Understanding Confinement via Holography

Transcript Excerpt

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Source Video Statement: This document is a direct transcript of an excerpt from the YouTube video `https://youtu.be/598kvRnhQqg`, titled *"[APCTP Colloquium] Strong Interactions, Confinement, and Strings | Prof. Igor Klebanov"*. The selected excerpt covers the discussion on utilizing holography and gauge/gravity duality as a toy model for understanding confinement.

[00:30:49] So one toy model which Jun Gi mentioned is the AdS/CFT correspondence. This is replacing essentially, say, $SU(N)$ Yang-Mills theory, which we believe is confining, by the maximalist supersymmetric theory. So it's then $SU(N)$ coupled to some scalar and fermion fields in the adjoint representation; it's called $\mathcal{N} = 4$ Super Yang-Mills theory.

[00:31:12] And about, like, almost 30 years ago there was great excitement that this theory appears also in the large N limit to be described by string theory on a particular 10-dimensional space. It's called $AdS_5 \times S^5$. So S^5 is just a sphere in six dimensions—a 5D sphere with five internal dimensions. But AdS_5 is a kind of hyperboloid.

[00:31:52] So this is a kind of funny picture of a hyperboloid which is tiled by cows to remind us that this is a toy model for the real world. So this theory actually is not confining. It's basically whatever gauge coupling you set, it does not run, and a miracle happens when the theory becomes strongly coupled because then the corresponding geometry becomes weakly coupled.

[00:32:20] So this is not the main point of my talk, so I will not explain it in detail, but this duality originated right around 30 years ago from studying wonderful objects in string theory called D-branes, whose relevance was realized by the late physicist Joe Polchinski. And they're basically particular defects where open strings serve as endpoints.

[00:32:47] So what happens is that there are particular three-dimensional D-branes, and when you stack N of them on top of each other, you get $SU(N)$ Super Yang-Mills theory. So this duality has been studied in, by now, tens of thousands of papers, and while it's not proven, it's been tested in great detail.

[00:33:13] But unfortunately, this in itself does not teach us about confinement because this theory is just not confining here. The gluons really are staying massless and there is no dimensional transmutation here; the beta function is exactly zero.

[00:33:32] But then there was a trick that came about very soon after the original AdS/CFT papers, which was to put these D-branes at the tip of a cone, the tip of a six-dimensional cone. This was, for example, done in my paper with Witten in 1998. And then the AdS/CFT duality basically, instead of the sphere S^5, you get the base of the cone.

[00:34:12] But very importantly, you can break supersymmetry down to $\mathcal{N} = 1$ for example, rather easily. And for $\mathcal{N} = 1$ supersymmetry, there are actually confining Yang-Mills theories. For example, the pure $\mathcal{N} = 1$ supersymmetric Yang-Mills theory is believed to be confining, and proving confinement in that theory is in itself a very worthy goal.

[00:34:40] So then one particularly nice space is something called the conifold. It's a very popular space to study in string theory because when you take a Calabi-Yau threefold cone, this one is described by a constraint on four complex variables that the sum of their squares is equal to zero. You can actually derive the metric on the base of this cone, and it looks like a circle bundle over a product of two two-spheres.

[00:35:16] So you have here two usual two-dimensional spheres and a circle angle bundled over them. The interesting thing is that this no longer has a topology of a five-sphere. It has a topology of a product of a two-sphere times a three-sphere ($S^2 \times S^3$), which will be very important actually for deforming this model to obtain a confining theory.

[00:35:43] So with Witten, we did a bunch of work to determine what gauge theory this is. It turns out to be $SU(N) \times SU(N)$ with some bifundamental chiral superfields. And this theory is just an $\mathcal{N} = 1$ superconformal field theory which in itself has been studied in detail, and it's dual to $AdS_5 \times T^{1,1}$ ($T^{1,1}$ is the name for the base of the cone).

[00:36:18] So then after a series of papers, actually it was figured out how to obtain dimensional transmutation in this gauge-gravity duality. Basically, what one needed to do is to, instead of only D3-branes, we add M D5-branes and wrap them over the two-sphere near the tip of the cone. That gives a nonconformal theory $SU(N+M) \times SU(N)$ which exhibits some complicated pattern of running.

[00:36:46] But then, with Strassler, we found a very explicit metric which is simple enough; it's a 10-dimensional metric which turns out to be dual to a confining gauge theory. We called it the warp-deformed conifold, because h is the warp factor. This ds_6^2 is the metric on the deformed conifold, and the deformation is the fact that you generate this ϵ^2 on the right side. It turns out that without this deformation, the metric is not nice; it has a naked singularity.

[00:37:32] Okay, so this is basically what happens. Like, the pure AdS metric as a function of r is just L^4/r^4 and it blows up at small r. The metric that we found earlier with a naked singularity that ignores the deformation behaves nicely for a while—so here the warp factor has this extra log and it behaves nicely for a while—but then it turns over and hits zero.

[00:38:02] And when h is zero, it means that this whole transverse space shrinks to zero, and there is a big trouble. But then, thanks to the appearance of this ϵ, which is a blown-up three-cycle, you actually get a perfectly nice monotonic warp factor.

[00:38:30] And this string that sits at this minimal radial coordinate at the bottom of this warp throat is the confining flux tube. It has a finite tension which is set by ϵ. So this is a kind of physicist's proof of confinement in this model, because unless you put in this ϵ deformation, the space is singular, but to make it non-singular and smooth, we needed to put in confinement basically.

[00:39:08] So this model has also been studied a lot. I mean, nowhere near as much as AdS_5, but it does describe confinement. For example, you can use it to compute quark-antiquark potential; it's a rather simple exercise. It's a semiclassical calculation and you see that it starts out as approximately Coulombic and then becomes linear.

[00:39:35] Qualitatively it's very similar to the $Q\bar{Q}$ potential obtained from lattice gauge theory. You see the interpolation between the Coulombic part and the linear part. So at least we get something qualitatively similar out of the space.

[00:39:52] But this theory, although it runs logarithmically, it's not really asymptotically free, so we would like to do better, but it's proven to be very hard. So a dream which I think still lives on is to do something similar for real large N QCD, not this somewhat contrived theory with two gauge groups.