Lagrangians with (2,0) Supersymmetry

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Introduction

Understanding M5-branes is a major challenge. It is a defining issue in M-Theory and it is important for QFT more generally.

We don't expect to have a traditional Lagrangian description:

- Modular anomalies = violation of diffeomorphism
- 'Tachikawa' test (twisted compactification of $SU(2n)$ theory leads to $SO(2n+1)$)
- No marginal deformations or even discrete limiting theories
- Reduction to 5D gives $g^2 \propto R$ not $g^2 \propto 1/R$
- No family of interacting renormalizable Lagrangians with Energy bounded from below.
- Difficulties with self-duality of the three-form and two-form gauge theory

But we are here because we like a challenge (or are stubborn)

On the other hand without some kind of Lagrangian or Hamiltonian construction it is difficult to see how to find a workable formulation of the $(2,0)$ theory or understand its robust relations to lower dimensional gauge theories.

Several proposals involve Lagrangians/Hamiltonians *e.g.*:

- DLCQ based on Instanton Quantum Mechanics
- Deconstruction based on 4D $\mathcal{N}=2$ SCFT Lagrangians
- 5D super-Yang-Mills as $(2,0)$ on S^1 of any radius.
- Various novel Lagrangians in 5D and/or 6D *e.g.* [Saemann's talk]

Maybe we have to learn how to piece these together to get a complete picture.

In this talk we will construct Lagrangians in six-dimensions with $(2,0)$ supersymmetry.

A recent general approach due to Sen offers a new window to self-duality and diffeomorphims which we will look through.

We will largely put aside all the no-go statements above and see how far we get. If only to test the boundaries. As with M2-branes one may hope that two M5-branes are more amenable than three or more ('Tachikawa Test').

The hope is that we will find interesting things and novel mathematics that are relevant to M-theory.

An Abelian (2,0) Action

In flat Minkowski space the action is

$$S = \int \left(\frac{1}{2} dB \wedge \star_\eta dB - 2H \wedge dB - \frac{1}{2}\partial_\mu X^I \partial^\mu X^I + \frac{i}{2}\bar{\Psi}\Gamma^\mu \partial_\mu \Psi \right)$$

- $H = \star_\eta H$
- H equation of motion sets $dB = \star_\eta dB$
- B equation of motion sets $d(H + \frac{1}{2}dB + \frac{1}{2}\star_\eta dB) = 0$
- and hence $dH = 0$

So two closed self-dual three-forms: H and

$$H_{(s)} = \frac{1}{2}(dB + \star_\eta dB) + H$$

Key idea [Sen]: ensure $H_{(s)}$ decouples.

We want to keep B decoupled, even from the metric:

$$S = \int \Big(\frac{1}{2} dB \wedge \star_\eta dB - 2H \wedge dB + H \wedge \tilde{\mathcal{M}}(H)$$
$$- \frac{1}{2} dX^I \wedge \star_g dX^I + \frac{i}{2}\bar{\Psi}\Gamma_\mu dx^\mu \wedge \star_g \nabla\Psi - \frac{1}{5} R X^I X^I \Big)$$

Now we find

$$d\Big(H - \tilde{\mathcal{M}}(H)\Big) = 0$$

and we define $\tilde{\mathcal{M}}$ so that

$$H_{(g)} = H - \tilde{\mathcal{M}}(H) = \star_g H_{(g)}$$

$H_{(g)}$ plays the role of the physical $\star_g$-self-dual three-form. One can also introduce sources for $H_{(g)}$, keeping $H_{(s)}$ decoupled - we will not consider this here but it can be included.

Geometrical Properties

Thus the metric dependence of the forms is contained in $\tilde{\mathcal{M}}$

To define $\tilde{\mathcal{M}}$ we have the following requirements

- $\tilde{\mathcal{M}}(H) = - \star_\eta \tilde{\mathcal{M}}(H)$
- $H_1 \wedge \tilde{\mathcal{M}}(H_2) = H_2 \wedge \tilde{\mathcal{M}}(H_1)$
- $\tilde{\mathcal{M}}(Q) = 0$ for $\star_\eta Q = -Q$
- if $H = \star_\eta H$ then $H - \tilde{\mathcal{M}}(H) = \star_g (H - \tilde{\mathcal{M}}(H))$

To construct $\tilde{\mathcal{M}}$ we consider a basis of three-forms

$$\{\omega_+^A\,, \omega_{-A}\}$$

$$\star_\eta \omega_+^A = \omega_+^A, \qquad \star_\eta \omega_{-A} = -\omega_{-A}$$

and hence we have, for some $\tilde{\mathcal{M}}^{AB}$,

$$\tilde{\mathcal{M}}(\omega_{-A}) = 0\,, \qquad \tilde{\mathcal{M}}(\omega_+^A) = \tilde{\mathcal{M}}^{AB}\omega_{-B}$$

Next we consider a basis of $\star_g$-self-dual three-forms:

$$\varphi^A = \mathcal{N}^A{}_B \omega_+^B + \mathcal{K}^{AB}\omega_{-B} \qquad \varphi^A = \star_g \varphi^A$$

and define

$$\tilde{\mathcal{M}}^{AB} = -(\tilde{\mathcal{N}}^{-1})^A{}_C \tilde{\mathcal{K}}^{CB}$$

so that if $H = H_A \omega_+^A$ then

$$\begin{aligned} H_{(g)} &= H - \tilde{\mathcal{M}}(H) \\ &= H_A \omega_+^A - H_A \tilde{\mathcal{M}}^{AB}\omega_{-B} \\ &= H_A \omega_+^A + H_A (\tilde{\mathcal{N}}^{-1})^A{}_C \tilde{\mathcal{K}}^{CB}\omega_{-B} \\ &= H_A (\tilde{\mathcal{N}}^{-1})^A{}_B \varphi^B \\ &= \star_g H_{(g)} \end{aligned}$$

Thus we have a map

$$\mathfrak{m}(H) = H - \tilde{\mathcal{M}}(H) \qquad \mathfrak{m}(H) = H_{(g)}$$

from $\star_\eta$-self-dual forms to $\star_g$-self-dual forms

There is a novel invariance under diffeomorphisms.

Consider $x^\mu \to x^\mu + \xi^\mu(x)$. Some calculations show that

$$\delta_\xi \tilde{\mathcal{M}}(H) = \frac{1}{2}(1-\star_\eta)\left[\xi(H) - \xi(\tilde{\mathcal{M}}(H)) + \tilde{\mathcal{M}}(\xi(H)) - \tilde{\mathcal{M}}(\xi(\tilde{\mathcal{M}}(H)))\right]$$

where $\xi(H) = \frac{1}{2}\nabla_\mu \xi^\lambda H_{\lambda\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$.

So $\tilde{\mathcal{M}}$ transforms a bit like a connection: if it vanishes in one frame it need not vanish in others.

In terms of the map $\mathfrak{m}$ we can write this as

$$\delta_\xi \tilde{\mathcal{M}}(H) = \frac{1}{2}(1-\star_\eta)\mathfrak{m}^{-1}(\xi(\mathfrak{m}(H))$$

How do B and H transform: They look like differential forms but they don't transform as differential forms: pseudo-forms.

Keep $H_{(s)}$ invariant:

$$\delta_\xi H = -\frac{1}{2} d\delta_\xi B - \frac{1}{2} \star_\eta d\delta_\xi B$$

Invariance of the action, up to a total derivative, determines:

$$\delta_\xi B = i_\xi H_{(g)} = \frac{1}{2} \xi^\lambda H_{(g)\lambda\mu\nu} dx^\mu \wedge dx^\nu$$

and hence

$$\begin{aligned} \delta_\xi H_{(g)} &= \delta_\xi H - \tilde{\mathcal{M}}(\delta_\xi H) - \delta_\xi \tilde{\mathcal{M}}(H) \\ &= -\xi(H_{(g)}) - i_\xi H_{(g)} - \frac{1}{2}(1 + \star_\eta) i_\xi dH_{(g)} + \tilde{\mathcal{M}}\left(i_\xi dH_{(g)}\right) \end{aligned}$$

We only recover the usual tensor transformation of $H_{(g)}$ on on-shell.

We can now compute the energy momentum tensor defined as the response to a variation in the metric:

$$\begin{aligned} T_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} \\ &= -4H \wedge \frac{\delta\tilde{\mathcal{M}}}{\delta g^{\mu\nu}}(H) \\ &= H^{(g)}_{\mu\lambda\rho} g^{\lambda\sigma} g^{\rho\tau} H^{(g)}_{\nu\sigma\tau} \end{aligned}$$

and the conserved energy (á la Noether):

$$E = \int d^5x \left(-\frac{1}{2} H^{(s)}_{0ij} H^{(s)}_{0ij} - \sqrt{-g} g^{0\mu} T_{\mu 0} \right)$$

Notably $H_{(s)}$ has the wrong sign

We can also compute the Hamiltonian.To cut a longer story short (see [Sen]):

The fields are B_{ij}, B_{0i} and H_{ijk}. Only B_{ij} has a conjugate momentum Π_{ij}. The others give constraints;

$$\partial_i \Pi_{ij} = 0 \qquad \text{imposed by } B_{0i}$$

$$\frac{1}{2}\varepsilon_{ijklm}\Pi_{lm} = H^{(g)}_{ijk}(H) + \frac{3}{2}\partial_{[i}B_{jk]} \qquad \text{imposed by imposed by } H_{ijk}$$

So we use the second equation to solve for H_{ijk} and work with

$$\Pi^{\pm}_{ij} = \frac{1}{2}\left(\Pi_{ij} \pm \frac{1}{4}\varepsilon_{ijklm}\partial_k B_{lm}\right)$$

that satisfy

$$\{\Pi^{\pm}_{ij}(x), \Pi^{\pm}_{kl}(y)\} = \pm\frac{1}{4}\varepsilon_{ijklm}\frac{\partial}{\partial x^m}\delta(x-y)$$

$$\{\Pi^{+}_{ij}(x), \Pi^{-}_{kl}(y)\} = 0$$

In particular we find

$$\Pi^+_{ij} = -\frac{1}{2} H^{(s)}_{0ij}$$
$$\Pi^-_{ij} = \frac{1}{2 \cdot 3!} \varepsilon_{ijklm} H^{(g)}_{klm} = \frac{1}{2} \sqrt{-g} H^{0ij}_{(g)}$$

The hamiltonian is (at least if $g_{0i} = 0$)

$$H = H_+ + H_-$$
$$H_- = \int d^5x {-2\Pi^+_{ij}\Pi^+_{ij}}$$
$$H_- = \int d^5x \; 4\Pi^-_{ij}\partial_i B_{0j} - \frac{2}{\sqrt{-g}} g_{00} g_{ik} g_{jl} \Pi^-_{ij}\Pi^-_{kl}$$

which agrees with the energy E that we computed above.

In the end all the expressions for $T_{\mu\nu}$ and $\Pi^{\pm}_{ij}$ are what we would expect from an action of the form $-dB \wedge \star_g dB$.

Sources

To include a source J we take

$$S_H = \int \Big(\frac{1}{2} dB \wedge \star_\eta dB - 2H \wedge dB$$
$$+ (H + J_+) \wedge \tilde{\mathcal{M}}(H + J_+) + 2H \wedge J_- - J_- \wedge J_+\Big)$$

and so

$$dH^J_{(g)} = dJ$$

but still

$$dH_{(s)} = d\left(\frac{1}{2} dB + \frac{1}{2} \star_\eta dB + H\right) = 0$$

We find similar expressions for diffeomorphisms, hamiltonian *etc.* as those above with

$$H_{(g)} \to H^J_{(g)} = H_{(g)} + J_+ - \tilde{\mathcal{M}}(J)$$

Supersymmetry

Recall our action is

$$S = \int \Big(\frac{1}{2}dB \wedge \star_\eta dB - 2H \wedge dB + H \wedge \tilde{\mathcal{M}}(H) \\ - \frac{1}{2}dX^I \wedge \star_g dX^I + \frac{i}{2}\bar{\Psi}\Gamma_\mu dx^\mu \wedge \star_g \nabla\Psi - \frac{1}{5}RX^I X^I \Big)$$

This is invariant under ($\nabla_\mu \epsilon = \frac{1}{6}\Gamma_\mu \Gamma^\nu \nabla_\nu \epsilon$)

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I \Psi \\ \delta B_{\mu\nu} &= -i\bar{\epsilon}\Gamma_{\mu\nu}\Psi \\ \delta H_{\mu\nu\lambda} &= \frac{3i}{2}\bar{\epsilon}\Gamma_{[\mu\nu}\nabla_{\lambda]}\Psi + \frac{3i}{2\cdot 3!}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}\eta^{\rho\alpha}\eta^{\sigma\beta}\eta^{\tau\gamma}\bar{\epsilon}\Gamma_{\alpha\beta}\nabla_\gamma\Psi \\ &\quad - \frac{i}{4}\nabla^\rho\bar{\epsilon}\Gamma_\rho\Gamma_{\mu\nu\lambda}\Psi - \frac{i}{4\cdot 3!}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}\eta^{\rho\alpha}\eta^{\sigma\beta}\eta^{\tau\gamma}\nabla^\omega\bar{\epsilon}\Gamma_\omega\Gamma_{\alpha\beta\gamma}\Psi \\ \delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\Gamma_{\mu\nu\lambda}(H - \tilde{\mathcal{M}}(H))^{\mu\nu\lambda}\epsilon \end{aligned}$$

In this case $H_{(s)} = \frac{1}{2}dB + \frac{1}{2}\star_\eta dB + H$ is a singlet

Example: Reduction on S^1

The simplest case to consider is $x^5 \sim x^5 + l$ and

$$g = \begin{pmatrix} \eta_5 & 0 \\ 0 & R^2 \end{pmatrix}$$

(**N.B.** R is dimensionless). A basis of three-forms is

$$\omega_+^A = \Omega^A \wedge dx^5 + \star_5 \Omega^A$$
$$\omega_{-A} = \Omega^A \wedge dx^5 - \star_5 \Omega^A$$

and $\star_g$ self-dual three-forms are given by:

$$\varphi^A = \Omega^A \wedge dx^5 + \frac{1}{R} \star_5 \Omega^A$$
$$= \frac{R+1}{2R} \omega_+^A + \frac{R-1}{2R} \omega_{-A}$$

so $\tilde{\mathcal{M}}^{AB} = -(R-1)/(R+1)\delta^{AB}$.

Thus we find $(a, b, = 1, 2, 3, 4)$

$$H_- = \int d^5x \Big(\frac{2}{R}\Pi^-_{ab}\Pi^-_{ab} + 4R\Pi^-_{a5}\Pi^-_{a5} + 4\Pi^-_{ab}\partial_a B_{b5} + \Pi^-_{a5}(\partial_a B_{05} - \partial_5 B_{0a}) \Big)$$

Let us set $\partial_5 = 0$ and solve the B_{a5} constraint by

$$\Pi^-_{ab} = -\frac{1}{4l}\varepsilon_{abcd}\partial_c A_d$$

and hence Π^-_{a5} is the conjugate momentum to A_a:

$$\{A_a(x), \Pi^-_{b5}(y)\} = \delta_{ab}\delta_4(x-y)$$

Thus

$$\partial_0 A_a = \{A_a, H\} = 8Rl\Pi^-_{a5} + l\partial_a B_{05}$$

and hence we arrive at 5D Maxwell:

$$L_- = \partial_0 A_a \Pi^-_{a5} - H_- = \frac{1}{8Rl}\int d^4x \left((\partial_0 A_a - l\partial_a B_{05})^2 - (\partial_a A_b - \partial_b A_a)^2 \right)$$

Example: M5 on a Riemann Surface

Subject to suitable boundary conditions, corresponding to intersecting branes, a single M5-brane wraps the Seiberg-Witten curve [Witten] of the associated gauge theory:

$$s = X^6 + iX^{10}\,, \qquad z = x^4 + ix^5 \qquad s = s(z;u)$$

where the u are moduli.

The induced metric on the M5-brane is

$$g = \begin{pmatrix} \eta_4 & 0 & 0 \\ 0 & 0 & (1+\partial s\bar{\partial}\bar{s})/2 \\ 0 & (1+\partial s\bar{\partial}\bar{s})/2 & 0 \end{pmatrix}$$

The low energy dynamics for the scalars of the M5-brane agrees with the SW effective action ($m = 0, 1, 2, 3$) [Howe,NL,West]:

$$\begin{aligned} S_s &= \int d^4x \int d^2z \; \partial_m s \partial^m \bar{s} \\ &= \int d^4x \int d^2z \frac{\partial s}{\partial u} \frac{\partial \bar{s}}{\partial \bar{u}} \partial_m u \partial^m \bar{u} \\ &= \int d^4x \mathrm{Im}(\tau \partial_m a \partial^m \bar{a}) \end{aligned}$$

Here $\lambda = (\partial s/\partial u)dz$ is the holomorphic one-form and

$$\frac{da}{du} = \oint_A \lambda \qquad \frac{da^D}{du} = \oint_B \lambda \qquad \tau = \frac{da^D}{da}$$

However obtaining the correct vector equations knowing only the equations of motion was quite involved [NL,West].

Now we can reduce the form part of action on the Riemann surface Σ defined by $s(z)$

We perform a standard KK reduction ansatz

$$H = \mathcal{F} \wedge \vartheta + \bar{\mathcal{F}} \wedge \bar{\vartheta}$$
$$B = C \wedge \vartheta + \bar{C} \wedge \bar{\vartheta}$$

where $\mathcal{F} = i \star_4 \mathcal{F}$ and $\vartheta = (du/da)\lambda$.

Since Σ is non-compact the 0-form and 2-form terms in the ansatz give divergent contributions and must be dropped.

For an H of this type $\star_g H = H$ and hence $\tilde{\mathcal{M}}(H) = 0$.

We find the four-dimensional form part of the action is

$$S_H = \int \Big((\tau - \bar{\tau}) \left(dC \wedge i \star d\bar{C} + 2\mathcal{F} \wedge d\bar{C} - 2\bar{\mathcal{F}} \wedge dC \right)$$
$$+ \frac{d\tau}{da}(-i \star d\bar{C} \wedge C \wedge da + 2\bar{\mathcal{F}} \wedge C \wedge da)$$
$$+ \frac{d\bar{\tau}}{d\bar{a}}(i \star dC \wedge \bar{C} \wedge d\bar{a} + 2\mathcal{F} \wedge \bar{C} \wedge d\bar{a}) \Big)$$

The equations of motion are

$$0 = (\tau - \bar{\tau})dC + d\tau \wedge C - i \star \Big((\tau - \bar{\tau})dC + d\tau \wedge C \Big)$$
$$0 = d\Big((\tau - \bar{\tau}) i \star dC + 2(\tau - \bar{\tau})\mathcal{F}_\beta + i \star d\tau \wedge C \Big)$$
$$+ d\bar{\tau} \wedge i \star dC + 2 d\bar{\tau} \wedge \mathcal{F}$$

We can substitute the first equation into the second to find

$$d\left((\tau - \bar{\tau})\mathcal{F} \right) + d\bar{\tau} \wedge (\mathcal{F} + \frac{1}{2}(i \star dC - dC)) = 0$$

This agrees with Seiberg-Witten if $\mathcal{F} = -\frac{1}{2} d\bar{C} - \frac{i}{2} \star d\bar{C}$.

A Non-abelian (2,0) Action

Next we want to construct a non-abelian (2,0) action.

We can construct a free theory by including a gauge field along with a Lagrange multiplier term that imposes flatness:

$$S = \int \Big[\frac{1}{4}\langle DB \wedge \star DB\rangle - \langle H \wedge DB\rangle - \frac{1}{2}\langle D_\mu X^I D^\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}\Gamma^\mu D_\mu \Psi\rangle + (\tilde{F} \wedge \tilde{W})\Big]$$

where $D = d - \tilde{A}$ and $\tilde{F} = d\tilde{A} - \tilde{A} \wedge \tilde{A}$ with

$$\delta\tilde{A}_\mu = 0$$

$$\delta\tilde{W}_{\mu\nu\lambda\rho}(\,\cdot\,) = 3i\bar{\epsilon}\Gamma_{[\mu\nu}[B_{\lambda\rho]}, \Psi, \,\cdot\,] + i\bar{\epsilon}\Gamma_{\mu\nu\lambda\rho}\Gamma^I[X^I, \Psi, \,\cdot\,]$$

Here the matter fields take values in a vector space $\mathcal{V}$ and the gauge field in a Lie-algebra $\mathcal{G}$ with a representation $\tilde{T}^r$ on $\mathcal{V}$.

- $\mathcal{V}$ has an inner-product $\langle \cdot, \cdot \rangle$
- $\mathcal{G}$ has an inner-product $(\cdot, \cdot)$

This leads to a three-algebra structure [Figueroa-O'Farrill, de Medeiros]:

$$[\cdot, \cdot, \cdot] : \mathcal{V} \otimes \mathcal{V} \otimes \mathcal{V} \to \mathcal{V}$$

$$[X, Y, Z] = \sum_r \langle X, \tilde{T}^r(Y) \rangle \tilde{T}_r(Z)$$

which implies the combatability conditions

$$(\tilde{T}, [U, V, \ \cdot \ ]) = \langle \tilde{T}(U), V \rangle = -\langle U, \tilde{T}(V) \rangle$$

$$[U, V, [X, Y, Z]] = [[U, V, X], Y, Z] + [X, [U, V, Y], Z] + [X, Y, [U, V, Z]]$$

In order to construct interactions we consider the $(2,0)$ system of [NL,Papageogakis] and introduce a non-dynamical vector field Y^μ with scaling dimension -1

$$D_\mu Y^\nu = 0 \qquad [Y^\mu, D_\mu(\,\cdot\,),\,\cdot'\,] = 0 \qquad [Y^\mu, Y^\nu,\,\cdot\,] = 0$$

Here the three-algebra is totally anti-symmetric and so we take $\mathcal{V} = \mathbb{R}^4$ leading to the gauge algebra $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$.

$$0 = D^2 X^I - \frac{i}{2}[Y^\sigma, \bar{\Psi}, \Gamma_\sigma \Gamma^I \Psi] + [Y^\sigma, X^J, [Y_\sigma, X^J, X^I]]$$

$$0 = D_{[\lambda} H_{\mu\nu\rho]} + \frac{1}{4}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}[Y^\sigma, X^I, D^\tau X^I] + \frac{i}{8}\varepsilon_{\mu\nu\lambda\rho\sigma\tau}[Y^\sigma, \bar{\Psi}, \Gamma^\tau \Psi]$$

$$0 = \Gamma^\rho D_\rho \Psi + \Gamma_\rho \Gamma^I [Y^\rho, X^I, \Psi]$$

$$0 = \tilde{F}_{\mu\nu}(\cdot) - [Y^\lambda, H_{\mu\nu\lambda},\,\cdot\,]$$

Now the flatness condition on $\tilde{F}$ is replaced by $\tilde{F} \sim [Y, H, \ ]$

So we adjust the Lagrange multiplier term to

$$\mathcal{L}_{\tilde{W}} = \langle H \wedge \tilde{W}(Y) \rangle + (\tilde{F} \wedge \tilde{W})$$

where $\tilde{W}(Y) = \frac{1}{3!} W_{\mu\nu\lambda\rho}(Y^\rho) dx^\mu \wedge dx^\nu \wedge dx^\lambda$ and make a guess

$$\begin{aligned} S_{guess} = \int \Big[& \frac{1}{4} \langle DB \wedge \star DB \rangle - \langle H \wedge (DB - \tilde{W}(Y)) \rangle + (\tilde{F} \wedge \tilde{W}) \\ & - \frac{1}{2} \langle D_\mu X^I D^\mu X^I \rangle - \frac{1}{4} \langle [Y^\mu, X^I, X^J][Y_\mu, X^I, X^J] \rangle \\ & + \frac{i}{2} \langle \bar{\Psi} \Gamma^\mu D_\mu \Psi \rangle + \frac{i}{2} \langle \bar{\Psi} \Gamma_\mu \Gamma^I [Y^\mu, X^I, \Psi] \rangle \Big] \end{aligned}$$

The matter terms clearly reproduce their correct equations.

This has introduced a source for H of the form $\tilde{W}(Y)$.

Alas this isn't quite right:

- self-dual part of $\tilde{W}(Y)$ is non-zero.
- $D^2 \sim \tilde{F} \neq 0$

After some more guess work we find [NL]

$$S = \int \Big[\frac{1}{4}\langle \mathcal{D}B \wedge \star\mathcal{D}B\rangle + \frac{1}{6}\langle \mathcal{D}B \wedge DB\rangle + \frac{1}{4}\langle \tilde{W}(Y) \wedge \star\tilde{W}(Y)\rangle$$
$$- \langle H \wedge (\mathcal{D}B - \tilde{W}(Y))\rangle - \frac{1}{2}\langle (\mathcal{D}B - \star\mathcal{D}B) \wedge \tilde{W}(Y)\rangle + (\tilde{F} \wedge \tilde{W})$$
$$- \frac{1}{2}\langle D_\mu X^I D^\mu X^I\rangle - \frac{1}{4}\langle [Y^\mu, X^I, X^J][Y_\mu, X^I, X^J]\rangle$$
$$+ \frac{i}{2}\langle \bar{\Psi}\Gamma^\mu D_\mu \Psi\rangle + \frac{i}{2}\langle \bar{\Psi}\Gamma_\mu \Gamma^I [Y^\mu, X^I, \Psi]\rangle\Big]$$

Here $\mathcal{D}_\mu = \partial_\mu - \tilde{\mathcal{A}}_\mu(\cdot)$ with

$$\tilde{\mathcal{A}}_\mu(\cdot) = \tilde{A}_\mu(\cdot) - \frac{1}{2}[B_{\mu\nu}, Y^\nu, \ \cdot\ ]$$

This reproduces all the equations of motion of the $(2,0)$ system.

In particular B and $\tilde{W}$ can be removed from the equations for the remaining fields.

It is invariant under $(2,0)$ supersymmetry:

$$\begin{aligned}
\delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\
\delta B_{\mu\nu} &= -i\bar{\epsilon}\Gamma_{\mu\nu}\Psi \\
\delta\Psi &= \Gamma^\mu\Gamma^I D_\mu X^I\epsilon + \frac{1}{2\cdot 3!}H_{\mu\nu\lambda}\Gamma^{\mu\nu\lambda}\epsilon - \frac{1}{2}\Gamma_\mu\Gamma^{IJ}[Y^\mu, X^I, X^J]\epsilon \\
\delta H_{\mu\nu\lambda} &= \frac{3}{2}(1+\star_\eta)i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi - i\bar{\epsilon}\Gamma_\rho\Gamma_{\mu\nu\lambda}\Gamma^I[Y^\rho, X^I, \Psi] \\
\delta\tilde{A}_\mu(\,\cdot\,) &= i\bar{\epsilon}\Gamma_{\mu\nu}[Y^\nu, \Psi, \,\cdot\,] \\
\delta\tilde{W}_{\mu\nu\lambda\rho}(\cdot) &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}[B_{\lambda\rho]}, \Psi, \,\cdot\,] + i\bar{\epsilon}\Gamma_{\mu\nu\lambda\rho}\Gamma^I[X^I, \Psi, \,\cdot\,]
\end{aligned}$$

Note that this is a reducible representation of supersymmetry:

$$\mathcal{H}_{(s)} = \frac{1}{2}(\mathcal{D}B - \tilde{W}(Y)) + \frac{1}{2}\star(\mathcal{D}B - \tilde{W}(Y)) + H$$
$$\tilde{\mathcal{A}}_{(s)\mu}(\cdot) = \tilde{A}_\mu(\cdot) - [B_{\mu\nu}, Y^\nu, \ \cdot\ ]$$

are singlets.

The interacting part is five-dimensional: $[Y^\mu D_\mu, \ , \ ] = 0$.

Coupling constant

$$g^2 = R_5 \left(\frac{\langle Y_\mu, Y^\mu \rangle}{R_5^2} \right)$$

Depending on the choice of Y one finds different five-dimensional theories.

- Y spacelike: (4+1)-dimensional super-Yang-Mills
- Y timelike: (5+0)-dimensional super-Yang-Mills
- Y null: novel non-Lorentzian theory ($G = \star G$):

$$S = \frac{1}{g^2}\mathrm{tr}\int d^4x\, dx^0 \Bigg(\frac{1}{2}F_{0i}F_{0i} + \frac{1}{2}F_{ij}G_{ij} - \frac{1}{2}\left(D_iX^I\right)\left(D_iX^I\right) \\ -\frac{i}{2}\bar{\Psi}\Gamma_- D_0\Psi + \frac{i}{2}\bar{\Psi}\Gamma_i D_i\Psi + \frac{1}{2}\bar{\Psi}\Gamma_-\Gamma^I[X^I,\Psi]\Bigg)$$

16 supersymmetries and 8 superconformal supersymmetries [NL, Owen][NL, Mouland].

Path integral reduces to instanton QM[Mouland]

An Ω-deformed version has an $SU(3,1)$ symmetry, 8 supersymmetries, 16 superconformal symmetries and an AdS_7 dual [NL, Lipstein,Richmond] [NL, Lipstein,Mouland,Richmond]

Conclusions

In this talk we adapted Sen's prescript for self-dual forms to the $(2,0)$ theory.

- Obtained a more geometrical formulation
- Obelian theory reproduces the dynamics of a single M5
- Presented an interacting non-abelian version which describes two M5-branes on an S^1

Comments

Interesting new geometrical structure for self-dual forms: $\tilde{\mathcal{M}}$. Diffeomorphisms are enabled unusually.

Extend to DBI-like M5's: Make $H - \tilde{\mathcal{M}}(H)$ non-linear? [Perry,Schwarz],[Howe,Sezgin West],[Pasti,Sorokin,Tonin]

Extend to $(1,0)$ theories [Sambtleben,Sezgin,Wimmer]

Is the appearance of a second connection $\tilde{\mathcal{D}}_\mu = D_\mu - \frac{1}{2}[B_{\mu\nu}, Y^\nu, \,\cdot\,]$ suggestive of some 2-form structure?

Better understanding of modular anomalies *vs* diffeomorphisms?

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