Kaluza-Klein Spectrometry for String Compactifications

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M-Theory and Mathematics
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with Bobev, Duboeuf, Eloy, Galli, Giambrone, Guarino, Josse, Nicolai, Petrini, Robinson, Samtleben, Sterckx, Trigiante, van Muiden + W.I.P

The importance of Kaluza-Klein spectra

$\Longrightarrow$

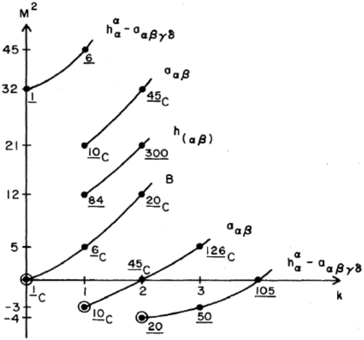

FIG. 2. Mass spectrum of scalars.

- AdS/CFT: conformal dimensions
- Stability of non-SUSY vacua

Computing Kaluza-Klein spectra is hard

- Free scalar on $\mathbb{R}^{D-1,1} \times S^1$:

$$0 = \partial_x^2 \phi(x,y) + \partial_y^2 \phi(x,y)\,,$$

$$\phi(x,y) = \sum_k \phi^{(k)}(x)\, e^{i\,k\,y/R}\,, \qquad m^2 = \frac{k^2}{R^2}\,.$$

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- SUGRA: (linearised) EoMs mix metric & fluxes $\Rightarrow$ eigenmodes?

$$\nabla_Q f^{QMNP} + \frac{1}{2} F^{QMNP} \nabla_Q h_R{}^R - \nabla_Q \left(h^{QR} F_R{}^{MNP} \right) - 3 \nabla^Q \left(h^{S[M} F_{QS}{}^{NP]} \right) = -\frac{1}{288} \epsilon^{MNPQ_1 \ldots Q_8} F_{Q_1 \ldots Q_4} f_{Q_5 \ldots Q_8}\,.$$

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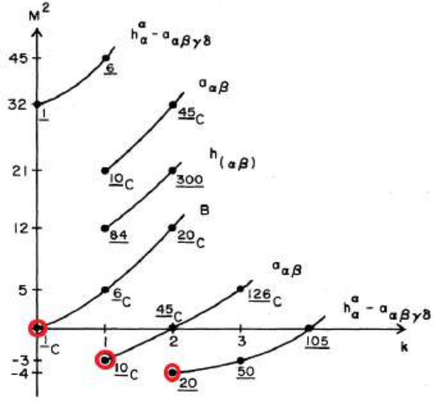
- Previously, only two cases understood:
  - Spin-2 fields [Bachas, Estes '11] ✓
  - $M_{int} = \frac{G}{H}$ ✓

Another tool: Consistent truncations

- Non-linear truncation to subset of KK-modes
- Solutions are solutions to higher-dim theory
- Compute subset of masses for any vacuum.
- Results can be misleading!



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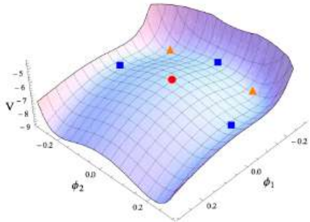

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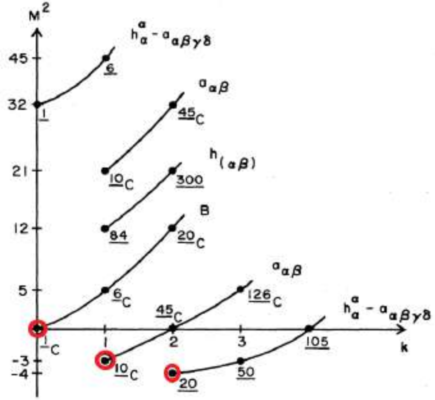

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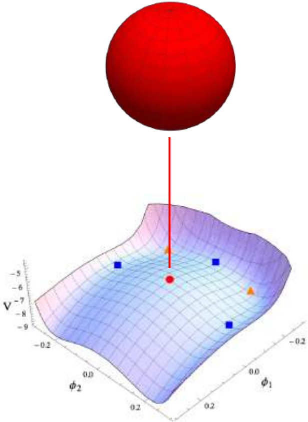

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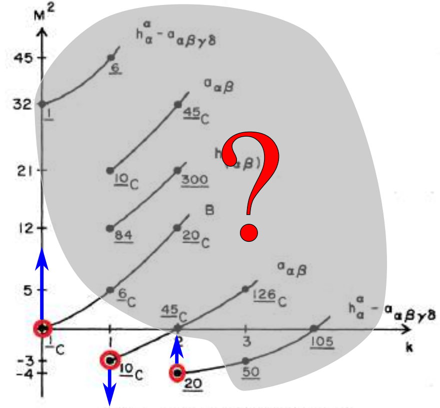

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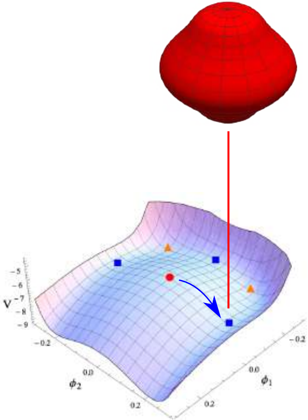
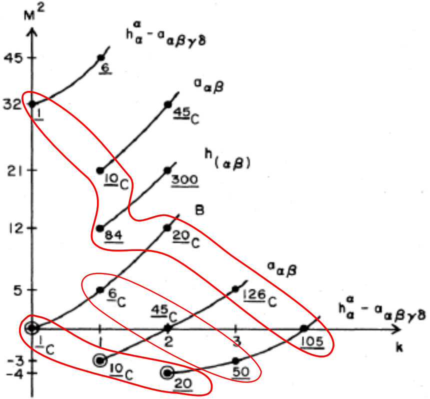


Another tool: Consistent truncations

[EM, Samtleben '20]
Extend this to full KK spectrum using Exceptional Field Theory!
Exploit hidden structures



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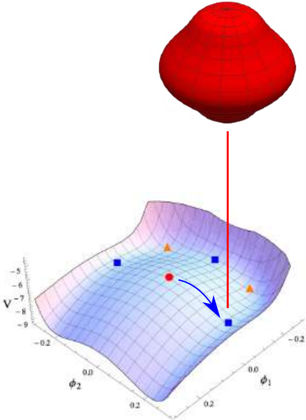
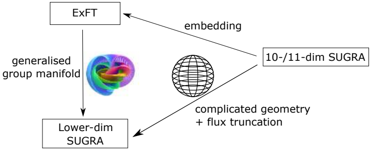
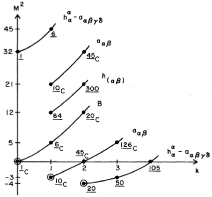


Exceptional Field Theory
& consistent truncations



Kaluza-Klein spectroscopy



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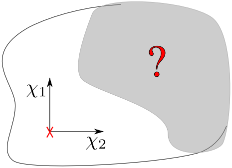
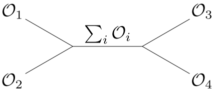
Applications



Higher-point couplings



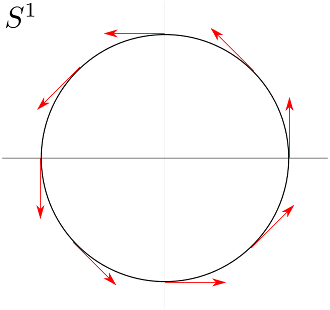
Consistent truncation

Non-linear embedding of lower-dimensional theory
into 10-/11-d supergravity

- All solutions of lower-d SUGRA $\rightarrow$ solutions of 10-/11-d SUGRA
- Non-linearity: highly non-trivial!
- Symmetry arguments crucial

Consistent truncation on group manifold

Symmetry arguments crucial for consistency, e.g. group manifold



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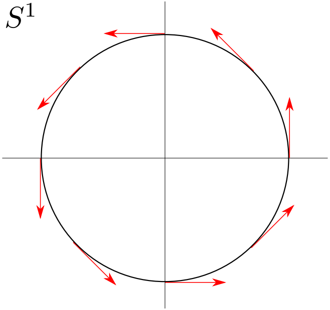


$$U_m{}^\mu \in \mathrm{GL}(D)$$

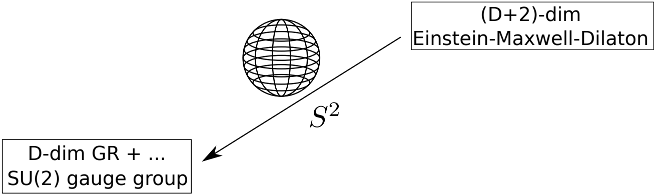
$$L_{U_m} U_n = f_{mn}{}^p U_p$$

$$g_{\mu\nu}(x, y) = g_{mn}(x)\,(U^{-1})_\mu{}^m(y)(U^{-1})_\nu{}^n(y)$$

Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+2}x \sqrt{|g|} \left(R_g - (\nabla\phi)^2 - e^{\alpha\phi} F^2 \right)$$



$$ds^2_{D+2} = Y^{\frac{1}{D}} \left(\Delta^{\frac{1}{D}} ds^2_D + g^{-2} \Delta^{-\frac{D-1}{D}} T^{-1}_{ij} \mathfrak{D}\mu^i \mathfrak{D}\mu^j \right) ,$$

$$e^{\sqrt{\frac{2(D)}{D+1}}\hat{\phi}} = \Delta^{-1} Y^{\frac{D-1}{D+1}} ,$$

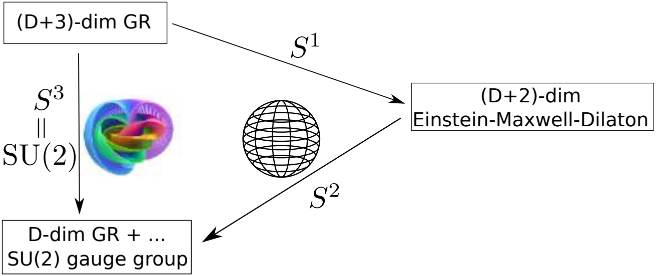
$$\hat{F}_2 = \frac{1}{2} \epsilon_{ijk} \left(g^{-1} \Delta^{-2} \mu^i \mathfrak{D}\mu^j \wedge \mathfrak{D}\mu^k - 2g^{-1} \Delta^{-2} \mathfrak{D}\mu^i \wedge \mathfrak{D} T_{jl} T_{km} \mu^l \mu^m - \Delta^{-1} F^{ij}_{(2)} T_{kl} \mu^l \right) .$$

[Cvetic, Lü, Pope '00]

Larger symmetry groups from generalising geometry

Symmetry argument for other consistent truncations?

$$S = \int d^{D+3}x \sqrt{|G|} \left(R_G \right)$$



$$U_m{}^\mu \in \mathrm{GL}(3)$$

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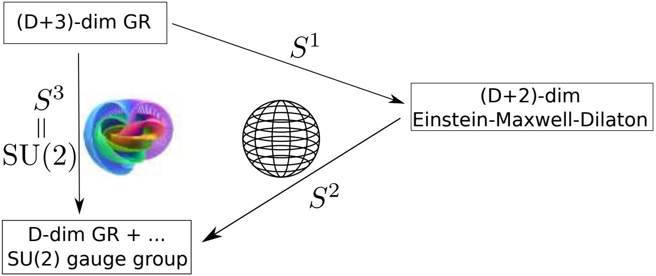

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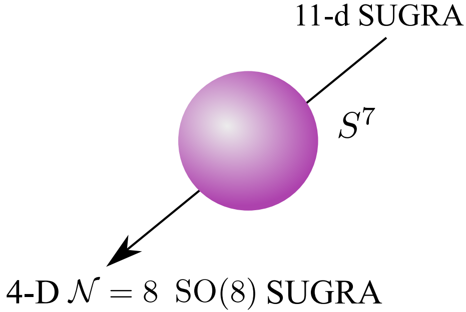
$$g_{\mu\nu}(x, y) = g_{mn}(x) \, (U^{-1})_{\mu}{}^{m}(y)(U^{-1})_{\nu}{}^{n}(y)$$

[Cvetic, Lü, Pope, Gibbons '03]

Consistent truncations beyond group manifolds

Consistent truncations of 10-d/11-d SUGRA beyond group manifolds?



[de Wit, Nicolai '82]

Exceptional Field Theory

[Siegel '93], [West '01], [Hull '07], [Hull, Zwiebach '09], [Berman, Perry '10], [Coimbra, Strickland-Constable, Waldram '11], [Hohm, Samtleben, '13], ...

Exceptional Field Theory: Unify metric + fluxes of supergravity

11-d SUGRA on $M_4 \times C_7$:

$$\{g\,,\ C_{(3)}\,,\ C_{(6)}\,,\ldots\} = \mathcal{M}_{MN} \in \frac{E_{7(7)}}{\mathrm{SU}(8)}\,.$$

Diffeo + gauge transf $\rightarrow$ generalised vector field $V^M \in \mathbf{56}$ of $E_{7(7)}$

Lie derivative $\rightarrow$ generalised Lie derivative

$$\mathcal{L}_V = V^M \partial_M - (\partial \times_{adj} V) = \text{diffeo} + \text{gauge transf}\,,$$

with $\partial_M = \left(\partial_i\,,\ \partial^{ij}\,,\ \partial^{ijklm}\,,\ \ldots\right) = (\partial_i\,,\ 0\,,\ \ldots\,,\ 0)$.

Exceptional Field Theory = reformulation of supergravity

Exceptional Field Theory: Reformulation of 10-/11-d supergravity

$$\{g\,,\ C_{(3)}\,,\ C_{(6)}\,,\ \ldots\} = \mathcal{M}_{MN}$$

$$L = R - \frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} + \ldots$$

with $F_{\mu\nu\rho\lambda} = 4\partial_{[\mu} C_{\nu\rho\lambda]}$.

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$$\begin{aligned} L &= R - \frac{1}{48} F_{\mu\nu\lambda\rho} F^{\mu\nu\lambda\rho} + \ldots \\ &= \mathcal{M}^{MN} \partial_M \mathcal{M}^{PQ} \partial_N \mathcal{M}_{PQ} + \ldots \end{aligned}$$

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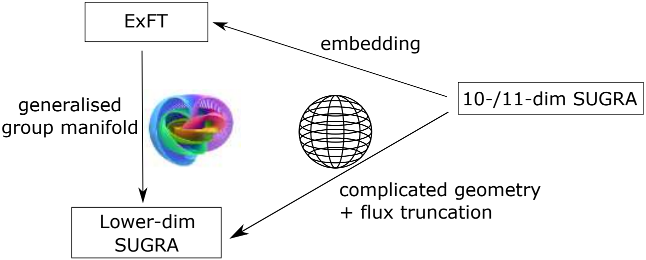
Generalised Lie derivative ⇒ generalised Ricci scalar

Similar for type II theories & other dimensions

Exceptional Field Theory and consistent truncations

Consistent truncations to max. gSUGRA captured by
"generalised group manifolds" in ExFT



$$U_A{}^M \in E_{7(7)}$$

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^C U_C$$

$$\mathcal{M}_{MN}(x, Y) = M_{AB}(x)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

Implications for AdS vacua

e.g. deformations of $\text{AdS}_4 \times S^7$, $\text{AdS}_5 \times S^5$, ...

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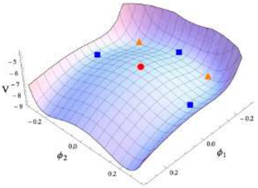
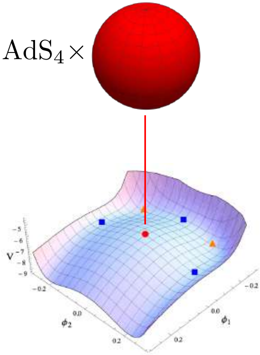


Implications for AdS vacua

e.g. deformations of $\mathrm{AdS}_4 \times S^7$, $\mathrm{AdS}_5 \times S^5$, ...

$$\mathcal{M}_{MN}(x, Y) = \delta_{AB}(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$



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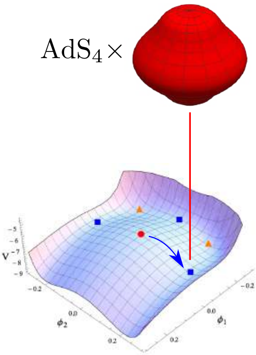

Warped compactifications with few/no remaining (super-)symmetries

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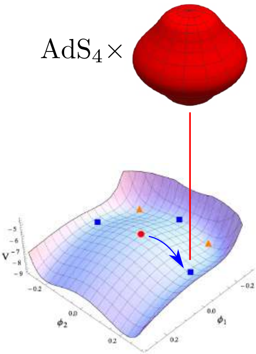


Warped compactifications with few/no remaining (super-)symmetries

"Hidden" group structure!

Kaluza-Klein spectroscopy

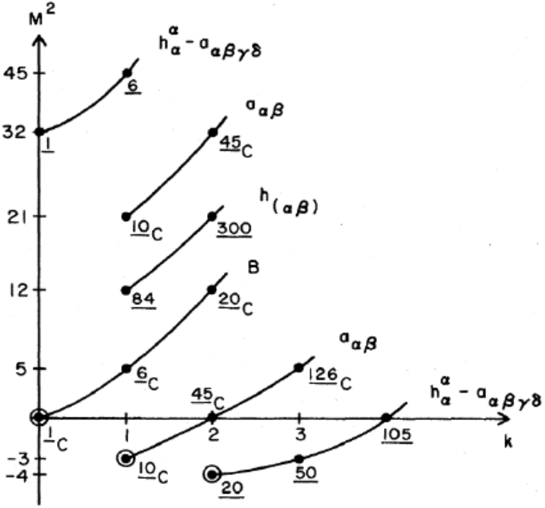
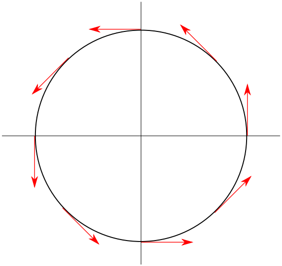


FIG. 2. Mass spectrum of scalars.

KK spectroscopy

${U_A}^M \in E_{7(7)}$ give basis for all fields



Only need scalar harmonics: $\mathcal{Y}_\Sigma$

c.f. $h_{ij}(x,y) = \sum_\ell h^{(\ell)}(x)\mathcal{Y}^{(\ell)}_{(ij)}(y), \quad c_{ijk}(x,y) = \sum_\ell c^{(\ell)}(x)\mathcal{Y}^{(\ell)}_{[ijk]}(y)$

"$\mathcal{N} = 8$ supermultiplet contains all SUGRA fields"

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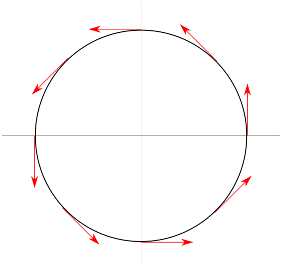

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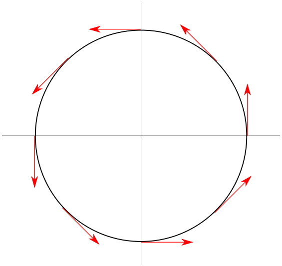

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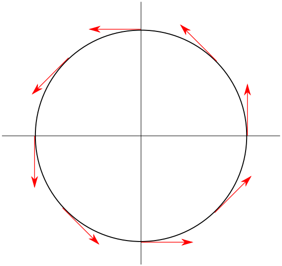

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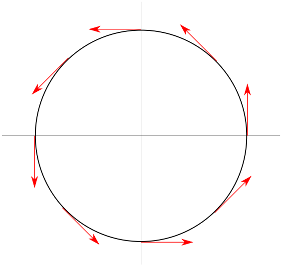

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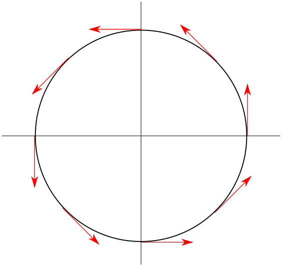

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KK Ansatz = consistent truncation $\otimes$ scalar harmonics

KK spectroscopy

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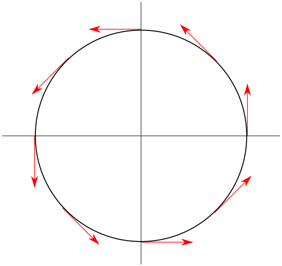

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$$j_{AB}{}^{\Sigma} \in \mathfrak{e}_{7(7)} \ominus \mathfrak{su}(8)$$

Immediate mass diagonalisation for any vacuum!

Mass matrix

- Lower-dim info:

$$\mathcal{L}_{U_A} U_B = X_{AB}{}^{C}\, U_C \,,$$

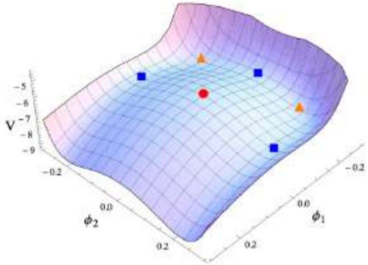
- Higher-dim info:

$$\mathcal{L}_{U_A} \mathcal{Y}_{\Sigma} = L_{K_A} \mathcal{Y}_{\Sigma} = \mathcal{T}_{A\,\Sigma}{}^{\Omega}\, \mathcal{Y}_{\Omega} \,.$$

Differential problem $\rightarrow$ **Algebraic** mass matrix

$$\mathbb{M}^2 \sim X^2 + X\,\mathcal{T} + \mathcal{T}^2 \,.$$

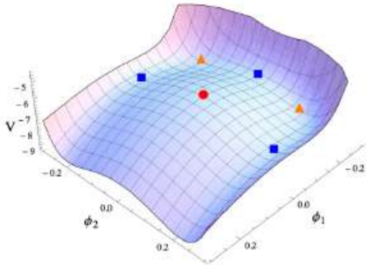
Harmonics



$\longleftrightarrow$

$$\text{KK Ansatz: } \underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$$

Harmonics

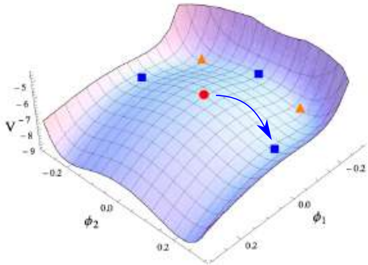


$\longleftrightarrow$

KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

c.f. traditional KK Ansatz: $\phi(x, y) = \phi^{\Sigma}(x)\ \underbrace{\mathcal{Y}_{\Sigma}(y)}_{\text{harmonics}}$

Harmonics



$\longleftrightarrow$

KK Ansatz: $\underbrace{\text{Consistent truncation}}_{\text{non-linear}} \otimes \underbrace{\text{scalar harmonics}}_{\text{linear}}$

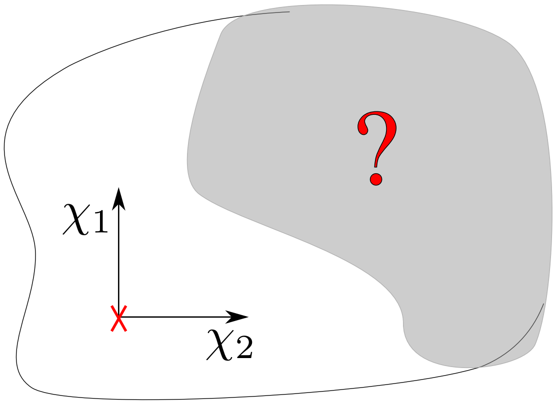
Use same harmonics as for max. symmetric point

Multiplication by $E_{7(7)}$ matrix, $M_{AB}(x)$!

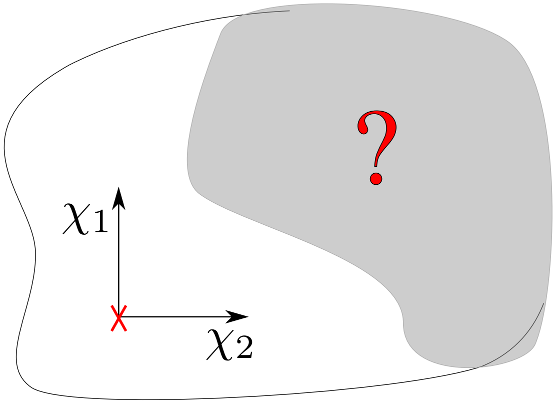
KK Spectroscopy Summary

- Only scalar harmonics of maximally symmetric point (round sphere)
- ExFT KK Ansatz $\Longrightarrow$ Differential problem $\rightarrow$ algebraic problem
- Compute full spectrum for any vacuum in consistent truncation
- Don't need explicit metric, fluxes!
- Spectrum for compactifications with few/no remaining (super-)symmetries

Applications

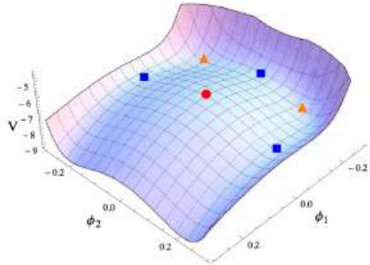


Applications



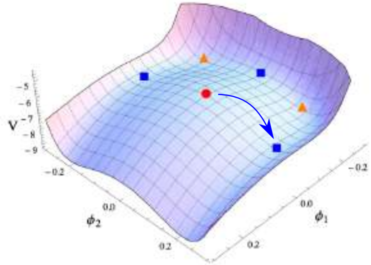
1. Non-SUSY AdS
2. Global properties of conformal manifold

Application to non-SUSY vacua



$\longleftrightarrow$

Application to non-SUSY vacua



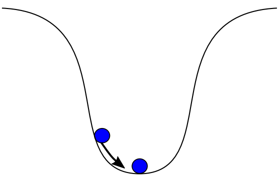
$\longleftrightarrow$

Can compute spectrum for non-SUSY vacua!

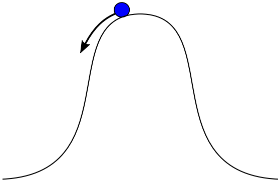
Stability of non-SUSY AdS vacua

Non-SUSY vacua typically suffer from instabilities

Stable



Unstable



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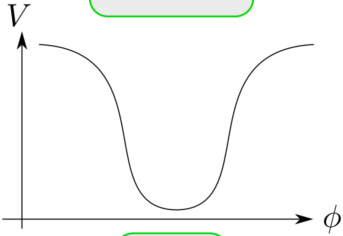
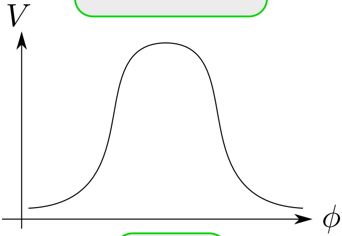


$m^2 > 0$

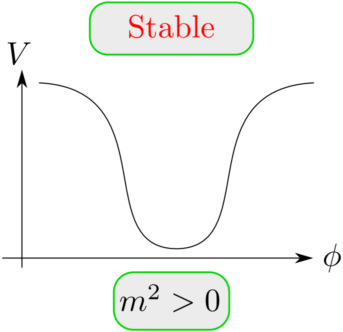
Unstable



$m^2 < 0$

Stability of non-SUSY AdS vacua

Non-SUSY vacua typically suffer from instabilities



In anti-de Sitter spacetime $m^2 < -m_{BF}^2$ for instability

Ex 1. Warning: Kaluza-Klein instability

Is zero-mode stability enough?

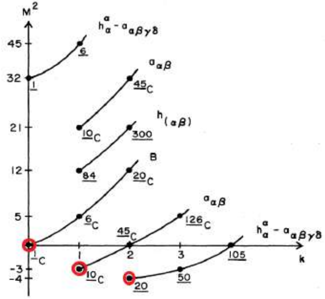
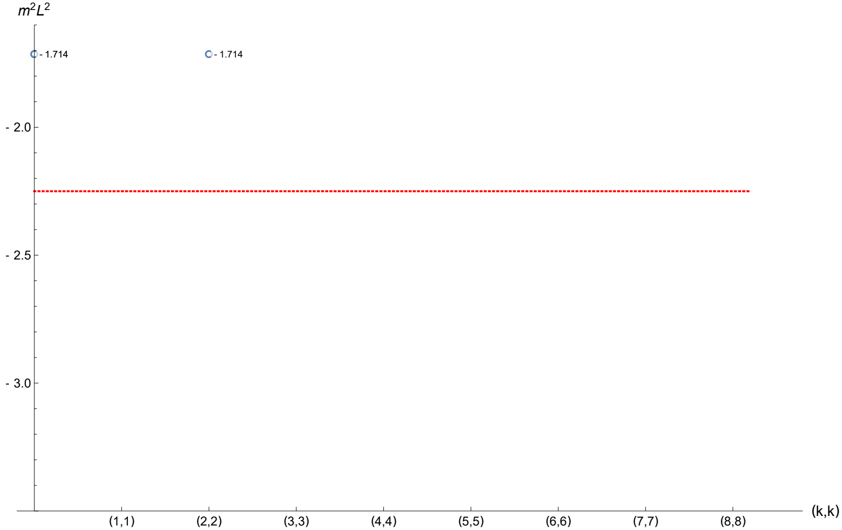


FIG. 2. Mass spectrum of scalars.

Single non-SUSY AdS_4 that is stable in 4-d truncation of 11-d SUGRA on S^7! [Warner '83], [Fischbacher, Pilch, Warner '10], [Comsa, Firsching, Fischbacher '19]

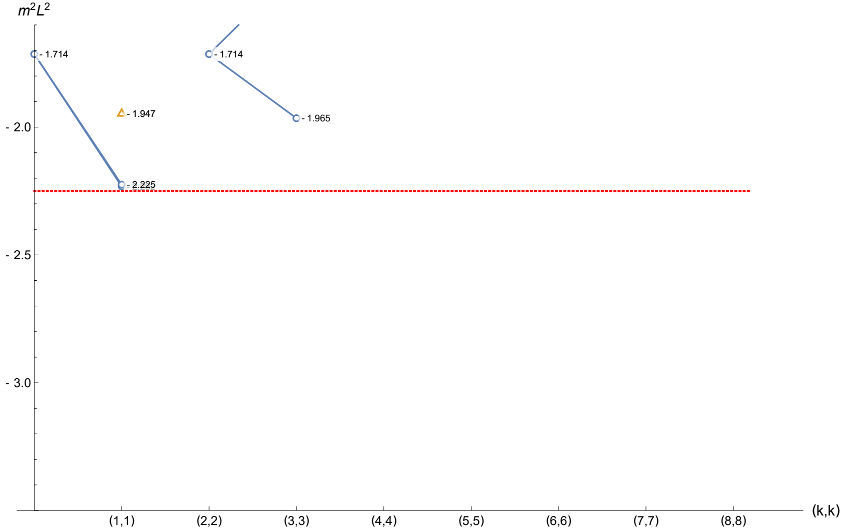
Ex 1. Tachyonic KK modes

Modes $\ell = 0$: $\mathcal{N} = 8$ supergravity multiplet [Fischbacher, Pilch, Warner '10]



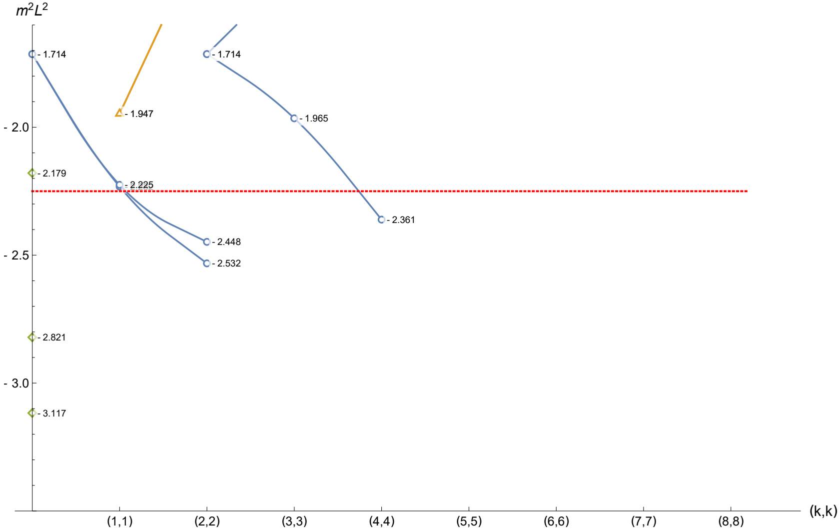
Ex 1. Tachyonic KK modes

Modes $\ell \leq 1$: still stable! [EM, Nicolai, Samtleben '20]



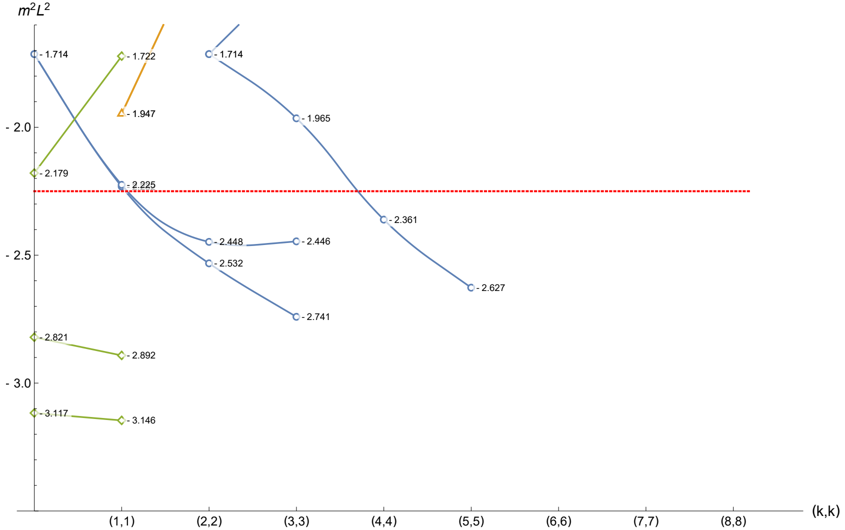
Ex 1. Tachyonic KK modes

Modes $\ell \leq 2$: **tachyons!** [EM, Nicolai, Samtleben '20]



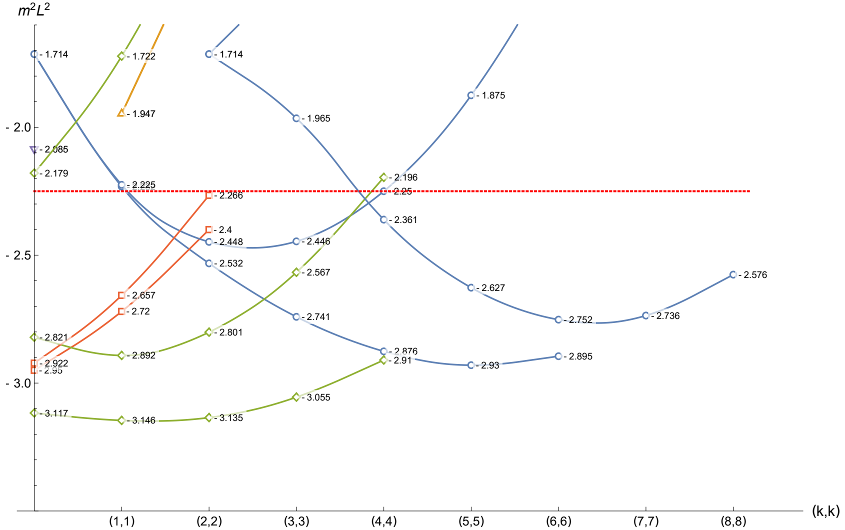
Ex 1. Tachyonic KK modes

Modes $\ell \leq 3$ [EM, Nicolai, Samtleben '20]



Ex 1. Tachyonic KK modes

Modes $\ell \leq 6$ [EM, Nicolai, Samtleben '20]



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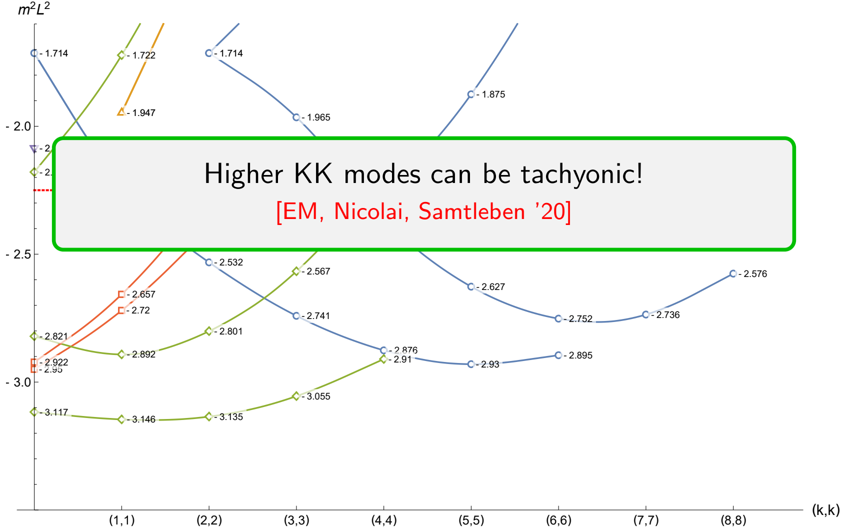
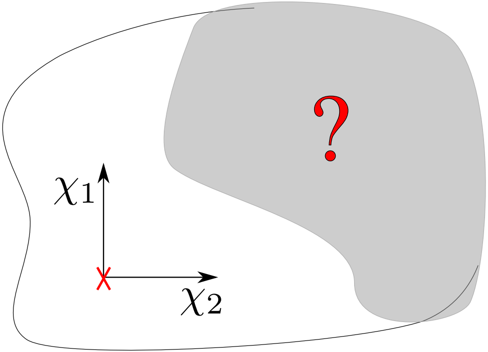


Higher KK modes can be tachyonic!
[EM, Nicolai, Samtleben '20]

Ex 2. Global properties of conformal manifolds

Moduli ⇔ (exactly) marginal deformations

$$L_{\rm CFT} \to L_{\rm CFT} + \chi_i \mathcal{O}^i$$



Kaluza-Klein masses ⇔ protected & **unprotected** anomalous dimensions

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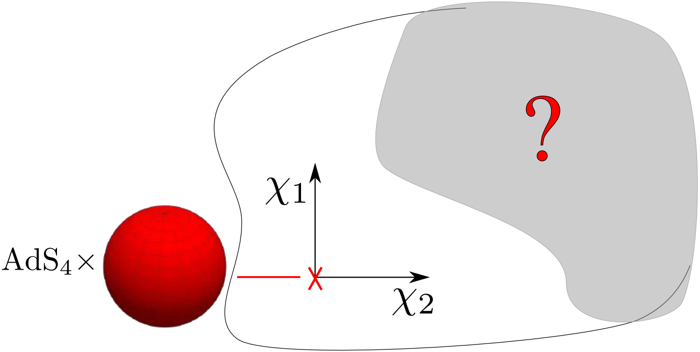

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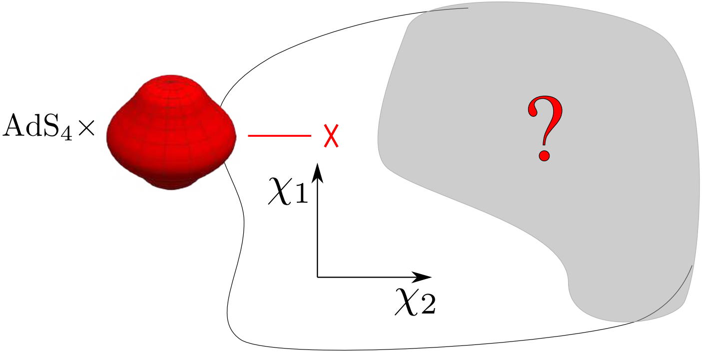
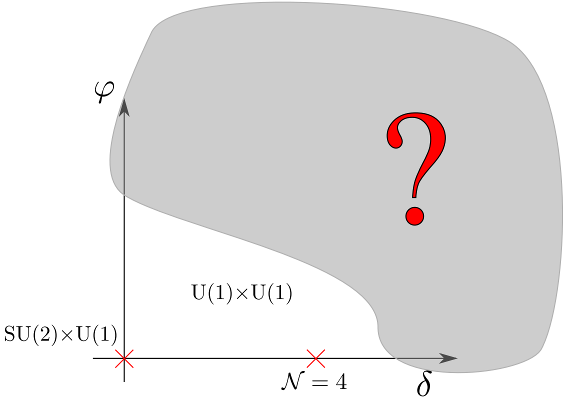


Kaluza-Klein masses ⇔ protected & **unprotected** anomalous dimensions

Ex 2. $\mathcal{N} = 2$ AdS$_4$ family

$[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ supergravity

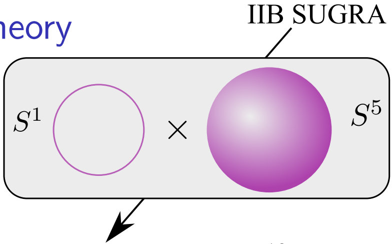
2 moduli $(\varphi, \delta) \in \mathbb{R}^2_{\geq 0}$ in 4-d theory $\Leftrightarrow$ $\mathcal{N} = 2$ conformal manifold
[Guarino, Sterckx, Trigiante '20], [Bobev, Gautason, van Muiden '21]



Expected to be compact e.g. [Perlmutter, Rastelli, Vafa, Valenzuela, '20]

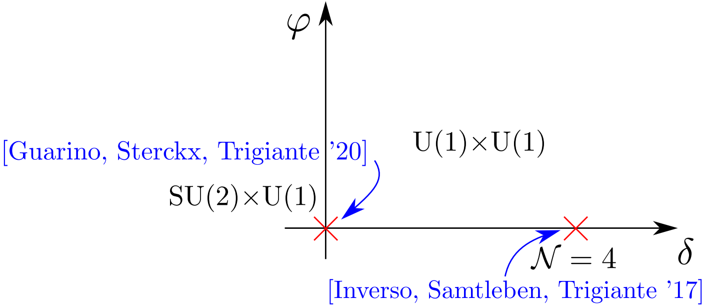
Ex 1. Uplift to IIB string theory

[Inverso, Samtleben, Trigiante '16]



4-D $[\mathrm{SO}(6) \times \mathrm{SO}(1,1)] \ltimes \mathbb{R}^{12}$ SUGRA

$\mathrm{AdS}_4 \times S^5 \times S^1$ "S-fold" of IIB

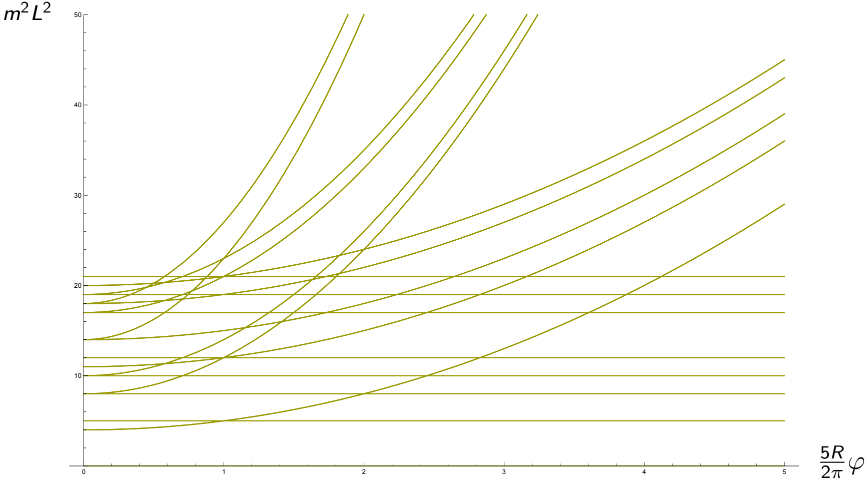


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} = 0$

[Giambrone, EM, Samtleben, Trigiante '21]

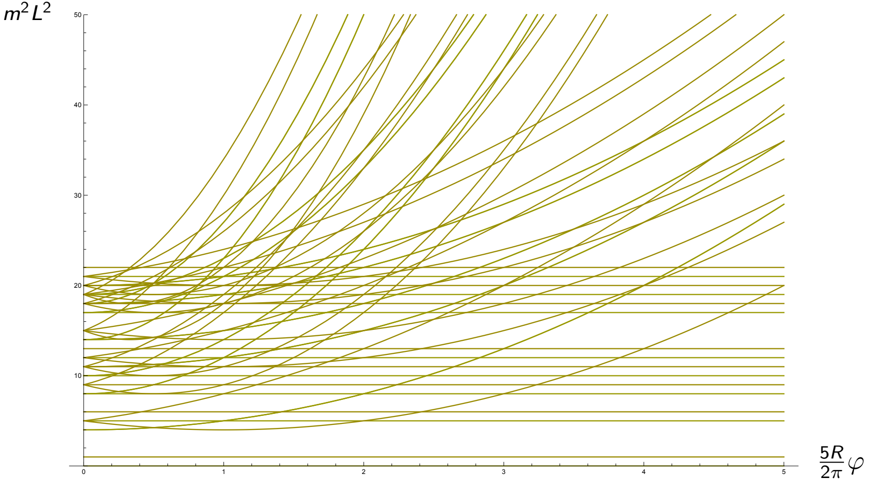


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$AdS_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 1$

[Giambrone, EM, Samtleben, Trigiante '21]

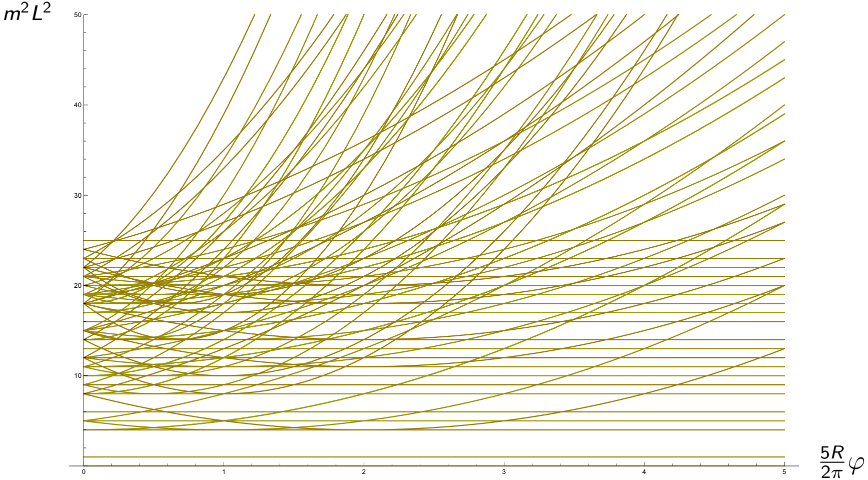


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 2$

[Giambrone, EM, Samtleben, Trigiante '21]

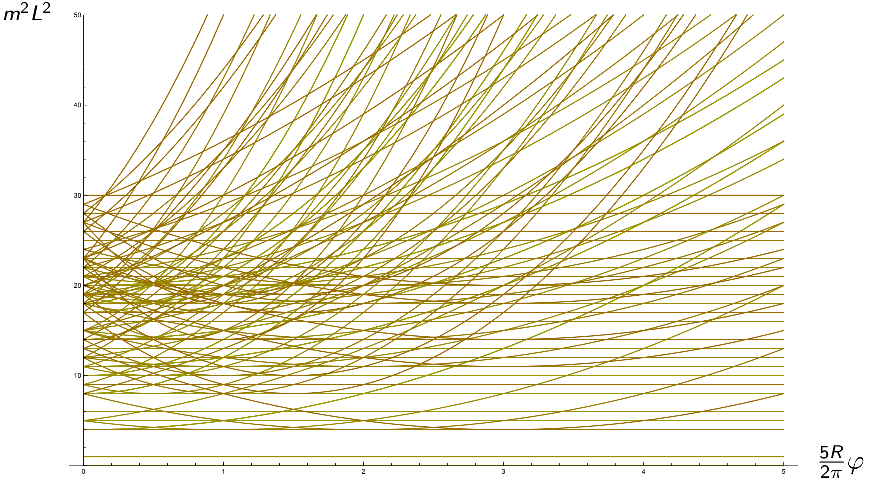


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 3$

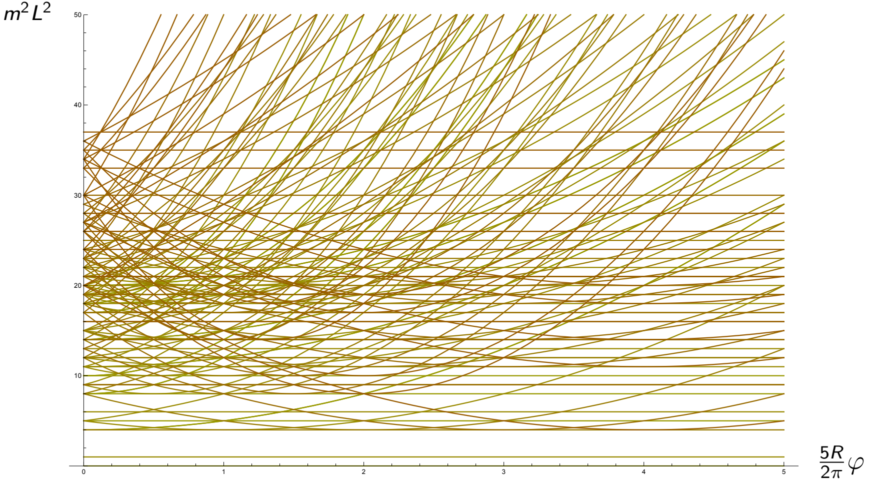
[Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

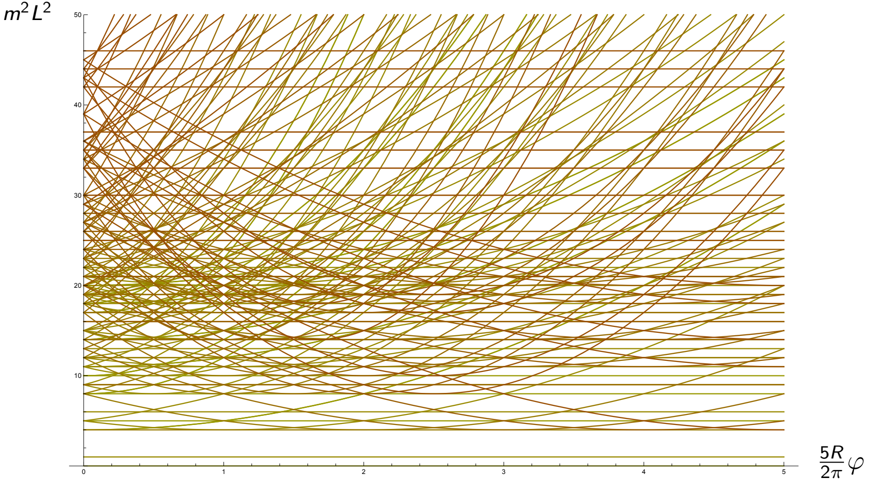
$\ell_{S^1} \leq 4$ [Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

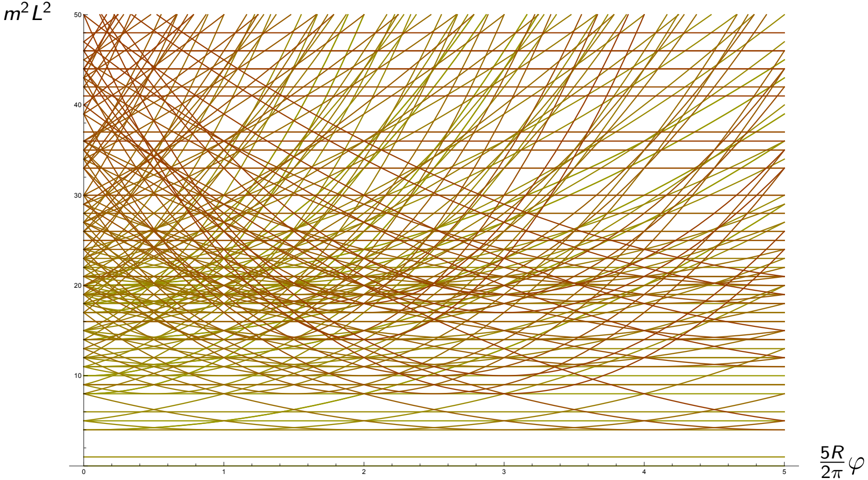
$\ell_{S^1} \leq 5$ [Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 6$ [Giambrone, EM, Samtleben, Trigiante '21]

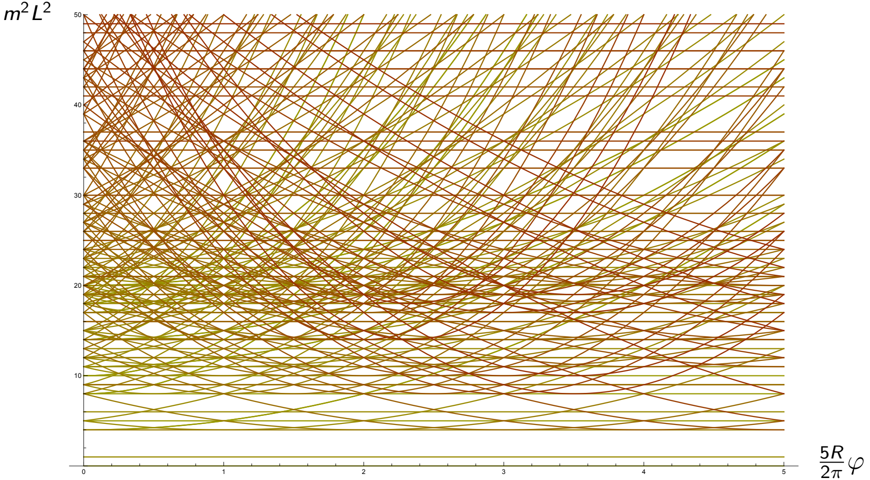


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 7$

[Giambrone, EM, Samtleben, Trigiante '21]

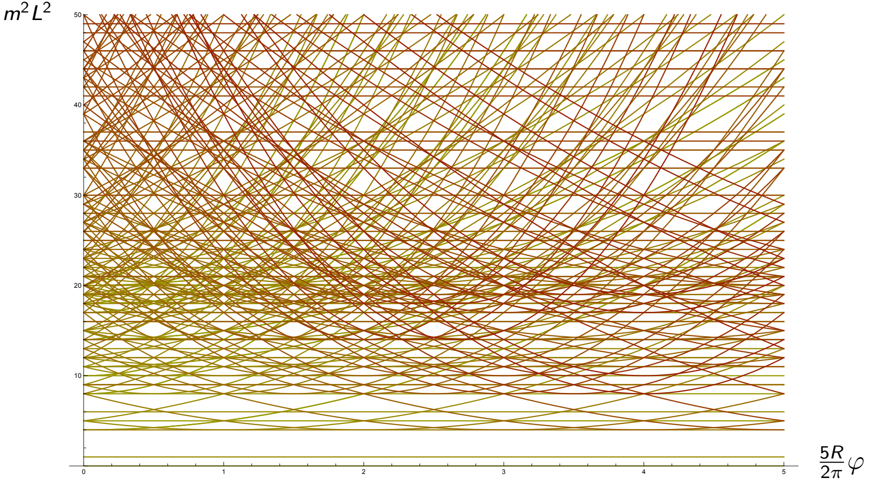


Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 8$

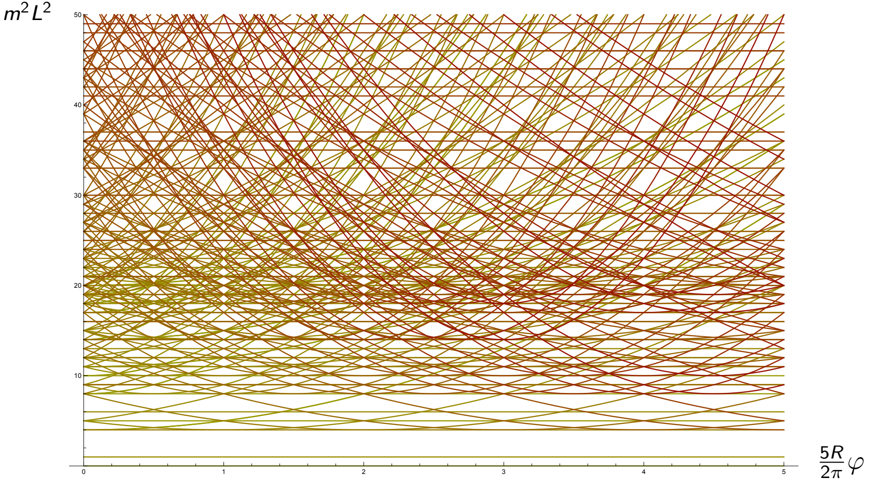
[Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

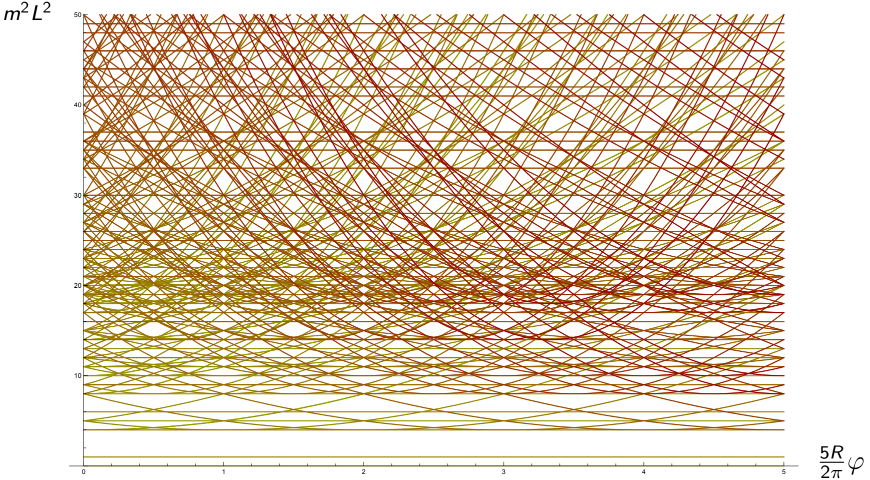
$\ell_{S^1} \leq 9$ [Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

$\mathrm{AdS}_4 \times S^5 \times S^1$ KK spectrum along φ direction

$\ell_{S^1} \leq 10$ [Giambrone, EM, Samtleben, Trigiante '21]



Ex 2. Global properties of the $\mathcal{N} = 2$ conformal manifold

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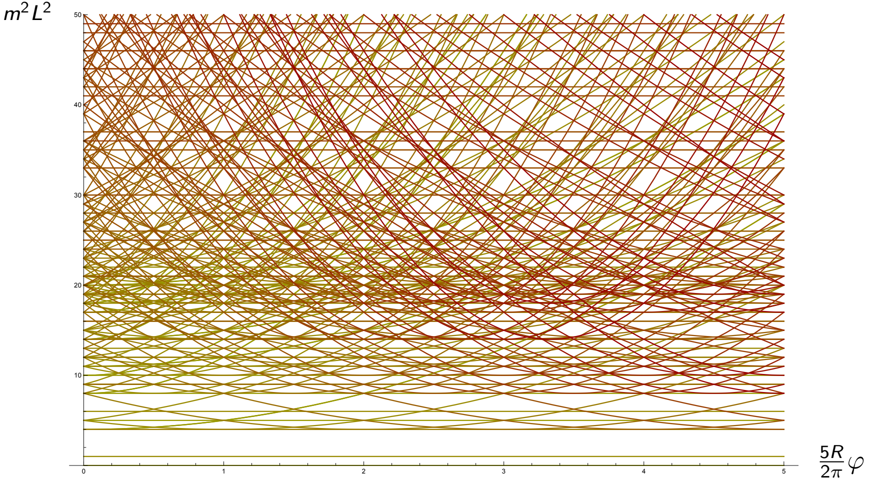
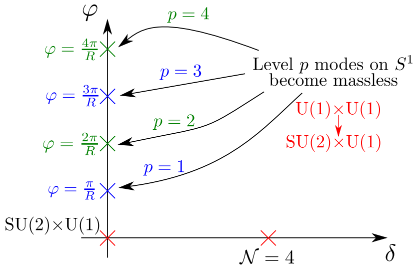


$\varphi \sim \varphi + \frac{2\pi}{R}$, R radius of S^1

Ex 2. Space invaders

Higher KK modes become massless when $\varphi = \frac{p\,\pi}{R}$, $p \in \mathbb{Z}$

[Giambrone, EM, Samtleben, Trigiante '21]



Spectrum identical for $\varphi = \frac{2\,p\,\pi}{R}$, $p \in \mathbb{Z}$

Spectrum differs for $\varphi = \frac{(2\,p+1)\,\pi}{R}$, $p \in \mathbb{Z}$

Ex 2. KK spectrum along $\mathcal{N} = 2$ conformal manifold

[Giambrone, EM, Samtleben, Trigiante '21]

- $\varphi \in \mathbb{R}^+$ a 4-d artefact $\longrightarrow \varphi \in [0, \frac{2\pi}{R})$ in 10 dimensions

- KK spectrum as fct of φ:

$$\Delta = \frac{1}{2} + \sqrt{\frac{17}{4} + \frac{1}{2}r^2 - J(J+1) - 2k(k+1) + \ell(\ell+4) + 4(\frac{\pi\, n}{R} - j\,\varphi)^2}\,.$$

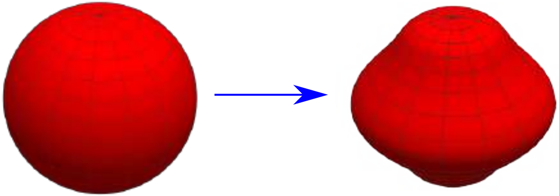
Lorentz spin: J
SU(2) spin: k
$U(1)_R$ charge: r
U(1) $\subset$ SU(2) Cartan: j
S^5 level: ℓ
S^1 level: n

- KK spectrum as fct of δ: non-compact? [Bobev, Gautason, van Muiden '21], [Cesàro, Larios, Varela '21] [Bobev, Gautason, van Muiden '23]

Ex 2. Geometric understanding of φ

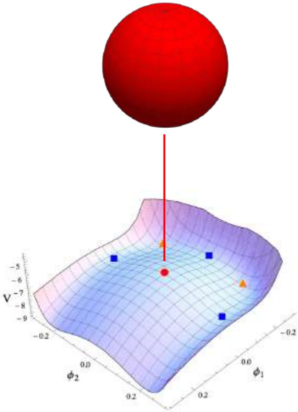
- Geometric origin: $\varphi \to \mathbb{C}$-structure deformation (locally diffeos)
- Universal feature of all vacua with S^1
  - Other S-fold vacua [Cesàro, Larios, Varela '22]
  - $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ [Eloy, Galli, EM '23]
  - $\mathrm{AdS}_3 \times M_3 \times T^4$ [Eloy, Larios '23]
- Supersymmetric or non-supersymmetric! [Guarino, Sterckx '21] [Giambrone, Guarino, EM, Samtleben, Sterckx, Trigiante '21]
- Fully perturbatively stable non-SUSY AdS vacua.
  Many protection mechanisms against other instabilities

KK Spectrometry beyond consistent truncations



KK spectrum beyond consistent truncations

Deformations not triggered by $\mathcal{N} = 8$ scalars?



KK spectrum beyond consistent truncations

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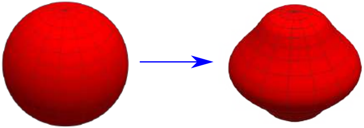
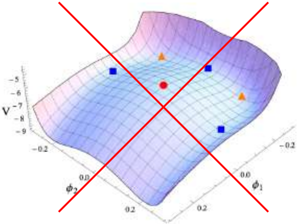




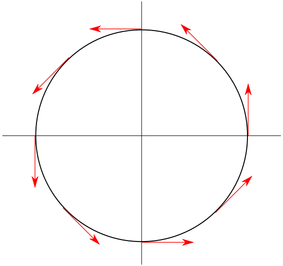
e.g. generic single-trace RG flows of $\mathcal{N} = 4$ SYM, ABJM

Generalised ~~Leibniz~~ parallelisability

[Duboeuf, EM, Samtleben '22]

${U_A}^M \in E_{7(7)}$ give basis for all fields

But, $\mathcal{L}_{U_A} U_B = {X_{AB}}^C(y) U_C$.



Only need scalar harmonics: $\mathcal{Y}_\Sigma$

Generalised ~~Leibniz~~ parallelisability

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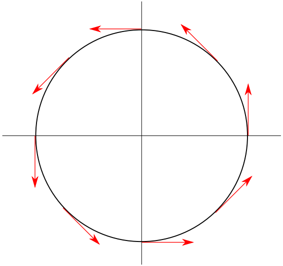


Only need scalar harmonics: $\mathcal{Y}_\Sigma$

$$\mathcal{M}_{MN}(x, Y) = (\delta_{AB} + {j_{AB}}^\Sigma(x)\mathcal{Y}_\Sigma)(U^{-1})_M{}^A(Y)(U^{-1})_N{}^B(Y)$$

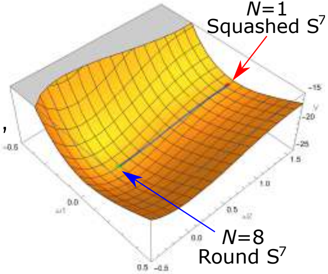
(Modified) ExFT mass matrices still apply!

KK spectrum of generic 10-d/11-d SUGRA deformations

- $\mathcal{N} = 1$ and $\mathcal{N} = 0$ $\mathrm{AdS}_4 \times S^7_{\mathrm{squashed}}$

  $S^7_{\mathrm{squashed}} = \frac{\mathrm{USp}(4)\times\mathrm{USp}(4)}{\mathrm{SU}(2)\times\mathrm{SU}(2)}$

  [Duff, Nilsson, Pope '83, '86], [Nilsson, Pope '83], [Bais, Nicolai, van Nieuwenhuizen '83], [Yamagishi '84], [Nilsson, Papadellaro, Pope '18], [Ekhammar, Nilsson '21], [Karlsson '21]



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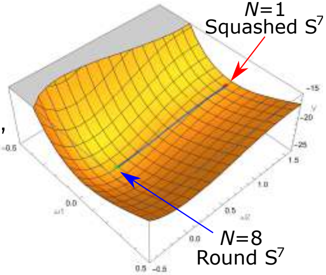


- Full spectrum for first time

$$L[J]\otimes[p,q,r]\otimes\{s\}: \quad \Delta = 1+\frac{5}{3}s+\frac{1}{3}\sqrt{(3J+2s^2)^2+5\,\mathcal{C}(p,q,r)}\,.$$

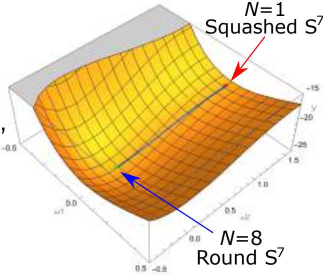
[Duboeuf, EM, Samtleben '22], [Duboeuf, Galli, EM, Samtleben '23]

KK spectrum of generic 10-d/11-d SUGRA deformations

- $\mathcal{N} = 1$ and $\mathcal{N} = 0$ AdS$_4 \times S^7_{\rm squashed}$

  $S^7_{\rm squashed} = \frac{\mathrm{USp}(4)\times\mathrm{USp}(4)}{\mathrm{SU}(2)\times\mathrm{SU}(2)}$

  [Duff, Nilsson, Pope '83, '86], [Nilsson, Pope '83], [Bais, Nicolai, van Nieuwenhuizen '83], [Yamagishi '84], [Nilsson, Papadellaro, Pope '18], [Ekhammar, Nilsson '21], [Karlsson '21]



- Full spectrum for first time

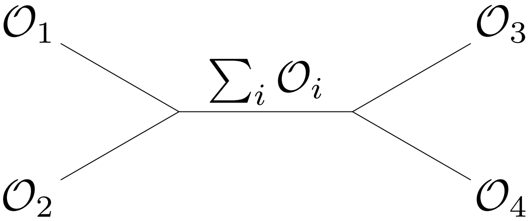
$$L[J] \otimes [p,q,r] \otimes \{s\} : \quad \Delta = 1 + \frac{5}{3}\,s + \frac{1}{3}\sqrt{(3J+2s^2)^2 + 5\,\mathcal{C}(p,q,r)}\,.$$

  [Duboeuf, EM, Samtleben '22], [Duboeuf, Galli, EM, Samtleben '23]

- Other examples, e.g. β-deformation of AdS$_5 \times S^5$

  [Cotellucci, Galli, Josse, EM, Petrini W.I.P]

Higher-point couplings



Higher-point couplings

- Expand fluctuations to higher order [Duboeuf, EM, Samtleben '23]

$$\mathcal{M}_{MN}(x,Y) = \exp({j_\alpha}^\Sigma {t^\alpha}_{AB} \mathcal{Y}_\Sigma)(U^{-1})_M{}^A (U^{-1})_N{}^B$$

- Plug into quadratic action $\Rightarrow$ n-point couplings

$$\mathcal{G}(j^{\alpha_1\Sigma_1}, j^{\alpha_2\Sigma_2}, \ldots, j^{\alpha_n\Sigma_n}) \sim c^{\Sigma_1\Sigma_2\ldots\Sigma_n} \equiv \int dy\, \mathcal{Y}^{\Sigma_1}\mathcal{Y}^{\Sigma_2}\ldots\mathcal{Y}^{\Sigma_n}$$

- Non-vanishing n-point interaction requires $c^{\Sigma_1\Sigma_2\ldots\Sigma_n}$ to exist!

Holds for all vacua of the truncation!

Extension of "consistent truncation" to full KK spectrum

Example: Vanishing of near-extremal correlators

- $\mathrm{AdS}_5 \times S^5$: chiral primaries $\mathcal{O}$ in $[m, 0, 0]$ of SO(6)
- SO(6) group theory: $[m_1, 0, 0] \otimes \ldots \otimes [m_n, 0, 0] \ni [0, 0, 0]$

$$\left(\sum_{j \neq i} m_j\right) - m_i < 0 \;\Rightarrow\; \text{vanishing coupling}$$

- ExFT analysis: $m = \ell + 2, \quad [\ell_1, 0, 0] \otimes \ldots \otimes [\ell_n, 0, 0] \ni [0, 0, 0]$

$$\left(\sum_{j \neq i} m_j\right) - m_i \leq 2(n-3) \;\Rightarrow\; \text{vanishing coupling}$$

- Proves conjectured vanishing of extremal and "near-extremal couplings" [D'Hoker, Erdmenger, Freedman, Perez-Victoria '00], [D'Hoker, Pioline '00]
- $\mathrm{AdS}_4 \times S^7$, $\mathrm{AdS}_7 \times S^4$, any vacua of the gSUGRA

Conclusions

ExFT: Compute full KK spectrum for warped compactifications with few/no remaining (super-)symmetries

- Danger of trusting lower-dimensional supergravity!
- Higher KK modes crucial for physics
  - Higher KK modes can trigger instabilities
  - Compactness of conformal manifold
  - Perturbatively stable non-SUSY AdS
- New holographic test & predictions: Comparison with index [Bobev, EM, Robinson, Samtleben, van Muiden '20]
- Structure of *n*-point couplings, explicit formulae extending "heroic efforts" [Lee, Minwalla, Rangamani, Seiberg '98], [Arutyunov, Frolov '99, '00]

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Outlook:

- Implications for holography? Structure of correlation functions?
- More general vacua?

Thank you!