Three-forms, and Axíons: String and Particle Physics Applications

Luis Ibáñez



European Research Council

SPLE Advanced Grant





Instituto de Física Teórica UAM-CSIC, Madrid Fayet Fest ENS, Paris, December 2016

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Of course:

One of the fathers of supersymmetry

Many crucial insights...

Spontaneous symmetry breaking in SUSY.... Supersymmetry breaking..... R-parity N=2 Fayet-Iliopoulos terms.....

SUSY phenomenology.....

Light dark matter

ultra light U(I)'s.....

What some of us had to study by 1980...

PHYSICS REPORT (Section C of Physics Letters) 32, No. 5 (1977) 249-334. NORTH-HOLLAND PUBLISHING COMPANY

SUPERSYMMETRY

P. FAYET* and S. FERRARA**

Laboratoire de Physique Théorique de l'École Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France

Received 2 July 1976

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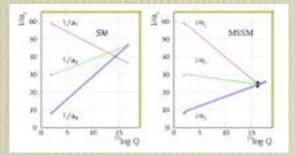
Abstract:

Supersymmetry transformations turn bosons into fermions and conversely. We discuss the algebraic aspects of the new structure, its role in relativistic quantum field theory and its possible applications to particle physics.

Suddenly 1981-1984 SUSY becomes popular !!

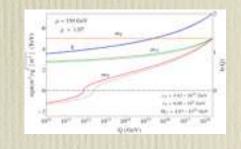
Emphasis on hierarchy problem

Gauge coupling unification



Coupling to supergravity induces soft terms: mSUGRA, CMSSM

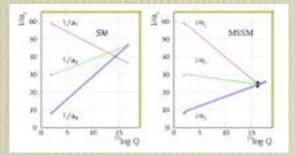
Radiative SU(2)xU(1) breaking



Suddenly 1981-1984 SUSY becomes popular !!

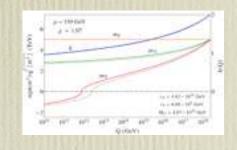
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Coupling to supergravity induces soft terms: mSUGRA, CMSSM

Radiative SU(2)xU(1) breaking



Still Pierre remains the father of the SUSY SM!

Later on, working in string theory, the inspiration of Pierre's work has been constant:

Two examples:

• Fayet-Iliopoulos terms in string-theory: important role in string compactification and D-brane physics and geometry

• R-parity appearing as a discrete gauge symmetry in string compactifications

I am going to talk about discrete shift symmetries:

 $\phi \rightarrow \phi + f$

.....apparently unrelated to SUSY....

...we will see are related to SUSY auxiliary fields from the point of view of string theory

Poincaré in 4 dimensions:

Spin: 0, 1/2, 1, 3/2, 2

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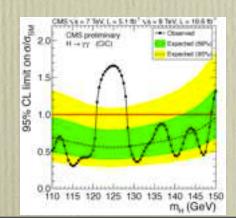
Fermions and Gauge bosons

Spin: 0, 1/2, 1, 3/2, 2

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Gravity is there

Spin: 0, 1/2, 1, 3/2, 2 Higgs found!!



Spin: 0, 1/2, 1, 3/2, 2



SUSY must exist!!

Somewhat analogous: **Bosons**: $Parity(+) : \phi, g_{\mu\nu}$ Higgs Gravity Parity(-) : C_0 , $C^{\mu\nu}$, $C^{\mu\nu}$

Axions Gauge Axions

Somewhat analogous: **Bosons**: $Parity(+) : \phi, g_{\mu\nu}$ Higgs Gravity Parity(-) : C_0 , $C^{\mu\nu}$, $C^{\mu\nu}$, $C^{\mu\nu\rho}$ Axions Gauge Axions

Somewhat analogous: **Bosons**: $Parity(+) : \phi, g_{\mu\nu}$ Higgs Gravity Parity(-) : C_0 , $C^{\mu\nu}$, $C^{\mu\nu}$, $C^{\mu\nu\rho}$ Axions Gauge Axions Usually ignored because it does not propagate but: Gives shift invariant masses to axions $F_4 = dC_3$ Contributes to c.c.:

Somewhat analogous: **Bosons**: $Parity(+) : \phi, g_{\mu\nu}$ Higgs Gravity Parity(-) : C_0 , $C^{\mu\nu}$, $C^{\mu\nu}$, Cινρ Axions Gauge Axions Usually ignored because it does not propagate but: Gives shift invariant masses to axions Landscape $F_4 = dC_3$ Contributes to c.c.: must exist !! 20



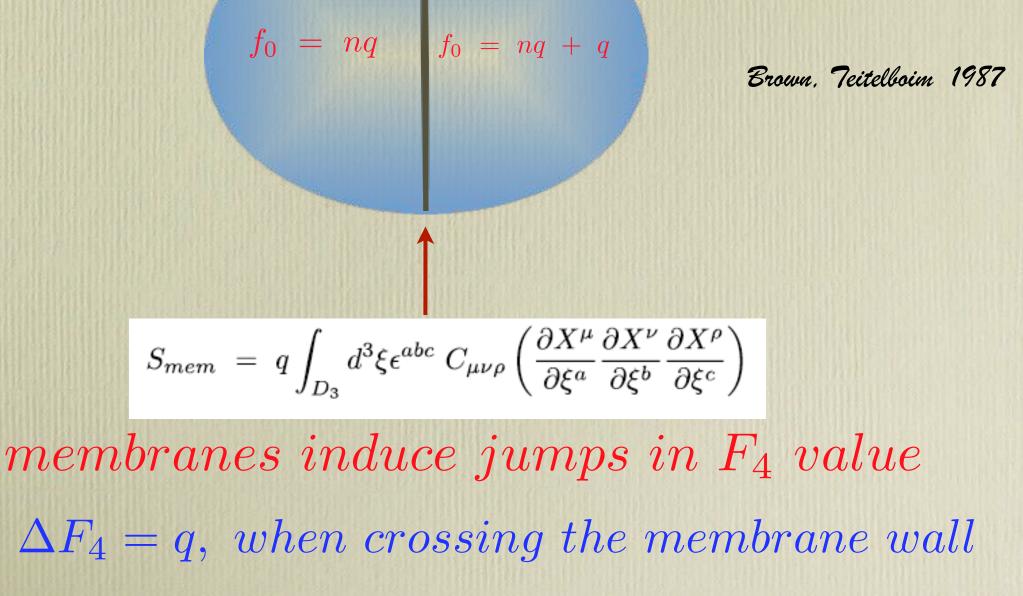
- The physics of Minkowski 3-forms
- Minkowski 3-forms in String Theory
- Applications: String Inflation Relaxion Higgs mass landscape

The physics of Minkowski 3-formsBosonic action of a 3-form field in 4d:
$$S = -\int d^4 x \sqrt{-g} \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}$$
 $S = -\int d^4 x \sqrt{-g} \frac{1}{48} F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}$ f_0 is constant $ropropagation...$ f_0 is constant $h_{cc} = \sum_i |F_4^i|^2$ If embedded in string theory: Bousso, Polchinski '00 $f_0 = nq$, $n \in \mathbb{Z}$ quantized in units of the membrane charge

Friday, December 9, 16

E

But 3-forms also couple to membranes:





1) Pseudoscalars invariant under shifts

$$\phi \rightarrow \phi + f$$

2) Perturbatively massless

3) Mass from non-perturbative instanton effects

Coupling axions to 3-forms

$$\mathcal{L} = -F_{4}^{2} + \mu\phi F_{4} + \dots$$

$$F_{4} = \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$$
Duali 05
Caloper. Sorbo 08:
wield:
Discrete gauge shift symmetry:

$$\phi \rightarrow \phi + \phi_{0}, q \rightarrow q - \mu\phi_{0}$$
It is a family (landscape) of
potentials parametrized 20 Y Q, μ

Coupling axions to 3-forms

$$\mathcal{L} = -F_4^2 + \mu \phi F_4 + \dots$$

$$F_4 = \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}$$
Axion mass consistent
with shift symmetry!
$$V_0 = \frac{1}{2}(q + \mu\phi)^2$$
Discrete gauge shift symmetry:
$$\phi \rightarrow \phi + \phi_0, q \rightarrow q - \mu\phi_0$$
It is a family (landscape) of
potentials parametrized by q, μ

One can formulate the same system in terms of a 2-form $B_{\mu\nu}$

$$\mathcal{L} = -F_4^2 - \frac{\mu^2}{2}|dB_2 - C_3|^2 + \dots * dB_2 = d\phi$$

 $C_3 \ eatsB_2 \ and \ becomes \ massive$ Invariant under the gauge transformation: $C_3 \ eatsB_2 \ and \ becomes \ massive$

$$B_2 \to B_2 + \Lambda_2$$
, $C_3 \to C_3 + d\Lambda_2$

These gauge invariances protect potential from uncontrolled corrections

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$$\delta V = V_0 \left(\frac{V_0}{M_{UV}^4}\right)^n$$

Kaloper, Sorbo 08;

$$\begin{array}{ll} (2h_{11}^{-}+2) \quad F_{4}'s \quad from \ RR \ sector: \\ F_{0}=-m \ , \quad F_{2}=\sum_{i}q_{i}\omega_{i} \ , \quad F_{4}=\underline{F_{4}^{0}}+\sum_{i}e_{i}\tilde{\omega}_{i} \\ F_{6}=\sum_{i}\underline{F_{4}^{i}}\omega_{i}+e_{0}dvol_{6} \ , \quad F_{8}=\sum_{i}\underline{F_{4}^{a}}\tilde{\omega}_{a} \ , \quad F_{10}=\underline{F_{4}^{m}}dvol_{6} \\ e_{0},e_{i},q_{i},m \ RR \ quantized \ fluxes \end{array}$$

 $(h_{21}^+ + 1) \quad H'_4s \quad from \ NS \ sector :$

$$H_7 = \sum_{I} \underline{H_4^{I}} \wedge \alpha_I \qquad H_3 = \sum_{I=0}^{h_{2,1}^-} h_I \beta_I \qquad h_I \ NS \ quantized \ fluxes$$

$$\begin{array}{ll} Axions : B_2 = \sum_i b_i \omega_i \ , \quad C_3 = \sum_I c_3^I \alpha_I \\ NS \end{array} \qquad \begin{array}{ll} \int_Y \omega_a \wedge \tilde{\omega}^\beta = \delta_a^\beta \ , \qquad \alpha, \beta \in \{1 \dots h_+^{(1,1)}\} \\ \int_Y \omega_a \wedge \tilde{\omega}^b = \delta_a^b \ , \qquad a, b \in \{1 \dots h_-^{(1,1)}\} \\ \int_Y \alpha_K \wedge \beta^L = \delta_K^L \ , \qquad K, L \in \{1 \dots h^{(2,1)} + 1\} \end{array}$$

$$(2h_{11}^{-}+2) \quad F_{4}'s \quad from \ RR \ sector :$$

$$F_{0} = -m, \quad F_{2} = \sum q_{i}\omega_{i} , \quad F_{4} = F_{4}^{0} + \sum_{i} e_{i}\tilde{\omega}_{i}$$

$$F_{6} = \sum_{i} \underbrace{F_{4}^{i}\tilde{\omega}_{i} + e_{0}dvol_{6}}_{e_{0},e_{i},q_{i},m} RR \ quantized \ fluxes$$

 $(h_{21}^+ + 1) \quad H'_4s \quad from \ NS \ sector :$

$$H_7 = \sum_{I} \underline{H_4^{I}} \wedge \alpha_I \qquad H_3 = \sum_{I=0}^{h_{2,1}^-} h_I \beta_I \qquad h_I \ NS \ quantized \ fluxes$$

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Bieleman, L. I., Valenzuela 15:

Full scalar potential in terms of 4-forms+local terms:

$$V = \frac{k}{2} |F_4^0|^2 + 2k \sum_{ij} g_{ij} F_4^i F_4^j + \frac{1}{8k} \sum_{ab} g_{ab} F_4^a F_4^b + k |F_4^m|^2 + \frac{1}{2s^2} \sum_{IJ} c_{IJ} H_4^I H_4^J + V_{loc}$$

Sort of generalized Kaloper-Sorbo structure:

$$V_{loc} = \sum_{a} \int_{\Sigma} T_a \sqrt{-g} \ e^{-\phi}$$

$$\begin{split} *_{4}F_{4}^{0} &= \frac{1}{k}(e_{0} + e_{i}b^{i} + \frac{1}{2}k_{ijk}q^{i}b^{j}b^{k} - \frac{m}{3!}k_{ijk}b^{i}b^{j}b^{k} - h_{0}c_{3}^{0} - h_{i}c_{3}^{i}) \\ & *_{4}F_{4}^{i} = \frac{g^{ij}}{4k}(e_{j} + k_{ijk}b^{j}q^{k} - \frac{m}{2}k_{ijk}b^{j}b^{k}) \\ & *_{4}F_{4}^{a} = 4kg^{ab}(q_{b} - mb_{b}) \\ & *_{4}F_{4}^{m} = -m \\ & *_{4}H_{4}^{I} = h^{I} \end{split}$$

All axion dependence goes through 3-forms 4-forms act as auxiliary fields

Generalized shift symmetries

transformations leave all 4 - formsinvariant for any IIA CY orientifold $V_{RR} + V_{NS}$ invariant

Some lessons:

The flux-induced scalar potential of Type IIA and IIB can be written as

$$V = \sum_{i} Z_{ij}(ReM_a) F_4^i F_4^j + \sum_{i} F_4^i \Theta_i(ImM_a) + V_{local}(ReM_a)$$

where all the dependence on axionic fields comes through couplings to Minkowski 3-form fields.

Ecs. motion: $*_4F_4^i = Z^{ij}\Theta^j(ReM_a, ImM_a)$

7. Valenzuela 15:
$$V_{4-forms} = \sum_{ij} Z_{ij} F_4^i F_4^j + V_{local}$$

Shift symmetries force potential axion dependence only through 4-forms

A N=1sugra formulation where auxiliary fields are 4forms seems appropriate...not much studied....

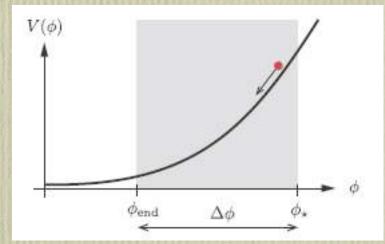
Gates et al. '81 Ovrut et al. '97 Louis et al. '13

Bieleman L.

Applications

1) Application to large field inflation

Chaotic Inflation Linde 88



 $V(\phi) = \mu^{4-p} \phi^p$

 $N_* \simeq \frac{1}{2n} \left(\frac{\phi_*}{M_n}\right)^2 \to trans - Planckian$

Is there a consistent string embeding?

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Silverstein, Westphal 08; McAllister, Silverstein, Westphal Kaloper, Sorbo 08 Marchesano, Shiu, Uranga , 14

Monodromy inflation

Simplest:

 $V_0 = \frac{1}{2} (q + \mu \phi)^2$ Chaotic

 $\delta V \simeq \frac{\sqrt{n}}{M_p^{n-4}} \qquad \delta V \simeq V_0 \left(\frac{V_0}{M_p^4}\right)^n \ll V_0$

Simplest:

$$V_0 = \frac{1}{2} \left(q + \mu \phi \right)^2 \qquad \text{Chaotic}$$

Quadratic potential is probably ruled out by Planck+BICEP !! There are however in general flattening effects: Silverstein, Westphal 08: E.g. if inflaton is aD – brane modulus :

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 $\delta V \simeq \frac{\sqrt{n}}{M_n^{n-4}} \quad \delta V \simeq V_0 \left(\frac{V_0}{M_n^4}\right)^n \ll V_0$

 $\mathcal{L}_{DBI} = -[1 + aV(\phi)]\partial_{\mu}\phi\partial^{\mu}\bar{\phi} - V(\phi)$

Gur-Ari, 13 $V \simeq \phi^{\prime\prime}$ L. ?., Marchesano, Valenzuela Bieleman, L. ?., Pedro, Valenzuela, Wieck 16;

 $V \simeq \phi^n \longrightarrow V' \simeq (\phi')^{2n/(n+2)}$ OK with lenzuela Planck-BICEP!

2) Cosmological Relaxation
$$\begin{split} V &= V(\mu\phi) + (-M^2 + \mu\phi)|h|^2 + \Lambda^4(h)\cos\left(\frac{\phi}{f}\right).\\ V(\mu\phi) &= \mu M^2\phi + \mu^2\phi^2 + \dots \end{split} \quad \begin{array}{l} \mathrm{M=cut-off} \end{split}$$
Slow roll dictated by $V(\mu\phi)$ $V(\phi)$ Higgs becomes massless Ó Higgs stopped by $\Lambda(h)^4 \cos\left(\frac{\phi}{f}\right)$. $\mu f M^2 \simeq \Lambda^4 (h=v)$ [Graham,Kaplan,Rajendran'15] $\mu \simeq \frac{\Lambda^4}{fM^2} \simeq 10^{-18} \left(\frac{10^{10} GeV}{f}\right) \left(\frac{M_W}{M}\right)^2 GeV$ tiny!! 33

Consistency problems for relaxation

Hierarchy traded for a tiny value of μ Technically natural due to axion ϕ shift symmetry

• Enormous trans-Planckian excursions of the axion: is the potential stable? A global shift symmetry not immune to gravitational corrections.

• If it is gauged, a non-vanishing axion potential $V(\mu\phi)$ explicitly breaks the gauge shift symmetry, which is inconsistent. [Gupta,Komargodski,Perez,Ubaldi'15]

Problems analogous to those of large field inflation: Can one build a consistent monodromy-like relaxion model?

A minimal 3-form relaxion model

L. I., Montero, Uranga, Valenzuela, 15

(no string theory needed here)

$$V = V_{SM} + V_{KS} - \eta F_4 |H|^2 + V_{cos}$$

$$V_{SM} = -m^2 |H|^2 + \lambda |H|^4 \qquad V_{KS} = F_4^2 - \mu \phi F_4$$

$$V = \tilde{\lambda} |H|^4 + (q + \mu \phi)^2 + 2\eta (-M^2 + \mu \phi) |H|^2 + V_{cos}$$

$$\int_{V(\mu \phi)} relaxion - Higgs coupling$$

$$Cut - off: M^2 = \frac{m^2}{2\eta} - q$$

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Features of relaxion monodromy

L. I., Montero, Uranga, Valenzuela, 15

• Shift gauge symmetry is respected by the relaxion potential.

Potential protected against Planck-supressed and loop corrections:

$$\delta V \simeq V_0 \left(\frac{V_0}{M^4}\right)^n \simeq V_0 \left(\frac{\Lambda^4}{M^4}\right)^n \ll V_0$$

• Scales:

 $F_4 = n\Lambda_k^2 \quad ; \quad \mu \simeq \frac{\Lambda_k^2}{2\pi f}$ $\mu \simeq 10^{-34} \ GeV \quad \longrightarrow \quad \Lambda_k \simeq 10^{-3} eV$

Anything to do with the c.c.?

3) A Higgs landscape A Hence, 4.9.16
Use 4-forms to construct a landscape of Higgss masses

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{2}(F_a)^2 - \frac{1}{2}(F_h)^2 + \phi(\mu_a F_a + \mu_h F_h) + \eta F_h |H|^2}{\sqrt{1-\frac{1}{2}(F_a)^2 - \frac{1}{2}(F_h)^2 + \phi(\mu_a F_a + \mu_h F_h) + \eta F_h |H|^2}} \\
\text{for string theory simulation of the symplectic string theory of the symplectic string theory$$

Can fine-tune the Higgs vev in steps: $m_h, m_a \in \mathbf{Z}$ $q_a = \mu f$ $\delta(\sigma^2) = \frac{\eta \mu f}{(2\lambda + \eta^2 \cos^2\theta)} \frac{q_a q_h}{q_a^2 + q_h^2} (m_a - m_h)$ $q_h = \mu^h f$ $\cos^2\theta = \frac{(\mu)^2}{\mu^2 + (\mu^h)^2}$ $\delta(\sigma^2) \simeq \eta \mu f = \eta q_a \leq m_H^2$ EW fine-tuning connected to 4-form quanta and coupling η Large family of SM vacua with different Higgs masses and vevs Antropic selection of correct EW vacua

Prediction: a Hierarxion Pseudoscalar with a mass:

$$4.7 \ \eta^{-3/2} \ 10^{-3} eV\left(\frac{10^{10} GeV}{f}\right) \left(\frac{m}{10^{10} GeV}\right)^{3/2} \lesssim m_{axion} \ \lesssim \ \eta^{-1} \ 10^{3} eV\left(\frac{10^{10} GeV}{f}\right) \tag{3.29}$$

For $\eta \simeq 1$ and $f \simeq m \simeq 10^{10}$ GeV one has $10^{-3} eV \lesssim m_{axion} \lesssim 10^3 eV$

one can have ultralight axions with $m_{axion} \simeq 10^{-17} \text{ eV}$

Difficult to identify with QCD axion though... Couplings to photons model dependent...

Possible to construct SUSY versions: the 4-forms are now part of the SUSY auxiliary fields

Conclusions

• 3-forms appear naturaly as new degrees of freedom in field theory. The field-strength is a 4-form which contributes to vacuum energy.

• 3-forms couple to membranes. Values of 4-forms change discretely while going through a membrane in units of the membrane charge.

• 3-forms can couple to axions and can give them a mass while mantaining the axion discrete shift symmetry. The scalar potential is necesarily a power expansion in 4-forms. Makes the axion potential stable.

 The field strength 4-forms appear naturally in String Theory from reduction of RR and NS higher dimensional antisymmetric tensors. The 4-forms are in bijection with internal fluxes and are quantized in units of membrane charges. • The full NS and RR axion potential can be written in terms of 4forms that act as auxiliary fields. In SUSY compactifications the 4forms behave like SUSY auxiliary fields of Kahler and c.s. chiral multiplets. $F_{aux}^{SUSY} \rightarrow \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}$

• Axions in string theory are 'monodromy axions' with associated 3forms. This makes the scalar potentials for axions stable even upon trans-Planckian trips. And makes string axions to be promising inflatons in large field.

 One can construct consistent 'relaxion models' involving 3-forms. They address some of the problems....
 First attempts to embed in string theory, challenging.

One can also construct a Higgs mass landscape from quantized 4-forms.



Thank you for

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your great physics!!

2 2021 ALIVE AND WELL



Instituto de Física Teórica UAM-CSIC Madrid, 28-30 September 2016

https://workshops.ift.uam-csic.es/susyaaw

SPEAKERS

3. Allanach (Cambridge U.)	
I. Baer (Oklahoma U.)	L. J.
3. Bélanger (LAPTH-Annecy)	A. K
). Buchmüller (Imperial Coll.)	J. Ly
1. Carena (Fermilab)	J. Ma
1. Cicoli (ICTP & Bologna U.)	F. M
I. Dreiner (Bonn U.)	P. R

Ellis (CERN & King's Coll.) J. Hall (Berkeley) Katz (CERN & Geneva U.) Lykken (Fermilab) March-Russell (Oxford U.) Moortgat (CMS-CERN) Ramond (Florida U.)

G. G. Ross (Oxford U.)
X. Tata (Hawaii U.)
D. Shih (Rutgers U.)
F. Staub (CERN)
A. Strumia (CERN & Pisa U.)
I. Vivarelli (ATLAS-Sussex U.)
A. Weiler (Munich)

DISCUSSION CONVENER: X. Tata (Hawaii U.)



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