form potential $\tilde{A}$ $\quad * F = d\tilde{A}$

Gauge symmetry $\quad \delta\tilde{A} = d\tilde{\lambda}$

Reducible: exact $\tilde{\lambda} = d\tilde{\alpha}$ don't act

$$Q[\tilde{\lambda}] = \int_{\Sigma} \tilde{\lambda} \wedge *\tilde{J} = \int_{\partial\Sigma} \tilde{\lambda} \wedge F$$

Conserved charges corresponding to Killing gauge parameter cohomology classes

Magnetic charges as electric charges of dual theory

$$dF = *\tilde{J}, \qquad d*F = *J$$

$$Q[\lambda] = \int_{\partial\Sigma} \lambda \wedge *F \qquad Q[\tilde{\lambda}] = \int_{\partial\Sigma} \tilde{\lambda} \wedge F \qquad d\lambda = 0, \quad d\tilde{\lambda} = 0$$

Charges conserved if sources $J, \tilde{J}$ have compact support

If $\tilde{J} = 0$ $Q[\lambda]$ is electric charge associated with $A \to A + d\lambda$, $Q[\tilde{\lambda}]$ is a magnetic charge

If $J = 0$ $Q[\tilde{\lambda}]$ is electric charge associated with $\tilde{A} \to \tilde{A} + d\tilde{\lambda}$, $Q[\lambda]$ is a magnetic charge

Charges “topological”: depend only on cohomology classes $[F]$, $[*F]$

Gravity Magnetic Charges: Noether Charges of Dual Graviton

Linearised gravity

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad G^L_{\mu\nu}(h) = T_{\mu\nu}$$

Gauge symmetry $\qquad \delta h_{\mu\nu} = \partial_{(\mu}\xi_{\nu)}$

Invariant field strength $\quad R_{\mu\nu\,\sigma\tau} = \partial_\mu \partial_\sigma h_{\nu\tau} + \ldots = -4\partial_{[\mu}h_{\nu][\sigma,\tau]}$

$$R_{[\mu\nu\,\sigma]\tau} = 0 \qquad R_{\mu\nu\,[\rho\sigma,\tau]} = 0$$

Killing vector $\qquad \partial_{(\mu}k_{\nu)} = 0$

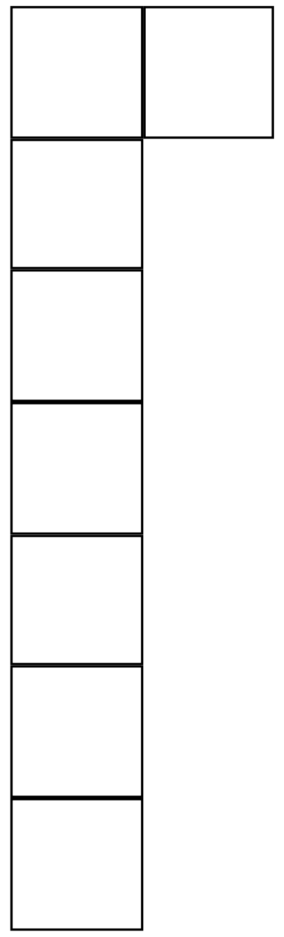
Noether charges $\qquad Q[k]$

Dual Graviton in D Dimensions

Dual graviton

$D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$

[n,1]



Two gauge symmetries.

Parameters:

$\alpha_{\mu_1\ldots\mu_{n-1}\,|\,\rho}$ [n-1,1]

$\beta_{\mu_1\ldots\mu_n}$ [n,0] n-form

$$\delta D_{\mu\nu\ldots\sigma\,|\,\rho} = \partial_{[\mu}\alpha_{\nu\ldots\sigma]\,|\,\rho} + \partial_\rho\beta_{\mu\nu\ldots\sigma} - \partial_{[\rho}\beta_{\mu\nu\ldots\sigma]}$$

Field strength $S_{\mu\nu\ldots\rho\,|\,\sigma\tau} = \partial_{[\mu}D_{\nu\ldots\rho]\,|\,[\sigma,\tau]}$ [n+1,2]

[Two types of Noether charge for two types of symmetery]

Killing Tensors

$$\delta D_{\mu\nu\ldots\sigma\,|\,\rho} = \partial_{[\mu}\alpha_{\nu\ldots\sigma]\,|\,\rho} + \partial_\rho\beta_{\mu\nu\ldots\sigma} - \partial_{[\rho}\beta_{\mu\nu\ldots\sigma]}$$

"Dual isometries" if

1) parameter α is [n-1,1] generalised Killing tensor $\kappa_{\mu_1\ldots\mu_{n-1}\,|\,\rho}$ satisfying

$$\partial_{[\mu}\kappa_{\nu\ldots\sigma]\rho} = 0$$

2) parameter β given by a Killing-Yano tensor, i.e, an n-form $\lambda_{\mu_1\ldots\mu_n}$ satisfying

$$\partial_\rho\lambda_{\mu\nu\ldots\sigma} - \partial_{[\rho}\lambda_{\mu\nu\ldots\sigma]} = 0$$

Noether charges $\quad Q[\kappa], \quad Q[\lambda]$

K-Charge

[n-1,1] generalised Killing tensor $\kappa_{\mu_1 \ldots \mu_{n-1} \mid \rho}$ satisfying

$$\partial_{[\mu}\kappa_{\nu \ldots \sigma]\rho} = 0$$

Particular solution

$$\kappa_{\mu_1 \ldots \mu_{n-1} \mid \nu} = \rho_{[\mu_1 \ldots \mu_{n-2}}\eta_{\mu_{n-1}]\nu}$$

where $\rho_{\mu_1 \ldots \mu_{n-2}}$ is closed (n-2)-form, $d\rho = 0$ n-2=D-5

Noether charge $Q[\rho]$ is linearised K-charge, depends only on cohomology class of ρ

Gravitational Duality

Dual to graviton: $S_{\mu_1\mu_2\ldots\mu_{n+1}\,|\,\nu\rho} = \frac{1}{2}\epsilon_{\mu_1\mu_2\ldots\mu_{n+1}\alpha\beta}R^{\alpha\beta}{}_{\nu\rho}$ $\qquad S = *\,R$

$$R_{\mu\,\nu} = 0 \qquad \leftrightarrow \qquad S_{[\mu_1\mu_2\ldots\mu_{n+1}\,\nu]\rho} = 0$$

$$R_{[\mu\nu\,\sigma]\tau} = 0 \qquad \leftrightarrow \qquad S'_{\mu_1\mu_2\ldots\mu_n\,\nu} = 0$$

Trace $\qquad S'_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = S_{\mu_1\mu_2\ldots\mu_n\rho\,|\,\nu}{}^{\rho}$

Double Trace $\qquad S''_{\mu_1\mu_2\ldots\mu_{n-1}} = S_{\mu_1\mu_2\ldots\mu_{n-1}\nu\rho\,|}{}^{\nu\rho}$

Field Equation

"Einstein" tensor

$$E_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = S'_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} - \frac{n}{2} S''_{[\mu_1\mu_2\ldots\mu_{n-1}}\eta_{\mu_n]\nu}$$

Identically conserved $\partial^{\mu_1} E_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = 0, \qquad \partial^{\nu} E_{\mu_1\mu_2\ldots\mu_{n-1}\mu_n\,|\,\nu} = 0$

Field equation

$$E_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$$

Conserved dual stress energy tensor

$$\partial^{\mu_1} U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = 0, \qquad \partial^{\nu} U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = 0$$

Sources

$T_{\mu\nu}$ source for $R_{\mu\,\nu}$ or $S_{[\mu_1\mu_2\ldots\mu_{n+1}\,\nu]\rho}$

$U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ source for $S'_{\mu_1\mu_2\ldots\mu_n\,\nu}$ or $R_{[\mu\nu\,\sigma]\tau}$

$T_{\mu\nu}$ gives regular $h_{\mu\nu}$, but Dirac strings in $D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$

$U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ gives regular $D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$, but Dirac strings in $h_{\mu\nu}$

In regions with $T_{\mu\nu} = 0$, can write theory in terms of $h_{\mu\nu}$

In regions with $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu} = 0$, can write theory in terms of $D_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$

Linearised KK monopole solution:

Superposition of 2 solutions $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ $X^\mu = (t, y, x^i)$

1) delta-function $T_{\mu\nu}$ source at $x_0 = 0$

$$h_{yy} = -U, \qquad h_{ij} = U\delta_{ij} \qquad U = \frac{M}{r}$$

2) delta-function $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ source at $x_0 = 0$

$$h_{iy} = h_{yi} = A_i \qquad F_{ij} = \varepsilon_{ijk}\partial^k W \qquad W = \frac{N}{r}$$

BPS if $V = W, M = N$

Linearised KK monopole solution:

1) delta-function $T_{\mu\nu}$ source at $x_0 = 0$ $\quad h_{yy} = -U, \quad h_{ij} = U\delta_{ij} \quad U = \frac{M}{r}$

2) delta-function $U_{\mu_1\mu_2\ldots\mu_n|\nu}$ source at $x_0 = 0$ $\quad h_{iy} = h_{yi} = A_i$

$$F_{ij} = \varepsilon_{ijk}\partial^k W \qquad W = \frac{N}{r}$$

Linearised KK monopole solution:

1) delta-function $T_{\mu\nu}$ source at $x_0 = 0$ $\quad h_{yy} = -U, \quad h_{ij} = U\delta_{ij} \quad U = \frac{M}{r}$

2) delta-function $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ source at $x_0 = 0$ $\quad h_{iy} = h_{yi} = A_i$

$F_{ij} = \varepsilon_{ijk}\partial^k W \quad W = \frac{N}{r}$

<u>Dualise</u> $\quad D_{ty|y} = W$

W 3D dual of A_i $\quad F_{ij} = \varepsilon_{ijk}\partial^k W$

Linearised KK monopole solution:

1) delta-function $T_{\mu\nu}$ source at $x_0 = 0$ $\qquad h_{yy} = -U, \qquad h_{ij} = U\delta_{ij} \qquad U = \dfrac{M}{r}$

<u>Dualise</u> $\qquad D_{ti|y} = B_i,\ D_{ti|j} = k_{ij}$

B_i, k_{ij} 3D duals of U

$\varepsilon_{ijk}\partial^k U = \partial_i B_j - \partial_j B_i \qquad \partial_i \partial_j U = \varepsilon_{ikl}\varepsilon_{jmn}\partial_k \partial_m k_{ln}$

2) delta-function $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ source at $x_0 = 0$ $\qquad h_{iy} = h_{yi} = A_i$

$F_{ij} = \varepsilon_{ijk}\partial^k W \qquad W = \dfrac{N}{r}$

Linearised KK monopole solution:

1) delta-function $T_{\mu\nu}$ source at $x_0 = 0$ $\quad h_{yy} = -U, \quad h_{ij} = U\delta_{ij} \quad U = \frac{M}{r}$

Dualise $\quad D_{ti|y} = B_i,\ D_{ti|j} = k_{ij}$

B_i, k_{ij} 3D duals of U

$\epsilon_{ijk}\partial^k U = \partial_i B_j - \partial_j B_i \quad \partial_i\partial_j U = \epsilon_{ikl}\epsilon_{jmn}\partial_k\partial_m k_{ln}$

2) delta-function $U_{\mu_1\mu_2\ldots\mu_n\,|\,\nu}$ source at $x_0 = 0$ $\quad h_{iy} = h_{yi} = A_i$

Dualise $\quad D_{ty|y} = W \quad F_{ij} = \epsilon_{ijk}\partial^k W \quad W = \frac{N}{r}$

W 3D dual of A_i

Dirac Strings for A_i, B_i

Linearised KK monopole solution 2

- Superposition: "electric" solution with charge M & "magnetic" one with charge N
- CHARGES: Mass Q[k]=M for $k^\mu = (1,\underline{0})$, K-charge $Q[\rho] = N$ for 0-form $\rho = 1$
- Each has Dirac string in one duality frame and not in other
- Can move positions of string singularities by gauge transformations
- Magnetic solution: can take 2 patches with $h^\pm_{iy} = A^\pm_i$, $\quad A^+ - A^- = Nd\phi$
- Transition function: $\xi_y = \phi$. Locally OK but globally problematic: $\phi \sim \phi + 2\pi$
- OK(?) if allow generalised gauge transformations cf $\delta A = \alpha$ with $d\alpha = 0$

KK Modes for (4,0) Theory

- Free (4,0) theory compactifies to linearised D=5 supergravity
- KK modes associated with linearised KK monopole
- Dirac strings depend on duality frame
- Singularity at $x^i = 0$ representing presence of "particle"
- Non-linear D=5 SUGRA: candidate solution given by GH metric for $x \neq 0$
- Dirac strings may not be so bad: signal of using wrong variables?
- Singularity at $x = 0$: resolve with *core* of 6D (4,0) theory?

Discussion

- New magnetic charges for linearised gravity from Noether charges for dual isometries of dual graviton. Includes linearised K-charge, extends to GR in $D \geq 5$
- Interacting (4,0) theory needs KK tower of K-charged BPS states in D=5 M-theory
- If can show there are no such states in M-theory, then conjecture falsified
- K-states are U-duality singlets, so duality doesn't help
- Would have liked non-singular soliton, as for YM/(2,0)
- Analysis of linearised theory suggests Dirac strings may be tolerable

- Singular D=5 solution OK if resolved in full D=6 theory
- cf M-theory: KK modes are $D0$-branes, which correspond to singular IIA supergravity solutions. But singularity resolved in D=11 pp-wave solution
- BPS states in M-theory correspond to supergravity solutions. If solution non-singular, then supergravity soliton strong evidence for existence of BPS state. If singular, need further input to understand whether state actually arises.
- Spin-off: gravitational duality, magnetic charges etc
- Irrespective of whether (4,0) conjecture true, gravitational duality, magnetic charges etc may have interesting consequences, especially for gravity in $D \geq 5$.