

Asymptotic of Young tableaux of bounded height

(Václav Kotěšovec, published Sep 12 2013)

In the [OEIS](#) I found several sequences "Number of standard Young tableaux of n cells and height $\leq k$ ". I computed recurrences and asymptotic expansions for all $k \leq 12$. For general case is my conjecture following:

$$A(n, k) \sim \frac{k^n}{\pi^{k/2}} * \left(\frac{k}{n}\right)^{\frac{k(k-1)}{4}} * \prod_{j=1}^k \Gamma\left(\frac{j}{2}\right)$$

where Γ is the [Gamma function](#).

k	OEIS sequence	$A(n, k)$ is asymptotic to
1		1
2	A001405	$\frac{2^{n+\frac{1}{2}}}{\sqrt{\pi n}}$
3	A001006	$\frac{3^{n+\frac{3}{2}}}{2\sqrt{\pi} n^{3/2}}$
4	A005817	$\frac{2^{2n+5}}{\pi n^3}$
5	A049401	$\frac{3 * 5^{n+5}}{8\pi n^5}$
6	A007579	$\frac{2^{n+\frac{11}{2}} 3^{n+\frac{17}{2}}}{\pi^{3/2} n^{15/2}}$
7	A007578	$\frac{45 * 7^{n+\frac{21}{2}}}{32 \pi^{3/2} n^{21/2}}$
8	A007580	$\frac{135 * 2^{3n+38}}{\pi^2 n^{14}}$
9	A212915	$\frac{175 * 9^{n+20}}{256 \pi^2 n^{18}}$
10	A212916	$\frac{1701 * 2^{n+\frac{35}{2}} 5^{n+\frac{49}{2}}}{\pi^{5/2} n^{45/2}}$
11	A229053	$\frac{40186125 * 11^{n+\frac{55}{2}}}{1024 \pi^{5/2} n^{55/2}}$
12	A229068	$\frac{30625 * 2^{2n+59} 3^{n+42}}{\pi^3 n^{33}}$

Table in Mathematica format:

```
Table[k^n/Pi^(k/2) * (k/n)^(k*(k-1)/4) * Product[Gamma[j/2], {j, 1, k}], {k, 1, 12}]
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Conjecture holds for all $k \leq 12$.

Product of Gamma function can be expressed also with help of [BarnesG function](#).

$$\prod_{j=1}^k \Gamma\left(\frac{j}{2}\right) = \frac{G\left(\frac{k}{2} + 1\right) G\left(\frac{k}{2} + \frac{1}{2}\right)}{G\left(\frac{1}{2}\right)}$$

See also sequences [A057863](#) and [A000178](#).

References:

- [1] [OEIS](#) - The On-Line Encyclopedia of Integer Sequences
- [2] Sequence [A182172](#) by Alois P. Heinz, 2012
- [3] Special programs under Mathematica by Václav Kotěšovec (2012): function "plinrec" search in the integer sequences linear recurrences with polynomial coefficients.
- [4] [Young Tableaux](#) (definition in MathWorld)

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