

Stringy black holes in five dimensions

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Overview

- General comments on black holes in string theory.
- Symmetry and anomaly arguments to explain *why* micro/macro entropies agree, including corrections.
- Explicit results for 5D black holes / strings in R^2 corrected sugra. Singularity resolution, corrections to entropy.

Black holes in string theory: where are we?

- > 10 yrs since Strominger/Vafa. Many examples, intricate checks, but really just a few basic themes.
- Two classes:
 - (1) exact entropy match (e.g. D1-D5-P)
 - (2) matching up to numerical factors (e.g. D3-branes, $3/4$)
- What's the distinction?
- Agreement $S_{BH} = S_{CFT}$ can seem "miraculous". Can we see agreement without computing both sides case by case?

The importance of AdS₃

- Cases with exact agreements have near horizon AdS₃ factor, and 1 + 1 CFT dual. Agreement follows from general properties.
- Micro side: key tool is Cardy formula

$$S = 2\pi \sqrt{\frac{c_L}{6} P_L} + 2\pi \sqrt{\frac{c_R}{6} P_R} + \dots$$

Just need $c_{L,R}$ to get large $P_{L,R}$ behavior. $c_{L,R}$ determined in vacuum.

- Gravity side: BH = BTZ = locally AdS₃. Same logic again leads to Cardy formula. $c_{L,R}$ again determined in vacuum.
- Can show that $c_{L,R}$ agree between two sides without having to compute them explicitly.

Anomaly inflow

- Use symmetries and anomalies to show that central charges must agree. Harvey/Minasian/Moore; P.K./Larsen
- Micro side: given D=1+1 CFT on brane worldvolume, susy relates c to R-symmetry anomaly

$$\partial_\mu j_R^\mu = k \epsilon^{\mu\nu} F_{\mu\nu} , \quad c = 6k$$

- Consider brane in ambient space with external gauge fields. Theory on brane is anomalous. Anomaly cancelled by inflow from bulk due to Chern-Simons terms $S_{CS} = kA \wedge dA$.
- Brane central charge determined from bulk CS term. Topological.
- Analogous argument determines c of non-susy half of CFT from gravitational anomaly.

Anomaly inflow cont.

- On gravity side assume existence of solution with same near horizon symmetries \Rightarrow AdS₃.
- In asymptotic region solution looks same as before. Same CS term and same inflow, now into AdS₃ region.
- AdS₃ theory not gauge invariant by itself due to inflow across boundary. Same inflow as before. Central charge must be the same.
- Only needed to use topological terms. Robust.
- In general, central charges have classical piece plus corrections. Argument shows that they agree exactly.
- In terms of entropy, implies matching of infinite series of corrections

$$S = \frac{1}{4}A(Q, P) + \sum c_n \frac{1}{Q^n}$$

Can be computed explicitly given full set of CS terms. Also works for non-BPS / near-extremal holes.

Other corrections

- Also have corrections to Cardy formula

$$S = S_{cardy} + \sum \left(\frac{1}{P_{L,R}} \right)^n$$

Not universal: depend on details of CFT. On gravity side requires knowledge of full action and non-perturbative effects.

- OSV conjecture

$$Z_{BH}(p^I, \phi^I) = \sum_{q_I} \Omega(p^I, q_I) e^{-\phi^I q_I}$$

$$Z_{BH} = |Z_{top}|^2$$

- Would like to Laplace transform to extract degeneracies. Various ambiguities need to be resolved.

Higher derivative corrections in 4D de Wit et. al.

- Type II on $CY_3 \Rightarrow \mathcal{N} = 2$ sugra.
- Gauge fields F^I with spectrum of electric and magnetic charges (q_I, p^I) corresponding to wrapped branes. Complex moduli Y^I obeying special geometry.
- Two-deriv. theory. Consider D4-D0 black hole with charges p^I and q_0 .

$$S = 2\pi \sqrt{\frac{1}{6} c_{IJK} p^I p^J p^K q_0}$$

- Infinite series of higher deriv. corrections. Subset captured by modified prepotential

$$F(Y^I) \rightarrow F(Y^I, W^2) = \sum_g F_g(Y^I) (W^2)^g$$

F_g = genus g top. string free energy.

- Gives corrections

$$L = \sum_g F_g(Y^I) (C_{\mu\nu\alpha\beta})^2 (F^I)^{2g-2} + \dots$$

- Susy manifest. Look for BPS solutions. Find entropy from attractor mechanism and Wald formula.
- For large CY_3 have

$$F(Y^I, W^2) = \frac{c_{IJK} Y^I Y^J Y^K}{Y^0} + \frac{c_{2I} Y^I}{Y^0} W^2 + \dots$$

- leads to BPS entropy.

$$S = 2\pi \sqrt{\frac{1}{6} (c_{IJK} p^I p^J p^K + c_{2I} p^I) q_0}$$

Agrees with microscopic counting from wrapped M5-branes [MSW](#).

- Agreement to all orders in c_2 . Surprising from 4D viewpoint.
- For non-BPS hole find disagreement at leading order in c_2 . Top. string misses some terms at 4-deriv order and beyond [Sahoo/Sen](#)
- Small BH: $CY_3 = K3 \times T^2$. Wrap p^1 D4-branes on $K3$ with q_0 D0-branes.

$$S = 2\pi \sqrt{\frac{24}{6} p^1 q_0}$$

Corresponds to $c_L = 24p^1$. Indeed this case is dual to p^1 wrapped heterotic strings carrying q_0 units of momentum.

5D perspective

- Many aspects clearer upon lifting to M-theory on $CY_3 \times S^1$. 4D black hole becomes a black string in 5D with $D = 1 + 1$ CFT dual.
- Also have access to 5D black holes and black rings.
- Charges correspond to

$$p^I = \text{M5 - branes on } I\text{th } 4 - \text{ cycle}$$

$$q_I = \text{M2 - branes on } I\text{th } 2 - \text{ cycle}$$

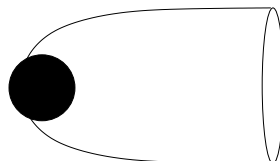
$$p^0 = \text{KK - monopole}$$

$$q_0 = \text{momentum along intersection}$$

- Taking $p^0 = 0$ but $p^I \neq 0$ gives magnetic black string with near horizon geometry $AdS_3 \times S^2 \times CY_3$.
- Setting $p^I = 0$ and $q_I \neq 0$ gives electric black hole with near horizon $AdS_2 \times S^3 \times CY_3$. No known micro description in general.

4D/5D connection

- Taub-NUT serves as an interpolator between 4D and 5D black holes:



- Take M-direction to be Taub-NUT fibre. Spin of 5D black hole becomes D0-charge. M2-charge become D2-charges.
- Leads to conjecture [Gaiotto/Strominger/Yin](#)

$$S_{5D}(q_I, J) = S_{4D}(q_I, q_0 = J)$$

- With OSV gives relation between 5D entropy and top. string.
- Simple relation holds at lowest order, but breaks down with higher derivs since Taub-NUT carries delocalized charges.

$$S \sim c_{2I} A^I \text{Tr}(R \wedge R) \quad \Rightarrow \quad q_I^{4D} = q_I^{5D} + \frac{c_{2I}}{24}$$

5D sugra

- R^2 sugra not as well studied in 5D.
- Of special interest is mixed gauge-gravity Chern-Simons term

$$S = \int c_{2I} A^I \wedge \text{Tr}(R \wedge R)$$

Coefficient known from M5-brane anomaly cancellation [Duff/Liu/Minasian](#)

- From previous arguments, susy completion should yield entropy corrections. In magnetic string case, just assuming existence of susy completion, plus near horizon AdS_3 , is enough to determine entropy.
- Need full susy action to find explicit solution and find corrected entropy for 5D black hole.

5D R^2 sugra

- Action found recently by [Hanaki, Ohashi, Tachikawa](#) . Start from superconformal theory and gauge fix to Poincare.
- Weyl multiplet

$$e_{\mu}^a, \psi_{\mu}, V_{\mu}, b_{\mu}, v^{ab}, \chi, D$$

- Vector multiplet

$$A_{\mu}^I, M^I, \Omega^I, Y^I$$

- Susy variations

$$\delta\psi_{\mu} = \left(\mathcal{D}_{\mu} + \frac{1}{2}v^{ab}\gamma_{\mu ab} - \frac{1}{3}\gamma_{\mu}\gamma \cdot v \right) \epsilon,$$

$$\delta\Omega^I = \left(-\frac{1}{4}\gamma \cdot F^I - \frac{1}{2}\gamma^a\partial_a M^I - \frac{1}{3}M^I\gamma \cdot v \right) \epsilon,$$

$$\delta\chi = \left(D - 2\gamma^c\gamma^{ab}\mathcal{D}_a v_{bc} - 2\gamma^a\epsilon_{abcde}v^{bc}v^{de} + \frac{4}{3}(\gamma \cdot v)^2 \right) \epsilon$$

5D action

- 2-deriv action

$$\begin{aligned}
 \frac{1}{2}\mathcal{L}_0 &= \partial^\alpha \mathcal{A}_i^\alpha \partial_\alpha \mathcal{A}_\alpha^i + \mathcal{A}^2 \left(\frac{1}{8}D - \frac{3}{16}R - \frac{1}{4}v^2 \right) \\
 &+ \mathcal{N} \left(\frac{1}{4}D + \frac{1}{8}R + \frac{3}{2}v^2 \right) + \mathcal{N}_I v^{ab} F_{ab}^I \\
 &+ \mathcal{N}_{IJ} \left(\frac{1}{8}F_{ab}^I F^{Jab} + \frac{1}{4}\partial_a M^I \partial^a M^J \right) + \frac{1}{48}c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde} .
 \end{aligned}$$

Integrating out auxiliary D , v^{ab} gives standard action

$$\begin{aligned}
 \mathcal{L}_0 &= -[-R + G_{IJ} \partial_a M^I \partial^a M^J + \frac{1}{2}G_{IJ} F_{ab}^I F^{Jab}] \\
 &\quad - \frac{1}{24}c_{IJK} A_a^I F_{bc}^J F_{de}^K \epsilon^{abcde} , \\
 G_{IJ} &= \frac{1}{2} (\mathcal{N}_I \mathcal{N}_J - \mathcal{N}_{IJ}) \\
 \mathcal{N} &= \frac{1}{6} c_{IJK} M^I M^J M^K , \quad \mathcal{N}_I = \partial_I \mathcal{N}
 \end{aligned}$$

- D equation gives constraint $\mathcal{N} = 1$.

4-deriv action (Hanaki, Ohashi, Tachikawa)

$$\begin{aligned}
 \mathcal{L}_1 = & \frac{c_{2I}}{24} \left(\frac{1}{16} \epsilon_{abcde} A^{Ia} C^{bcfg} C^{de}_{fg} + \frac{1}{8} M^I C^{abcd} C_{abcd} + \frac{1}{12} M^I D^2 \right. \\
 & + \frac{1}{6} F^{Iab} v_{ab} D + \frac{1}{3} M^I C_{abcd} v^{ab} v^{cd} + \frac{1}{2} F^{Iab} C_{abcd} v^{cd} + \frac{8}{3} M^I v_{ab} \hat{D}^b \hat{D}_c v^{ac} \\
 & + \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_a v_{bc} + \frac{4}{3} M^I \hat{D}^a v^{bc} \hat{D}_b v_{ca} - \frac{2}{3} M^I \epsilon_{abcde} v^{ab} v^{cd} \hat{D}_f v^{ef} \\
 & + \frac{2}{3} F^{Iab} \epsilon_{abcde} v^{cd} \hat{D}_f v^{ef} + F^{Iab} \epsilon_{abcde} v^c \hat{D}^d v^{ef} \\
 & \left. - \frac{4}{3} F^{Iab} v_{ac} v^{cd} v_{db} - \frac{1}{3} F^{Iab} v_{ab} v^2 + 4M^I v_{ab} v^{bc} v_{cd} v^{da} - M^I (v^2)^2 \right)
 \end{aligned}$$

Believed to be complete set of 4-deriv terms.

- D equation now modifies special geometry constraint: $\mathcal{N} \neq 1$.
- Solving equations of motion directly obviously very difficult. But constructing BPS solutions tractable since susy variations are uncorrected.

BPS solutions

- First look for spherically symmetric solutions.
- Form Killing vector from Killing spinor

$$K^\mu = \bar{\epsilon} \gamma^\mu \epsilon$$

$$K^\mu K_\mu \geq 0 \Rightarrow K^\mu \text{ timelike or null } (K^0 \neq 0).$$

- Leads to two types of solutions

$$K^\mu = \begin{cases} \text{timelike} & \text{electric 5D BH } AdS_2 \times S^3 \\ \text{null} & \text{magnetic 5D black string } AdS_3 \times S^2 \end{cases}$$

- Ansatz

$$\begin{aligned} \text{electric : } ds^2 &= e^{4U_1} dt^2 - e^{-2U_2} dx^i dx^i \\ \text{magnetic : } ds^2 &= e^{2U_1} (dt^2 - dx_4^2) - e^{-4U_2} dx^i dx^i \end{aligned}$$

Spherical symmetry then fixes all fields up to radial dependence.

BPS equations

- Imposing susy fixes v , F^I and D . e.g., magnetic case

$$\begin{aligned}v &\sim e^{-2U} \partial_r U \epsilon_2 \\ F^I &\sim r^2 \partial_r (M^I e^{-2U}) \epsilon_2 \\ D &\sim e^{4U} \nabla^2 U\end{aligned}$$

- Maxwell equation yields M^I in terms of harmonic functions. e.g., magnetic case

$$M^I e^{-2U} = H^I = 1 + \frac{p^I}{2r}$$

- Final ingredient is constraint from D equation

$$0 = e^{-6U} + \frac{1}{6} c_{IJK} H^I H^J H^K + c_{2I} (\nabla H^I \nabla U + 2M^I \nabla^2 U)$$

Nonlinear ODE requiring numerical treatment.

Properties of solutions: asymptotic behavior

- Asymptotic behavior is a bit subtle. In 2-deriv theory solution approaches unique vacuum

$$e^{-2U} \sim 1 + \frac{c}{r^{(2)}} , \quad r \rightarrow \infty$$

But four derivative theory has spurious solution for small fluctuations around Minkowski

$$\delta U \sim A \sin(kr) , \quad k^{-1} \sim l_{Pl}$$

- Solutions are unphysical: don't show up in string spectrum. Can be removed by field redefinition:

$$U \rightarrow \tilde{U} = U + \frac{1}{k^2} U''$$

- Expect generic localized solution to match onto finite A oscillatory solution. Indeed happens. But happens far away, so not really worrisome. Same as in 4D story. [Sen; Hubeny/Maloney/Rangamani](#)

Properties of solutions: near horizon

- Structure of near horizon region understood from enhanced susy

$$\delta(\text{fermion}) = (\text{stuff} = 0)\epsilon$$

Forces geometry to be $\text{AdS}_{2,3} \times S^{3,2}$ and moduli to take attractor values.

- Find

$$\text{magnetic :} \quad \ell_A = 2\ell_S = \left(\frac{1}{6} c_{IJK} p^I p^J p^K + \frac{1}{12} c_{2I} p^I \right)^{1/3}$$

$$\text{electric :} \quad \ell_S = 2\ell_A = \left(\frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K - \frac{1}{12} c_{2I} p^I \right)^{1/3}$$

with

$$\frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I + \frac{1}{8} c_{2I}$$

- Exhibits resolution of naked singularity. Setting $cp^3 = 0$ but $c_{2I} p^I \neq 0$ gives smooth would-be singular solution.

Entropy

- In presence of higher derivatives area law is replaced by Wald entropy in general

$$S \sim \int_{hor} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu} \epsilon_{\alpha\beta}$$

Can be tedious to evaluate.

- Major simplifications for black holes with near horizon AdS. Can use extremization approach.
- Magnetic AdS₃ case: have predictions for corrections to $c_{L,R}$ from anomaly argument. Can now check explicitly.
- c-extremization: given any action with near horizon AdS₃ × S², define

$$c = -6\ell_A^3 \ell_S^2 \mathcal{L}$$

Regard as function of unknown $\ell_{A,S}$ and other fields, but at fixed charge. Equations of motion reduce to extremizing c .

c-extremization cont.

- At extremum

$$c = \frac{1}{2}(c_L + c_R)$$

Show by relating CFT trace anomaly $T_{\mu}^{\mu} = \frac{c}{24}R$ to divergent part of bulk action.

- Evaluation of on-shell action indeed reproduces result predicted from anomalies.
- Corrected entropy now follows form Cardy formula
- Further prediction: any additional higher derivative terms will have to cancel out in c-function.

Entropy of electric solutions

- Given AdS_2 , most convenient to use Sen's entropy function. Extremize

$$S = \pi \ell_A^2 \ell_S^3 \left(F_{tr}^I \frac{\partial \mathcal{L}}{\partial F_{tr}^I} - \mathcal{L} \right)$$

- 2-deriv result

$$S = 2\pi \ell_S^3 = 2\pi \left(\frac{1}{6} c_{IJK} \hat{M}^I \hat{M}^J \hat{M}^K \right)$$

with $\frac{1}{2} c_{IJK} \hat{M}^J \hat{M}^K = q_I$.

- Modifications due to 4-deriv terms are surprisingly simple. Geometrically,

$$S = 2\pi \mathcal{N} \ell_s^3, \quad \mathcal{N} = 1 + \frac{c_{2I} M^I}{12\ell_s^2}$$

In 2-deriv theory $\mathcal{N} = 1$ corresponds to volume of CY_3 . Assuming same here (ambiguous), still have $S = \frac{A}{4}$.

- In terms of charges, only change is shift in charges

$$q_I \rightarrow q_I + \frac{1}{8} c_{2I}$$

- Strictly valid to first order in c_2 .

Comparison to previous results

- Vafa computed microscopic corrections to entropy of electric BH on elliptically fibred CY_3 . Gets same leading order correction as we find

$$q \cdot q \rightarrow q \cdot q + \frac{1}{4} c_2 \cdot q$$

- Huang et. al. recently numerically computed asymptotic M2 degeneracy from GV invariants. Found agreement with $\frac{1}{8} c_{2I}$ shift.
- Top-string and 4D \rightarrow 5D lift gives same entropy expressed in terms of \hat{M}^I , but with a different charge shift [Strominger et. al](#)

$$q_I \rightarrow q_I + \frac{1}{6} c_{2I}$$

- To resolve this we constructed black holes on Taub-NUT. Only change is in Gauss' law

$$\nabla \cdot E^I = \frac{c_{2I}}{8 \cdot 24} (R_{ijkl} R^{ijkl})_{TN}$$

Delocalized charge $q_I = \frac{c_{2I}}{24}$ accounts for different shifts.

Spinning black holes

- Can find higher derivative BMPV solution. Attractor moduli now depend on angular momentum ($\hat{j} = J/\ell_s^3$)

$$\frac{1}{2}c_{IJK}\hat{M}^J\hat{M}^K = q_I + c_{2I}\left(\frac{1}{8} - \frac{1}{6}\hat{j}^2\right)$$

and the entropy is

$$S = 2\pi\sqrt{1 - \hat{j}^2}\left(\frac{1}{6}c_{IJK}\hat{M}^I\hat{M}^J\hat{M}^K + \frac{1}{6}\hat{j}^2c_{2I}\hat{M}^I\right)$$

- Naive result using 4D/5D now off by

$$q_0 = J + \frac{1}{12}c_{2I}M^I\frac{e^0}{(1 + (e^0)^2)}$$

Black rings

- Full R^2 black ring geometry not found yet.
- Entropy can be computed in terms of near horizon data. Near horizon geometry governed by same attractor as magnetic string.

$$S = 2\pi \frac{(p^3 + \frac{1}{6} c_{2I} p^I)}{e^0}$$

- Can also write this as

$$S = (2 - \mathcal{N}) \frac{A}{4G_5}$$

- e^0 is a near horizon quantity. Need to trade it for ring charges.

Conclusions

- 5D higher derivative corrections under good control. Should help in pinning down microscopic description of 5D black holes on generic CY_3 , and of black rings.
- To do: full black ring solutions, non-BPS solutions, ...