Lagrangians with (2,0) Supersymmetry

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**Introduction**

Understanding M5-branes is a major challenge. It is a defining issue in M-Theory and it is important for QFT more generally.

We don’t expect to have a traditional Lagrangian description:

- Modular anomalies = violation of diffeomorphism
- ‘Tachikawa’ test (twisted compactification of $SU(2n)$ theory leads to $SO(2n + 1)$)
- No marginal deformations or even discrete limiting theories
- Reduction to 5D gives $g^2 \propto R$ not $g^2 \propto 1/R$
- No family of interacting renormalizable Lagrangians with Energy bounded from below.
- Difficulties with self-duality of the three-form and two-form gauge theory

But we are here because we like a challenge (or are stubborn)
On the other hand without some kind of Lagrangian or Hamiltonian construction it is difficult to see how to find a workable formulation of the $(2, 0)$ theory or understand its robust relations to lower dimensional gauge theories.

Several proposals involve Lagrangians/Hamiltonians e.g.:

- DLCQ based on Instanton Quantum Mechanics
- Deconstruction based on 4D $\mathcal{N} = 2$ SCFT Lagrangians
- 5D super-Yang-Mills as $(2, 0)$ on $S^1$ of any radius.
- Various novel Lagrangians in 5D and/or 6D e.g. [Saemann’s talk]

Maybe we have to learn how to piece these together to get a complete picture.
In this talk we will construct Lagrangians in six-dimensions with $(2, 0)$ supersymmetry.

A recent general approach due to Sen offers a new window to self-duality and diffeomorphisms which we will look through.

We will largely put aside all the no-go statements above and see how far we get. If only to test the boundaries. As with M2-branes one may hope that two M5-branes are more amenable than three or more (‘Tachikawa Test’).

The hope is that we will find interesting things and novel mathematics that are relevant to M-theory.
An Abelian (2,0) Action

In flat Minkowski space the action is

$$S = \int \left( \frac{1}{2} dB \wedge \ast_{\eta} dB - 2H \wedge dB - \frac{1}{2} \partial_{\mu} X^{I} \partial^{\mu} X^{I} + \frac{i}{2} \bar{\Psi} \Gamma_{\mu} \partial_{\mu} \Psi \right)$$

- $H = \ast_{\eta} H$
- $H$ equation of motion sets $dB = \ast_{\eta} dB$
- $B$ equation of motion sets $d(H + \frac{1}{2} dB + \frac{1}{2} \ast_{\eta} dB) = 0$
- and hence $dH = 0$

So two closed self-dual three-forms: $H$ and

$$H_{(s)} = \frac{1}{2} (dB + \ast_{\eta} dB) + H$$

Key idea [Sen]: ensure $H_{(s)}$ decouples.
We want to keep $B$ decoupled, even from the metric:

$$S = \int \left( \frac{1}{2} dB \wedge \star_\eta dB - 2H \wedge dB + H \wedge \tilde{\mathcal{M}}(H) \\
- \frac{1}{2} dx^I \wedge \star_g dx^I + \frac{i}{2} \bar{\Psi} \Gamma_\mu dx^\mu \wedge \star_g \nabla \Psi - \frac{1}{5} RX^I X^I \right)$$

Now we find

$$d \left( H - \tilde{\mathcal{M}}(H) \right) = 0$$

and we define $\tilde{\mathcal{M}}$ so that

$$H(g) = H - \tilde{\mathcal{M}}(H) = \star_g H(g)$$

$H(g)$ plays the role of the physical $\star_g$-self-dual three-form. One can also introduce sources for $H(g)$, keeping $H(s)$ decoupled - we will not consider this here but it can be included.
Geometrical Properties

Thus the metric dependence of the forms is contained in $\mathcal{N}$

To define $\mathcal{N}$ we have the following requirements

- $\mathcal{N}(H) = - \star_\eta \mathcal{N}(H)$
- $H_1 \wedge \mathcal{N}(H_2) = H_2 \wedge \mathcal{N}(H_1)$
- $\mathcal{N}(Q) = 0$ for $\star_\eta Q = -Q$
- if $H = \star_\eta H$ then $H - \mathcal{N}(H) = \star_g (H - \mathcal{N}(H))$

To construct $\mathcal{N}$ we consider a basis of three-forms

$$\{\omega^A_+, \omega^-_A\}$$

$$\star_\eta \omega^A_+ = \omega^A_+, \quad \star_\eta \omega^-_A = -\omega^-_A$$

and hence we have, for some $\mathcal{N}^{AB}$,

$$\mathcal{N}(\omega^-_A) = 0, \quad \mathcal{N}(\omega^A_+) = \mathcal{N}^{AB} \omega^-_B$$
Next we consider a basis of $*_g$-self-dual three-forms:

$$\varphi^A = \mathcal{N}^A_B \omega^B + \mathcal{K}^{AB} \omega_{-B} \quad \varphi^A = *_g \varphi^A$$

and define

$$\mathcal{M}^{AB} = - (\mathcal{N}^{-1})^A_C \tilde{\mathcal{K}}^{CB}$$

so that if $H = H_A \omega^A_+$ then

$$H(g) = H - \mathcal{M}(H)$$

$$= H_A \omega^A_+ - H_A \mathcal{M}^{AB} \omega_{-B}$$

$$= H_A \omega^A_+ + H_A (\mathcal{N}^{-1})^A_C \tilde{\mathcal{K}}^{CB} \omega_{-B}$$

$$= H_A (\mathcal{N}^{-1})^A_B \varphi^B$$

$$= *_g H(g)$$

Thus we have a map

$$m(H) = H - \mathcal{M}(H) \quad m(H) = H(g)$$

from $*_\eta$-self-dual forms to $*_g$-self-dual forms
There is a novel invariance under diffeomorphisms.

Consider $x^\mu \rightarrow x^\mu + \xi^\mu (x)$. Some calculations show that

$$\delta_\xi \tilde{\mathcal{M}} (H) = \frac{1}{2} (1 - * \eta) \left[ \xi (H) - \xi (\tilde{\mathcal{M}} (H)) + \tilde{\mathcal{M}} (\xi (H)) - \tilde{\mathcal{M}} (\xi (\tilde{\mathcal{M}} (H))) \right]$$

where $\xi (H) = \frac{1}{2} \nabla_\mu \xi^\lambda H_{\lambda \nu \rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$.

So $\tilde{\mathcal{M}}$ transforms a bit like a connection: if it vanishes in one frame it need not vanish in others.

In terms of the map $m$ we can write this as

$$\delta_\xi \tilde{\mathcal{M}} (H) = \frac{1}{2} (1 - * \eta) m^{-1} (\xi (m(H)))$$
How do $B$ and $H$ transform: They look like differential forms but they don’t transform as differential forms: pseudo-forms.

Keep $H_{(s)}$ invariant:

$$\delta_{\xi} H = -\frac{1}{2} d\delta_{\xi} B - \frac{1}{2} \ast \eta \ d\delta_{\xi} B$$

Invariance of the action, up to a total derivative, determines:

$$\delta_{\xi} B = i_{\xi} H(g) = \frac{1}{2} \xi^{\lambda} H(g)_{\lambda \mu \nu} dx^\mu \wedge dx^\nu$$

and hence

$$\delta_{\xi} H(g) = \delta_{\xi} H - \mathcal{M}(\delta_{\xi} H) - \delta_{\xi} \mathcal{M}(H)$$

$$= -\xi(H(g)) - i_{\xi} H(g) - \frac{1}{2} (1 + \ast \eta) i_{\xi} dH(g) + \mathcal{M}(i_{\xi} dH(g))$$

We only recover the usual tensor transformation of $H(g)$ on on-shell.
We can now compute the energy momentum tensor defined as the response to a variation in the metric:

$$
T_{\mu \nu} = - \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{\mu \nu}}
$$

$$
= -4 H \wedge \frac{\delta \tilde{M}}{\delta g^{\mu \nu}} (H)
$$

$$
= H^{(g)}_{\mu \lambda \rho} g^{\lambda \sigma} g^{\rho \tau} H^{(g)}_{\nu \sigma \tau}
$$

and the conserved energy (á la Noether):

$$
E = \int d^5 x \left( -\frac{1}{2} H^{(s)}_{0i j} H^{(s)}_{0i j} - \sqrt{-g} g^{0 \mu} T_{\mu 0} \right)
$$

Notably $H_{(s)}$ has the wrong sign
We can also compute the Hamiltonian. To cut a longer story short (see [Sen]):

The fields are $B_{ij}$, $B_{0i}$ and $H_{ijk}$. Only $B_{ij}$ has a conjugate momentum $\Pi_{ij}$. The others give constraints;

\[
\partial_i \Pi_{ij} = 0 \quad \text{imposed by } B_{0i}
\]
\[
\frac{1}{2} \varepsilon_{ijklm} \Pi_{lm} = H^{(g)}_{ijk}(H) + \frac{3}{2} \partial[i B_{jk}] \quad \text{imposed by imposed by } H_{ijk}
\]

So we use the second equation to solve for $H_{ijk}$ and work with

\[
\Pi_{ij}^\pm = \frac{1}{2} \left( \Pi_{ij} \pm \frac{1}{4} \varepsilon_{ijklm} \partial_k B_{lm} \right)
\]

that satisfy

\[
\{\Pi_{ij}^\pm (x), \Pi_{kl}^\pm (y)\} = \pm \frac{1}{4} \varepsilon_{ijklm} \frac{\partial}{\partial x_m} \delta(x - y)
\]
\[
\{\Pi_{ij}^+(x), \Pi_{kl}^-(y)\} = 0
\]
In particular we find

\[ \Pi^+_{ij} = -\frac{1}{2} H^{(s)}_{0ij} \]

\[ \Pi^-_{ij} = \frac{1}{2 \cdot 3!} \varepsilon_{ijklm} H^{(g)}_{klm} = \frac{1}{2} \sqrt{-g} H^{0ij}_{(g)} \]

The hamiltonian is (at least if \( g_{0i} = 0 \))

\[ H = H_+ + H_- \]

\[ H_- = \int d^5 x -2\Pi^+_{ij} \Pi^+_{ij} \]

\[ H_- = \int d^5 x \ 4\Pi^-_{ij} \partial_i B_{0j} - \frac{2}{\sqrt{-g}} g_{00} g_{ik} g_{jl} \Pi^-_{ij} \Pi^-_{kl} \]

which agrees with the energy \( E \) that we computed above.

In the end all the expressions for \( T_{\mu\nu} \) and \( \Pi^\pm_{ij} \) are what we would expect from an action of the form \( -dB \wedge \ast_g dB \).
Sources

To include a source $J$ we take

$$S_H = \int \left( \frac{1}{2} dB \wedge *_\eta dB - 2H \wedge dB \\ + (H + J_+) \wedge \tilde{M}(H + J_+) + 2H \wedge J_- - J_- \wedge J_+ \right)$$

and so

$$dH^J_{(g)} = dJ$$

but still

$$dH_{(s)} = d \left( \frac{1}{2} dB + \frac{1}{2} *_\eta dB + H \right) = 0$$

We find similar expressions for diffeomorphisms, hamiltonian etc. as those above with

$$H_{(g)} \to H^J_{(g)} = H_{(g)} + J_+ - \tilde{M}(J)$$
Supersymmetry

Recall our action is

\[ S = \int \left( \frac{1}{2} dB \wedge \star_\eta dB - 2H \wedge dB + H \wedge \tilde{\mathcal{M}}(H) \right. \]

\[ \left. - \frac{1}{2} dX^I \wedge \star_g dX^I + \frac{i}{2} \bar{\Psi} \Gamma_\mu dx^\mu \wedge \star_g \nabla \Psi - \frac{1}{5} RX^I X^I \right) \]

This is invariant under \((\nabla_\mu \epsilon = \frac{1}{6} \Gamma_\mu \Gamma^\nu \nabla_\nu \epsilon)\)

\[ \delta X^I = i \epsilon \Gamma^I \Psi \]

\[ \delta B_{\mu \nu} = -i \epsilon \Gamma_{\mu \nu} \Psi \]

\[ \delta H_{\mu \nu \lambda} = \frac{3i}{2} \epsilon \Gamma_{[\mu \nu} \nabla_\lambda] \Psi + \frac{3i}{2} \epsilon \mu \nu \lambda \rho \sigma \tau \eta^\rho \alpha \eta^\sigma \beta \eta^\tau \gamma \epsilon \Gamma_{\alpha \beta} \nabla \gamma \Psi \]

\[ - \frac{1}{4} \nabla^\rho \epsilon \Gamma^\rho \Gamma_{\mu \nu \lambda} \Psi - \frac{i}{4 \cdot 3!} \epsilon \mu \nu \lambda \rho \sigma \tau \eta^\rho \alpha \eta^\sigma \beta \eta^\tau \gamma \nabla^\omega \epsilon \Gamma^\omega \Gamma_{\alpha \beta \gamma} \Psi \]

\[ \delta \Psi = \Gamma^\mu \Gamma^I \partial_\mu X^I \epsilon + \frac{1}{3!} \Gamma_{\mu \nu \lambda} (H - \tilde{\mathcal{M}}(H))^{\mu \nu \lambda} \epsilon \]

In this case \(H_{(s)} = \frac{1}{2} dB + \frac{1}{2} \star_\eta dB + H\) is a singlet
Example: Reduction on $S^1$

The simplest case to consider is $x^5 \sim x^5 + l$ and

$$g = \begin{pmatrix} \eta_5 & 0 \\ 0 & R^2 \end{pmatrix}$$

(N.B. $R$ is dimensionless). A basis of three-forms is

$$\omega_+^A = \Omega^A \wedge dx^5 + \star_5 \Omega^A$$

$$\omega_-^A = \Omega^A \wedge dx^5 - \star_5 \Omega^A$$

and $\star_g$ self-dual three-forms are given by:

$$\varphi^A = \Omega^A \wedge dx^5 + \frac{1}{R} \star_5 \Omega^A$$

$$\varphi^A = \frac{R + 1}{2R} \omega_+^A + \frac{R - 1}{2R} \omega_-^A$$

so $\tilde{\mathcal{M}}^{AB} = -(R - 1)/(R + 1) \delta^{AB}$. 
Thus we find \((a, b, = 1, 2, 3, 4)\)

\[
H_- = \int d^5x\left(\frac{2}{R} \Pi^-_{ab} \Pi^-_{ab} + 4R \Pi^-_{a5} \Pi^-_{a5} + 4\Pi^-_{ab} \partial_a B_b + \Pi^-_{a5}(\partial_a B_{05} - \partial_5 B_{0a})\right)
\]

Let us set \(\partial_5 = 0\) and solve the \(B_{a5}\) constraint by

\[
\Pi^-_{ab} = -\frac{1}{4l} \varepsilon_{abcd} \partial_c A_d
\]

and hence \(\Pi^-_{a5}\) is the conjugate momentum to \(A_a\):

\[
\{A_a(x), \Pi^-_{b5}(y)\} = \delta_{ab}\delta_4(x - y)
\]

Thus

\[
\partial_0 A_a = \{A_a, H\} = 8Rl\Pi^-_{a5} + l \partial_a B_{05}
\]

and hence we arrive at 5D Maxwell:

\[
L_- = \partial_0 A_a \Pi^-_{a5} - H_-
\]

\[
= \frac{1}{8Rl} \int d^4x \left((\partial_0 A_a - l \partial_a B_{05})^2 - (\partial_a A_b - \partial_b A_a)^2\right)
\]
Example: M5 on a Riemann Surface

Subject to suitable boundary conditions, corresponding to intersecting branes, a single M5-brane wraps the Seiberg-Witten curve [Witten] of the associated gauge theory:

\[
s = X^6 + iX^{10}, \quad z = x^4 + ix^5 \quad s = s(z; u)
\]

where the \( u \) are moduli.

The induced metric on the M5-brane is

\[
g = \begin{pmatrix}
\eta_4 & 0 & 0 \\
0 & 0 & (1 + \partial s \bar{\partial} \bar{s})/2 \\
0 & (1 + \partial s \bar{\partial} \bar{s})/2 & 0
\end{pmatrix}
\]
The low energy dynamics for the scalars of the M5-brane agrees with the SW effective action \((m = 0, 1, 2, 3)\) [Howe,NL,West]:

\[
S_{s} = \int d^{4}x \int d^{2}z \, \partial_{m}s \partial^{m}\bar{s} \\
= \int d^{4}x \int d^{2}z \frac{\partial \bar{s}}{\partial u} \frac{\partial s}{\partial \bar{u}} \partial_{m}u \partial^{m}\bar{u} \\
= \int d^{4}x \text{Im}(\tau \partial_{m}a \partial^{m}\bar{a})
\]

Here \(\lambda = (\partial s/\partial u)dz\) is the holomorphic one-form and

\[
\frac{da}{du} = \int_{A} \lambda \quad \quad \frac{da^{D}}{du} = \int_{B} \lambda \quad \quad \tau = \frac{da^{D}}{da}
\]

However obtaining the correct vector equations knowing only the equations of motion was quite involved [NL,West].
Now we can reduce the form part of action on the Riemann surface $\Sigma$ defined by $s(z)$

We perform a standard KK reduction ansatz

\[ H = \mathcal{F} \wedge \vartheta + \bar{\mathcal{F}} \wedge \bar{\vartheta} \]
\[ B = C \wedge \vartheta + \bar{C} \wedge \bar{\vartheta} \]

where $\mathcal{F} = i \star_4 \mathcal{F}$ and $\vartheta = (du/da)\lambda$.

Since $\Sigma$ is non-compact the 0-form and 2-form terms in the ansatz give divergent contributions and must be dropped.

For an $H$ of this type $\star_9 H = H$ and hence $\tilde{\mathcal{M}}(H) = 0$. 
We find the four-dimensional form part of the action is

\[
S_H = \int \left( (\tau - \bar{\tau}) \left( dC \wedge i \star d\bar{C} + 2\mathcal{F} \wedge d\bar{C} - 2\bar{\mathcal{F}} \wedge dC \right) \\
+ \frac{d\tau}{da} \left( -i \star d\bar{C} \wedge C \wedge da + 2\bar{\mathcal{F}} \wedge C \wedge da \right) \\
+ \frac{d\bar{\tau}}{d\bar{a}} \left( i \star dC \wedge \bar{C} \wedge d\bar{a} + 2\mathcal{F} \wedge \bar{C} \wedge d\bar{a} \right) \right)
\]

The equations of motion are

\[
0 = (\tau - \bar{\tau})dC + d\tau \wedge C - i \star \left( (\tau - \bar{\tau})dC + d\tau \wedge C \right)
\]

\[
0 = d \left( (\tau - \bar{\tau})i \star dC + 2(\tau - \bar{\tau})\mathcal{F}_\beta + i \star d\tau \wedge C \right) \\
+ d\bar{\tau} \wedge i \star dC + 2d\bar{\tau} \wedge \mathcal{F}
\]

We can substitute the first equation into the second to find

\[
d \left( (\tau - \bar{\tau})\mathcal{F} \right) + d\bar{\tau} \wedge \left( \mathcal{F} + \frac{1}{2} \left( i \star dC - dC \right) \right) = 0
\]

This agrees with Seiberg-Witten if \( \mathcal{F} = -\frac{1}{2} d\bar{C} - \frac{i}{2} \star d\bar{C} \).
A Non-abelian (2,0) Action

Next we want to construct a non-abelian (2,0) action.

We can construct a free theory by including a gauge field along with a Lagrange multiplier term that imposes flatness:

\[ S = \int \left[ \frac{1}{4} \langle DB \wedge \star DB \rangle - \langle H \wedge DB \rangle - \frac{1}{2} \langle D_\mu X^I D^\mu X^I \rangle + \frac{i}{2} \langle \bar{\Psi}_\Gamma^\mu D_\mu \Psi \rangle + (\tilde{F} \wedge \tilde{W}) \right] \]

where \( D = d - \tilde{A} \) and \( \tilde{F} = d\tilde{A} - \tilde{A} \wedge \tilde{A} \) with

\[ \delta \tilde{A}_\mu = 0 \]

\[ \delta \tilde{W}_{\mu \nu \lambda \rho} (\cdot) = 3i\bar{\epsilon}_\Gamma [\mu \nu [B_{\lambda \rho} , \Psi , \cdot ] + i\bar{\epsilon}_\Gamma \mu \nu \lambda \rho \Gamma^I [X^I , \Psi , \cdot ] \]
Here the matter fields take values in a vector space \( \mathcal{V} \) and the gauge field in a Lie-algebra \( \mathcal{G} \) with a representation \( \tilde{T}^r \) on \( \mathcal{V} \).

- \( \mathcal{V} \) has an inner-product \( \langle \cdot, \cdot \rangle \)
- \( \mathcal{G} \) has an inner-product \( (\cdot, \cdot) \)

This leads to a three-algebra structure [Figueroa-O’Farrill, de Medeiros]:

\[
\left[ \cdot, \cdot, \cdot \right]: \mathcal{V} \otimes \mathcal{V} \otimes \mathcal{V} \to \mathcal{V}
\]

\[
[X, Y, Z] = \sum_r \langle X, \tilde{T}^r(Y) \rangle \tilde{T}_r(Z)
\]

which implies the combatability conditions

\[
(\tilde{T}, [U, V, \cdot]) = \langle \tilde{T}(U), V \rangle = -\langle U, \tilde{T}(V) \rangle
\]

\[
[U, V, [X, Y, Z]] = [[U, V, X], Y, Z] + [X, [U, V, Y], Z] + [X, Y, [U, V, Z]]
\]
In order to construct interactions we consider the \((2, 0)\) system of [NL, Papageogakis] and introduce a non-dynamical vector field \(Y^\mu\) with scaling dimension \(-1\)

\[
D_\mu Y^\nu = 0 \quad [Y^\mu, D_\mu (\cdot), \cdot] = 0 \quad [Y^\mu, Y^\nu, \cdot] = 0
\]

Here the three-algebra is totally anti-symmetric and so we take \(\mathcal{V} = \mathbb{R}^4\) leading to the gauge algebra \(\mathfrak{su}(2) \oplus \mathfrak{su}(2)\).

\[
0 = D^2 X^I - \frac{i}{2} [Y^\sigma, \bar{\Psi}, \Gamma_\sigma \Gamma^I \Psi] + [Y^\sigma, X^J, [Y^\sigma, X^J, X^I]]
\]

\[
0 = D[\lambda H_{\mu \nu \rho}] + \frac{1}{4} \varepsilon_{\mu \nu \lambda \rho \sigma \tau} [Y^\sigma, X^I, D^\tau X^I] + \frac{i}{8} \varepsilon_{\mu \nu \lambda \rho \sigma \tau} [Y^\sigma, \bar{\Psi}, \Gamma^\tau \Psi]
\]

\[
0 = \Gamma^\rho D_\rho \Psi + \Gamma_\rho \Gamma^I [Y^\rho, X^I, \Psi]
\]

\[
0 = \tilde{F}_{\mu \nu}(\cdot) - [Y^\lambda, H_{\mu \nu \lambda}, \cdot]
\]
Now the flatness condition on $\tilde{F}$ is replaced by $\tilde{F} \sim [Y, H, ]$

So we adjust the Lagrange multiplier term to

$$\mathcal{L}_{\tilde{W}} = \langle H \wedge \tilde{W}(Y) \rangle + (\tilde{F} \wedge \tilde{W})$$

where $\tilde{W}(Y) = \frac{1}{3!} W_{\mu \nu \lambda} (Y^\rho) dx^\mu \wedge dx^\nu \wedge dx^\lambda$ and make a guess

$$S_{\text{guess}} = \int \left[ \frac{1}{4} \langle DB \wedge * DB \rangle - \langle H \wedge (DB - \tilde{W}(Y)) \rangle + (\tilde{F} \wedge \tilde{W}) ight.$$

$$- \frac{1}{2} \langle D_\mu X^I D^\mu X^I \rangle - \frac{1}{4} \langle [Y^\mu, X^I, X^J] [Y^\mu, X^I, X^J] \rangle$$

$$+ \frac{i}{2} \langle \bar{\Psi} \Gamma^\mu D_\mu \Psi \rangle + \frac{i}{2} \langle \bar{\Psi} \Gamma^I [Y^\mu, X^I, \Psi] \rangle \right]$$

The matter terms clearly reproduce their correct equations.

This has introduced a source for $H$ of the form $\tilde{W}(Y)$. 
Alas this isn’t quite right:

- self-dual part of $\tilde{W}(Y)$ is non-zero.
- $D^2 \sim \tilde{F} \neq 0$

After some more guess work we find [NL]

\[
S = \int \left[ \frac{1}{4} \langle DB \wedge \ast DB \rangle + \frac{1}{6} \langle DB \wedge DB \rangle + \frac{1}{4} \langle \tilde{W}(Y) \wedge \ast \tilde{W}(Y) \rangle \
- \langle H \wedge (DB - \tilde{W}(Y)) \rangle - \frac{1}{2} \langle (DB - \ast DB) \wedge \tilde{W}(Y) \rangle + (\tilde{F} \wedge \tilde{W}) \
- \frac{1}{2} \langle D_\mu X^I D^\mu X^I \rangle - \frac{1}{4} \langle [Y^\mu, X^I, X^J][Y_\mu, X^I, X^J] \rangle \
+ \frac{i}{2} \langle \bar{\Psi} \Gamma^\mu D_\mu \Psi \rangle + \frac{i}{2} \langle \bar{\Psi} \Gamma_\mu \Gamma^I [Y^\mu, X^I, \Psi] \rangle \right]
\]

Here $D_\mu = \partial_\mu - \tilde{A}_\mu(\cdot)$ with

$$\tilde{A}_\mu(\cdot) = \tilde{A}_\mu(\cdot) - \frac{1}{2} [B_{\mu \nu}, Y^\nu, \cdot]$$
This reproduces all the equations of motion of the \((2, 0)\) system.

In particular \(B\) and \(\tilde{W}\) can be removed from the equations for the remaining fields.

It is invariant under \((2, 0)\) supersymmetry:

\[
\delta X^I = i \bar{\epsilon} \Gamma^I \Psi \\
\delta B_{\mu \nu} = -i \bar{\epsilon} \Gamma_{\mu \nu} \Psi \\
\delta \Psi = \Gamma^\mu \Gamma^I D_\mu X^I \epsilon + \frac{1}{2 \cdot 3!} H_{\mu \nu \lambda} \Gamma^{\mu \nu \lambda} \epsilon - \frac{1}{2} \Gamma_{\mu} \Gamma^{I J} [Y^\mu, X^I, X^J] \epsilon \\
\delta H_{\mu \nu \lambda} = \frac{3}{2} (1 + \star_\eta) i \bar{\epsilon} \Gamma_{[\mu \nu} D_{\lambda]} \Psi - i \bar{\epsilon} \Gamma_\rho \Gamma_{\mu \nu \lambda} \Gamma^I [Y^\rho, X^I, \Psi] \\
\delta \tilde{A}_\mu (\cdot) = i \bar{\epsilon} \Gamma_{\mu \nu} [Y^\nu, \Psi, \cdot] \\
\delta \tilde{W}_{\mu \nu \lambda \rho} (\cdot) = 3 i \bar{\epsilon} \Gamma_{[\mu \nu} [B_{\lambda \rho}], \Psi, \cdot] + i \bar{\epsilon} \Gamma_{\mu \nu \lambda \rho} \Gamma^I [X^I, \Psi, \cdot]
\]
Note that this is a reducible representation of supersymmetry:

\[
\mathcal{H}_{(s)} = \frac{1}{2}(DB - \tilde{W}(Y)) + \frac{1}{2} \star (DB - \tilde{W}(Y)) + H
\]

\[
\tilde{A}_{(s)\mu}(\cdot) = \tilde{A}_\mu(\cdot) - [B_{\mu\nu}, Y^\nu, \cdot]
\]

are singlets.

The interacting part is five-dimensional: \([Y^\mu D_\mu, \cdot, \cdot] = 0\).

Coupling constant

\[
g^2 = R_5 \left( \frac{\langle Y_\mu, Y^\mu \rangle}{R_5^2} \right)
\]

Depending on the choice of \(Y\) one finds different five-dimensional theories.
• Y spacelike: (4+1)-dimensional super-Yang-Mills
• Y timelike: (5+0)-dimensional super-Yang-Mills
• Y null: novel non-Lorentzian theory \((G = \star G)\):

\[
S = \frac{1}{g^2} \text{tr} \int d^4x \, dx^0 \left( \frac{1}{2} F_{0i} F_{0i} + \frac{1}{2} F_{ij} G_{ij} - \frac{1}{2} (D_i X^I) (D_i X^I) \\
- \frac{i}{2} \bar{\Psi} \Gamma \cdots D_0 \Psi + \frac{i}{2} \bar{\Psi} \Gamma_i D_i \Psi + \frac{1}{2} \bar{\Psi} \Gamma \cdots \Gamma^I [X^I, \Psi] \right)
\]

16 supersymmetries and 8 superconformal supersymmetries [NL, Owen][NL, Mouland].

Path integral reduces to instanton QM[Mouland]

An \(\Omega\)-deformed version has an \(SU(3, 1)\) symmetry, 8 supersymmetries, 16 superconformal symmetries and an \(AdS_7\) dual [NL, Lipstein,Richmond] [NL, Lipstein,Mouland,Richmond]
Conclusions

In this talk we adapted Sen’s prescript for self-dual forms to the (2, 0) theory.

- Obtained a more geometrical formulation
- Obelian theory reproduces the dynamics of a single M5
- Presented an interacting non-abelian version which describes two M5-branes on an $S^1$
Comments

Interesting new geometrical structure for self-dual forms: $\tilde{\mathcal{M}}$. Diffeomorphisms are enabled unusually.

Extend to DBI-like M5’s: Make $H - \tilde{\mathcal{M}}(H)$ non-linear? [Perry, Schwarz], [Howe, Sezgin West], [Pasti, Sorokin, Tonin]

Extend to $(1, 0)$ theories [Samtleben, Sezgin, Wimmer]

Is the appearance of a second connection $\tilde{D}_\mu = D_\mu - \frac{1}{2} [B_{\mu \nu}, Y^\nu, \cdot ]$ suggestive of some 2-form structure?

Better understanding of modular anomalies vs diffeomorphisms?
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