# APPENDIX

## TWO FRAGMENTS FROM LEIBNIZ

(Translated from the Latin of Gehrhardt's text, *Die Philosophischen Schriften von G. W. Leibniz, Band VII, "Scientia Generalis. Characteristica,"* XIX and XX.)

These two fragments represent the final form of Leibniz's "universal calculus": their date is not definitely known, but almost certainly they were written after 1685. Of the two, XX is in all respects superior, as the reader will see, but XIX also is included because it contains the operation of "subtraction" which is dropped in XX. Leibniz's comprehension of the fact that $+$ and $-$ (or, in the more usual notation, "multiplication" and "division") are not simple inverses in this calculus, and his appreciation of the complexity thus introduced, is the chief point of interest in XIX. The distinction of "subtraction" (in intension) and negation, is also worthy of note. It will be observed that, in both these fragments, $A + B$ (or $A \oplus B$) may be interpreted in two ways: (1) As "both $A$ and $B$" in intension; (2) as "either $A$ or $B$", the class made up of the two classes $A$ and $B$, in extension. The "logical" illustrations mostly follow the first interpretation, but in XX (see esp. *scholium to defs.* 3, 4, 5, *and* 6) there are examples of the application to logical classes in extension. The illustration of the propositions by the relations of line-segments also exhibits the application to relations of extension. Attention is specifically called to the parallelism between relations of intension and relations of extension in the remark appended to prop. 15, in XX. The *scholium to axioms* 1 *and* 2, in XX, is of particular interest as an illustration of the way in which Leibniz anticipates later logistic developments.

The Latin of the text is rather careless, and constructions are sometimes obscure. Gehrhardt notes (p. 232) that the manuscript contains numerous interlineations and is difficult to read in many places.

### XIX

#### Non inelegans specimen demonstrandi in abstractis[1]

*Def.* 1. Two terms are the *same* (*eadem*) if one can be substituted for the other without altering the truth of any statement (*salva veritate*). If we have $A$ and $B$, and $A$ enters into some true proposition, and the substitution of $B$ for $A$ wherever it appears, results in a new proposition which is likewise true, and if this can be done for every such proposition, then $A$ and $B$ are said to be the *same;* and conversely, if $A$ and $B$ are the same, they can be substituted for one another as I have said. Terms which are the same are also called *coincident* (*coincidentia*); $A$ and $A$ are, of course, said to be the same, but if $A$ and $B$ are the same, they are called *coincident.*

*Def.* 2. Terms which are not the same, that is, terms which cannot always be substituted for one another, are *different* (*diversa*). *Corollary.* Whence also, whatever terms are not different are the same.

*Charact.* 1.[2] $A = B$ signifies that $A$ and $B$ are the *same*, or *coincident.*

*Charact.* 2.[3] $A \neq B$, or $B \neq A$, signifies that $A$ and $B$ are *different.*

*Def.* 3. If a plurality of terms taken together coincide with one, then any one of the plurality is said to *be in* (*inesse*) or to *be contained in* (*contineri*) that one with which they

[1] This title appears in the manuscript, but Leibniz has afterward crossed it out. Although pretentious, it expresses admirably the intention of the fragment, as well as of the next.

[2] We write $A = B$ where the text has $A \infty B$.

[3] We write $A \neq B$ where the text has $A$ non $\infty B$.