Inflationary Cosmology from Supergravity

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Contents

Abstract

The supergravity models for cosmological inflation and dark matter are introduced by assuming their (super)gravitational origin. Inflation is known to be sensitive to quantum gravity needed for its ultraviolet completion. Supergravity

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C. Bambi et al. (eds.), *Handbook of Quantum Gravity*,
https://doi.org/10.1007/978-981-99-7681-2_51

is considered as the meeting point of high-energy physics and cosmology, as well as the bridge between classical and quantum gravities. String theory is a mathematically consistent theory of quantum gravity that requires space-time supersymmetry. Modified supergravity theories are introduced as the extensions of the Starobinsky model of inflation based on modified gravity. Physical applications of the supergravity models include viable single-field and multi-field inflation, high-scale spontaneous supersymmetry breaking, primordial black hole production, and dark matter genesis. Supermassive gravitino particles and primordial black holes are the viable candidates for the dark matter originating from supergravity. High-precision measurements of the cosmic microwave background radiation by satellite missions and detection of the gravitational waves, induced by primordial black hole formation, by the future space-based gravitational interferometers can be viewed as the possible observational tests of the cosmological models based on supergravity theory.

Keywords

Inflation · Supergravity · Dark matter · Primordial black holes

Introduction

The oldest signal from the past to the present is given by the cosmic microwave background (CMB) emerged when electromagnetic radiation decoupled from matter about 380,000 years after the birth of the universe. It leads to the observational wall against any electromagnetic probe because photons did not propagate freely before that time due to Thompson scattering. Extracting observables from the earlier universe may only be possible from gravitational waves (GW) or neutrino sources.

Information about the CMB spectrum, its anisotropies, and fluctuations is available due to the satellite missions in the past (COBE, WMAP, Planck). Theoretical explanation of the CMB observations is possible by assuming a very short era of cosmological inflation in the early universe. The cosmological paradigm of inflation assumes an accelerated (quasi-de Sitter) expansion of the universe with a "graceful exit," which greatly amplified microscopic fluctuations that became seeds of the large-scale structure of the universe we observe today. From the viewpoint of particle physics, inflation was the most powerful particle accelerator in Nature, well beyond the standard model (SM). The CMB gives us the great (though small) window into very high-energy physics beyond the SM. The inflationary paradigm also solves the old (flatness, horizon, initial conditions) problems of the standard (Friedmann) cosmology.

The simplest mechanisms of inflation employ the canonical scalar field called *inflaton* whose scalar potential defines a single-field inflation model. However, the physical nature of inflation and the origin of its scalar potential are still unknown. Matching CMB observations leaves many viable models of inflation, so that their discrimination requires adding other principles. One of such principles, namely, the possible *gravitational origin* of inflation, is extended to supergravity and dark matter (DM) in this chapter.

In section "Starobinsky Inflation," we review the Starobinsky model of inflation having the gravitational origin. More general inflation models motivated by the primordial black hole (PBH) formation are defined in section "Generalized Models of Inflation and PBH." The supergravity setup is given in section "Supergravity and Inflation." A minimal embedding of inflation to supergravity is given in section "Minimal Embedding of Inflation to Supergravity." Section "Starobinsky-Type Supergravity and PBH Production" is devoted to PBH production in modified supergravity models of inflation. The GW induced by the PBH production are studied in section "Induced Gravitational Waves." Section "Adding Matter and Spontaneous SUSY Breaking" is devoted to spontaneous supersymmetry (SUSY) breaking with PBH production. Gravitino production in the Polonyi-Starobinsky supergravity and the massive gravitino DM are considered in section "Gravitino DM Genesis." Our conclusion is section "Conclusion."

Starobinsky Inflation

The gravitational origin of inflation implies that only gravitational interactions can be used for its description. This leads to modified gravity and the Starobinsky inflation model [1]. In this section, the Starobinsky model of inflation is reviewed, without following historical developments.

A spatially (flat) homogeneous and isotropic (1+3)-dimensional universe at large scales (beyond 100 Mpc) is described by the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2_{\text{FLRW}} = -dt^2 + a^2(d\vec{x})^2, \tag{1}$$

where the function $a(t)$ is called the cosmic scale factor. Cosmological inflation is referred to the very early (after 10^{-36} s and before 10^{-32} s) accelerating universe with

$$\ddot{a}(t) > 0 \tag{2}$$

and a "graceful exit" quickly after inflation. The Hubble radius (causal distance) H^{-1}/a, where $H = \dot{a}/a$ is the Hubble function, was decreasing during inflation:

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0. \tag{3}$$

The single-field mechanism of inflation uses the scalar field called inflaton, whose potential energy drives inflation. The corresponding "quintessence" action reads

$$S_{\text{single}}[g_{\mu\nu},\varphi]=\frac{M_{\text{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}R-\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi+V(\varphi)\right], \tag{4}$$

where we have introduced the reduced Planck mass $M_{\text{Pl}}=1/\sqrt{8\pi G_{\text{N}}}\approx 2.4\times 10^{18}$ GeV.

The Starobinsky model [1] is defined by the modified gravity action

$$S_{\text{Star.}}=\frac{M_{\text{Pl}}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left(R+\frac{1}{6m^2}R^2\right), \tag{5}$$

where we have introduced the inflaton mass parameter m. The action (5) reduces to the Einstein-Hilbert gravitational action in the low curvature regime. In the high curvature regime relevant to inflation, the action (5) reduces to the no-scale R^2 gravity action with the positive dimensionless coupling constant $M_{\text{Pl}}^2/(12m^2)$. It guarantees the absence of ghosts and tachyons in the model.

The R^2-gravity equations of motion in the FLRW universe have a quasi-de Sitter attractor solution with the “graceful exit,” whose leading term during slow-roll inflation is

$$H(t)\approx\left(\frac{m}{6}\right)^2(t_{\text{end}}-t), \tag{6}$$

for $m(t_{\text{end}}-t)\gg 0$. This solution spontaneously breaks the scale invariance of the R^2-gravity and, hence, implies the existence of the associated Nambu-Goldstone boson (inflaton) called scalaron. The easiest way to extract the scalaron φ from the higher-derivative gravity theory (5) is via the known (classical) equivalence between the modified $F(R)$ gravity theories and the scalar-tensor gravity theories [2]. The equivalence is realized via the Legendre-Weyl transform:

$$\varphi=\sqrt{\frac{3}{2}}M_{\text{Pl}}\ln F'(\chi)\quad\text{and}\quad g_{\mu\nu}\to\frac{2}{M_{\text{Pl}}^2}F'(\chi)g_{\mu\nu},\quad \chi=R, \tag{7}$$

leading to the inflaton scalar potential

$$V(\chi)=\left(\frac{M_{\text{Pl}}^2}{2}\right)\frac{\chi F'(\chi)-F(\chi)}{F'(\chi)^2} \tag{8}$$

and the canonical kinetic term for φ minimally interacting with gravity, as in Eq. (4).

The inverse transformation reads

$$R=\left[\frac{\sqrt{6}}{M_{\text{Pl}}}V'(\varphi)+\frac{4V(\varphi)}{M_{\text{Pl}}^2}\right]\exp\left(\sqrt{\frac{2}{3}}\varphi/M_{\text{Pl}}\right), \tag{9}$$

$$F = \left[\frac{\sqrt{6}}{M_{\mathrm{Pl}}} V'(\varphi) + \frac{2V(\varphi)}{M_{\mathrm{Pl}}^2} \right] \exp\left(2\sqrt{\frac{2}{3}}\varphi/M_{\mathrm{Pl}} \right), \tag{10}$$

defining the function $F(R)$ in the parametric form for a given inflaton scalar potential $V(\varphi)$. The flatness of the inflaton potential (needed for slow roll during a sufficient duration of inflation) amounts to the smallness of the first term against the second one in the square brackets of Eqs. (9) and (10). For instance, ignoring the first term leads to $F \propto R^2$.

The exact inflaton potential $V(\varphi)$ of the Starobinsky model (5) according to Eqs. (7) and (8) is given by

$$V(\varphi) = \frac{3}{4} M_{\mathrm{Pl}}^2 m^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\mathrm{Pl}} \right) \right]^2. \tag{11}$$

The potential (11) is a sum of the induced cosmological constant and the exponentially small corrections for large values of φ, where the potential has a plateau of positive height.

The parameter m of the Starobinsky model is fixed by the observed CMB amplitude as

$$m \approx 3 \cdot 10^{13}\,\mathrm{GeV} \quad \text{or} \quad \frac{m}{M_{\mathrm{Pl}}} \approx 1.3 \cdot 10^{-5}. \tag{12}$$

The ultraviolet (UV) cutoff of the quantized $(R + \alpha R^2)$ gravity is given by M_{Pl}, as is clear from expanding the nonrenormalizable scalar potential (11) in powers of φ. This feature leads to expected protection of the Starobinsky inflation on the scale $H_{\mathrm{inf.}} \sim 10^{14}$ GeV against quantum gravity corrections on the scale M_{Pl}.

The duration of slow-roll inflation is measured by e-folds running backward with time:

$$N = -\ln(a/a_{\mathrm{end}}) \approx \frac{1}{M_{\mathrm{Pl}}^2} \int_{\varphi_{\mathrm{end}}}^{\varphi_*} \frac{V}{V'} d\varphi, \tag{13}$$

where φ_* is the inflaton value at the horizon crossing and φ_{end} is the inflaton value at the end of inflation when one of the slow-roll parameters,

$$\varepsilon_V(\varphi) = \frac{M_{\mathrm{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \quad \text{and} \quad \eta_V(\varphi) = M_{\mathrm{Pl}}^2 \left| \frac{V''}{V} \right|, \tag{14}$$

is no longer small, being close to one.

Metric perturbations on the FLRW background are given by

$$g_{ij}(x) = a^2(t) \left[(1 - 2\mathcal{R})\delta_{ij} + h_{ij} \right], \tag{15}$$

where $\mathcal{R}(x)$ describes scalar perturbations and $h_{ij}(x)$ describes tensor perturbations or primordial GW. Linearizing the action (4) on the FLRW background with respect to scalar perturbations yields the Mukhanov-Sasaki equation [3,4]

$$\mathcal{R}''_{\vec{k}} + 2\frac{z'}{z}\mathcal{R}'_{\vec{k}} + k^2\mathcal{R}_{\vec{k}} = 0, \tag{16}$$

where $\mathcal{R}_{\vec{k}}$ is the 3D Fourier transform of $\mathcal{R}(x)$, $z = a\dot{\varphi}/H$, $k^2 = \vec{k}^2$, and the primes denote the derivatives with respect to conformal time τ defined by $ad\tau = dt$. Changing the variables as $u_{\vec{k}} = z\mathcal{R}_{\vec{k}}$ yields the harmonic oscillator equation

$$u''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right)u_{\vec{k}} = 0, \tag{17}$$

with the time-dependent frequency.

The power spectrum of scalar perturbations is given by

$$P_s(k) = \frac{k^3}{2\pi^2}\left|\mathcal{R}_{\vec{k}}\right|^2, \tag{18}$$

while its tilt n_s is defined by

$$n_s(k) - 1 = \frac{d\ln P_s}{d\ln k}. \tag{19}$$

As regards CMB, the tilt n_s is measured at the horizon crossing with $k^* = 0.05\,\mathrm{Mpc}^{-1}$. Similar definitions apply to the power spectrum $P_g(k)$ of tensor perturbations, leading to the tensor-to-scalar ratio $r = P_g/P_s$.

The Starobinsky model (5) is known to be the excellent model of inflation, in very good agreement with CMB measurements. The tilt of scalar perturbations is given by $n_s \approx 1 + 2\eta_V - 6\varepsilon_V \approx 0.9649 \pm 0.0042$ (with 68% CL), while the tensor-to-scalar ratio is given by $r \approx 16\varepsilon_V < 0.036$ (with 95% CL), according to the 2021 data [5,6]. The Starobinsky model gives $r \approx 12/N^2 \approx 0.004$ and $n_s \approx 1 - 2/N$, with the best fit at $N = 55$.

After rewriting the scalar potential (11) to the form of a mass term by the field redefinition,

$$\sqrt{\frac{3}{2}}M_{\mathrm{Pl}}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\varphi/M_{\mathrm{Pl}}\right)\right] = \phi, \tag{20}$$

one gets a noncanonical kinetic term of the field ϕ, with the pre-factor having a singularity at $\phi_{\mathrm{cr.}} = \sqrt{3/2}M_{\mathrm{Pl}}$ and the critical exponent $\sqrt{\frac{2}{3}}$ defining the universality class of the Starobinsky-like inflation models. This approach is known as the *pole* inflation.

The CMB data gives a small window into high-energy physics of inflation, while no strong observational constraints are available for the scales beyond the inflationary scale or a few orders of magnitude smaller.

Generalized Models of Inflation and PBH

Primordial density fluctuations in the early universe (during or after inflation) may be responsible for PBH seeds when their amplitude is larger by the factor of 10^7 compared to the CMB amplitude. The very idea of PBH was proposed a long time ago by Zel'dovich and Novikov [7] and also by Hawking [8]. PBH may survive in the current universe and thus are the candidates for (non-particle) DM [9]. The PBH idea attracted a lot of attention in connection to GW [10–12].

The PBH-DM is the viable alternative to the explanations of DM in particle physics, such as DM being composed of the weakly interacting massive particles (WIMP) like neutralino or axions [13]. Supergravity offers yet another possibility that DM is composed of the supergravitationally interacting massive particles (SGIMP) such as massive gravitinos [14]. Given part of DM in the form of PBH, one should search for DM signals in cosmological data rather than in direct detection on colliders or via indirect detection in astro-particle physics. Another reason for PBH studies is due to observational progress in lensing, cosmic rays, GW detection, and CMB radiation measurements; see, for example, Ref. [10] for a review of observational constraints on PBH. PBH may also offer a solution to several astrophysical puzzles, for example, the existence of supermassive black holes.

There are many possible mechanisms that may catalyze the formation of PBH in the early universe, for instance, (i) gravitational instabilities induced by scalar fields [15] in single-field or multi-field inflation, (ii) bubble collisions from first-order phase transitions [16–18], and (iii) formation of critical topological defects such as cosmic strings [19] and domain walls [20, 21]. The leading scenario at present is given by (i).

PBH can also be considered as a probe of very high-energy physics and quantum gravity, "even if they never formed" [22]. Several phenomenological scenarios were proposed for PBH formation and, especially, for PBH generation after inflation in the early universe, under the assumption that PBH significantly contribute to DM; see, for example, Refs. [23–27] and the references therein. The *whole* PBH-DM appears to allow only two limited windows for the PBH masses, either around 10^{-15} or around 10^{-12} of the solar mass. These masses are much less the masses of the black holes whose mergers resulted in the GW observed by the LIGO detector [28].

Since the current absence of observed non-Gaussianities and isocurvature perturbations in the CMB data [29], the single-field models were distinguished in the literature about inflation and PBH formation, also due to their simplicity. The Starobinsky model is one of the most favored models of single-field inflation, but its sharp predictions for the tilts n_s and r may be ruled out by future CMB measurements. The Starobinsky model also does not allow PBH production.

Getting arbitrary values of r is possible by using the *alpha-attractor* generalizations [30] of the Starobinsky model. For instance, replacing the critical exponent $\sqrt{\frac{2}{3}}$ in the Starobinsky potential (11) by $\sqrt{\frac{2}{3\alpha}}$ with arbitrary $\alpha > 0$ gives the new (called E-type) inflationary models having the potential

$$V_\alpha(\varphi) = V_0\left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha}}\varphi/M_{\rm Pl}\right)\right]^2 . \tag{21}$$

The key feature of the alpha-attractors is the value of the tensor-to-scalar ratio (on CMB scales)

$$r_\alpha \approx \frac{12\alpha}{N_e^2}, \tag{22}$$

while they have $n_s = 1 - 2/N_e$ (on CMB scales) as in the Starobinsky model.

The alpha-attractors can be generalized to the (T-type) inflaton potentials [30]

$$V_{\tilde{\alpha},f}(\varphi) = f^2\left(\tanh\frac{\kappa\varphi}{\sqrt{6\alpha}}\right) \tag{23}$$

with a monotonically increasing (during slow roll) function f, and $\kappa = M_{\rm Pl}^{-1}$. In these models, slow-roll inflation occurs for large positive values of the canonical inflation field φ with the approximate scalar potential ($\kappa = 1$)

$$V(\varphi) = f_\infty^2 - 4f_\infty f'_\infty\, e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + O\left(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}\right), \tag{24}$$

where we have introduced the parameters $f_\infty = f|_{\varphi\to\infty}$ and $f'_\infty = \partial_\varphi f\big|_{\varphi\to\infty}$. The constant in front of the second term in Eq. (24) can be adjusted at will by a constant shift of the inflaton field, so that the potential (24) can be simplified to

$$V(\varphi) = V_0\left(1 - 2e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\right) + O\left(e^{-2\sqrt{\frac{2}{3\alpha}}\varphi}\right), \tag{25}$$

thus establishing the asymptotic equivalence to the E-type alpha-attractors on CMB scales.

The enhancement of the power spectrum of scalar perturbations (needed for PBH formation) can be achieved by engineering *a near-inflection point* in the inflaton potential [31, 32]. Details of the PBH production in single-field inflation are dependent upon a choice of the inflaton potential and require fine-tuning of the parameters. For instance, it can be realized via Taylor expansion of the function f, when keeping the first three terms as [33],

$$V_{\mathrm{T}}(\phi) = V_0 \left[1 + c_1 \tanh \frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}} + c_2 \tanh^2 \frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}} + c_3 \tanh^3 \frac{\kappa\phi}{\sqrt{6\tilde{\alpha}}} \right]^2 , \tag{26}$$

after fine-tuning of the parameters V_0 and c_i for $i = 1, 2, 3$. The same goal can be achieved by a similar deformation of the E-type scalar potential (21) as [34]

$$V_{\mathrm{E}}(\varphi) = V_0 \left[1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi/M_{\mathrm{Pl}}} + \beta e^{-2\sqrt{\frac{2}{3\alpha}}\varphi/M_{\mathrm{Pl}}} - \gamma e^{-3\sqrt{\frac{2}{3\alpha}}\varphi/M_{\mathrm{Pl}}} \right]^2 \tag{27}$$

with the tuned parameters V_0, β, γ. The generated PBH masses appear in the mass window of $10^{17} \nabla \cdot 10^{20}$ g [33, 34].

There are no fundamental reasons for the absence of non-Gaussianities and isocurvature perturbations; they just have to be below observational limits. For instance, PBH production may be a generic feature of two-field inflation with a sharp turn of inflationary trajectory [35]. The required growth of primordial fluctuations can be achieved by tachyonic instabilities of scalars, similar to the waterfall phase of hybrid inflation [36].

A multi-field action for describing inflation is given by

$$\begin{aligned} S_{\mathrm{multi}}[g_{\mu\nu}, \phi_a] = {} & \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^4x \sqrt{-g} R \\ & - \int \mathrm{d}^4x \sqrt{-g} \left[\frac{1}{2} G^{ab}(\phi) g^{\mu\nu} \partial_\mu \phi_a \partial_\nu \phi_b + V(\phi) \right] \end{aligned} \tag{28}$$

in terms of several (real) scalar fields $\phi_a, a = 1, 2, \ldots, n$. The scalar kinetic terms in Eq. (28) have the form of the nonlinear sigma-model (NLSM) [37] with the metric G^{ab} in the field space.

Supergravity and Inflation

Multi-field models of inflation and PBH formation increase physical degrees of freedom and possible interactions, which reduces their predictive power. It is, therefore, important to impose some fundamental symmetry principles to restrict physics beyond the SM of elementary particles. Supersymmetry (SUSY) is the fundamental symmetry unifying elementary particles of different spin into irreducible multiplets, restricting their interactions and independent coupling constants. In the context of gravity, one needs *local* SUSY, i.e., supergravity.

Supergravity is also a good framework to study inflation and theoretical origin of PBH at the more fundamental level than general relativity because local SUSY transformations imply general coordinate transformations. Supergravity is also a bridge from classical gravity to quantum gravity when the latter is given by string theory because string theory requires SUSY for its consistency.

Despite the absence of experimental confirmation of SUSY on TeV-scales at the Large Hadron Collider (LHC), SUSY remains one of the leading candidates for new physics beyond SM of elementary particles for the scales beyond 100 TeV. SUSY has to be spontaneously or softly broken at the collider energies. The scale of SUSY breaking is unknown.

We confine ourselves to $N = 1$ supergravity in four space-time dimensions because it is chiral that is necessary for particle phenomenology and CP violation. SUSY and supergravity have many attractive theoretical features such as the following :

- SUSY unifies bosons and fermions.
- Supergravity includes general relativity.
- SUSY grand unified theories (super-GUT) lead to a perfect unification of electroweak and strong interactions.
- the spectrum of matter-coupled supergravities with spontaneously broken SUSY has the natural DM candidate given by the lightest SUSY particle (LSP) provided that R-parity is conserved.
- SUSY leads to the cancellation of the quadratic UV divergences in Feynman graphs and protection of chiral (F-type) actions in quantum perturbation theory
- Supergravity is necessary to consistent coupling of spin-3/2 particles to gravity.
- Supergravity arises as the low-energy effective action of superstrings and M-theory.

However, the high-scale SUSY implies that it cannot stabilize the fundamental scales (the hierarchy problem), such as the electroweak scale and the GUT scale. In summary, SUSY and supergravity are healthy and are not ruled out by observations despite the absence of any signs of their presence by the LHC and in the sky so far.

SUSY requires superpartners for each SM particle and equal numbers of bosonic and fermionic degrees of freedom, as well as supersymmeteric actions. To match observations and connect to the SM, SUSY has to be (spontaneously) broken. The formal technology of local supersymmetry is based on (i) the superconformal tensor calculus [38] and (ii) the curved superspace [39]. Both approaches are equivalent but technically involved. The $N = 1$ superspace technology is geometrical and offers *manifest* SUSY of supersymmetric actions. Being applied to inflation, it means that the bosonic actions (4) and (5) have to be locally supersymmetrized. It is worth mentioning that the modified gravity action (5) includes the higher derivatives so that its supersymmetrization goes beyond the textbooks. In addition, inflation has a positive energy (or a positive height of the potential) and thus breaks SUSY. Therefore, there must be a Goldstone spin-1/2 fermion called *goldstino* that is associated with spontaneous SUSY breaking during inflation.

Both inflaton and goldstino have to belong to (irreducible) supermultiplets or superfields. Usually, both superfields are chosen to be *chiral* with the maximal spin 1/2 [40, 41], which requires complexification of the inflaton and the need of two chiral superfields having four real scalars leading to multi-field inflation.

Slow-roll inflation in supergravity is usually realized by *engineering* the scalar potential V in terms of a Kähler potential K and a superpotential W of the chiral superfields $\mathbf{\Phi}^i$. The standard Lagrangian for the chiral superfields coupled to supergravity is given by a sum of the D-term and F-term as follows ($M_{\rm Pl} = 1$):

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[\frac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})e^{-K(\mathbf{\Phi}^i,\overline{\mathbf{\Phi}}^i)/3} + W(\mathbf{\Phi}^i)\right] + \text{h.c.}, \tag{29}$$

where $\mathcal{E}$ is the chiral density superfield, $\mathcal{R}$ is the chiral curvature superfield, $\mathcal{D}_\alpha$ *and* $\overline{\mathcal{D}}_{\dot\alpha}$ are the superspace covariant derivatives, $\mathcal{D}^2 \equiv \mathcal{D}^\alpha\mathcal{D}_\alpha$, and $\overline{\mathcal{D}}^2 \equiv \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}^{\dot\alpha}$; see Ref. [39] for the standard notation of supergravity in curved superspace. A non-holomorphic Kähler potential $K(\Phi^i, \overline{\Phi}^i)$ and a holomorphic superpotential $W(\Phi^i)$ define the model and uniquely determine its scalar sector in the form (28).

A chiral superfield in terms of its field components reads

$$\mathbf{\Phi}(x,\Theta) = \Phi(x) + \sqrt{2}\Theta\chi(x) + \Theta^2 F(x). \tag{30}$$

After eliminating the auxiliary fields (F) via their algebraic equations of motion and going to Einstein frame by a Weyl transform of metric, the bosonic part of the Lagrangian (29) is given by

$$e^{-1}\mathcal{L} = \frac{1}{2}R - K_{i\bar{j}}\partial_m\Phi^i\partial^m\overline{\Phi}^j - e^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\overline{W} - 3|W|^2\right), \tag{31}$$

where we have used the same notation for the superfields and their leading field components, together with

$$K_{i\bar{j}} \equiv \frac{\partial^2 K}{\partial\Phi^i\partial\overline{\Phi}^j}, \quad K^{i\bar{j}} \equiv K^{-1}_{i\bar{j}}, \quad D_iW \equiv \frac{\partial W}{\partial\Phi^i} + W\frac{\partial K}{\partial\Phi^i}. \tag{32}$$

Therefore, we have

$$V = e^K\left(K^{i\bar{j}}D_iWD_{\bar{j}}\overline{W} - 3|W|^2\right). \tag{33}$$

The Θ-expansion of the chiral superfields $\mathcal{E}$ and $\mathcal{R}$ of supergravity in terms of their field components is given by

$$2\mathcal{E} = e\left[1 + i\Theta\sigma^m\overline{\psi}_m + \Theta^2(6\overline{X} - \overline{\psi}_m\bar{\sigma}^{mn}\overline{\psi}_n)\right], \tag{34}$$

$$\mathcal{R} = X + \Theta\left(-\frac{1}{6}\sigma^m\overline{\sigma}^n\psi_{mn} - i\sigma^m\overline{\psi}_m X - \frac{i}{6}\psi_m b^m\right) +$$
$$+\ \Theta^2\left(-\frac{1}{12}R - \frac{i}{6}\overline{\psi}^m\overline{\sigma}^n\psi_{mn} - 4X\overline{X} - \frac{1}{18}b_m b^m + \frac{i}{6}\nabla_m b^m + \right.$$
$$\left. +\ \frac{1}{2}\overline{\psi}_m\overline{\psi}^m X + \frac{1}{12}\psi_m\sigma^m\overline{\psi}_n b^n - \frac{1}{48}\varepsilon^{abcd}(\overline{\psi}_a\overline{\sigma}_b\psi_{cd} + \psi_a\sigma_b\overline{\psi}_{cd})\right), \quad (35)$$

where $e \equiv \det(e^a_m)$, $\psi_{mn} \equiv \tilde{D}_m\psi_n - \tilde{D}_n\psi_m$, and $\tilde{D}_m\psi_n \equiv (\partial_m + \omega_m)\psi_n$. The chiral superfield $\mathcal{E}$ is the SUSY extension of $e = \sqrt{-g}$, and $\mathcal{R}$ is the SUSY extension of the (Ricci) scalar curvature R. The real vector b_m and complex scalar X are known in the supergravity literature as the (old-minimal) set of fields needed to complete the off-shell supergravity multiplet with a closed algebra of SUSY transformations, i.e., independently upon an action.

Single-field inflation is possible by identifying inflaton with one of the scalars while suppressing other scalars during inflation by assigning heavy masses to them (beyond the Hubble value). There is no need to learn supergravity theory for that because Eq. (33) is enough. There are several problems with this approach in supergravity. First, it is the so-called η-problem related to the e^K factor in the scalar potential that is too steep in the case of the canonical Kähler potential and does not have the required flatness during inflation or, equivalently, leads to a large slow-roll parameter η. Second, there is no good reason for a choice of the inflationary trajectory in the (scalar) multi-field space. Third, potential instabilities of the inflationary trajectory (because of inflaton mixing with other scalars) may easily spoil single-field inflation and thus fail to achieve the desired number of e-foldings. Though all those problems are solvable, it requires careful engineering of the potentials K and W, which implies rather low predictive power. Though a superpotential W is (perturbatively) protected against quantum corrections, it is not the case for a Kähler potential K.

It is therefore natural to adopt an economical approach in supergravity by minimizing a number of the physical degrees of freedom involved and by using the Starobinsky model as the starting point. Starobinsky-like supergravities can be introduced either as the locally supersymmetric extensions of the $(R + R^2)$ gravity action (5) or as the supergravity extensions of the quintessence model (4) with the inflaton potential (11). The modified supergravity models [42–44] can be reformulated in terms of the standard (Einstein) supergravity coupled to chiral and vector superfields, similar to the relation between modified $F(R)$ gravity and scalar-tensor gravity, which can also relate inflation to PBH and DM genesis. Several realizations of these ideas are described in the next sections.

Minimal Embedding of Inflation to Supergravity

In this section, the simplest minimal models of supergravity-based inflation are introduced, where (i) inflaton is assigned to a massive *vector* supermultiplet having only one physical scalar or (ii) the Starobinsky model (5) is embedded to supergravity by using a *nilpotent* chiral superfield having Volkov-Akulov goldstino.

Single-Field Inflation in Supergravity

The minimal supergravity framework [45–48] allows one to embed any inflaton potential given by square of a real function. The inflaton field complexification in a chiral supermultiplet can be avoided by assigning inflaton to a massive vector supermultiplet V that has a *single* physical scalar. The scalar potential of a vector multiplet is given by the D-term instead of the F-term, while any desired values of the CMB observables (n_s and r) are possible. The manifestly supersymmetric Lagrangian is given by

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left\{\frac{3}{8}(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R})e^{-\frac{2}{3}J} + \frac{1}{4}W^\alpha W_\alpha\right\} + \text{h.c.}, \tag{36}$$

where $W_\alpha \equiv -\frac{1}{4}(\overline{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{D}_\alpha V$ is the Abelian superfield strength of the vector superfield V and J(V) is arbitrary real function.

The bosonic part (ignoring all contributions of fermions) of the Lagrangian in Einstein frame (after Weyl rescaling and elimination of the auxiliary fields) reads [45, 46]

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}J''\partial_m C\partial^m C - \frac{1}{2}J''B_m B^m - \frac{g^2}{2}J'^2, \tag{37}$$

where $C = V|$ is the real (scalar) inflaton field, $F_{mn} = \partial_m B_n - \partial_n B_m$, $J = J(C)$, and g is the coupling constant. The input for model building is given by a real function J (instead of K and W). Ghost freedom requires $J''(C) > 0$.

For instance, the scalar potential of the Starobinsky inflation model is obtained by

$$J(C) = \frac{3}{2}(C - \ln C) \quad \text{and} \quad C = \exp\left(\sqrt{2/3}\varphi\right), \tag{38}$$

in terms of the canonical inflaton φ. In the case of the generalized alpha-attractors (23), one gets the nonlinear differential equations

$$\frac{g}{\sqrt{2}}\frac{dJ}{dC} = f\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right) \quad \text{and} \quad \left(\frac{d\varphi}{dC}\right)^2 = \frac{d^2J}{dC^2}. \tag{39}$$

The master function $J(V)$ can be replaced by the function $\tilde{J}(He^{2V}\overline{H})$, where we have introduced the (charged) Higgs chiral superfield H and have chosen $g = 1$ for simplicity. The arguments of the function $\tilde{J}$ and, hence, the function $\tilde{J}$ itself are both invariant under the gauge transformations

$$H \to e^{-iZ}H, \quad \overline{H} \to e^{i\overline{Z}}\overline{H}, \quad V \to V + \frac{i}{2}(Z - \bar{Z}), \tag{40}$$

whose gauge parameter Z is also a chiral superfield. The original theory is recovered in the supersymmetric gauge $H = 1$.

One can choose another (Wess-Zumino-type) supersymmetric gauge $V = V_1$, where V_1 describes the irreducible *massless* vector gauge supermultiplet minimally coupled to the *dynamical* Higgs-type chiral multiplet H. The standard super-Higgs mechanism [39] appears with the canonical function $J = \frac{1}{2}He^{2V}\bar{H}$ that corresponds to a linear function $\tilde{J}$ [47, 48]. In the case under consideration, the supersymmetric $U(1)$ gauge theory in terms of the superfields H and V_1 coupled to supergravity defines the analogue of Higgs inflation that is equivalent to the Starobinsky inflation by construction because both arise in the two different gauges of the same gauge theory. The chiral (Higgs) superfield H is charged with respect to $U(1)$, being the gauge degree of freedom that can be eaten up by the vector gauge supermultiplet becoming massive.

Given the inflaton potential (37), its minima are given by solutions to $J'(C) = 0$ because $J''(C) > 0$, that, in turn, implies only Minkowski vacua with restored SUSY and unrealistic phenomenology after inflation. Spontaneous SUSY breaking can be achieved by adding the so-called *alternative* Fayet-Iliopoulos terms [49, 50]; see section "Polonyi-Starobinsky (PS) Supergravity".

Volkov-Akulov-Starobinsky Supergravity

A *generic* embedding of inflation to supergravity requires two superfields, the one including inflaton φ and another one including goldstino G because of SUSY breaking. The number of physical degrees of freedom can be reduced by demanding the chiral goldstino superfield S to be nilpotent, $S^2 = 0$ [51, 52]. The expansion of S in terms of its field components reads

$$S = \frac{GG}{2F} + \sqrt{2}\Theta G + \Theta^2 F, \tag{41}$$

where the leading scalar component is the goldstino bilinear GG.

The constrained goldstino superfield S is used to define the Volkov-Akulov-Starobinsky (VAS) supergravity by the setup [53]

$$K = -3\log\left(T + \bar{T} - \bar{S}S\right) \quad \text{and} \quad W = \lambda + \beta S + \gamma ST \tag{42}$$

with real parameters λ, β, *and* γ, the inflaton chiral superfield T, and the nilpotent goldstino superfield S. This construction is less minimal than that in the preceding subsection because the chiral superfield T includes besides inflaton φ its physical pseudo-scalar superpartner τ called *sinflaton*. The effective bosonic Lagrangian in the parametrization

$$T = \frac{T_0}{2}\left[e^{\sqrt{\frac{2}{3}}\varphi} + i\sqrt{\frac{2}{3}}\tau\right], \quad T_0 = -2\beta/\gamma \equiv \langle T\rangle\,, \tag{43}$$

reads

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}e^{-2\sqrt{\frac{2}{3}}\varphi}(\partial_\mu\tau)^2 - \frac{\gamma^2}{12}\left(1 - e^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - \frac{\gamma^2}{18}e^{-2\sqrt{\frac{2}{3}}\varphi}\tau^2 \tag{44}$$

and has a Minskowski minimum at $\varphi = 0$ with the Starobinsky potential and the formal masses $m_\varphi = m_\tau = \gamma/3$. During inflation when $\varphi \gg 1$, the sinflaton dynamics is suppressed by the exponential terms in Eq. (44). After inflation, when inflaton is settled at the minimum, SUSY is broken due to the nonvanishing vacuum expectation value of the F-field component of the inflaton superfield T, with the gravitino mass

$$m_{3/2} = \left\langle e^{K/2}\,|W|\right\rangle = \frac{\gamma^{3/2}\lambda}{2\sqrt{2}\beta^{3/2}}. \tag{45}$$

The nilpotency constraint is no longer valid at the minimum because the vacuum expectation value of the F-component of S becomes zero. It can be avoided by adding a linear term in T to the superpotential (42), leading to a de Sitter vacuum after inflation [54].

The higher-derivative Starobinsky-like supergravity model *dual* to the "quintessence" supergravity model (42) also exists [53, 54]. The supersymmetric Lagrangian with the Kähler potential and the superpotential given in Eq. (42) read

$$\begin{aligned}\mathcal{L} &= \int d^2\Theta 2\mathcal{E}\left[\frac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})(T + \bar{T} - \bar{S}S) + \lambda + \beta S + \gamma ST\right] + \text{h.c.} \\ &= \int d^2\Theta 2\mathcal{E}\left[-\frac{3}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})\bar{S}S + \lambda + \beta S - T(6\mathcal{R} - \gamma S)\gamma ST\right] + \text{h.c.},\end{aligned} \tag{46}$$

where we have used the superspace identity [39]

$$\int d^2\Theta 2\mathcal{E}(\overline{\mathcal{D}}^2 - 8\mathcal{R})(T + \bar{T}) + \text{h.c.} = -16\int d^2\Theta 2\mathcal{E}\mathcal{R}T + \text{h.c.} \tag{47}$$

Varying the action (46) with respect to T yields

$$S = \frac{6}{\gamma}\mathcal{R} \tag{48}$$

and, hence, the scalar curvature chiral superfield $\mathcal{R}$ is also nilpotent, $\mathcal{R}^2 = 0$, because of $S^2 = 0$. Substituting the solution (48) back into the action (46) leads to the Starobinsky-like supergravity with the Lagrangian

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[-3\mathcal{R} - \frac{27}{2\gamma^2}(\overline{\mathcal{D}}^2 - 8\mathcal{R})\bar{\mathcal{R}}\mathcal{R} + \lambda + \Lambda\mathcal{R}^2\right] + \text{h.c.}, \tag{49}$$

where we have fixed $\beta = -\gamma/2$ in order to get the proper normalization of the first (Einstein) supergravity term and have introduced the Lagrange multiplier chiral superfield Λ in order to enforce the nilpotency condition $\mathcal{R}^2 = 0$. This condition eliminates the scalar and pseudo-scalar degrees of freedom in the scalar curvature chiral superfield $\mathcal{R}$, the goldstino arises from the gauge-invariant gravitino field strength, and the axion is traded for the longitudinal mode $\mathcal{D}\cdot b$ of the pseudo-vector field b_m [53]. The parameter γ is fixed by the scalaron mass, while the parameter λ is related to the scale of SUSY breaking.

Starobinsky-Type Supergravity and PBH Production

The Starobinsky-type modified supergravity without nilpotent superfields is defined by the Lagrangian [43, 55]

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[-\frac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})N(\mathcal{R},\bar{\mathcal{R}}) + \mathcal{F}(\mathcal{R})\right] + \text{h.c.} \tag{50}$$

with two arbitrary potentials $N(\mathcal{R},\bar{\mathcal{R}})$ (real) and $\mathcal{F}(\mathcal{R})$ (holomorphic), where $\mathcal{R}$ is the chiral scalar curvature superfield. The Lagrangian (50) is a generic off-shell (locally) supersymmetric extension of the modified $(R + R^2)$ gravity with four real scalars (including scalaron), all belonging to a single (off-shell) supergravity multiplet described by the chiral superfield $\mathcal{R}$ having the space-time scalar curvature R, the gravitino field strength, a real pseudo-vector b_m, and a complex scalar X as its field components. The fields b_m and X are known in the supergravity literature as the old-minimal set of the auxiliary fields needed to complete the supergravity multiplet with a closed algebra of SUSY transformations. In the modified (Starobinsky-type) supergravity, these "auxiliary" fields become *dynamical* or propagating because of the higher derivatives in the Lagrangian (50). No higher powers of R actually appear beyond the linear and quadratic terms. The first (D-type) term and the second (F-type) term in Eq. (50) are similar to the two terms in Eq. (29); however, they do not represent matter, being depended upon the supergravity fields only. The standard

(Einstein) supergravity action [39] is the extension of the Einstein-Hilbert term linear in R, which is recovered in the special case of $N = 0$ and $\mathcal{F} = -3\mathcal{R}$.

Let us expand the functions N and $\mathcal{F}$ in Taylor series and keep only a few leading terms as [56]

$$N = \frac{12}{M^2}|\mathcal{R}|^2 - \frac{72}{M^4}\zeta|\mathcal{R}|^4 - \frac{768}{M^6}\gamma|\mathcal{R}|^6, \tag{51}$$

$$\mathcal{F} = -3\mathcal{R} + \frac{3\sqrt{6}}{M}\delta\mathcal{R}^2, \tag{52}$$

where M is the scalaron mass, with the parameters ζ, γ, and δ. The M^2 enters as the overall factor in the scalar potential and thus does not change its shape. In the case of $\zeta = \gamma = \delta = 0$, one gets the simplest supersymmetric extension of $(R+R^2)$ gravity. However, it leads to a tachyonic instability along the inflationary trajectory and the scalar potential unbounded from below. This can be avoided by introducing the extra term $\zeta|\mathcal{R}|^4$ as in Eq. (51), whose parameter has the lower bound [44,57]. The model (50) with $\gamma = \delta = 0$ and $\zeta > 1/54$ is known the simplest phenomenologically viable extension of Starobinsky inflation in the old-minimal supergravity without nilpotent superfields [58].

By extending the model further, either via N with $\gamma \neq 0$, $\delta = 0$ or via $\mathcal{F}$ with $\gamma = 0$, $\delta \neq 0$, it is possible to achieve an enhancement of the power spectrum of scalar perturbations at a scale smaller the inflationary scale, which is necessary to produce PBH seeds [56]. Focusing on the effective dynamics of two real scalars (when the others are stabilized), the enhancement of the power spectrum is achieved due to a saddle point in the two-field scalar potential, which creates a short period of the "ultra-slow-roll" (USR) inflation (actually, during USR, inflaton rolls faster than during SR [59]). The USR regime leads to a violation of the slow-roll conditions. The SR stage is driven by scalaron, whereas the USR stage is driven by a combination of both scalars.

Let us call the model with $\gamma \neq 0$ and $\delta = 0$ as the γ-extension and the model with $\delta \neq 0$ and $\gamma = 0$ as the δ-extension. According to Ref. [56], the γ-extension exhibits the attractor behavior, in the sense that the shape of the scalar potential becomes less sensitive to changes in γ when the value of γ increases. The enhancement of the power spectrum can be achieved when $\gamma \geq \mathcal{O}(1)$ and ζ is tuned around the saddle point value in order to control the duration of USR stage ΔN_2 – the longer it lasts, the larger the power spectrum peak grows. As for the δ-extension, no attractor behavior is found, though the desired power spectrum peak is possible in the two parameter regions – the one is around $\delta = 0.1$, and another one is around $\delta = 0.6$ – while the parameter ζ controls the duration of the USR stage here as well.

The relevant part of the Lagrangian is calculable by parametrizing the leading field component of the curvature superfield as

$$\mathcal{R}|_{\Theta=0} = \frac{M}{\sqrt{24}}e^{-ia}\sigma, \tag{53}$$

and setting $b_m = a = 0$, where b_m is the real vector of an old-minimal supergravity multiplet and the real scalars σ and a are the radial and angular modes of $\mathcal{R}|$, respectively. After using the standard Legendre-Weyl transform to eliminate the R^2-term, the bosonic part (ignoring all contributions of fermions) of the Lagrangian in the Einstein frame reads

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - \frac{3M^2}{2}Be^{-\sqrt{\frac{2}{3}}\varphi}(\partial\sigma)^2 - \frac{1}{4B}\left(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\right)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U, \tag{54}$$

where φ is the scalaron and the functions $A \equiv A(\sigma)$, $B \equiv B(\sigma), andU \equiv U(\sigma)$ are given by

$$\begin{aligned} A(\sigma) &= 1 - \delta\sigma + \frac{1}{6}\sigma^2 - \frac{11}{24}\zeta\sigma^4 - \frac{29}{54}\gamma\sigma^6, \\ B(\sigma) &= \frac{1}{3}M^{-2}(1 - \zeta\sigma^2 - \gamma\sigma^4), \\ U(\sigma) &= \frac{1}{2}M^2\sigma^2\left(1 + \frac{1}{2}\delta\sigma - \frac{1}{6}\sigma^2 + \frac{3}{8}\zeta\sigma^4 + \frac{25}{54}\gamma\sigma^6\right). \end{aligned} \tag{55}$$

The Kähler potential and the superpotential of the Einstein supergravity *dual* to the modified supergravity defined by Eqs. (51) and (52) are given by

$$K = -3\log\left[T + \overline{T} - \frac{1}{3}N(S,\bar{S})\right], \quad W = 3MST + \mathcal{F}(S), \tag{56}$$

where T and S are the chiral (super)fields, and the functions

$$N(S,\bar{S}) = 3\left(|S|^2 - \frac{3}{2}\zeta|S|^4 - 4\gamma|S|^6\right), \quad \mathcal{F}(S) = 3MS\left(\frac{\sqrt{6}}{4}\delta S - \frac{1}{2}\right), \tag{57}$$

have been obtained from Eqs. (51) and (52) by replacing $\mathcal{R} = MS/2$. In particular, Eq. (53) gives $S = e^{-ia}\sigma/\sqrt{6}$. The scalaron φ in the dual picture is given by

$$e^{\sqrt{\frac{2}{3}}\varphi} = T + \overline{T} - \frac{1}{3}N(S,\bar{S}). \tag{58}$$

Setting $\operatorname{Im} T = a = 0$ gives the Lagrangian (54).

The mass of PBH created as a result of the primordial power spectrum enhancement followed by gravitational collapse of large density perturbations can be estimated from the peak as follows [23]:

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})}\exp\left[2(N_{\rm end} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{60}}\varepsilon(t)H(t)dt\right], \tag{59}$$

where t_{peak} is the time when the wavenumber corresponding to the power spectrum peak (k_{peak}) exits the horizon and t_{60} is the time when k_{60} exits the horizon.

The PBH density fraction in DM can be roughly estimated by using the standard (Press-Schechter) formalism [60]. The underlying formulae for the PBH mass $\tilde{M}_{\text{PBH}}(k)$, the production rate $\beta_f(k)$, and the density contrast $\sigma(k)$ coarse-grained over k are [61,62]

$$\tilde{M}_{\text{PBH}} \simeq 10^{20}\left(\frac{7\times 10^{12}}{k\ \text{Mpc}}\right)^2 \text{g}, \quad \beta_f(k) \simeq \frac{\sigma(k)}{\sqrt{2\pi}\delta_c} e^{-\frac{\delta_c^2}{2\sigma^2(k)}}, \tag{60}$$

$$\sigma^2(k) = \frac{16}{81}\int \frac{dq}{q}\left(\frac{q}{k}\right)^4 e^{-q^2/k^2} P_\zeta(q),$$

respectively, where the parameter δ_c is the density threshold for PBH formation. The PBH mass is estimated as the horizon mass at the time when the co-moving momentum k reenters the horizon. The PBH-to-DM density fraction is given by [61,62]

$$\frac{\Omega_{\text{PBH}}(k)}{\Omega_{\text{DM}}} \equiv f(k) \simeq \frac{1.4\times 10^{24}\beta_f(k)}{\sqrt{\tilde{M}_{\text{PBH}}(k)\text{g}^{-1}}}. \tag{61}$$

Three specific examples in the Starobinsky supergravity were proposed and studied in Ref. [56]: one γ-extension and two δ-extensions. The δ-extensions were motivated by the existence of two suitable parameter regions, where $\delta \simeq 0.1$ and $\delta \simeq 0.6$ yield different shapes of the power spectrum (broad and narrow, respectively). The parameter sets of all three examples are given in Table 1, and the corresponding power spectra P_ζ and PBH density fractions $f(M)$ (numerically computed in Ref. [56]) are shown in Fig. 1, with the normalization of the wavenumber $k_{\text{exit}} = 0.05\ \text{Mpc}^{-1}$, where k_{exit} is the scale that leaves the horizon around 54 e-folds before the end of inflation. The parameter ζ is fixed by the choice of ΔN_2 at given γ and δ. In cases I, II, and III, one finds ζ as -2.374, 0.032, and 0.102, respectively.

To demonstrate the end of SR and the beginning of USR, Fig. 2 shows the evolution of the SR parameters ε_H and η_H in the case II. The SR parameters are defined by

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2}, \quad \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}. \tag{62}$$

Table 1 The selected parameter values and the corresponding CMB tilts n_s and r computed at $\Delta N = 54$ e-folds before the end of inflation (including the USR e-folds ΔN_2)

	γ	δ	ΔN_2	δ_c	n_s	r
Case I	1.5	0	20	0.4	0.942	0.009
Case II	0	0.09	19	0.47	0.946	0.008
Case III	0	0.61	20	0.4	0.946	0.007

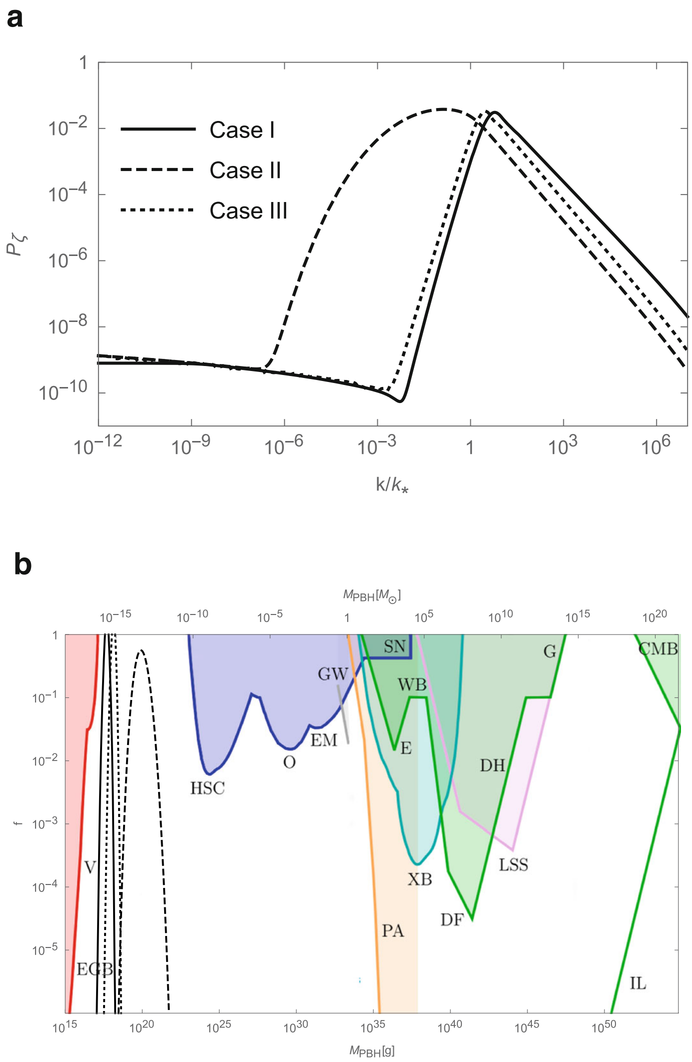

Fig. 1 (**a**) The power spectra in the examples of Table 1. Here, k_* represents the end of SR and the beginning of USR. (**b**) The respective PBH density fractions, where the background observational constraints on PBH are taken from Ref. [12]. In both plots, case I is denoted by the solid line, case II by the dashed line, and case III by the dotted line

The end of SR can be defined by the local maximum of ε_H (or, alternatively, by $\eta_H = 1$), and it is shown in Fig. 2 by the dashed vertical line.

According to Table I, the spectral tilt n_s in case I is ruled out by 3σ from the CMB data [29], whereas in cases II and III the value of n_s is within the current 3σ constraints. The PBH fraction in case II of Fig. 1 implies that this case is more

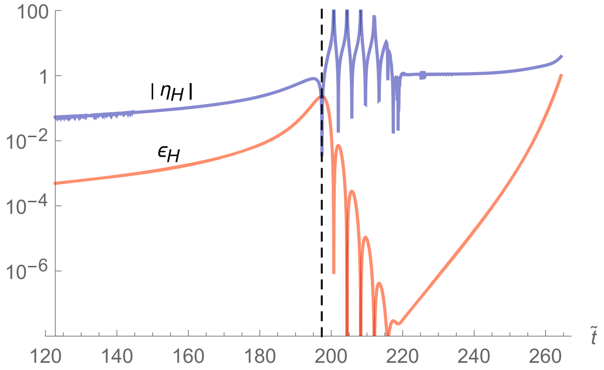

Fig. 2 The evolution of the slow-roll parameters ε_H and $|\eta_H|$ in case II around the start of the USR regime with respect to the normalized time $\tilde{t}$

flexible for accommodating slightly larger n_s. It happens because the PBH fraction in case II peaks at the center of the allowed window, and it is still possible to move the peak further to the left, thus lowering the PBH masses. More general cases with the nonvanishing γ and δ do not increase $M_{\rm PBH}$ and n_s [63].

Induced Gravitational Waves

In order to confirm or falsify the proposed supergravity models of inflation and PBH production by observations, one may look at detection of the stochastic gravitational waves (GW) induced by PBH formation. Let us estimate the energy density of the induced GW in the examples of Table 1 by following Ref. [64].

The present-day GW density function (spectrum) $\Omega_{\rm GW}$ is given by [65, 66]

$$\frac{\Omega_{\rm GW}(k)}{\Omega_r} = \frac{c_g}{72}\int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s \left[\frac{(s^2-\frac{1}{3})(d^2-\frac{1}{3})}{s^2+d^2}\right]^2 P_\zeta(kx)P_\zeta(ky)\left(I_c^2+I_s^2\right), \tag{63}$$

where the constant $c_g \approx 0.4$ in the case of the standard model (SM) and $c_g \approx 0.3$ in the case of the minimal supersymmetric standard model (MSSM); see Ref. [67] for details.

The present-day value of the radiation density Ω_r is equal to $h^2\Omega_r \approx 2.47 \times 10^{-5}$, according to measurements of CMB temperature [68]. Here, h is the reduced (present-day) Hubble parameter that we take as $h = 0.67$ (ignoring the Hubble tension). The variables x, y are related to the integration variables s, d as

$$x = \frac{\sqrt{3}}{2}(s+d), \quad y = \frac{\sqrt{3}}{2}(s-d), \tag{64}$$

while the functions I_c and I_s of $x(s,d)$ and $y(s,d)$ are given by [65,66]

$$I_c = -4\int_0^\infty \mathrm{d}\eta \sin\eta \big\{2T(x\eta)T(x\eta) + \big[T(x\eta)+x\eta T'(x\eta)\big]\big[T(y\eta)+y\eta T'(y\eta)\big]\big\} \tag{65}$$

and

$$I_s = 4\int_0^\infty \mathrm{d}\eta \cos\eta \big\{2T(x\eta)T(x\eta) + \big[T(x\eta)+x\eta T'(x\eta)\big]\big[T(y\eta)+y\eta T'(y\eta)\big]\big\}, \tag{66}$$

where the T-function is defined by

$$T(k\eta) = \frac{9}{(k\eta)^2}\left[\frac{\sqrt{3}}{k\eta}\sin\left(\frac{k\eta}{\sqrt{3}}\right) - \cos\left(\frac{k\eta}{\sqrt{3}}\right)\right], \tag{67}$$

in terms of the conformal time η.

The integrations in I_c and I_s can be performed analytically, and the results are [65]

$$I_c = -36\pi\,\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1), \tag{68}$$

$$I_s = -36\frac{s^2+d^2-2}{(s^2-d^2)^2}\left[\frac{s^2+d^2-2}{s^2-d^2}\log\left|\frac{d^2-1}{s^2-1}\right| + 2\right], \tag{69}$$

where θ is the Heaviside step function.

Using the formulae above, the GW density can be numerically computed from a given power spectrum. Let us consider the power spectra in the cases of Table 1, where PBH are merely a part of DM because the cases with $f_{\rm tot} = 1$ have quite similar power spectra though with slightly larger peaks. By using the power spectra of Fig. 1 (on the left side), the density $\Omega_{\rm GW}(k)$ in terms of frequency $k = 2\pi f$ is plotted in Fig. 3 together with the expected sensitivity curves for several space-based GW experiments planned in the future. Here, the power-law integrated curves [69] have been used and applied to the LISA noise model [70, 71]. To draw the sensitivity curves, the parameters and the noise models for TianQin [72], Taiji [73], and DECIGO [74] have been used.

It follows from Fig. 3 that the upcoming space-based GW experiments are expected to be sensitive enough to detect the stochastic GW background predicted by some inflation models proposed in the preceding sections, where PBH may account for a significant fraction (or all) of DM. Figure 3 shows that the supergravity models produce GW peaking in the frequency range $10^{-3}\nabla \cdot 10^{-1}$ Hz expected to be accessible by LISA, TianQin, Taiji, and DECIGO gravitational interferometers.

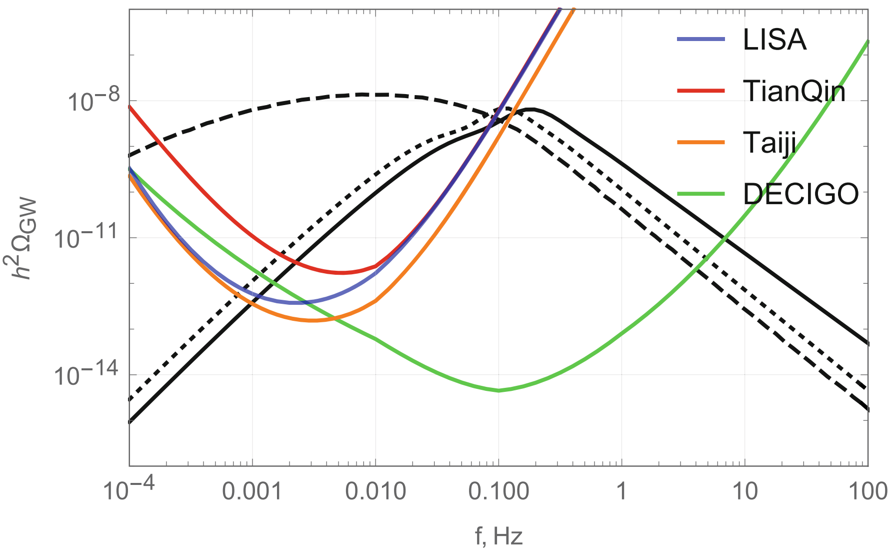

Fig. 3 The density of stochastic GW induced by the power spectrum enhancement in the supergravity models: case I (solid black curve), case II (dashed black curve), and case III (dotted black curve). The expected sensitivity curves for the space-based GW experiments are represented by the different colors

Adding Matter and Spontaneous SUSY Breaking

The Starobinsky-type supergravity models considered in section "Starobinsky-Type Supergravity and PBH Production" have only Minkowski vacua where SUSY is restored. An improved model of the Starobinsky-type supergravity coupled to a chiral matter superfield can *simultaneously* describe inflation, PBH formation, present DM, and spontaneous SUSY breaking after inflation in a Minkowski vacuum [75].

The manifestly supersymmetric curved superspace Lagrangian of a chiral matter superfield Φ coupled to the modified Starobinsky-type supergravity is given by

$$\mathcal{L} = \int d^2\Theta 2\mathcal{E}\left[-\frac{1}{8}(\overline{\mathcal{D}}^2 - 8\mathcal{R})(N + J) + \mathcal{F} + \Omega\right] + \text{h.c.} \tag{70}$$

It is parametrized by four arbitrary potentials: two non-holomorphic ones, $N = N(\mathcal{R}, \overline{\mathcal{R}})$ and $J = J(\Phi, \overline{\Phi})$, and two holomorphic ones, $\mathcal{F} = \mathcal{F}(\mathcal{R})$ and $\Omega = \Omega(\Phi)$, being the functions of the chiral scalar curvature superfield $\mathcal{R}$ of supergravity and the chiral superfield Φ of matter. When Φ enters J and Ω without mixing with $\mathcal{R}$, it is called the *minimal* coupling of the modified supergravity to chiral matter, as in Eq. (70).

Eliminating the auxiliary F-field of Φ in terms of the leading scalar field components $\phi \equiv \Phi|$ and $X \equiv \mathcal{R}|$ yields

$$F = -J^{-1}_{,\phi\bar{\phi}}(2\overline{X} J_{,\bar{\phi}} + \overline{\Omega}_{,\bar{\phi}}), \tag{71}$$

where the subscripts with commas denote the derivatives with respect to the given (field) arguments.

The simplest model of SUSY breaking in the Einstein supergravity with a single chiral matter superfield is known as the *Polonyi* model where supergravity is minimally coupled to a chiral matter superfield Φ with the canonical kinetic term and the superpotential given by a *linear* polynomial in Φ. It leads to a Minkowski vacuum with spontaneously broken SUSY at any scale. In phenomenological applications, the Polonyi superfield is usually affiliated with the hidden (heavy) matter sector that interacts with the observable matter (like the SM) only gravitationally. Then SUSY breaking is supposed to be transferred to the observable sector at the electroweak scale by gravity mediation.

The bosonic part (ignoring all contributions of fermions) of the Lagrangian reads [75]

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos.}} = &-\tfrac{1}{12}\big(\mathcal{F}_{,X} + \overline{\mathcal{F}}_{,\bar{X}} + 2N_{,X}X + 2N_{,\bar{X}}\overline{X} - 8N_{,X\bar{X}}X\overline{X} + 2N + 2J - \tfrac{1}{9}N_{,X\bar{X}}b_m b^m\big)R \\ &+ \tfrac{1}{144}N_{,X\bar{X}}R^2 - N_{,X\bar{X}}\partial X\partial\overline{X} - J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi} - \tfrac{i}{3}b_m(N_{,X}\partial^m X + J_{,\phi}\partial^m\phi - \text{c.c.}) \\ &+ \tfrac{i}{6}\big(\mathcal{F}_{,X} - \overline{\mathcal{F}}_{,\bar{X}} + 2N_{,X}X - 2N_{,\bar{X}}\overline{X} - \tfrac{i}{6}N_{,X\bar{X}}D_m b^m\big)D_m b^m \\ &- \tfrac{1}{2}\big(\mathcal{F}_{,X} + \overline{\mathcal{F}}_{,\bar{X}} + 2N_{,X}X + 2N_{,\bar{X}}\overline{X} - 4N_{,X\bar{X}}X\overline{X} + 2N + 2J \\ &\quad - \tfrac{1}{18}N_{,X\bar{X}}b_m b^m\big)(8X\overline{X} + \tfrac{1}{9}b_m b^m) + 6X(\overline{\mathcal{F}} + \overline{\Omega}) + 6\overline{X}(\mathcal{F} + \Omega) \\ &+ 12X\overline{X}(N + J) - J^{-1}_{,\phi\bar{\phi}}|2XJ_{,\phi} + \Omega_{,\phi}|^2. \end{aligned} \tag{72}$$

When ignoring also the pseudo-vector field, $b_m = 0$, the Lagrangian above can be rewritten to the form

$$e^{-1}\mathcal{L}_{\text{bos.}} = \frac{A}{2}R + \frac{B}{12M^2}R^2 - \frac{12B}{M^2}\partial X\partial\overline{X} - J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi} - U, \tag{73}$$

with the specific scalar potential U, where we have defined

$$A \equiv -\tfrac{1}{6}\big(\mathcal{F}_{,X} + \overline{\mathcal{F}}_{,\bar{X}} + 2N_{,X}X + 2N_{,\bar{X}}\overline{X} - 8N_{,X\bar{X}}X\overline{X} + 2N + 2J\big), \tag{74}$$

$$B \equiv \tfrac{1}{12}M^2 N_{,X\bar{X}}, \tag{75}$$

$$\begin{aligned} U \equiv\ & 4X\overline{X}\big(\mathcal{F}_{,X} + \overline{\mathcal{F}}_{,\bar{X}} + 2N_{,X}X + 2N_{,\bar{X}}\overline{X} - 4N_{,X\bar{X}}X\overline{X} - N - J\big) \\ & - 6X(\overline{\mathcal{F}} + \overline{\Omega}) - 6\overline{X}(\mathcal{F} + \Omega) + J^{-1}_{,\phi\bar{\phi}}|2XJ_{,\phi} + \Omega_{,\phi}|^2. \end{aligned} \tag{76}$$

As can be seen from Eq. (76) when $X = 0$, a Minkowski vacuum requires $\Omega_{,\phi} = 0$. This leads to $F = 0$ from Eq. (71). Hence, in order to break SUSY in a Minkowski vacuum, one needs a nonvanishing vacuum expectation value (VEV) of X, $\langle X \rangle \neq 0$. If at the onset of inflation the fields ϕ and X are stabilized around zero, as is the case in the models under consideration, there will be a nontrivial multi-field dynamics at smaller (than CMB) scales when the Jordan frame potential $U(X, \overline{X}, \phi, \bar{\phi})$ starts to control the dynamics (assuming it is initially suppressed), and the inflationary trajectory turns toward the nonvanishing VEV of X and ϕ. Therefore, the canonical Kähler potential and the linear superpotential of the standard Polonyi model have to be generalized for spontaneous SUSY breaking in the modified supergravity.

The Lagrangian (73) in the Jordan frame can be transformed to the dual (scalar-tensor) Lagrangian with the scalaron field φ in the Einstein frame. One finds [75]

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos.}} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}\Big(\frac{12B}{M^2}\partial X\partial\overline{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}\Big) \\ -\frac{3M^2}{4B}\Big(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\Big)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U. \end{aligned} \tag{77}$$

The potentials in the minimalistic (Starobinsky supergravity) setup are given by

$$\mathcal{F} = -3X, \quad N = \frac{12}{M^2}X\overline{X} - \frac{72}{M^4}\zeta(X\overline{X})^2, \tag{78}$$

just needed for the proper embedding of the Starobinsky ($R + R^2$) gravity model of inflation into the modified supergravity; see section "Starobinsky-Type Supergravity and PBH Production." In particular, the parameter M is proportional to the scalaron mass m_φ as $m_\varphi^2 = M^2/\langle B \rangle$ after assuming that $\langle Ae^{-\sqrt{2/3}\varphi} \rangle = 1$ and $\langle U \rangle = 0$, where the angle brackets denote the vacuum expectation values (VEV). The extra (second) term in N with the real parameter $\zeta > 0$ is needed for stabilization of the inflationary trajectory and the vacuum [44].

After the rescalings

$$X \to MX/\sqrt{12}, \quad \Omega \to M\Omega/\sqrt{3}, \tag{79}$$

the Lagrangian takes the form

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos.}} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}(B\partial X\partial\overline{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}) \\ -\frac{3M^2}{4B}\Big(1 - Ae^{-\sqrt{\frac{2}{3}}\varphi}\Big)^2 - e^{-2\sqrt{\frac{2}{3}}\varphi}U. \end{aligned} \tag{80}$$

where the functions A, B, and U read

$$A = 1 + \tfrac{1}{3}(X\overline{X} - J) - \tfrac{11}{6}\zeta(X\overline{X})^2, \qquad B = 1 - 2\zeta X\overline{X},$$
$$U = M^2\left[X\overline{X}\left(1 - \tfrac{1}{3}J\right) - \tfrac{1}{3}(X\overline{X})^2 + \tfrac{3}{2}\zeta(X\overline{X})^3 - X\overline{\Omega} - \overline{X}\Omega + \tfrac{1}{3}J^{-1}_{,\phi\bar{\phi}}|XJ_\phi + \Omega_{,\phi}|^2\right]. \tag{81}$$

The proper extension of the Polonyi model is given by [75]

$$J = \phi\bar{\phi} - \tfrac{\lambda}{2}(\phi\bar{\phi})^2, \tag{82}$$

$$\Omega = b\phi + \tfrac{c}{2}\phi^2 + \tfrac{f}{3}\phi^3, \tag{83}$$

with four real parameters (λ, b, c, f). The Lagrangian (80) in this case reads

$$e^{-1}\mathcal{L} = \frac{1}{2}R - \frac{1}{2}\partial\varphi\partial\varphi - e^{-\sqrt{\frac{2}{3}}\varphi}\left(B\partial X\partial\overline{X} + J_{,\phi\bar{\phi}}\partial\phi\partial\bar{\phi}\right) - V \tag{84}$$

with the scalar potential in the Einstein frame as

$$V = \frac{3M^2}{4B}(1 - Ay)^2 + y^2 U, \tag{85}$$

where we have introduced the notation

$$y \equiv e^{-\sqrt{2/3}\varphi}. \tag{86}$$

A, B, and U are the functions of X, $\overline{X}$ and ϕ, $\bar{\phi}$ with

$$X = \tfrac{1}{\sqrt{2}}(\sigma + i\hat{\sigma}), \quad \phi = \tfrac{1}{\sqrt{2}}(\rho + i\hat{\rho}), \tag{87}$$

where the hats are used to denote the imaginary parts (pseudo-scalars).

This model can accommodate the power spectrum enhancement (peak) leading to PBH as the whole DM or as a significant part of DM for the tuned values of the parameters given in Table 2.

Table 2 The parameters c, f, and λ for three selected values of b and $\Delta N_2 = 20$ and the corresponding predictions for the inflation tilts. The scalaron mass M is in Planck units

b	c	f	λ	n_s	$r_{\max}$	M
0.01	1.8×10^{-9}	-0.007098	0.42	0.9464	0.0081	1.87×10^{-5}
0.1	1.7×10^{-7}	-0.03863	0.27	0.9463	0.0082	1.88×10^{-5}
1	1.4×10^{-5}	-0.1717	0.19	0.9434	0.0092	2.00×10^{-5}

Table 3 The SUSY breaking VEV of the auxiliary F-fields, the gravitino masses $m_{3/2}$, and the masses of pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$ for the three values of the parameter b

b	$\frac{\lvert\langle F^T\rangle\rvert}{MM_{\rm Pl}}$	$\frac{\lvert\langle F^S\rangle\rvert}{MM_{\rm Pl}}$	$\frac{\lvert\langle F^\phi\rangle\rvert}{MM_{\rm Pl}}$	$\frac{\langle m_{3/2}\rangle}{M}$	$\frac{m_{\hat{\sigma}}}{M}$	$\frac{m_{\hat{\rho}}}{M}$
1	0.11	0.624	1.631	1.121	0.25	1.21
0.1	6×10^{-5}	0.048	0.155	0.092	0.77	0.19
0.01	3×10^{-8}	0.048	0.014	0.007	0.82	0.02

Table 4 The parameters, the inflation observables, and the PBH masses in three examples A, B, and C. The mass parameter varies from $M \approx 1.9 \times 10^{-5} M_{\rm Pl}$ to $2 \times 10^{-5} M_{\rm Pl}$

set	b	c	f	λ	n_s	$r_{\rm max}$	ΔN_2	$\hat{f}_{\rm tot}$	$M_{\rm PBH}$
A	1	7.25×10^{-6}	-0.32037	0.251	0.9460	0.0084	18.27	1	10^{18} g
B	0.3	6.88×10^{-7}	-0.09812	0.2442	0.9460	0.0082	20.15	0.54	10^{18} g
C	0.1	7.66×10^{-8}	-0.03462	0.252	0.9434	0.0090	21.92	1	10^{18-20} g

The corresponding numerical results for the F-fields and the gravitino masses $m_{3/2}$ in the Minkowski vacuum are shown in Table 3, together with the masses of the pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$, demonstrating that the latter are not destabilized after inflation.

As regards a numerical calculation of the PBH masses and PBH-to-DM density fraction by using the Press-Schechter formalism, the results are illustrated by Table 4.

The corresponding PBH fractions are given in Fig. 4 for the parameter sets A (the blue curve in Fig. 4) and B (the orange curve in Fig. 4). The set A produces $\hat{f}_{\rm tot} = 1$, with n_s on the margin of the 3σ CMB constraint, whereas the set B leads to $\hat{f}_{\rm tot} = 0.54$ with the same value of n_s. The parameter set C is excluded by 3σ.

In this supergravity model, the scalaron φ is the driver of the first stage of inflation where the CMB scale exits the horizon, whereas the second stage of inflation is driven by a combination of σ (the real part of X) and ρ (the real part of the chiral matter scalar ϕ). The beginning of the second inflationary stage gives rise to an enhancement (peak) of the power spectrum. In order to achieve the required enhancement of $O(10^7)$ in the power spectrum of scalar perturbations for a substantial PBH production, the additional term proportional to $|\phi|^4$ is needed in the Kähler potential.

SUSY is spontaneously broken in the vacuum after inflation, while one of the parameters can be fixed to achieve the vanishing cosmological constant. The masses of the pseudo-scalars $\hat{\sigma}$ and $\hat{\rho}$ (from X and ϕ, respectively) around the vacuum are close to the inflationary Hubble scale. During inflation, the scalars $\hat{\sigma}$ and $\hat{\rho}$ are stable and have larger effective masses. There is also another real scalar (sinflaton) that also has a large (not tachyonic) effective mass during and after inflation. In the initial higher-curvature formulation of this supergravity model, the extra scalar is related to the divergence $D_m b^m$ of the vector field b_m belonging to the old-minimal supergravity multiplet. There are six real scalars in total, with three of them being stabilized and the other three being participating in inflation.

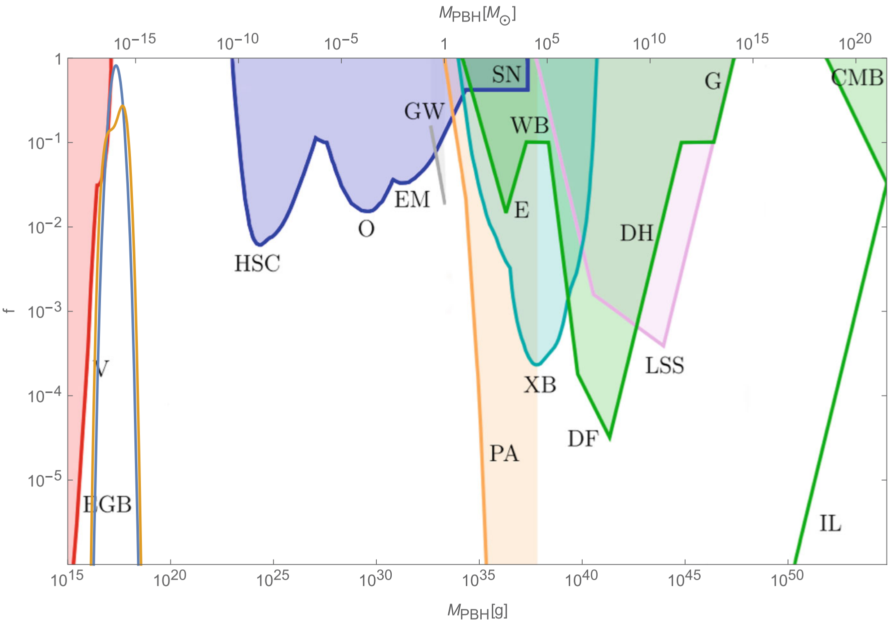

Fig. 4 The PBH-to-DM fraction for the parameter sets A (the blue curve) and B (the orange curve) in Table 4. The background of observational constraints is taken from Ref. [12]: evaporation (red), lensing (blue), gravitational waves (gray), various dynamical effects (green), accretion (light blue), large-scale structure (pink), and CMB distortions (orange)

The produced PBH may describe the whole DM or its significant fraction. Current observational constraints on PBH allow only a limited mass range for the whole PBH-DM between 10^{16} g and 10^{21} g. The spectral tilt n_s is rather low, though it is within the 3σ CMB constraint when the parameter b (a free parameter in our model) is not smaller than $\mathcal{O}(10^{-1})$.

The scale of SUSY breaking is high, about $\mathcal{O}(10^{13})$ GeV, being under the GUT and inflation scales, which is reflected in the gravitino masses shown in Table 3.

The second-order GW background induced by the enhanced scalar perturbations was numerically computed in Ref. [75] confirming those GW may be accessible by the future space-based GW experiments, as is expected from the (low-mass) PBH-DM scenarios [66]. For instance, in the case of LISA, the induced GW frequency should not be far away from 3.4 mHz, which implies the PBH masses of the order $10^{-12} M_\odot \sim 10^{21}$ g [66] that is close to the PBH masses one gets from the modified supergravity models.

Gravitino DM Genesis

In the preceding sections, all fermionic fields were ignored in cosmological applications of supergravity. However, they can also play the important role in cosmology. The fundamental consequence of supergravity is the existence of the spin-3/2 particle called *gravitino* that is the superpartner of graviton with vanishing charges. After SUSY breaking via super-Higgs mechanism, gravitino particles get masses and can be viable DM candidates called SGIMP (supergravitationally interacting massive particles). Their stability (or metastability) can be secured by demanding preservation of the R-parity, when gravitino is identified with the lightest SUSY particle (LSP). In this section, we introduce the Polonyi-Starobinsky (PS) supergravity [14] and describe production of Polonyi and gravitino particles by Schwinger effect at the scales close to the scale of Starobinsky inflation. The Polonyi mass is slightly higher two gravitino masses, so that Polonyi particles are unstable and decay into two gravitinos. Part of (or whole) cold DM composed of gravitinos can be achieved. This DM production channel may be complementary to the PBH production and implies the *composite* nature of DM built from massive gravitinos and PBH. In this scenario, the parameter space of the inflaton potential is directly related to the dark matter one, providing a new unifying framework of inflation and dark matter genesis. Due to the superheavy masses of gravitinos, no constraints from the Big Bang nucleosynthesis (BBN) arise, while the gravitino overproduction problem can also be avoided. This framework can be embedded into the "flipped" SUSY GUT theories inspired by heterotic string compactifications on Calabi-Yau threefolds [76], thus unifying particle physics with quantum gravity.

Polonyi-Starobinsky (PS) Supergravity

The PS supergravity is obtained by combining chiral matter superfields described by the Lagrangian (29) with the minimal supergravity described by the Lagrangian (36), with Starobinsky's inflaton belonging a massive vector supermultiplet. The Lagrangian is given by

$$\mathcal{L} = \int d^2\theta 2\mathcal{E} \left\{ \frac{3}{8}(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R})e^{-\frac{1}{3}(K+2J)} + \frac{1}{4}W^{\alpha}W_{\alpha} + \mathcal{W} \right\} + \text{h.c.} \tag{88}$$

After eliminating the auxiliary fields, one finds the bosonic part of the Lagrangian (88) as [47]

$$e^{-1}\mathcal{L} = -\frac{1}{2}R - K_{A\bar{A}}\partial_m A\partial^m \bar{A} - \frac{1}{4}F_{mn}F^{mn} - \frac{1}{2}J''\partial_m C\partial^m C - \frac{1}{2}J''B_m B^m - \mathcal{V}, \tag{89}$$

where the capital Latin subscripts denote the derivatives with respect to A ands $\bar{A}$ and the primes denote the derivatives with respect to C. The scalar potential reads

$$\mathcal{V} = \frac{g^2}{2}J'^2 + e^{K+2J}\left\{ K^{-1}_{A\bar{A}}(\mathcal{W}_A + K_A\mathcal{W})(\overline{\mathcal{W}}_{\bar{A}} + K_{\bar{A}}\overline{\mathcal{W}}) - \left(3 - 2\frac{J'^2}{J''}\right)\mathcal{W}\overline{\mathcal{W}} \right\}, \tag{90}$$

generalizing the standard formula (33).

The bosonic field components of the supergravity superfields are

$$2\mathcal{E}| = e, \quad \mathcal{D}\mathcal{D}(2\mathcal{E})| = 4e\overline{M}, \tag{91}$$
$$\mathcal{R}| = -\frac{1}{6}M, \quad \mathcal{D}\mathcal{D}\mathcal{R}| = -\frac{1}{3}R + \frac{4}{9}M\overline{M} + \frac{2}{9}b_m b^m - \frac{2}{3}i\mathcal{D}_m b^m,$$

in terms of the vierbein determinant $e \equiv \det e_m^a$, the space-time scalar curvature R, and the auxiliary fields given by the complex scalar M, and the real pseudo-vector b_m. The superspace covariant derivatives are used to define the field components here, unlike section "Supergravity and Inflation."

The bosonic field components of the chiral superfields are defined by

$$\Phi| = A, \quad \mathcal{D}_\alpha\mathcal{D}_\beta\Phi| = -2\varepsilon_{\alpha\beta}F, \quad \overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha\Phi| = -2i\sigma_{\alpha\dot\alpha}{}^m\partial_m A, \tag{92}$$
$$\overline{\mathcal{D}}\overline{\mathcal{D}}\mathcal{D}\mathcal{D}\Phi| = 16\Box A + \frac{32}{3}ib_a\partial^a A + \frac{32}{3}FM,$$
$$V| = C, \quad \mathcal{D}_\alpha\mathcal{D}_\beta V| = \varepsilon_{\alpha\beta}X, \quad \overline{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha V| = \sigma_{\alpha\dot\alpha}{}^m(B_m - i\partial_m C),$$
$$\mathcal{D}_\alpha W^\beta| \equiv -\frac{1}{4}\mathcal{D}_\alpha(\overline{\mathcal{D}}\overline{\mathcal{D}} - 8\mathcal{R})\mathcal{D}^\beta V = \frac{1}{2}\sigma_{\alpha\dot\alpha}{}^m\bar\sigma^{\dot\alpha\beta n}(\mathcal{D}_m\partial_n C + iF_{mn}) + \delta_\alpha{}^\beta(D + \frac{1}{2}\Box C),$$
$$\overline{\mathcal{D}}\overline{\mathcal{D}}\mathcal{D}\mathcal{D}V| = \frac{16}{3}b^m(B_m - i\partial_m C) + 8\Box C - \frac{16}{3}MX + 8D,$$

in terms of the physical fields (A, C, B_m), the auxiliary fields (F, X, D), and the vector field strength $F_{mn} = \mathcal{D}_m B_n - \mathcal{D}_n B_m$.

As is clear from Eq. (89), the absence of ghosts requires $J''(C) > 0$. The Kähler potential and the superpotential of the standard Polonyi model are

$$K = \Phi\overline{\Phi}, \qquad \mathcal{W} = \mu(\Phi + \beta), \tag{93}$$

with the parameters μ and β. Unlike section "Volkov-Akulov-Starobinsky Supergravity," no nilpotency condition is imposed on the chiral superfield Φ. The model includes the single-field (C) inflationary model, whose D-type scalar potential is given by

$$V(C) = \frac{g^2}{2}(J')^2. \tag{94}$$

The Minkowski vacuum conditions (after inflation) are satisfied at $J' = 0$, which implies

$$\langle A\rangle = \sqrt{3} - 1 \qquad \text{and} \qquad \beta = 2 - \sqrt{3}. \tag{95}$$

This solution describes a stable Minkowski vacuum with spontaneous SUSY breaking at arbitrary scale $\langle F\rangle = \mu$. The related gravitino mass is given by

$$m_{3/2} = \mu e^{2-\sqrt{3}+\langle J \rangle}. \tag{96}$$

There is also a complex (Polonyi) scalar of mass

$$M_A = 2\mu e^{2-\sqrt{3}} \geq 2m_{3/2} \tag{97}$$

and a massless fermion in the physical spectrum. The inequality in Eq. (97) is saturated in the original Polonyi model, but it is not the case in the model under consideration because $\langle J \rangle < 0$.

The D-type scalar potential associated with the Starobinsky model arises when [46]

$$J(C) = \frac{3}{2}(C - \ln C) \tag{98}$$

that implies

$$J'(C) = \frac{3}{2}\left(1 - C^{-1}\right) \qquad \text{and} \qquad J''(C) = \frac{3}{2}\left(C^{-2}\right) > 0. \tag{99}$$

According to Eq. (89), the inflaton field ϕ with the canonical kinetic term is related to the field C by the field redefinition

$$C = \exp\left(\sqrt{2/3}\phi\right). \tag{100}$$

Thus, one gets the scalar potential of the Starobinsky model,

$$V_{\text{Star.}}(\phi) = \frac{9g^2}{8}\left(1 - e^{-\sqrt{2/3}\phi}\right)^2. \tag{101}$$

In order to break SUSY spontaneously, adding the standard Fayet-Iliopoulos (FI) term does not work because it leads to the gauged R-symmetry [77] and, hence, to highly restricted superpotentials. However, there exists the *alternative* FI term, without gauging the R-symmetry. It reads [78]

$$\mathcal{L}_{\text{FI}} = 8\xi \int d^4\theta\, E \frac{W^2 \overline{W}^2}{\mathcal{D}^2 W^2 \overline{\mathcal{D}}^2 \overline{W}^2} \mathcal{D}^\alpha W_\alpha \tag{102}$$

with the constant real parameter ξ. This term is manifestly SUSY- and gauge-invariant and does not include the higher derivatives of the field components; however, it has the inverse powers of the auxiliary field D (up to the forth order) in the fermionic sector only. When all fermions are ignored, the scalar D enters the bosonic action as a quadratic polynomial. The Kähler-Weyl gauge invariance is broken by the FI term (102) but can be restored by further modifications [49, 50].

With the J-function and the FI term (102), the scalar potentials get modified as

$$V_D = \frac{g^2}{2}\left[J' + \xi e^{\frac{1}{3}(K+2J)}\right]^2, \tag{103}$$

$$V_F = \mu^2 e^{\bar{A}A+2J}\left\{|\bar{A}A + A\beta + 1|^2 - \left(3 - 2\frac{J'^2}{J''}\right)|A+\beta|^2\right\}. \tag{104}$$

Demanding the V_D to reproduce the Starobinsky potential yields the first-order nonlinear differential equation

$$\frac{dJ}{dC} + \xi e^{\frac{1}{3}(K+2J)} = -\frac{3}{2}\left(1 + \frac{1}{C}\right). \tag{105}$$

Since the Polonyi field A should stay in its minimum at $A = \langle A\rangle$, one introduces the effective (field-dependent) alternative FI term $\tilde{\xi}(A,\bar{A}) = \xi e^{\frac{1}{3}K(A,\bar{A})}$ together with its VEV, $\tilde{\xi} = \xi e^{\frac{1}{3}K(\langle A\rangle,\langle\bar{A}\rangle)}$. Then, Eq. (105) takes the form

$$\frac{dJ}{dC} + \tilde{\xi} e^{\frac{2}{3}J} = -\frac{3}{2}\left(1 + \frac{1}{C}\right). \tag{106}$$

Without the FI term, $\tilde{\xi} = 0$, one finds the asymptotic behavior $J \sim -\frac{3}{2}C > 0$ for large negative C, which causes an instability. The instability can be removed when the function J would approach a constant instead, because large negative values of C exactly correspond to a plateau (slow roll) of Starobinsky inflation. Indeed, Eq. (106) can be easily integrated for $\left|C^{-1}\right| \ll 1$, with the result

$$J(C) \approx J_\infty - \frac{3}{2}\ln\left(1 - e^{C-C_0}\right), \tag{107}$$

where C_0 is the integration constant, and we have used the notation

$$J_\infty = \frac{3}{2}\ln\left(\frac{3}{-2\tilde{\xi}}\right). \tag{108}$$

As is clear from Eqs. (107) and (108), demanding

$$\xi < 0 \tag{109}$$

implies $\tilde{\xi} < 0$ also, whereas the function J fast approaches the constant J_∞ from above, with C taking large negative values. It is worth noticing that $J_\infty = 0$ at the "critical" value $\tilde{\xi} = -3/2$.

The FI-modified inflationary scalar potential of PS supergravity during slow roll is

$$\mathcal{V} = \frac{9}{8}g^2 M_{\rm Pl}^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)^2 + \mu^2 M_{\rm Pl}^{-2} \exp\left(M_{\rm Pl}^{-2}\bar{A}A + 2J_\infty\right)$$
$$\times \left\{|\bar{A}A + A\beta + M_{\rm Pl}^2|^2 - 3M_{\rm Pl}^2\left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}}\right)|A+\beta|^2\right\} \equiv V_D + V_F, \tag{110}$$

where the dependence upon the reduced Planck mass $M_{\rm Pl}$ is restored. At large values of ϕ (and fixed $\bar{A}$, A), the V_F goes to zero, while $V_D \to 9g^2 M_P^4/8$.

Polonyi and Gravitino Dynamics

Since the equations of motion in PS supergravity are very complicated, let us keep only the leading order with respect to the inverse Planck mass, temporarily neglect the couplings of Polonyi and gravitino particles to inflaton, and return them back in the end of this section. Then the effective action of the Polonyi field in the FLRW background, in the co-moving coordinates, reads

$$I[A] = \int dt \int d^3x \frac{a^3}{2}\left(\dot{A}^2 - \frac{1}{a^2}(\nabla A)^2 - M_A^2 A^2 - \zeta R A^2\right), \tag{111}$$

where ζ is the non-minimal coupling constant of the Polonyi field to gravity, A is the Polonyi field, M_A stands for its mass, R is the Ricci scalar, and a is the FLRW scale factor.

The mode decomposition of the Polonyi field reads

$$A(\mathbf{x}) = \int d^3k(2\pi)^{-3/2}a^{-1}(\eta)\left[b_k h_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}} + b_k^\dagger h_k^*(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\right], \tag{112}$$

where the conformal time coordinate η and the creation/annihilation operators $b, b^\dagger$ have been introduced, while the coefficient functions h, h^+ have been normalized as follows:

$$h_k h_k'^* - h_k' h_k^* = i. \tag{113}$$

Because of Eqs. (111) and (112), the equations of motion for the modes are

$$h_k''(\eta) + \omega_k^2(\eta)h_k(\eta) = 0, \quad \text{where} \quad \omega_k^2 = 5\frac{a''}{a} + k^2 + M_A^2 a^2, \tag{114}$$

and $h'' = d^2h/d\eta^2$. Equation (114) can be conveniently rescaled by using some reference scales $a(\eta_*) \equiv a_*$ and $H(\eta_*) = H_*$ as follows:

$$h''_{\tilde{k}}(\tilde{\eta}) + (\tilde{k}^2 + b^2\tilde{a}^2)h_{\tilde{k}}(\tilde{\eta}) = 0\,, \tag{115}$$

in terms of the rescaled quantities

$$\tilde{\eta} = \eta a_* H_*, \quad \tilde{a} = a/a_*, \quad \tilde{k} = k/(H_* a_*).$$

The leading order of the gravitino action is given by the massive Rarita-Schwinger action,

$$I[\psi] = \int d^4x\, e\, \bar{\psi}_\sigma \mathcal{R}^\sigma\{\psi\}\,, \tag{116}$$

where the gravitino kinetic operator has been introduced,

$$\mathcal{R}^\sigma\{\psi\} = m_{3/2}\gamma^{\sigma\nu}\psi_\nu + i\gamma^{\sigma\nu\rho}\mathcal{D}_\nu\psi_\rho, \tag{117}$$

with the supercovariant derivative

$$\mathcal{D}_\mu\psi_\nu = -\Gamma^\rho_{\mu\nu}\psi_\rho + \partial_\mu\psi_\nu + \frac{1}{4}\omega_{\mu ab}\gamma^{ab}\psi_\nu, \tag{118}$$

in the γ-notation $\gamma^{\mu_1\cdots\mu_n} = \gamma^{[\mu_1}\ldots.\gamma^{\mu_n]}$ with unit weight of the antisymmetrized product.

Since the supergravity torsion is of the second order with respect to the inverse Planck mass, we can ignore it in the leading order approximation. The $\Gamma^\rho_{\mu\nu}$ can be represented by the standard symmetric Christoffel symbols that are actually cancelled from the Rarita-Schwinger action (116). The Rarita-Schwinger action leads to the gravitino equation of motion:

$$(i\not{\mathcal{D}} - m_{3/2})\psi_\mu - \left(i\mathcal{D}_\mu + \frac{m_{3/2}}{2}\gamma_\mu\right)\gamma\cdot\psi = 0. \tag{119}$$

In the flat FLRW background, Eq. (119) reduces to

$$i\gamma^{mn}\partial_m\psi_n = -\left(m_{3/2} + i\frac{a'}{a}\gamma^0\right)\gamma^m\partial_m\psi, \tag{120}$$

where

$$\omega_{\mu ab} = 2\dot{a}a^{-1}e_{\mu[a}e^0_{b]}, \quad e^a_\mu = a(\eta)\delta^a_\mu, \quad m_{3/2} = m_{3/2}(\eta). \tag{121}$$

A solution to Eq. (120) is

$$\psi_\mu(x) = \int d^3\mathbf{p}(2\pi)^{-3}(2p_0)^{-1} \sum_\lambda \{e^{i\mathbf{k}\cdot\mathbf{x}} b_\mu(\eta,\lambda) a_{k\lambda}(\eta) + e^{-i\mathbf{k}\cdot\mathbf{x}} b^C_\mu(\eta,\lambda) a^\dagger_{k\lambda}(\eta)\}. \tag{122}$$

The equations of motion for the 3/2-helicity gravitino modes have the same form as in Eq. (114), namely,

$$b''_\mu(\eta,\lambda) + \hat{C}(k,a) b'_\mu(\eta,\lambda) + \omega^2(k,a) b_\mu(\eta,\lambda) = 0, \tag{123}$$

where we have introduced the notation

$$\hat{C}(k,a) b'_\mu(\eta,\lambda) = -2i\gamma^{\nu i} k_i \gamma_{\nu\eta} \partial^\eta b_\mu - 2\gamma_\nu (m_{3/2} + i\frac{a'}{a}\gamma^0) i\gamma^{\nu\eta}\partial_\eta b_\mu, \tag{124}$$

$$\omega^2(k,a)/2 = k^2 + m^2_{3/2} + 2i\frac{a'}{a}\gamma^0 m_{3/2} - \left(\frac{a'}{a}\right)^2. \tag{125}$$

Following the standard procedure in the case of Dirac and Klein-Gordon equations, the mode equations of motion can be reformulated to

$$P_\nu P^\nu b_\mu(\eta,\lambda) = 0, \tag{126}$$

where we have introduced the projector operator

$$P^\nu = i\gamma^{\nu\eta}\partial_\eta - \gamma^{\nu i} k_i - \left(m_{3/2} + i\frac{a'}{a}\gamma^0\right)\gamma^\nu = 0\,. \tag{127}$$

Equation (123) can be rescaled in the same way as Eq. (115).

The gravitino interaction with matter can be described via the *effective* gravitino mass $M_{3/2}$ that depends on the matter fields in the Rarita-Schwinger equation and satisfies $m_{3/2} = \langle M_{3/2} \rangle$. The effective gravitino mass is generated from the following interactions of gravitino with inflaton ϕ and Polonyi scalar $\tilde{A}$ (having $\left\langle \tilde{A} \right\rangle = 0$):

$$\begin{aligned} &M_{3/2}(\phi,\tilde{A}) = \\ &\quad \mu M_{\rm Pl}^{-1} \exp\left[(1/\sqrt{6}) M_{\rm Pl}^{-1}\phi + M_{\rm Pl}^{-2}(\bar{\tilde{A}}\tilde{A} + \alpha\bar{\tilde{A}} + \alpha\tilde{A} + \alpha^2)\right](\tilde{A} + \alpha + \beta), \end{aligned} \tag{128}$$

where we have introduced $\alpha \equiv \langle A \rangle$, with $A = \alpha + \tilde{A}$, and have restored the dependence on the Planck mass because of its relevance for phenomenological applications.

Polonyi and Gravitino Production

The dynamics of the gravitino and Polonyi fields during inflation necessarily leads to their quantum production. The number density of produced particles is calculated by using the Bogoliubov transformation

$$h_k^{\eta_1}(\eta) = \alpha_k h_k^{\eta_0}(\eta) + \beta_k h_k^{*\eta_0}(\eta)\,. \tag{129}$$

This transformation is performed from the vacuum solution selected by the boundary conditions at $\eta = \eta_{in}$, corresponding to the initial time of inflation, to the final time $\eta = \eta_f$, when the particle creation process from inflation stops. In the inflationary epoch, the dynamical regime is $a'/a^2 \ll M_{\rm Pl}$ and $M_{\rm Pl}\, ba/k \ll 1$. This implies that one can consider the extremes as $\eta_{in} = -\infty$ and $\eta_f = +\infty$ and then perform the WKB semiclassical approximation. By assuming these boundary conditions, the energy density of the Polonyi particles produced during inflation reads

$$\rho_A(\eta) = M_A n_A(\eta) = M_A H^3_{\rm inf} \left(\frac{1}{\tilde{a}(\eta)}\right)^3 \mathcal{P}_A \quad , \tag{130}$$

where

$$\mathcal{P}_A = \frac{1}{2\pi^2} \int_0^\infty d\tilde{k}\tilde{k}^2 |\beta_{\tilde{k}}|^2. \tag{131}$$

Similar equations are valid for massive gravitinos, with the power spectrum

$$\mathcal{P}_\psi = \frac{1}{2\pi^2} \int_0^\infty d\tilde{k}\tilde{k}^2 |b_\mu b^{C\mu}|. \tag{132}$$

The inflaton mass sets the characteristic energy scale for the Hubble parameter calculated at a fixed cosmological time $t \equiv t_f$,

$$H^2(t_f) \simeq m_\phi^2, \quad \rho(t_f) \simeq m_\phi^2 M_{\rm Pl}^2\,.$$

The formula for the Polonyi particles (energy density and Polonyi mass) produced during inflation was proposed in Ref. [14]:

$$(\Omega_A h^2/\Omega_R h^2) \simeq \frac{8\pi}{3}\left(\frac{M_A}{M_{Pl}}\right)\left(\frac{T_{\rm reh}}{T_0}\right)\frac{n_A(t_f)}{M_{Pl} H^2(t_f)}, \tag{133}$$

where M_A is the Polonyi mass, $\Omega_R h^2 \simeq 4.31 \times 10^{-5}$ is the radiation energy density at today's temperature T_0, and $\Omega_A h^2$ is the energy density of the produced Polonyi fields, all in units of the critical energy density. The similar estimate is valid for gravitino particles also. Equation (133) is motivated by the relation

$$\frac{\rho_A(t_0)}{\rho_R(t_0)} = \frac{\rho_A(t_{\rm reh})}{\rho_R(t_{\rm reh})}\left(\frac{T_{\rm reh}}{T_0}\right), \tag{134}$$

where ρ_A is the Polonyi energy density, ρ_R is the energy density of radiation, and $T_{\rm reh}$ and T_0 are the temperature of the universe calculated at reheating $t_{\rm reh}$ and today t_0 time scales, respectively. It is reasonable to assume that Polonyi particles are mainly produced after the de Sitter phase t_e, when the transition to the coherent oscillation phase starts. Given this assumption, the inflaton and Polonyi energy densities are redshifted with almost the same dilution rate. Their co-scaling relations hold until the reheating epoch finishes and the radiation dominated stage begins. Then most of the energy density is converted into radiation, i.e.,

$$\rho_R \simeq \rho_c = \frac{3H^2 M_{Pl}^2}{8\pi}. \tag{135}$$

This implies the following relation:

$$\frac{\rho_A(t_{\rm reh})}{\rho_R(t_{\rm reh})}\left(\frac{\rho_A(t_e)}{M_{Pl}^2 H^2(t_e)}\right)^{-1} \simeq \frac{8\pi}{3}, \tag{136}$$

where $H(t_e)$ is the Hubble parameter at a fixed time $t \equiv t_e$. Equation (133) follows from Eq. (136).

According to Eq. (130), when relating Hubble scale, Polonyi mass, and the *desiderata* Polonyi energy density, there is about $8th$-order-of-magnitude suppression of the energy density. The normalized power spectrum $\mathcal{P}_A$ cannot provide such suppression with our values for M_A and $H_{\rm inf}$. However, it comes from the dilution factor $(\tilde{a})^{-3} = (a_f/a_i)^{-3}$ in Eq. (130).

The semi-analytical estimates for Eq. (130) indicate that almost all Polonyi particles are produced in an excursion of the inflaton field around $\phi_e \equiv \phi(t_e)$ with $\Delta\phi \simeq 0.2$. The value of the dilution factor can be estimated from

$$(a(t_f)/a(t_i))^{-3} = \exp\left[-24\pi \int_{\phi_f}^{\phi_i} d\phi\, V^{-1}(\phi) V_{,\phi}(\phi)\right] = \exp(-\Delta\Phi), \tag{137}$$

where we have defined $\Delta\Phi = \Phi(\phi(t_i)) - \Phi(\phi(t_f))$, having in mind that $\phi(t_i) > \phi(t_f)$ and

$$\Phi(\phi) = 48\pi\sqrt{2/3}e^{-\sqrt{2\phi/3}}(1 - e^{-\sqrt{2\phi/3}})^{-1}. \tag{138}$$

After integrating over the effective particle production region $\Delta\phi$, we find $\Delta\Phi = 18.2$, i.e.,

$$(a(t_f)/a(t_i))^{-3} \simeq \exp(-18.2) \simeq 10^{-8}. \tag{139}$$

The $a(t_f)$ refers to a cosmological time t_f close to reheating. The cosmological time t_i when particle production effectively started is not far from the t_i because the inflaton field has an excursion of merely $\Delta\Phi = \Phi(t_i) - \Phi(t_f) \simeq 20$ that is proportional to $\Delta t = t_f - t_i$. But the relation between $a(t)$ and the cosmological time t is exponential. This is the origin of the very large exponential suppression (of the eighth order) between a_f and a_i despite the fact that the effective time of particle production is very short. From the physical point of view, particles produced during t_f are diluted with the factor exponentially larger then $a(t_i)$. These results are in agreement with the common expectations that particle production is most efficient toward the end of inflation [79].

Gravitino Mass

To get the gravitino and Polonyi masses, we add a few more cosmological assumptions about relevant parameters of the reheating process and, in particular, about the reheating temperature $T_{\rm reh}$ in the scenario based on the Starobinsky inflation. This implies that all cosmological parameters can be fixed by specifying the e-folding number N_e that is in the range between 50 and 60, due to the CMB bounds. For a more precise estimate of the cold DM abundance, we choose $N_e = 55$. This leads to the precise set of inflation parameters as

$$\begin{aligned} &n_s = 0.964, \quad r = 0.004, \quad m_{\rm inf} = 3.2 \cdot 10^{13}\,{\rm GeV}, \\ &H_{\rm inf} = \pi M_{\rm Pl}\sqrt{P_g/2} = 1.4 \cdot 10^{14}\,{\rm GeV}. \end{aligned} \tag{140}$$

Well below the inflation scale, the low-energy effective field theory is given by the standard model (SM) that has the effective number of d.o.f. as $g_* = 106.75$. Let us assume that all the SM particles originated from perturbative inflaton decay via the (Starobinsky) universal reheating mechanism, whose reheating temperature is known [80]:

$$T_{\rm reh} = \left(\frac{90}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_{tot} M_P} = 3 \cdot 10^9\,{\rm GeV}\,. \tag{141}$$

This value is in agreement with the successful leptogenesis mechanism of Ref. [81].

On the other hand, the reheating temperature for heavy gravitinos is given by [82]

$$T_{\rm reh} = 1.5 \cdot 10^8\,{\rm GeV} \left(\frac{80}{g_*}\right)^{1/4} \left(\frac{m_{3/2}}{10^{12}\,{\rm GeV}}\right)^{3/2}\,. \tag{142}$$

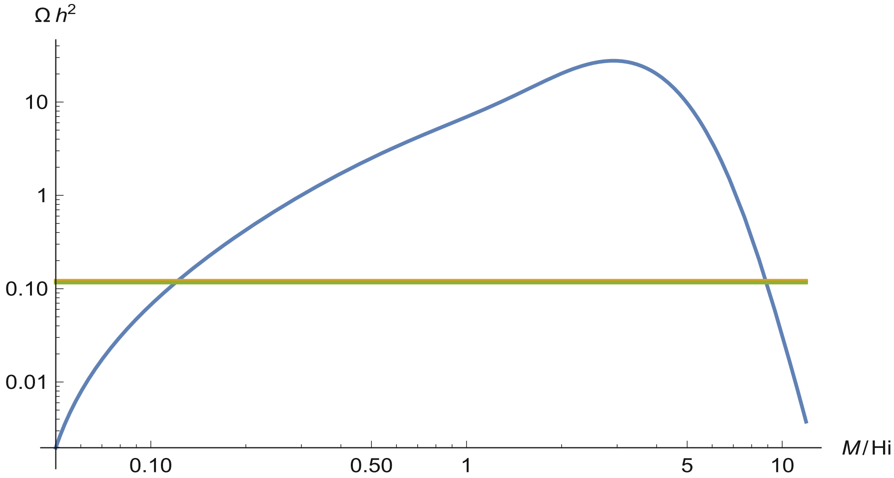

Fig. 5 Numerical simulations of the produced gravitino mass density (normalized) as a function of the Polonyi mass parameter are displayed in blue, in the parameter range compatible with inflation, reheating, and leptogenesis (at the reference point $N_e = 55$): $n_s = 0.964$, $r = 0.004$, $m_{\rm inf} = 3.2 \cdot 10^{13}$ GeV, $H_{\rm inf} = 1.4 \cdot 10^{14}$ GeV, and $T_{\rm reh} = 3 \cdot 10^9$ GeV. The right amount of the whole cold DM, $\Omega_{3/2}h^2 = \Omega_{DM}h^2 = 0.11$ (in orange), is generated when the Polonyi mass is $M_A \approx 2m_{3/2} = (1.54 \pm 0.2) \times 10^{13}$ GeV

Combining Eqs. (141) and (142), we get the gravitino and Polonyi masses as

$$m_{3/2} = (7.7 \pm 0.8) \cdot 10^{12}\,\text{GeV} \quad \text{and} \quad M_A = 2e^{-\langle J \rangle} m_{3/2} \approx 2m_{3/2}. \tag{143}$$

These masses are compatible with the whole abundance of cold DM, according to the numerical estimates in Fig. 5, which lends further support toward the conjecture in Eq. (133).

Decay and Annihilation Channels

With the cold DM made of massive gravitinos, it is important to evaluate the competition between Schwinger effect and decay channels. Polonyi particles decay into gravitino pairs with the decay rate [83]

$$\Gamma(A \to \psi_{3/2}\psi_{3/2}) \simeq \frac{3}{288\pi} M_A^3/m_{3/2}^2 \simeq 2.6 \times 10^{-2} m_{3/2}. \tag{144}$$

This channel is a direct consequence of the gravitino mass generation mechanism from the nonvanishing Polonyi vacuum expectation value. Since $m_{3/2} = 7.7 \cdot 10^{12}$ GeV, it implies $\Gamma \ll H_{\rm inf}$. Hence, the decay time of Polonyi into gravitinos is much larger the production time during inflation: $\tau_{A\to\psi_{3/2}\psi_{3/2}} \gg \tau_{\rm inflation}$. Decays of Polonyi particles into gravitinos are, therefore, negligible during inflation, while gravitino and Polonyi particles are independently generated during the inflationary epoch. After reheating, all Polonyi particles rapidly decay into gravitinos. As a result, the Polonyi number density n_S is completely converted into a contribution to the gravitino number density as $\Delta n_\Psi = 2n_A$.

The gravitino-inflaton coupling arises from the Weyl rescaling of the vierbein in the gravitino action. On the other hand, the gravitino kinetic term does not provide any contribution because of the conformal flatness of the FLRW universe. This means that the only source of the gravitino genesis is given by the effective gravitino mass term,

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2} e^{G_{\text{tot}}/2} \bar{\psi}_\mu \gamma^{\mu\nu} \psi_\nu,\, G_{\text{tot}} = K + \ln|W|^2 + 2J, \tag{145}$$

while the gravitino mass is the expectation value $m_{3/2} = \left\langle e^{G_{\text{tot}}/2} \right\rangle$. The perturbative decay rate of the inflaton ϕ into a pair of gravitinos is [83]

$$\Gamma_{\phi\to\psi_{3/2}\psi_{3/2}} = \frac{\left|G_\phi\right|^2}{288\pi} \frac{m_{\text{inf}}^5}{m_{3/2}^2 M_{Pl}^2}. \tag{146}$$

In the model under consideration, the factor G_ϕ vanishes at the minimum of the inflaton scalar potential. This implies that the perturbative production of the gravitinos from the inflaton decays is suppressed. Another contribution comes from the Polonyi production due to the inflaton decays. This decay channel is kinematically allowed since inflaton is heavier than Polonyi particle in our model. The perturbative decay rate of inflaton into a pair of Polonyi scalars — see, for example, Ref. [41] for a review – reads

$$\Gamma_{\phi\to AA} = \frac{1}{192\pi} \frac{m_{\text{inf}}^3}{M_{Pl}^2}. \tag{147}$$

One may expect that the non-perturbative preheating can be significantly increased due to a broad parametric resonance [84]. Inflaton and Polonyi fields are mixed as

$$\mathcal{L}_{\phi\phi\to AA} = \lambda\phi^2 \bar{A} A \tag{148}$$

that follows from an expansion of the scalar potential with respect to both the scalar fields ϕ and A in (90) with the J-function (38). The broad parametric resonance could lead to an enhancement of the perturbative production of the Polonyi particles, up to the factor of $O(10^5)$ – see, for example, Ref. [85] for numerical calculations of the broad resonance effects in supergravity. A calculation yields

$$\lambda = 10 e^{7-2\sqrt{3}} \frac{\mu^2}{M_{Pl}^2}, \tag{149}$$

i.e., the coupling constant λ is of the order $\mathcal{O}(10^{-7})$ or less in PS supergravity. This small value fully compensates any possible enhancement of the Polonyi production by the broad parametric resonance. In other words, the Polonyi production from the inflaton decays is a perturbative process in the case of study. In summary, the gravitino production from inflaton decays is sub-leading versus the Schwinger effect.

Conclusion

It was demonstrated in this chapter that the proposed extensions of the Starobinsky inflation model in supergravity can describe inflation in agreement with CMB measurements, as well as DM in the form of PBH and/or supermassive gravitino particles, all having the supergravitational origin, with a small number of free parameters. The viable inflationary models in supergravity can be based on either single-field or multi-field inflation. The power spectrum enhancement and the ultra-slow-roll phase, needed for PBH production, can be engineered via tachyonic instabilities of scalar fields in the double-inflation scenario; see also Ref. [86] for the similar approach in supergravity. Being the more fundamental theory of gravity (versus general relativity), supergravity has higher predictive power for phenomenology of the early universe. In turn, observational cosmology can be considered as a *probe* of supergravity and high-energy physics beyond the SM.

The key theoretical tools also include *manifest* local supersymmetry provided by curved superspace and required for consistency of the phenomenological models, modified supergravity, specific mechanisms of spontaneous SUSY breaking, and the super-Higgs effect. It leads to high-scale SUSY and the gravitino mass of $\mathcal{O}(10^{12})$ GeV just below the inflationary scale. This value is consistent with the known value of Higgs mass and the two-loop calculations of the renormalization group equations relating the MSSM scale with the high scale of SUSY in the case of the gaugino coupling mixing parameter $\tan\beta$ close to one [87]. Having part of DM in the form of the massive LSP gravitinos produced at the end of inflation also implies that none of the SM superpartners was produced after inflationary reheating, which means no hope to detect sparticles in colliders.

Supergravity can be embedded into compactified superstring/M-theory that gives its ultraviolet completion in quantum gravity. For instance, high-scale SUSY breaking, heavy particle DM, and connection to MSSM can be realized in the heterotic M-theory [88].

Significant part of DM may be in the form of PBH, with the PBH masses being in the range between 10^{17} g and 10^{21} g. The whole DM as the PBH is possible too, though after significant fine-tuning of the parameters. These PBH masses corresponding to the asteroid-size black holes are beyond the lower bound provided by Hawking radiation but are much lower the solar mass and the black hole masses discovered by the LIGO experiment [89].

PBH formation leads to GW because large scalar overdensities act as a source of stochastic GW background. Those GW may be detected by the future space-based GW interferometers such as LISA [70], TAIJI (old ALIA) [73], TianQin [72], and

DECIGO [74]. The supergravity models also predict the GW stochastic background radiation that is sensitive to the inflationary parameters and the PBH mass spectrum. The NANOGrav Collaboration data [90] hints to the PBH as part of DM too [91].

Fine-tuning in the supergravity models amounts to fixing the parameter $M \sim 10^{-5} M_{\mathrm{Pl}}$ as the (Starobinsky) inflaton mass and the dimensionless parameter ζ needed for enough e-folds. The PBH masses found are compatible with all astrophysical and cosmological constraints [12].

The main takeaway from this chapter is that inflation, PBH formation, PBH/gravitino DM, and SUSY breaking can be unified in the supergravity framework and directly affect each other, leading to the intriguing unifying picture of inflation and DM, in which their parameter spaces are linked to each other. This scenario also suggests interesting phenomenology in the ultra-high-energy cosmic rays because superheavy Polonyi particles may also decay into the SM particles, as the secondaries, in top-bottom decays.

Supergravity is often regarded as an extension of gravity at super-high-energy scales. With the whole DM being composed of superheavy LSP gravitinos, the only experimentally verifiable signature of SUSY would be just the DM. However, the new scalars of supergravity can play the active role during inflation, catalyze PBH formation, and produce GW radiation. The interactions of those scalars are dictated by local SUSY and are not assumed *ad hoc*, so that the supergravity models have the predictive power that can be falsified in future experiments. It gives us the reason to believe that indirect footprints of SUSY may be detected from GW physics rather than from high-energy particle colliders.

Funding

The author was supported by Tokyo Metropolitan University, the Japan Society for the Promotion of Science under the Grant No. 22K03624 and the World Premier International Research Center Initiative (WPI), MEXT, Japan.

Cross-References

- ▶ D-Branes
- ▶ Moduli Stabilization in String Theory
- ▶ Nonminimal Higgs Inflation and Initial Conditions in Cosmology
- ▶ Quantum General Relativity and Effective Field Theory
- ▶ Simple Supergravity

Acknowledgments The author thanks his research collaborators Hiroyuki Abe, Andrea Addazi, Yermek Aldabergenov, Shuntaro Aoki, Auttakit Chatrabhuti, Ruben Campos Delgado, Ryotaro Ishikawa, Masao Iihoshi, Vsevolod Ivanov, Daniel Frolovsky, S. James Gates Jr., Anirudh Gundhi, Sho Kaneda, Maxim Khlopov, Sergey Kruglov, Antonino Marciano, Hiroshi Nakada, Ekaterina Pozdeeva, Sultan Saburov, Alexei A. Starobinsky, Christian Steinwachs, Takahiro Terada, Shinji

Tsujikawa, Sergey Vernov, Yuki Wakimoto, and Natsuki Watanabe. Without them, this chapter would never appear.

The author is also grateful to Gia Dvali, Guillem Domènech, David I. Kaiser, Alex Kehagias, Edward W. Kolb, Florian Kühnel, Olaf Lechtenfeld, Burt Ovrut, Shi Pi, Misao Sasaki, and Ilya Shapiro for discussions and correspondence.

References

1. A. A. Starobinsky, A new type of isotropic cosmological models without singularity. Phys. Lett. B **91**(1), 99–102 (1980)
2. K.-I. Maeda, Towards the Einstein-Hilbert action via conformal transformation. Phys. Rev. D **39**, 3159 (1989)
3. V.F. Mukhanov, Gravitational instability of the Universe filled with a scalar field. JETP Lett. **41**, 493–496 (1985)
4. M. Sasaki, Large scale quantum fluctuations in the inflationary Universe. Prog. Theor. Phys. **76**, 1036 (1986)
5. BICEP, Keck Collaboration, P.A.R. Ade et al., Improved constraints on primordial gravitational waves using planck, WMAP, and BICEP/Keck observations through the 2018 observing season. Phys. Rev. Lett. **127**(15), 151301 (2021). `arXiv:2110.00483 [astro-ph.CO]`
6. M. Tristram et al., Improved limits on the tensor-to-scalar ratio using BICEP and Planck data. Phys. Rev. D **105**(8), 083524 (2022). `arXiv:2112.07961 [astro-ph.CO]`
7. I. Novikov, Y. Zeldovic, Cosmology. Ann. Rev. Astron. Astrophys. **5**, 627–649 (1967)
8. S. Hawking, Gravitationally collapsed objects of very low mass. Mon. Not. R. Astron. Soc. **152**, 75 (1971)
9. J.D. Barrow, E.J. Copeland, A.R. Liddle, The cosmology of black hole relics. Phys. Rev. D **46**, 645–657 (1992)
10. M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, Primordial black holes – perspectives in gravitational wave astronomy. Class. Quant. Grav. **35**(6), 063001 (2018). `arXiv:1801.05235 [astro-ph.CO]`
11. S.V. Ketov, M.Y. Khlopov, Cosmological probes of supersymmetric field theory models at superhigh energy scales. Symmetry **11**(4), 511 (2019)
12. B. Carr, K. Kohri, Y. Sendouda, J. Yokoyama, Constraints on primordial black holes. Rept. Prog. Phys. **84**(11), 116902 (2021). `arXiv:2002.12778 [astro-ph.CO]`
13. M. Dine, W. Fischler, The not so harmless axion. Phys. Lett. B **120**, 137–141 (1983)
14. A. Addazi, S.V. Ketov, M.Y. Khlopov, Gravitino and Polonyi production in supergravity. Eur. Phys. J. C **78**(8), 642 (2018). `arXiv:1708.05393 [hep-ph]`
15. M. Khlopov, B. Malomed, I. Zeldovich, Gravitational instability of scalar fields and formation of primordial black holes. Mon. Not. R. Astron. Soc. **215**, 575–589 (1985)
16. R. Konoplich, S. Rubin, A. Sakharov, M. Khlopov, Formation of black holes in first-order phase transitions as a cosmological test of symmetry-breaking mechanisms. Phys. Atom. Nucl. **62**, 1593–1600 (1999)
17. M. Khlopov, R. Konoplich, S. Rubin, A. Sakharov, First-order phase transitions as a source of black holes in the early universe. Grav. Cosmol. **6**, 153–156 (2000)
18. A. Addazi, A. Marcianò, R. Pasechnik, Probing trans-electroweak first order phase transitions from gravitational waves. MDPI Phys. **1**(1), 92–102 (2019). `arXiv:1811.09074 [hep-ph]`
19. A. Vilenkin, Y. Levin, A. Gruzinov, Cosmic strings and primordial black holes. JCAP **11**, 008 (2018). `arXiv:1808.00670 [astro-ph.CO]`
20. K.M. Belotsky, V.I. Dokuchaev, Y.N. Eroshenko, E.A. Esipova, M.Y. Khlopov, L.A. Khromykh, A.A. Kirillov, V.V. Nikulin, S.G. Rubin, I.V. Svadkovsky, Clusters of primordial black holes. Eur. Phys. J. C **79**(3), 246 (2019). `arXiv:1807.06590 [astro-ph.CO]`
21. J. Liu, Z.-K. Guo, R.-G. Cai, Primordial black holes from cosmic domain walls. Phys. Rev. D **101**(2), 023513 (2020). `arXiv:1908.02662 [astro-ph.CO]`

22. B.J. Carr, Primordial black holes as a probe of cosmology and high energy physics. Lect. Notes Phys. **631**, 301–321 (2003). arXiv:astro-ph/0310838
23. S. Pi, Y.-l. Zhang, Q.-G. Huang, M. Sasaki, Scalaron from R^2-gravity as a heavy field. JCAP **05**, 042 (2018). arXiv:1712.09896 [astro-ph.CO]
24. C. Germani, I. Musco, Abundance of primordial black holes depends on the shape of the inflationary power spectrum. Phys. Rev. Lett. **122**(14), 141302 (2019). arXiv:1805.04087 [astro-ph.CO]
25. J. Fumagalli, S. Renaux-Petel, J.W. Ronayne, L.T. Witkowski, Turning in the landscape: a new mechanism for generating primordial black holes. Phys. Lett. B 841 (2023) 137921. arXiv:2004.08369 [hep-th].
26. G.A. Palma, S. Sypsas, C. Zenteno, Seeding primordial black holes in multifield inflation. Phys. Rev. Lett. **125**(12), 121301 (2020). arXiv:2004.06106 [astro-ph.CO]
27. R.-G. Cai, S. Pi, M. Sasaki, Gravitational waves induced by non-Gaussian scalar perturbations. Phys. Rev. Lett. **122**(20), 201101 (2019). arXiv:1810.11000 [astro-ph.CO]
28. LIGO Scientific, Virgo Collaboration, B.P. Abbott et al., GWTC-1: a gravitational-wave transient catalog of compact binary mergers observed by LIGO and Virgo during the first and second observing runs. Phys. Rev. X **9**(3), 031040 (2019). arXiv:1811.12907 [astro-ph.HE]
29. Planck Collaboration, Y. Akrami et al., Planck 2018 results. X. Constraints on inflation. Astron. Astrophys. **641**, A10 (2020). arXiv:1807.06211 [astro-ph.CO]
30. R. Kallosh, A. Linde, D. Roest, Superconformal inflationary α-attractors. JHEP **11**, 198 (2013). arXiv:1311.0472 [hep-th]
31. J. Garcia-Bellido, E. Ruiz Morales, Primordial black holes from single field models of inflation. Phys. Dark Univ. **18**, 47–54 (2017). arXiv:1702.03901 [astro-ph.CO]
32. H. Motohashi, W. Hu, Primordial black holes and slow-roll violation. Phys. Rev. D **96**(6), 063503 (2017). arXiv:1706.06784 [astro-ph.CO]
33. I. Dalianis, A. Kehagias, G. Tringas, Primordial black holes from α-attractors. JCAP **01**, 037 (2019). arXiv:1805.09483 [astro-ph.CO]
34. D. Frolovsky, S.V. Ketov, S. Saburov, E-models of inflation and primordial black holes. Front. Phys. **10**, 1005333 (2022). arXiv:2207.11878 [astro-ph.CO]
35. M. Braglia, D.K. Hazra, F. Finelli, G.F. Smoot, A.A. Starobinsky, Generating PBHs and small-scale GWs in two-field models of inflation. JCAP **8** 001 (2020). arXiv:2005.02895 [astro-ph.CO]
36. J. Garcia-Bellido, A.D. Linde, D. Wands, Density perturbations and black hole formation in hybrid inflation. Phys. Rev. D **54**, 6040–6058 (1996). arXiv:astro-ph/9605094
37. S.V. Ketov, *Quantum Nonlinear Sigma Models: From Quantum Field Theory to Supersymmetry, Conformal Field Theory, Black Holes and Strings* (Springer, Berlin/Germany, 2000)
38. D.Z. Freedman, A. Van Proeyen, *Supergravity*, vol. 5 (Cambridge University Press, Cambridge, 2012)
39. J. Wess, J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, Princeton, NJ, USA, 1992)
40. M. Yamaguchi, Supergravity based inflation models: a review. Class. Quant. Grav. **28**, 103001 (2011). arXiv:1101.2488 [astro-ph.CO]
41. S.V. Ketov, Supergravity and early Universe: the meeting point of cosmology and high-energy physics. Int. J. Mod. Phys. A **28**, 1330021 (2013). arXiv:1201.2239 [hep-th]
42. S.V. Ketov, A.A. Starobinsky, Inflation and non-minimal scalar-curvature coupling in gravity and supergravity. JCAP **08**, 022 (2012). arXiv:1203.0805 [hep-th]
43. S.V. Ketov, T. Terada, Old-minimal supergravity models of inflation. JHEP **12**, 040 (2013). arXiv:1309.7494 [hep-th]
44. A. Addazi, S.V. Ketov, Energy conditions in Starobinsky supergravity. JCAP **03**, 061 (2017). arXiv:1701.02450 [hep-th]
45. F. Farakos, A. Kehagias, A. Riotto, On the Starobinsky model of inflation from supergravity. Nucl. Phys. B **876**, 187–200 (2013). arXiv:1307.1137 [hep-th]
46. S. Ferrara, R. Kallosh, A. Linde, M. Porrati, Minimal supergravity models of inflation. Phys. Rev. D **88**(8), 085038 (2013). arXiv:1307.7696 [hep-th]

47. Y. Aldabergenov, S.V. Ketov, SUSY breaking after inflation in supergravity with inflaton in a massive vector supermultiplet. Phys. Lett. B **761**, 115–118 (2016). arXiv:1607.05366 [hep-th]
48. Y. Aldabergenov, S.V. Ketov, Higgs mechanism and cosmological constant in $N = 1$ supergravity with inflaton in a vector multiplet. Eur. Phys. J. C **77**(4), 233 (2017). arXiv:1701.08240 [hep-th]
49. Y. Aldabergenov, S.V. Ketov, Removing instability of inflation in Polonyi–Starobinsky supergravity by adding FI term. Mod. Phys. Lett. A **91**(05), 1850032 (2018). arXiv:1711.06789 [hep-th]
50. Y. Aldabergenov, S.V. Ketov, R. Knoops, General couplings of a vector multiplet in $N = 1$ supergravity with new FI terms. Phys. Lett. B **785**, 284–287 (2018). arXiv:1806.04290 [hep-th]
51. D.V. Volkov, V.P. Akulov, Is the neutrino a goldstone particle? Phys. Lett. B **46**, 109–110 (1973)
52. Z. Komargodski, N. Seiberg, From linear SUSY to constrained superfields. JHEP **09**, 066 (2009). arXiv:0907.2441 [hep-th]
53. I. Antoniadis, E. Dudas, S. Ferrara, A. Sagnotti, The Volkov–Akulov–Starobinsky supergravity. Phys. Lett. B **733**, 32–35 (2014). arXiv:1403.3269 [hep-th]
54. Y. Aldabergenov, Volkov–Akulov–Starobinsky supergravity revisited. Eur. Phys. J. C **80**(4), 329 (2020). arXiv:2001.06617 [hep-th]
55. S. Cecotti, Higher derivative supergravity is equivalent to standard supergravity coupled to matter. 1. Phys. Lett. **B190**, 86–92 (1987)
56. Y. Aldabergenov, A. Addazi, S.V. Ketov, Primordial black holes from modified supergravity. Eur. Phys. J. C **80**(10), 917 (2020). arXiv:2006.16641 [hep-th]
57. R. Kallosh, A. Linde, Superconformal generalizations of the Starobinsky model. JCAP **1306**, 028 (2013). arXiv:1306.3214 [hep-th]
58. S.V. Ketov, On the equivalence of Starobinsky and Higgs inflationary models in gravity and supergravity. J. Phys. A **53**(8), 084001 (2020). arXiv:1911.01008 [hep-th]
59. K. Dimopoulos, Ultra slow-roll inflation demystified. Phys. Lett. B **775**, 262–265 (2017). arXiv:1707.05644 [hep-ph]
60. W.H. Press, P. Schechter, Formation of galaxies and clusters of galaxies by selfsimilar gravitational condensation. Astrophys. J. **187**, 425–438 (1974)
61. K. Inomata, M. Kawasaki, K. Mukaida, Y. Tada, T.T. Yanagida, Inflationary primordial black holes as all dark matter. Phys. Rev. D **96**(4), 043504 (2017). arXiv:1701.02544 [astro-ph.CO]
62. K. Inomata, M. Kawasaki, K. Mukaida, T.T. Yanagida, Double inflation as a single origin of primordial black holes for all dark matter and LIGO observations. Phys. Rev. D **97**(4), 043514 (2018). arXiv:1711.06129 [astro-ph.CO]
63. R. Ishikawa, S.V. Ketov, Exploring the parameter space of modified supergravity for double inflation and primordial black hole formation. Class. Quant. Grav. **39**(1), 015016 (2022). arXiv:2108.04408 [astro-ph.CO]
64. Y. Aldabergenov, A. Addazi, S.V. Ketov, Testing primordial black holes as dark matter in supergravity from gravitational waves. Phys. Lett. B **814**, 136069 (2021). arXiv:2008.10476 [hep-th]
65. J.R. Espinosa, D. Racco, A. Riotto, A cosmological signature of the SM Higgs instability: gravitational waves. JCAP **09**, 012 (2018). arXiv:1804.07732 [hep-ph]
66. N. Bartolo, V. De Luca, G. Franciolini, A. Lewis, M. Peloso, A. Riotto, Primordial black hole dark matter: LISA serendipity. Phys. Rev. Lett. **122**(21), 211301 (2019). arXiv:1810.12218 [astro-ph.CO]
67. G. Domènech, Scalar induced gravitational waves review. Universe **7**(11), 398 (2021). arXiv:2109.01398 [gr-qc]
68. J.C. Mather, D. Fixsen, R. Shafer, C. Mosier, D. Wilkinson, Calibrator design for the COBE far infrared absolute spectrophotometer (FIRAS). Astrophys. J. **512**, 511–520 (1999). arXiv:astro-ph/9810373
69. E. Thrane, J.D. Romano, Sensitivity curves for searches for gravitational-wave backgrounds. Phys. Rev. D **88**(12), 124032 (2013). arXiv:1310.5300 [astro-ph.IM]

70. LISA Collaboration, P. Amaro-Seoane et al., Laser interferometer space antenna. (2017) arXiv:1702.00786 [astro-ph.IM]
71. T.L. Smith, R. Caldwell, LISA for cosmologists: calculating the signal-to-noise ratio for stochastic and deterministic sources. Phys. Rev. D **100**(10), 104055 (2019). arXiv:1908.00546 [astro-ph.CO]
72. TianQin Collaboration, J. Luo et al., TianQin: a space-borne gravitational wave detector. Class. Quant. Grav. **33**(3), 035010 (2016). arXiv:1512.02076 [astro-ph.IM]
73. X. Gong et al., Descope of the ALIA mission. J. Phys. Conf. Ser. **610**(1), 012011 (2015). arXiv:1410.7296 [gr-qc]
74. H. Kudoh, A. Taruya, T. Hiramatsu, Y. Himemoto, Detecting a gravitational-wave background with next-generation space interferometers. Phys. Rev. D **73**, 064006 (2006). arXiv:gr-qc/0511145
75. Y. Aldabergenov, A. Addazi, S.V. Ketov, Inflation, SUSY breaking, and primordial black holes in modified supergravity coupled to chiral matter. Eur. Phys. J. C **82**(8), 681 (2022). arXiv:2206.02601 [astro-ph.CO]
76. J. Ellis, H.-J. He, Z.-Z. Xianyu, Higgs inflation, reheating and gravitino production in no-scale supersymmetric GUTs. JCAP **08**, 068 (2016). arXiv:1606.02202 [hep-ph]
77. D.Z. Freedman, Supergravity with axial gauge invariance. Phys. Rev. D **15**, 1173 (1977)
78. N. Cribiori, F. Farakos, M. Tournoy, A. van Proeyen, Fayet-Iliopoulos terms in supergravity without gauged R-symmetry. JHEP **04**, 032 (2018). arXiv:1712.08601 [hep-th]
79. D.J.H. Chung, E.W. Kolb, A.J. Long, Gravitational production of super-Hubble-mass particles: an analytic approach. JHEP **01**, 189 (2019). arXiv:1812.00211 [hep-ph]
80. A. Vilenkin, Classical and quantum cosmology of the Starobinsky inflationary model. Phys. Rev. D **32**, 2511 (1985)
81. S. Antusch, A.M. Teixeira, Towards constraints on the SUSY seesaw from flavour-dependent leptogenesis. *JCAP* **02**, 024 (2007). arXiv:hep-ph/0611232
82. K.S. Jeong, F. Takahashi, A Gravitino-rich Universe. JHEP **01**, 173 (2013). arXiv:1210.4077 [hep-ph]
83. M. Endo, F. Takahashi, T.T. Yanagida, Inflaton decay in supergravity. Phys. Rev. D **76**, 083509 (2007). arXiv:0706.0986 [hep-ph]
84. L. Kofman, A.D. Linde, A.A. Starobinsky, Reheating after inflation. Phys. Rev. Lett. **73**, 3195–3198 (1994). arXiv:hep-th/9405187
85. S.V. Ketov, S. Tsujikawa, Consistency of inflation and preheating in F(R) supergravity. Phys. Rev. D **86**, 023529 (2012). arXiv:1205.2918 [hep-th]
86. S.R. Geller, W. Qin, E. McDonough, D.I. Kaiser, Primordial black holes from multifield inflation with nonminimal couplings. Phys. Rev. D **106**(6), 063535 (2022). arXiv:2205.04471 [hep-th]
87. G.F. Giudice, A. Strumia, Probing high-scale and split supersymmetry with Higgs mass measurements. Nucl. Phys. B **858**, 63–83 (2012). arXiv:1108.6077 [hep-ph]
88. S. Dumitru, B.A. Ovrut, FIMP dark matter in heterotic M-theory. JHEP **09**, 068 (2022). arXiv:2204.13174 [hep-ph]
89. KAGRA, LIGO Scientific, VIRGO Collaboration, B. Abbott et al., Prospects for observing and localizing gravitational-wave transients with advanced LIGO, advanced Virgo and KAGRA. Living Rev. Rel. **21**(1), 3 (2018). arXiv:1304.0670 [gr-qc]
90. NANOGrav Collaboration, Z. Arzoumanian et al., The NANOGrav 12.5 yr data set: search for an isotropic stochastic gravitational-wave background. Astrophys. J. Lett. **905**(2), L34 (2020). arXiv:2009.04496 [astro-ph.HE]
91. V. De Luca, G. Franciolini, A. Riotto, NANOGrav data hints at primordial black holes as dark matter. Phys. Rev. Lett. **126**(4), 041303 (2021). arXiv:2009.08268 [astro-ph.CO]