Topological Quantum Field Theories: Knots and Links in Three-dimensions and Black Holes in $3+1$ Dimensions

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On the other hand, topological (global) properties are independent of the metric.

For example, the size and shape of a knot in a three dimensional manifold do depend on the metric, its 'knotedness' does not.

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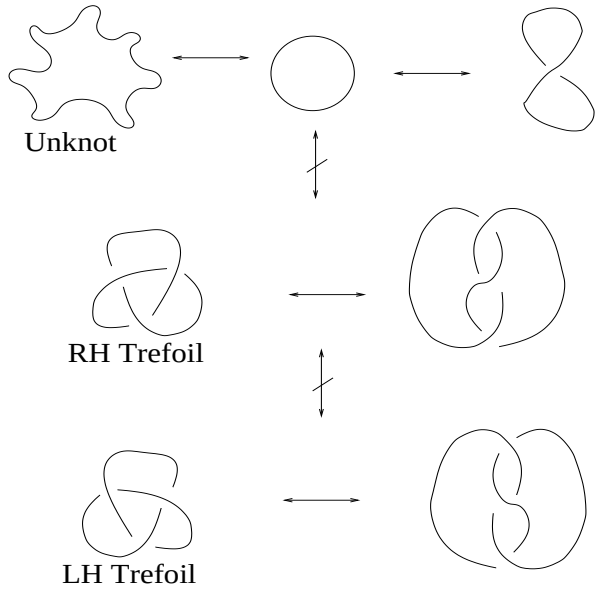

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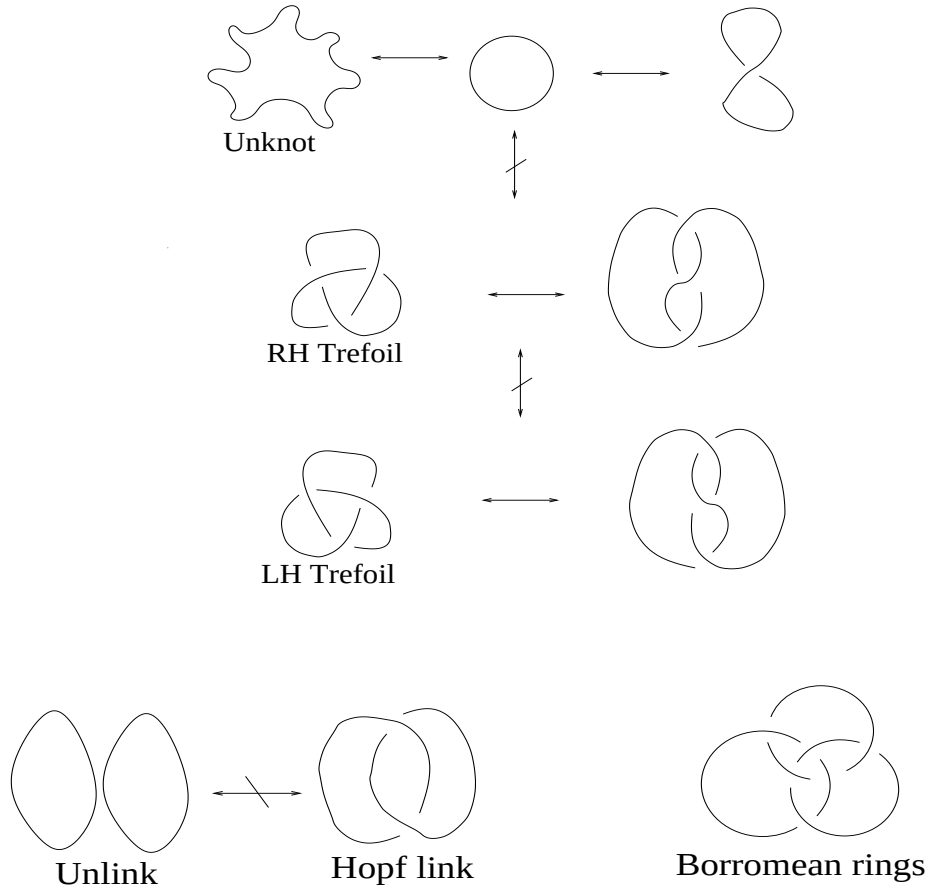

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Thus, for any compact gauge group $\mathcal{G}$, we have a new polynomial knot/link invariant.

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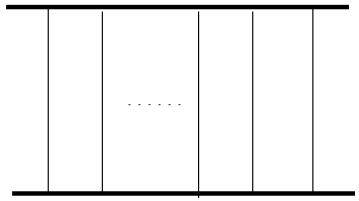

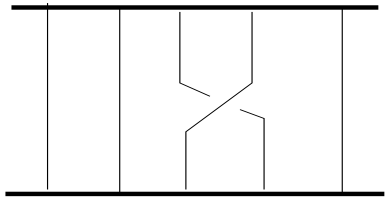

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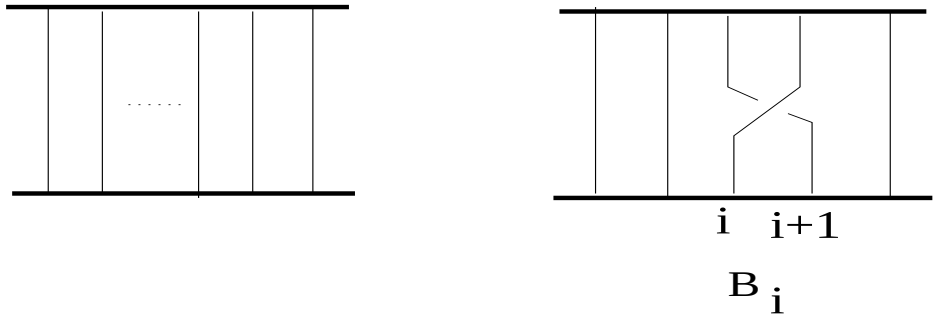

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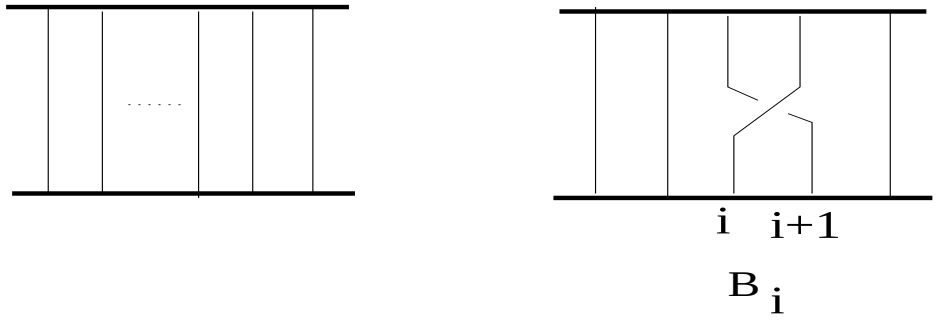

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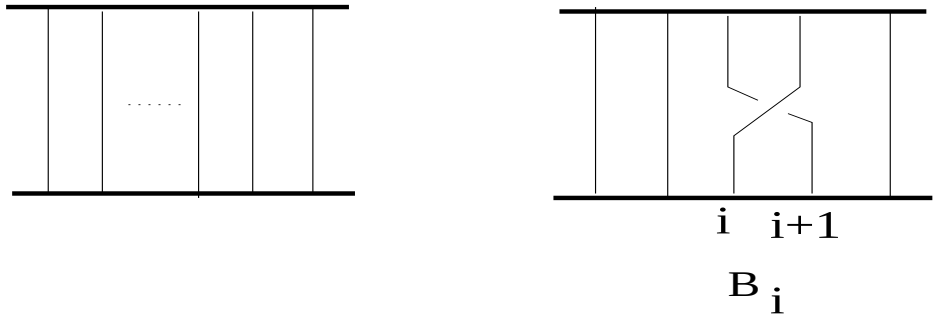

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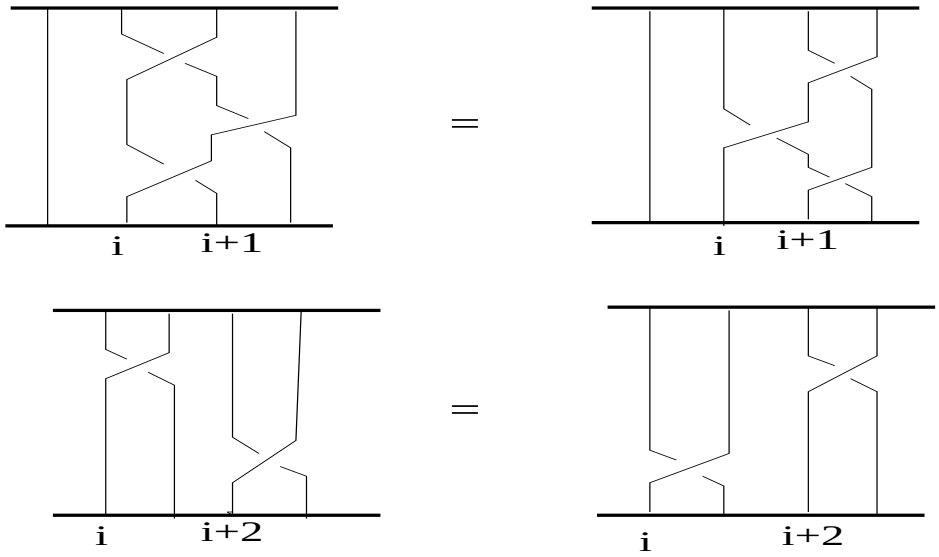

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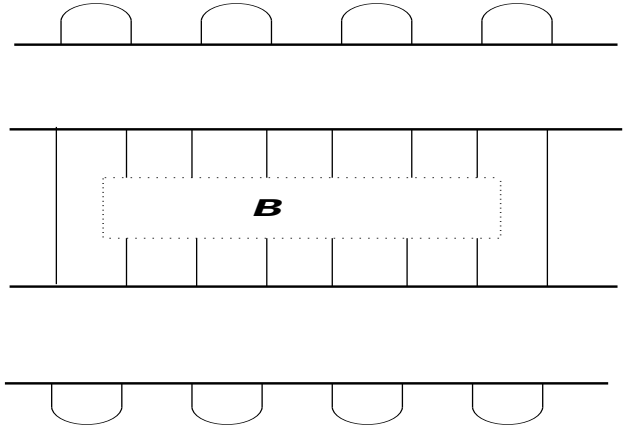

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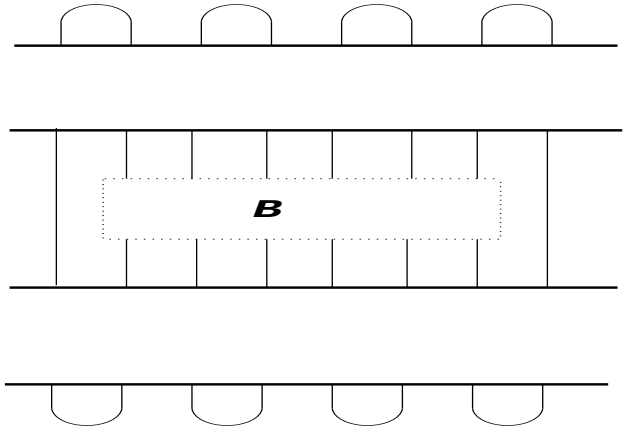

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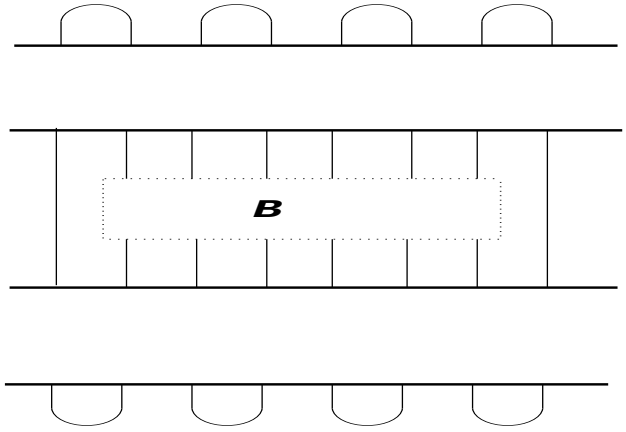

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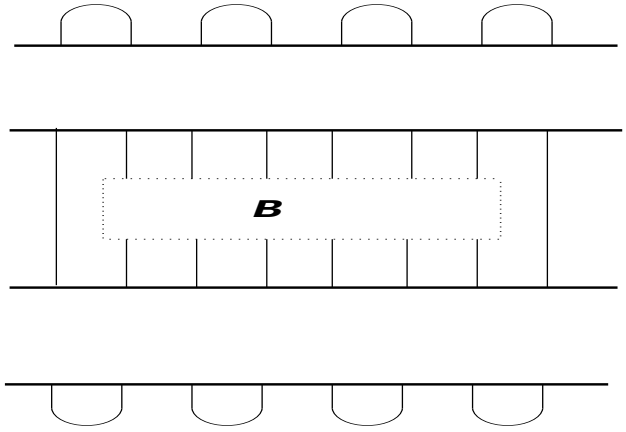

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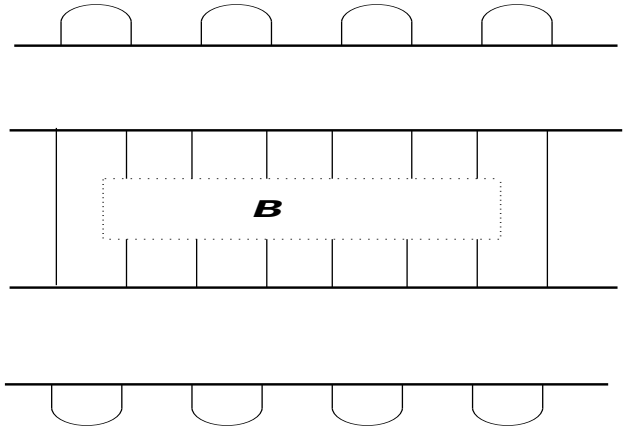

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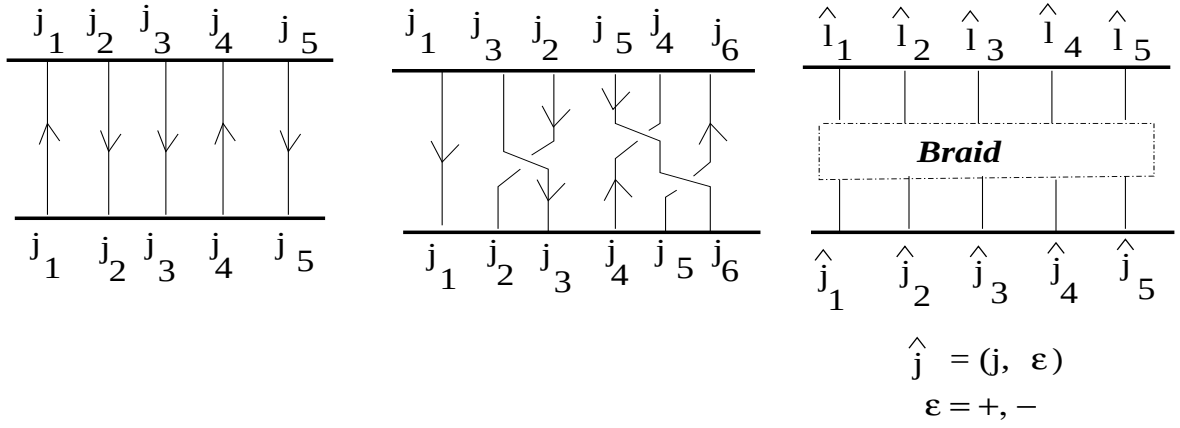

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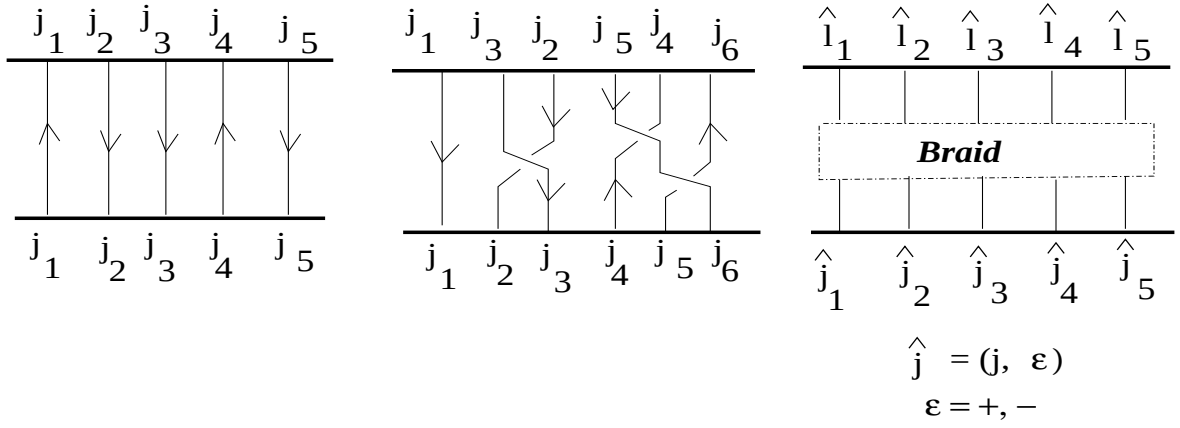

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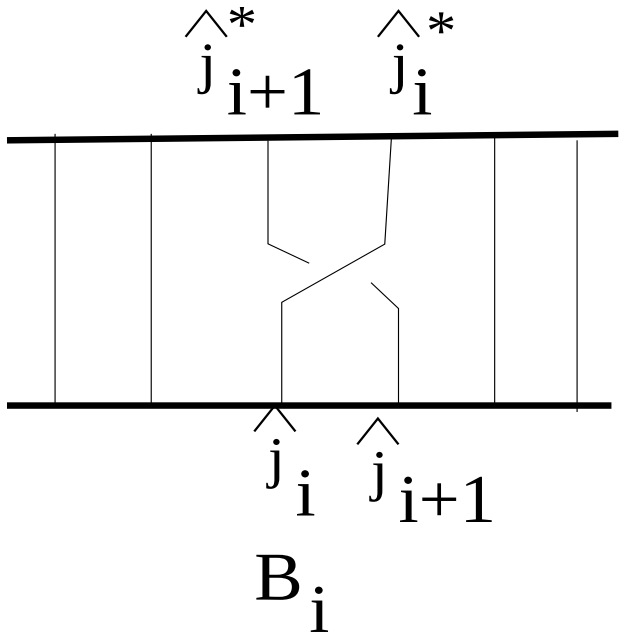

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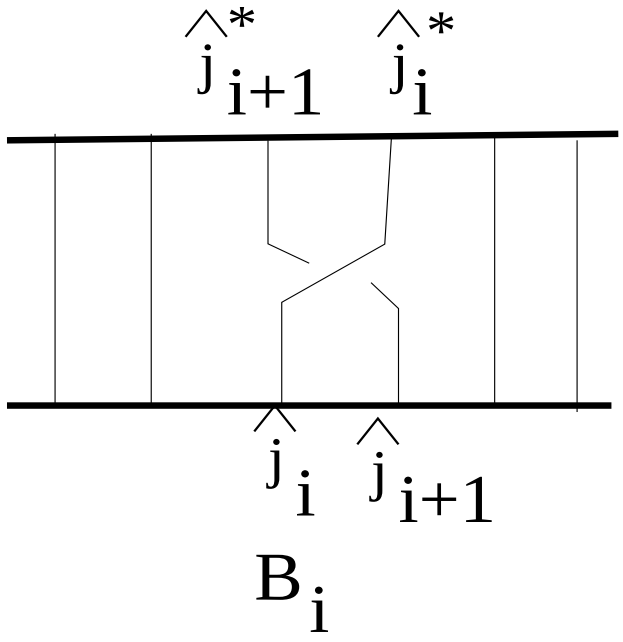

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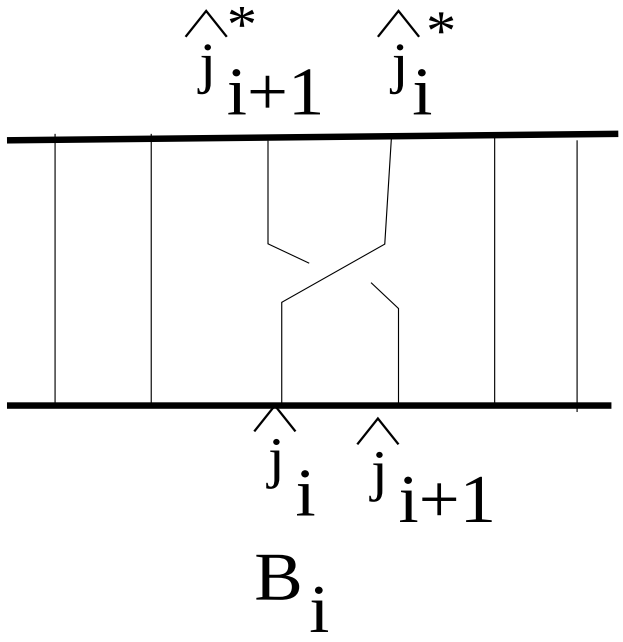

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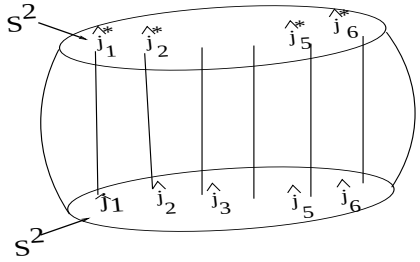

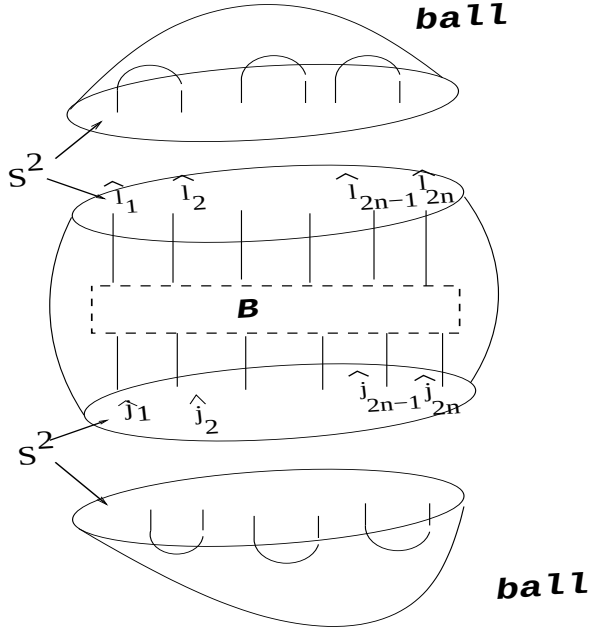

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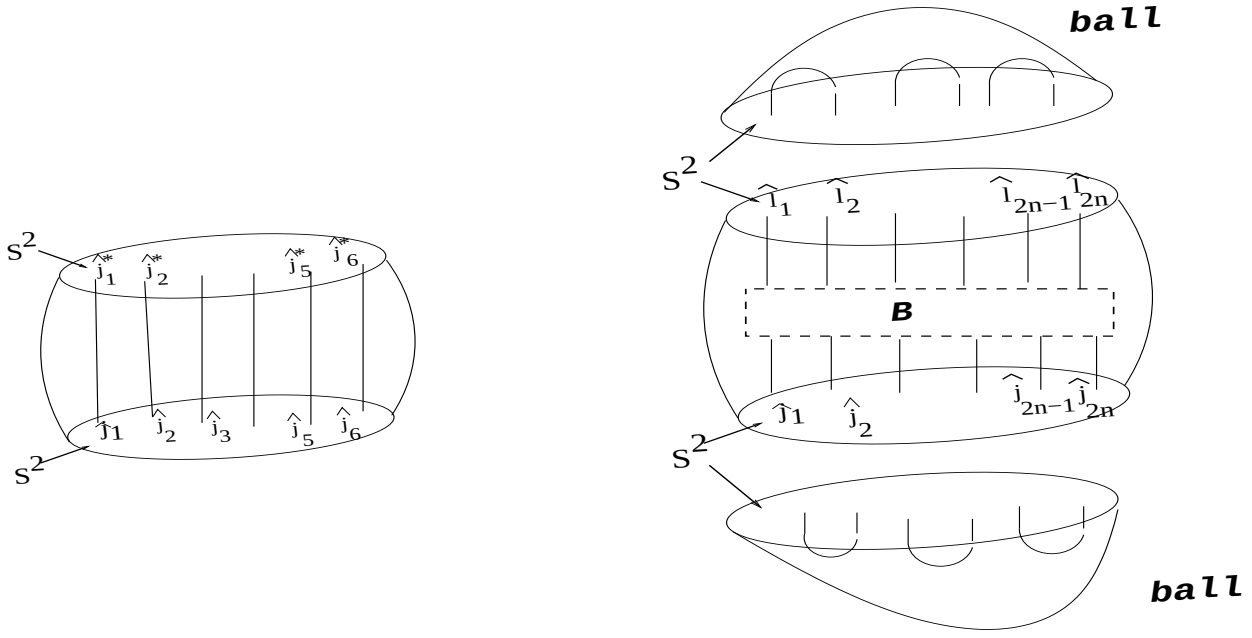

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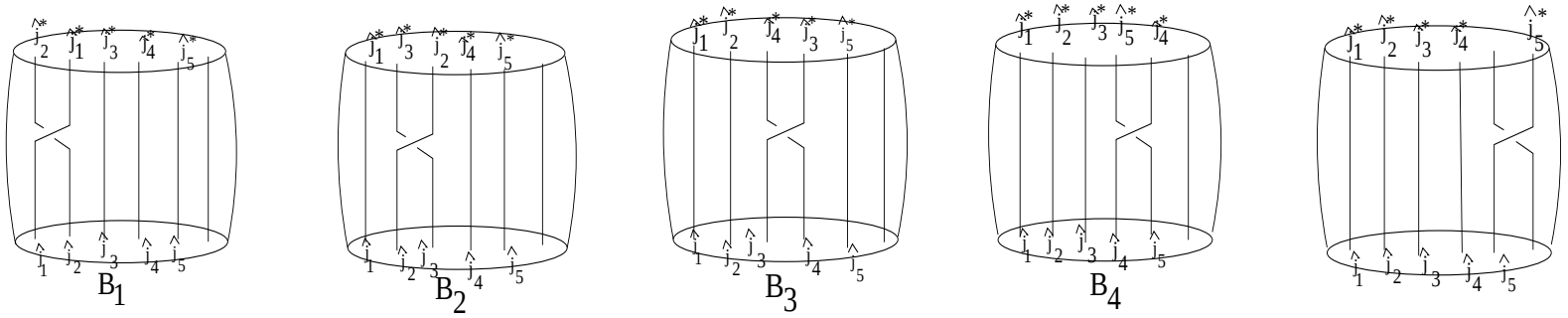

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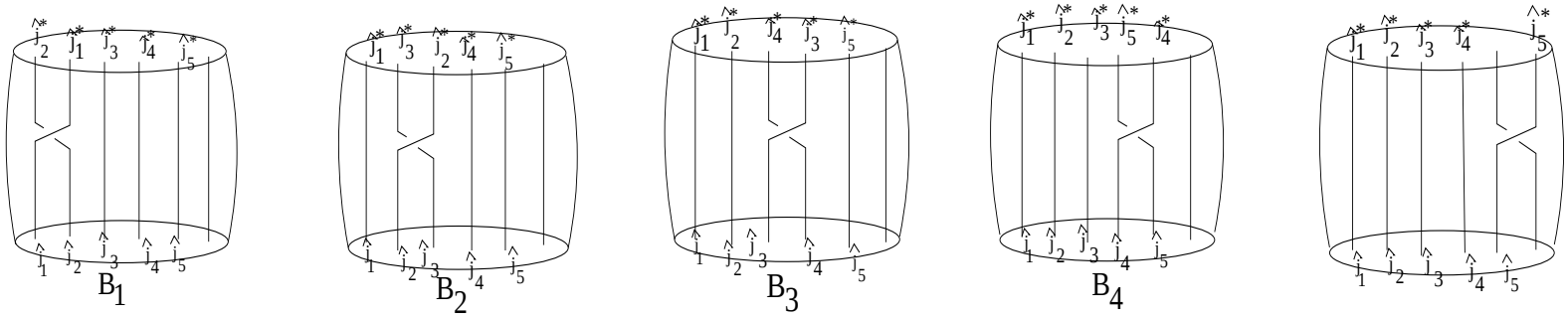

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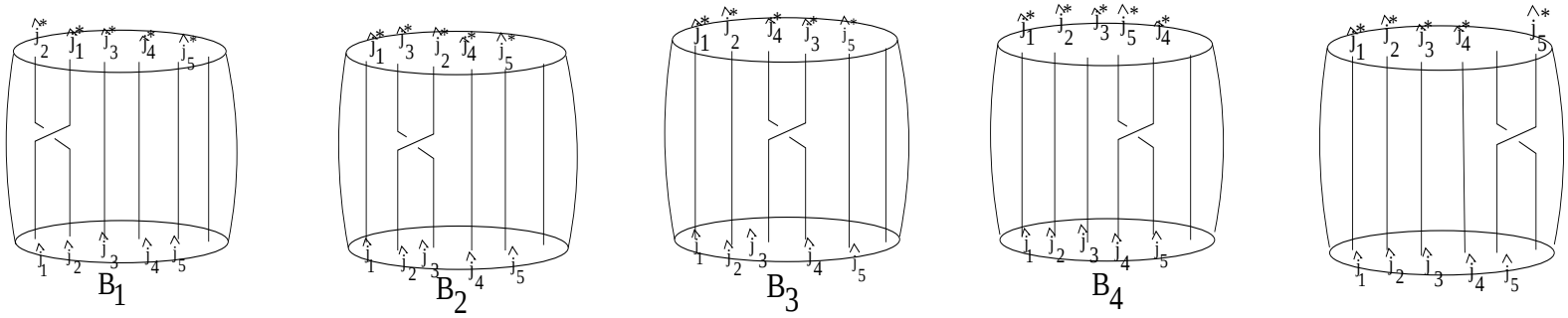

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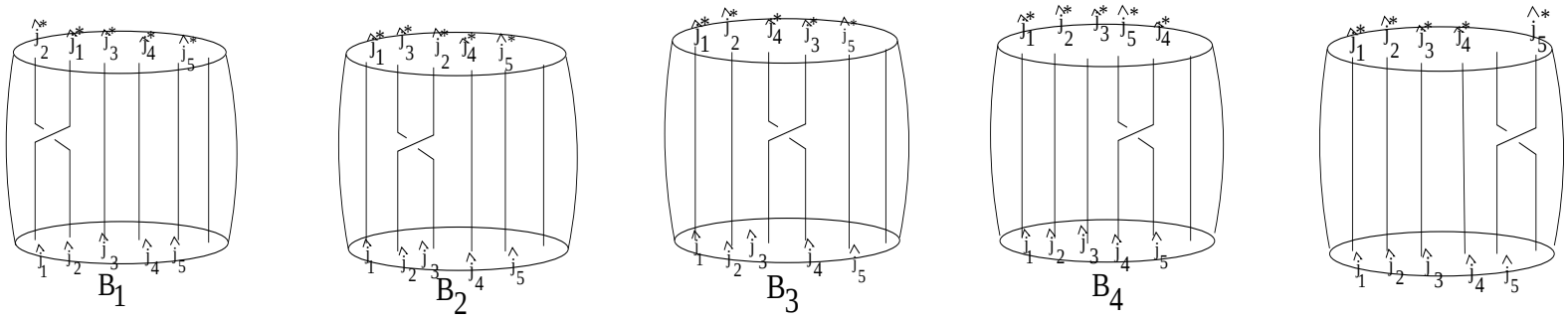

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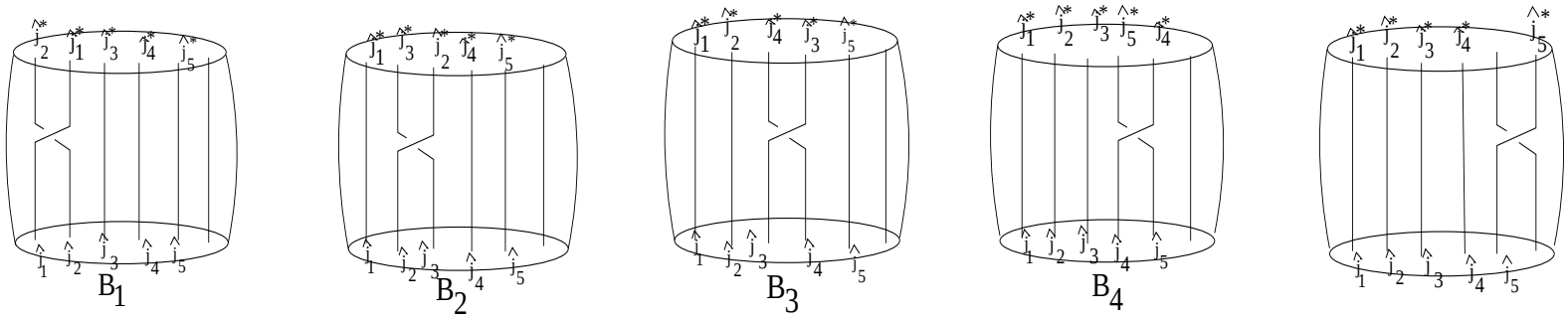

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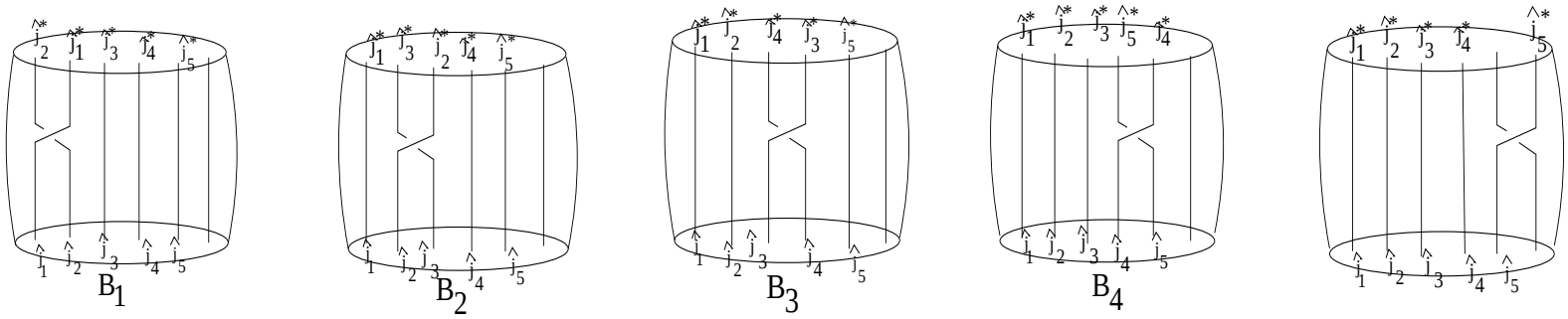

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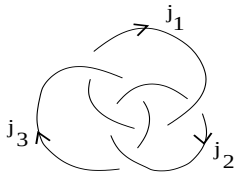
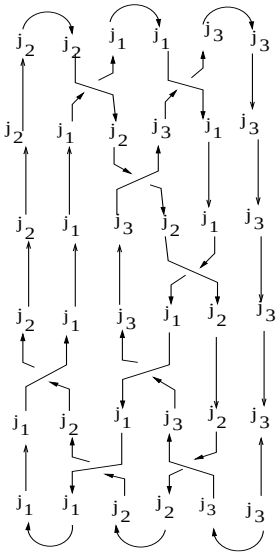


IMSc Logo


Plat

$$V_{j_1 j_2 j_3}[IMSc\ \ logo]$$

$$= [2j_1+1][2j_2+1][2j_3+1] \sum \left(\lambda^{(-)}_{q_1}(j_1 j_2)\right)^{-1} \lambda^{(-)}_{q_2}(j_2 j_3) \lambda^{(+)}_{p_0}(j_1 j_2) \left(\lambda^{(-)}_{p_1}(j_1 j_3)\right)^{-1}$$

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$$S_{CS} = \frac{k}{2\pi} \int_\Delta \epsilon^{abc} \left(A^i_a \partial_b A^i_c + \frac{1}{3} \epsilon^{ijk} A^i_a A^j_b A^k_c \right) + \int_\Delta J^{ai} A^i_a$$

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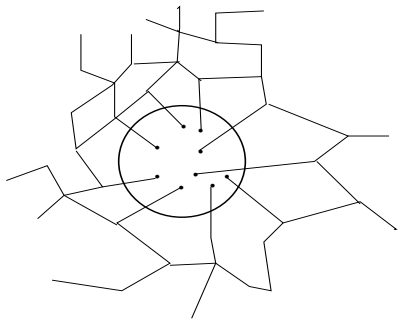
Black hole micro-states

Quantizing this CS theory then provides a framework to count the horizon micro-states.

Quantum theory is described by quantizing the $SU(2)$ CS theory in the bdy Δ with bulk solder form $J^i = \Sigma^i_{\theta\phi} \equiv \frac{1}{2}\epsilon^{ijk} e^j_\theta e^k_\phi$ as the external source.

LQG is described by spin networks made from functionals of Wilson line integral as configuration operators. The corresponding momentum operators are the flux operators constructed from the solder forms: $\int_{S^2} d^2\sigma \Sigma^i_{\theta\phi}$.



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Now the micro-states can be counted by simply counting the number of states satisfying this constraint, either in bulk theory or in the boundary CS theory.

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$$\mathcal{N}_{\mathcal{P}}(j_1, j_2, ..., j_p) = \frac{2}{k+2} \sum_{r=0}^{k/2} \frac{\prod_{l=1}^{p} \sin\left(\frac{(2j_l+1)(2r+1)\pi}{k+2}\right)}{\left[\sin\left(\frac{(2r+1)\pi}{k+2}\right)\right]^{p-2}}.$$

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$$A_{\rm H} = 8\pi\gamma \sum_{l=1,2,...,p} \sqrt{j_l(j_l+1)}; \qquad \ell_P = 1.$$

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All these facts suggest a universality of this entropy formula.

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$SU(2)$ CS theory also provides a description of the black hole micro-states, yielding an entropy formula with a possibly universal coefficient, $-3/2$, for the logarithmic area correction to the Bekenstein-Hawking law.