On a worldvolume realization of M-theory in 26+1 dimensions

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Based on joint collaboration with **Mike Rios** and **David Chester** :

- *Geometry of Exceptional SYM Theories*, PRD **99** (2019) 4, 046004 = [Rios,AM,Chester '18];
- *Exceptional Super Yang-Mills in 27+3 and Worldvolume M-theory*, PLB **808** (2020) 135674 = [Rios,AM,Chester '19];
- *Monstrous M-Theory*, arXiv: 2008.06742 [hep-th] = [AM,Rios,Chester '20].

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Let us consider the **Albert algebra**, realized as a matrix algebra of 3×3 Hermitian matrices over the octonions $\mathbb{O}$: this is the largest of finite-dimensional, simple rank-3 **Jordan algebras** [Jordan, Wigner, von Neumann '34].

$$\mathbf{J}_3^8 \equiv \mathbf{J}_3\left(\mathbb{O}\right) \ni \begin{pmatrix} r_1 & o_1 & \overline{o}_2 \\ * & r_2 & o_3 \\ * & * & r_3 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{8}_v & \mathbf{8}_s \\ * & \mathbf{1} & \mathbf{8}_c \\ * & * & \mathbf{1} \end{pmatrix} \text{ of } tri(\mathbb{O}) = so(8)$$

By fixing a rank-1 idempotent $\rho \in \mathbb{R}$ [*Peirce decomposition*], one obtains

$$J_3(\mathbb{O}) \rightarrow J_2(\mathbb{O}) \oplus \mathbb{R} \oplus \mathbb{O}^{\oplus 2} \Longleftrightarrow \begin{pmatrix} \mathbf{1} & \mathbf{8}_v & \mathbf{8}_s \\ * & \mathbf{1} & \mathbf{8}_c \\ * & * & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{8}_v & 0 \\ * & \mathbf{1} & 0 \\ 0 & 0 & \mathbf{1} \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & \mathbf{8}_s \\ 0 & 0 & \mathbf{8}_c \\ * & * & 0 \end{pmatrix}$$

corresponding, in terms of representations of the corresponding (reduced) structure groups :

$$\begin{aligned} E_{6(-26)} &\rightarrow Spin(9,1) \otimes SO_{1,1}; \\ \mathbf{27} &= \mathbf{10}_{-2} \oplus \mathbf{1}_4 \oplus \mathbf{16}_1 . \end{aligned}$$

$J_2(\mathbb{O})$ is the rank-2 Jordan algebra over the octonions $\mathbb{O}$, and it is nothing but the *spin factor* $\mathbf{S}_{9,1}$, which in general is a rank-2 Jordan algebra with quadratic form of signature $9+1$.

Let us consider the **Albert algebra**, realized as a matrix algebra of 3×3 Hermitian matrices over the octonions $\mathbb{O}$: this is the largest of finite-dimensional, simple rank-3 **Jordan algebras** [Jordan, Wigner, von Neumann '34].

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The 11th dimension pertaining to the uplift to $10+1$ M-theory corresponds to the rank increasing from 2 to 3, by the addition of a 3rd rank-1 idempotent $\rho \in \mathbb{R}$.

At the level of Lie algebras of the various symmetry groups of the Jordan algebras, one obtains :

$$\mathfrak{der} \; : \; \mathfrak{f}_{4(-52)} = \mathfrak{so}_9 \oplus \mathbf{16};$$

$$\mathbf{16} \simeq \underset{\text{Moufang plane}}{T\left(\mathbb{O}P^2\right)} \; ;$$

$$\mathfrak{str}_0 \; : \; \mathfrak{e}_{6(-26)} = \mathbf{16}'_{-3} \oplus (\mathfrak{so}_{9,1} \oplus \mathbb{R})_0 \oplus \mathbf{16}_3;$$

3-grading : $\left(\mathbf{16}'_{-3}, \mathbf{16}_3\right)$ is a **Jordan pair**≠pair of JA's

$$\mathbf{16}' \oplus \mathbf{16} \simeq \underset{\text{split-complex Rosenfeld plane}}{T\left(\left(\mathbb{C}_s \otimes \mathbb{O}\right) P^2\right)} \; ;$$

$$\mathfrak{conf} \; : \; \mathfrak{e}_{7(-25)} = \mathbf{1}_{-2} \oplus \mathbf{32}^{(\prime)}_{-1} \oplus (\mathfrak{so}_{10,2} \oplus \mathbb{R})_0 \oplus \mathbf{32}^{(\prime)}_1 \oplus \mathbf{1}_2;$$

5-grading, contact type : $\mathbf{32}^{(\prime)}$ (**MW spinor**) is a the reduced **Freudenthal TS** over $J_3(\mathbb{H})$

$$\mathbf{32}^{(\prime)} \oplus \mathbf{32}^{(\prime)} \simeq \underset{\text{split-quaternionic Rosenfeld plane}}{T\left(\left(\mathbb{H}_s \otimes \mathbb{O}\right) P^2\right)} \; ;$$

$$\mathfrak{qconf} \; : \; \mathfrak{e}_{8(-24)} = \mathfrak{so}_{12,4} \oplus \mathbf{128}^{(\prime)} = \mathbf{14}_{-2} \oplus \mathbf{64}'_{-1} \oplus (\mathfrak{so}_{11,3} \oplus \mathbb{R})_0 \oplus \mathbf{64}_1 \oplus \mathbf{14}_2; \quad ;$$

5-grading, ext. Poincaré type : $\mathbf{64}^{(\prime)}$ (**MW spinor**) may realize a non-chiral **Kantor pair** over $J_3(\mathbb{H})$

$$\mathbf{128}^{(\prime)} = \mathbf{64}' \oplus \mathbf{64} \simeq \underset{\text{split-octonionic Rosenfeld plane}}{T\left(\left(\mathbb{O}_s \otimes \mathbb{O}\right) P^2\right)}$$

In some papers dating back to '97 and '98, Bars, Sezgin, Nishino, Rudychev and Sundell constructed SYM theories beyond $9+1$ space-time dimensions, namely in $10+2$, $11+3$ and $12+4$. The multi-time interpretation is delicate, and we will not enter in such subtleties here; let us only mention that the enhancement of the number of timelike dimensions in the sequence

$$\mathfrak{so}_{9,1} \to \mathfrak{so}_{10,2} \to \mathfrak{so}_{11,3} \to \mathfrak{so}_{12,4}$$

was considered as *multi-particle symmetry* : a single particle enjoys $\mathfrak{so}_{9,1}$ symmetry, while two, three and four particles acquire enhanced $\mathfrak{so}_{10,2}$, $\mathfrak{so}_{11,3}$ and $\mathfrak{so}_{12,4}$ symmetry, respectively. Also, in such an enhancement *Lorentz covariance is spoiled*, since putting all particles but one *on-shell* yields constant momenta that appear as null-vectors.

$\mathfrak{so}_{11,3}$ is the space-time, purely bosonic, symmetry Lie algebra of the $\mathcal{N} = (1,0)$ SYM in 11 + 3 space-time dimensions [Sezgin '97] [Bars '97] [Nishino '98] We will call it Sezgin-Bars-Nishino (SBN) superalgebra

By generalizing such results, the following global $\mathcal{N} = (1,0)$ chiral supersymmetry algebras in various dimensions were found ($\mathbf{n} \in \mathbb{N} \cup \{0\}$)

1. $D = (9 + 8\mathbf{n}) + 1$: [Rios, AM, Chester '19]

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu)_{\alpha\beta} P_\mu + (\gamma^{\mu_1 \cdots \mu_5})_{\alpha\beta} Z_{\mu_1 \ldots \mu_5} + \ldots + (\gamma^{\mu_1 \cdots \mu_{5+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{5+4\mathbf{n}}}.$$

2. $D = (10 + 8\mathbf{n}) + 2$:

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + (\gamma^{\mu_1 \cdots \mu_6})_{\alpha\beta} Z_{\mu_1 \ldots \mu_6} + \ldots + (\gamma^{\mu_1 \cdots \mu_{6+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{6+4\mathbf{n}}}.$$

3. $D = (11 + 8\mathbf{n}) + 3$:

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1 \cdots \mu_7})_{\alpha\beta} Z_{\mu_1 \ldots \mu_7} + \ldots + (\gamma^{\mu_1 \cdots \mu_{7+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{7+4\mathbf{n}}}.$$

4. $D = (12 + 8\mathbf{n}) + 4$:

$$\{Q_\alpha, Q_\beta\} = \eta_{\alpha\beta} Z + (\gamma^{\mu_1 \cdots \mu_4})_{\alpha\beta} Z_{\mu_1 \ldots \mu_4} + (\gamma^{\mu_1 \cdots \mu_8})_{\alpha\beta} Z_{\mu_1 \ldots \mu_8} + \ldots + (\gamma^{\mu_1 \cdots \mu_{8+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{8+4\mathbf{n}}}.$$

- $Z_{\mu_1\ldots\mu_p}$ are the bosonic p-form generators;
- $\gamma^{\mu_1\cdots\mu_p} \equiv \gamma^{\mu_1\cdots\mu_p} C^{-1}$, where C is the charge conjugation matrix;
- the maximal rank γ-matrices have a definite duality property, and hence the corresponding bosonic generator is taken to be self-dual;

1. $D = (9 + 8\mathbf{n}) + 1$: [Rios, AM, Chester '19]

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4. $D = (12 + 8\mathbf{n}) + 4$:

$$\{Q_\alpha, Q_\beta\} = \eta_{\alpha\beta} Z + (\gamma^{\mu_1\cdots\mu_4})_{\alpha\beta} Z_{\mu_1\ldots\mu_4} + (\gamma^{\mu_1\cdots\mu_8})_{\alpha\beta} Z_{\mu_1\ldots\mu_8} + \ldots + (\gamma^{\mu_1\cdots\mu_{8+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{8+4\mathbf{n}}}.$$

- the rhs's of anti-commutators below span the full symmetric space $\psi \otimes_s \psi$, where ψ is the MW spinor in the dimension under consideration;

- among the chiral superalgebras below, only the one in $D = (9+4\mathbf{n})+(1+4\mathbf{n})$ space-time dimensions is a *proper* Poincaré superalgebra, containing the momentum operator P_μ in the rhs;

1. $D = (9+8\mathbf{n})+1$: [Rios, AM, Chester '19]

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu)_{\alpha\beta} P_\mu + (\gamma^{\mu_1 \cdots \mu_5})_{\alpha\beta} Z_{\mu_1 \ldots \mu_5} + \ldots + (\gamma^{\mu_1 \cdots \mu_{5+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{5+4\mathbf{n}}}.$$

2. $D = (10+8\mathbf{n})+2$:

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + (\gamma^{\mu_1 \cdots \mu_6})_{\alpha\beta} Z_{\mu_1 \ldots \mu_6} + \ldots + (\gamma^{\mu_1 \cdots \mu_{6+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{6+4\mathbf{n}}}.$$

3. $D = (11+8\mathbf{n})+3$:

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1 \cdots \mu_7})_{\alpha\beta} Z_{\mu_1 \ldots \mu_7} + \ldots + (\gamma^{\mu_1 \cdots \mu_{7+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1 \ldots \mu_{7+4\mathbf{n}}}.$$

4. $D = (12+8\mathbf{n})+4$:

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- the symmetry property of the γ-matrices and the reality and conjugation properties of spinor representations are defined by two parameters: $D = s + t \mod(8)$ and $\rho = s - t \mod(8)$, where mod(8) denotes the *Bott periodicity*.

1. $D = (9 + 8\mathbf{n}) + 1$: [Rios, AM, Chester '19]

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2. $D = (10 + 8\mathbf{n}) + 2$:

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3. $D = (11 + 8\mathbf{n}) + 3$: **n**=0 : SBN superalgebra

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1\cdots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \ldots + (\gamma^{\mu_1\cdots\mu_{7+4\mathbf{n}}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{7+4\mathbf{n}}}.$$

SBN sequence

4. $D = (12 + 8\mathbf{n}) + 4$:

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Q : *can the SBN superalgebra be realized on a brane worldvolume ?*
We need a 11-brane in (*at least*) 3 timelike dimensions.

A1 : If the 11-brane is a *central extension* of a minimal, chiral superalgebra in $D = s + 3$, the smallest (odd) s is $s = 19 = 11 + 8$ ($\mathbf{n} = 1$ in *SBN sequence*) :

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + \underbrace{(\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7}}_{\text{electric 7-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_{11}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{11}}}_{\text{magnetic 11-brane}}$$

(1,0) superalgebra in 19+3 [Rios, AM, Chester '18, '19]

However, in $(1, 0)$ minimal, chiral superalgebra in $D = 19 + 3$ the 11-brane is the *magnetic* dual of the *electric* 7-brane.

A2 : If one wants to have the 11-brane as an **electric** *central extension* of a minimal, chiral superalgebra in $D = s+3$, the smallest (odd) s is $s = 27 = 11+16$ ($\mathbf{n} = 2$ in *SBN sequence*) :

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \underbrace{(\gamma^{\mu_1\ldots\mu_{11}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{11}}}_{\text{electric 11-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_{15}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{15}}}_{\text{magnetic 15-brane}}$$

(1,0) superalgebra in 27+3 [Rios, AM, Chester '18, '19]

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \underbrace{(\gamma^{\mu_1\ldots\mu_{11}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{11}}}_{\text{electric 11-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_{15}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{15}}}_{\text{magnetic 15-brane}}$$

(1,0) superalgebra in 27+3 [Rios, AM, Chester '18, '19]

In $D = 27+3$, the electric 11-brane has a **multi-time worldvolume**, with signature $11+3$, which can be used to provide a **worldvolume realization** for the $11+3$ SYM of [Bars '97] and [Sezgin '97]. In other words, the multi-time worldvolume of the *electric* 11-brane in $27+3$ can support the **SBN superalgebra** in $11+3$:

$$\{Q_\alpha, Q_\beta\} = \underbrace{(\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho}}_{\text{electric 3-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7}}_{\text{magnetic 7-brane}}$$

(1,0) SBN superalgebra in 11+3

The SBN superalgebra in $11+3$ gives rise to M-superalgebra ($\mathcal{N} = 1$ in $10+1$) (and thus to IIA $(1,1)$ superalgebra in $9+1$), **but also to** IIB $(2,0)$ superalgebra in $9+1$ (through $\mathcal{N} = 2$ in $10+1$) :

$$\begin{array}{ccccccc}
 & & & & & & (1,0)_{10+2} \\
 & & & & & & \downarrow \\
\mathrm{SBN} \equiv (1,0)_{11+3} & \longrightarrow & (1,1)_{11+1} & \longrightarrow & (1,0)_{11+1} & \longrightarrow & \mathrm{M} \equiv (\mathcal{N}=1)_{10+1} \\
 & & \downarrow & & & & \downarrow \\
 & & (\mathcal{N}=2)_{10+1} & & & & \mathrm{IIA} \equiv (1,1)_{9+1} \\
 & & \downarrow & & & & \downarrow \\
 & & \mathrm{IIB} \equiv (2,0)_{9+1} & \longrightarrow & \longrightarrow & \longrightarrow & (1,0)_{9+1}
\end{array}$$

[Rudychev, Sezgin, Sundell '97]

Conjecture : String dualities (*at least* the ones involving M-theory, type IIA and type IIB) may be traced back to transitions among orbits of the stratification of the MW semispinor representation space **64** under the non-transitive action of $Spin(11,3)$. Such transformations belong to the pseudo-Riemannian, *maximal* and homogeneous non-symmetric space

$$\frac{SL_{64}(\mathbb{R})}{Spin_{11,3}}, \ \dim_{\mathbb{R}} = 4,004, \ \chi := nc - c = 88.$$

The SBN superalgebra in $11+3$ gives rise to M-superalgebra ($\mathcal{N} = 1$ in $10+1$) (and thus to IIA $(1,1)$ superalgebra in $9+1$), **but also to** IIB $(2,0)$ superalgebra in $9+1$ (through $\mathcal{N} = 2$ in $10+1$) :

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 & & \downarrow & & & & \downarrow \\
 & & (\mathcal{N}=2)_{10+1} & & & & \mathrm{IIA} \equiv (1,1)_{9+1} \\
 & & \downarrow & & & & \downarrow \\
 & & \mathrm{IIB} \equiv (2,0)_{9+1} & \longrightarrow & \longrightarrow & \longrightarrow & (1,0)_{9+1}
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[Rudychev, Sezgin, Sundell '97]

Conjecture : String dualities (*at least* the ones involving M-theory, type IIA and type IIB) may be traced back to transitions among orbits of the stratification of the MW semispinor representation space **64** under the non-transitive action of $Spin(11,3)$.

Note : so(11,3) and **64** occur in the 0- and \pm1- graded parts Of the 5-grading of e8(-24) of extended Poincaré type, as discussed in [Cantarini-Ricciardo-Santi]

This is interesting, for the following reasons :

- $(D_7, \mathbf{64})$ is a θ-group action (in the sense of [Vinberg '76]), thus it has a *finite number* of nilpotent orbits. Moreover, the ring of invariant polynomials is 1-dimensional, and it is *freely generated* (i.e., with *no syzygies*) by I_8, an homogeneous polynomial of degree 8.
- The polynomial I_8 has been recently related to a remarkable rank-14 matrix factorization over **64** or **64'** [Abuaf & Manivel '19].
- The semidirect product $D_7 \ltimes \mathbf{64}$ appeared in relation to a mysterious algebra X_2 in the charting of Vogel's plane [Mkrtchyan '12].

$$\begin{array}{ccccccc}
 & & & & & & (1,0)_{10+2} \\
 & & & & & & \downarrow \\
\mathrm{SBN} \equiv (1,0)_{11+3} & \longrightarrow & (1,1)_{11+1} & \longrightarrow & (1,0)_{11+1} & \longrightarrow & \mathrm{M} \equiv (\mathcal{N}=1)_{10+1} \\
 & & \downarrow & & & & \downarrow \\
 & & (\mathcal{N}=2)_{10+1} & & & & \mathrm{IIA} \equiv (1,1)_{9+1} \\
 & & \downarrow & & & & \downarrow \\
 & & \mathrm{IIB} \equiv (2,0)_{9+1} & \longrightarrow & \longrightarrow & \longrightarrow & (1,0)_{9+1}
\end{array}$$

[Rudychev, Sezgin, Sundell '97]

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1\cdots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \underbrace{(\gamma^{\mu_1\cdots\mu_{11}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{11}}}_{\text{electric 11-brane}} + \underbrace{(\gamma^{\mu_1\cdots\mu_{15}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{15}}}_{\text{magnetic 15-brane}}$$

(1,0) superalgebra in 27+3 [Rios, AM, Chester '19]

Thus, we have found a **worldvolume realization/support** of $11+3$ *SBN superalgebra* (which is the higher-dimensional *avatar* of **both** *M*-superalgebra in $10+1$ and IIB superalgebra in $9+1$) by means of the largest *electric* *p*-brane central extension of an element (namely, for $\mathbf{n}=2$) of the *SBN sequence of superalgebras* (i.e., in terms of the *electric* 11-brane of $(1,0)$ minimal chiral superalgebra in $27+3$).

$$(1,0)_{10+2}$$
$$\downarrow$$
$$\mathrm{SBN} \equiv (1,0)_{11+3} \longrightarrow (1,1)_{11+1} \longrightarrow (1,0)_{11+1} \longrightarrow \mathrm{M} \equiv (\mathcal{N}=1)_{10+1}$$
$$(1,1)_{11+1} \downarrow (\mathcal{N}=2)_{10+1} \downarrow \mathrm{IIB} \equiv (2,0)_{9+1}$$
$$\mathrm{M} \equiv (\mathcal{N}=1)_{10+1} \downarrow \mathrm{IIA} \equiv (1,1)_{9+1} \downarrow (1,0)_{9+1}$$
$$\mathrm{IIB} \equiv (2,0)_{9+1} \longrightarrow \longrightarrow \longrightarrow (1,0)_{9+1}$$

[Rudychev, Sezgin, Sundell '97]

Hence one can consider the reduction from

$$\underbrace{\{Q_\alpha, Q_\beta\}}_{(1,0)\text{ superalgebra in }27+3} = (\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho} + (\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7} + \underbrace{(\gamma^{\mu_1\ldots\mu_{11}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{11}}}_{\text{electric 11-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_{15}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{15}}}_{\text{magnetic 15-brane}}$$

down to

$$\underbrace{\{Q_\alpha, Q_\beta\}}_{\mathcal{N}=1\text{ superalgebra in }26+1} = (\gamma^{\mu})_{\alpha\beta} P_\mu + (\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + (\gamma^{\mu_1\ldots\mu_5})_{\alpha\beta} Z_{\mu_1\ldots\mu_5} + (\gamma^{\mu_1\ldots\mu_6})_{\alpha\beta} Z_{\mu_1\ldots\mu_6}$$
$$+ (\gamma^{\mu_1\ldots\mu_9})_{\alpha\beta} Z_{\mu_1\ldots\mu_9} + \underbrace{(\gamma^{\mu_1\ldots\mu_{10}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{10}}}_{\text{electric 10-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_{13}})_{\alpha\beta} Z_{\mu_1\ldots\mu_{13}}}_{\text{magnetic 13-brane}},$$

consistent with the representation theoretical counting

$$\underbrace{\mathbf{8,192} \otimes_s \mathbf{8,192}}_{8{,}192\cdot 8{,}193/2=33{,}558{,}528} = \underbrace{\mathbf{27}}_{\text{1-form }P_\mu} \oplus \underbrace{\mathbf{351}}_{\text{2-form M2}} \oplus \underbrace{\mathbf{80,730}}_{\text{5-form M5}} \oplus \underbrace{\mathbf{296,010}}_{\text{6-form M6}} \oplus \underbrace{\mathbf{4,686,825}}_{\text{9-form M9}} \oplus \underbrace{\mathbf{8,436,285}}_{\text{10-form M10}} \oplus \underbrace{\mathbf{20,058,300}}_{\text{13-form M13}},$$

Considering the worldvolume of the maximal electric p-brane extension of such superalgebras, this reduction corresponds to reducing *SBN superalgebra*

$$\{Q_\alpha, Q_\beta\} = \underbrace{(\gamma^{\mu\nu\rho})_{\alpha\beta} Z_{\mu\nu\rho}}_{\text{electric 3-brane}} + \underbrace{(\gamma^{\mu_1\ldots\mu_7})_{\alpha\beta} Z_{\mu_1\ldots\mu_7}}_{\text{magnetic 7-brane}}$$

SBN (1,0) superalgebra in 11+3, **supported** by the WV of the **electric** 11-brane of (1,0) superalgebra in 27+3

down to *M-superalgebra*

$$\{Q_\alpha, Q_\beta\} = (\gamma^\mu)_{\alpha\beta} P_\mu + \underbrace{(\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu}}_{\text{electric 2-brane M2}} + \underbrace{(\gamma^{\mu_1\ldots\mu_5})_{\alpha\beta} Z_{\mu_1\ldots\mu_5}}_{\text{magnetic 5-brane M5}},$$

$\mathcal{N}$=1 M-superalgebra in 10+1, **supported** by the WV of the **electric** 10-brane of $\mathcal{N}$=1 superalgebra in 26+1

consistent with the representation theoretical counting

$$\underbrace{\mathbf{32} \otimes_s \mathbf{32}}_{32\cdot 33/2=528} = \underbrace{\mathbf{11}}_{\text{1-form } P_\mu} \oplus \underbrace{\mathbf{55}}_{\text{2-form M2}} \oplus \underbrace{\mathbf{462}}_{\text{5-form M5}}.$$

Considering the worldvolume of the maximal electric p-brane extension of such superalgebras, this reduction corresponds to reducing *SBN superalgebra*

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This puts forward the following *conjecture* [Rios, AM, Chester '19] :
***M*-theory can be realized as a *worldvolume theory* of an electric 10-brane in a higher $(26+1)$-dimensional space-time**

Since in $26+1$ an electric 10-brane is **dual** to a *magnetic* 13-brane, the conjecture seemingly implies the existence of a mysterious **"dual *M*-theory"**, realized as a **worldvolume theory of a *magnetic* 13-brane in a higher $(26+1)$-dimensional space-time.**

Q : Why $26+1$ signature is interesting?

A1 : Because in $26+1$ space-time dimensions it has been formulated the **bosonic *M*-theory**
[Horowitz & Susskind '01]

Horowitz and Susskind conjectured there exists a strong coupling limit of bosonic string theory that generalizes the relation between M-theory and superstring theory, called bosonic M-theory The main evidence for the existence of such a $D=26+1$ theory comes from the dilaton and its connection to the coupling constant, with the dilaton entering the action for the massless sector of bosonic string theory as

$$S=\int d^{26}x\sqrt{-g}e^{-2\phi}\left[R+4\nabla_{\mu}\phi\nabla^{\mu}\phi-\frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}\right],$$

in a way similar to type IIA string theory, as if representing the compactification scale of a Kaluza-Klein reduction from $D=26+1$ space-time dimensions with $\mathbf{324}\rightarrow\mathbf{299}+\mathbf{24}+\mathbf{1}$ graviton decomposition. However, while in type IIA string theory the existence of a vector boson in the string spectrum implies an S^1 compactification, in closed bosonic string theory there is no massless vector. For this reason, an $S^1/\mathbb{Z}_2$ orbifold compactification of bosonic M-theory was proposed as its origin The bosonic string is then a stretched membrane across the interval; the orbifold breaks translation symmetry, thus the massless vector does not appear.

In *"Monstrous M-theory"* [AM, Rios, Chester '20], a **purely bosonic** gravity theory in $26+1$ has been proposed, such that it contains Horowitz & Susskind's bosonic M-theory as a subsector, and it relates, upon reduction to $25+1$, to the smallest representation of the **Monster group** $\mathbb{M}$, of dimension $\mathbf{1}$ (dilaton ϕ in $25+1$) and $\mathbf{196,883}$. Such a theory has been named **Monstrous M-theory**, or **M^2-theory**.

M^2-theory is purely bosonic. A certain subsector of it, coupled to a Rarita-Schwinger 1-form spinor field in $26+1$, enjoys the same number of massless bosonic and fermionic degrees of freedom ($B=F$), a necessary condition for supersymmetry to exist. A Lagrangian has been conjectured in [AM, Rios, Chester '20] for such a theory, but **no further evidence for supersymmetry so far**.

If one proceeds as it is done in M-theory in $10+1$, then $B \neq F$.

To recap :

- By generalizing some results by [Sezgin '97], [Bars '97] and [Nishino '98], we considered the minimal, chiral $(1, 0)$ non-standard global superalgebra in $27 + 3$ space-time dimensions, which can be centrally extended by an electric 11-brane and its 15-brane magnetic dual [Rios, AM, Chester '18].

- We proposed the (multi-time) worldvolume of the 11-brane itself as support for the $(1, 0)$ SYM theory in $11 + 3$ space-time dimensions introduced in [Sezgin '97] and [Bars '97].

- As discussed in [Rudychev, Sezgin, Sundell '97], the $(1, 0)$ superalgebra in $11 + 3$ dimensions reduces to

  i) the $\mathcal{N} = 1$ M-superalgebra in $10+1$ (and thus to the maximal non-chiral $(1, 1)$ type IIA superalgebra in $9 + 1$);

  and to

  ii) the maximal chiral $(2, 0)$ type IIB superalgebra in $9 + 1$.

- Thus, we proposed the reduced (single-time) 10-brane worldvolume theory in $10+1$ space-time dimensions as a **worldvolume realization of M-theory** (this also entails the existence of a would-be "*dual* worldvolume M-theory" realized as a worldvolume theory in $13+1$) [Rios, AM, Chester '19].

- The space-time reduction $27+3 \longrightarrow 26+1$ induces a $11+3 \longrightarrow 10+1$ reduction for the worldvolume of the largest electric brane which centrally extends the corresponding minimal superalgebra, and thus it yields a natural map from

  from bosonic M-theory [Horowitz & Susskind '01] in $D=26+1$

  to to $D=10+1$ M-theory.

- The worldvolume picture may provide a quite natural explation of the origin of the $E_8 \otimes E_8$ heterotic string (through anomaly cancellation); indeed, the Horava-Witten work on heterotic M-theory [Horava & Witten '96] requires a manifold with boundary in $10+1$ dimensions, which occurs naturally if the $(10+1)$-dimensional manifold is itself a brane worldvolume with boundary.

- Thus, we proposed the reduced (single-time) 10-brane worldvolume theory in $10+1$ space-time dimensions as a **worldvolume realization of M-theory** (this also entails the existence of a would-be "*dual* worldvolume M-theory" realized as a worldvolume theory in $13+1$) [Rios, AM, Chester '19].

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Remark. By suitably changing the space-time signatures, the same conclusions can be obtained for the other versions of M-theory, named M'-theory (in $6+5$) and M^*-theory (in $10+2$) [Hull '98]; indeed, besides $10+1$, the signatures $6+5$ and $10+2$ are the only other signatures which allow for a **real** 32-dimensional spinor representation of the spin group in $D=11$.

Q : How can the 16 transverse dimensions be described ?

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A : They sit in the vector module **16** of $Spin(16)$. By virtue of *Dynkin's "anomalous" embedding* [Dynkin '57, Ramond '03], the smallest subgroup of $Spin(16)$ such that the **16** of $Spin(16)$ stays irreducible is its maximal, non-symmetric subgroup $Spin(9)$:

$$\begin{aligned} Spin(16) &\supset Spin(9); \\ \mathbf{16} &= \mathbf{16}, \end{aligned}$$

which is a consequence of the self-conjugatedness of the spinor irrepr. of the spinor irrepr. of $Spin(9)$ (i.e., $\exists!\mathbf{1} \in \mathbf{16} \otimes_s \mathbf{16}$). Thus, the "minimal" approach to the 16 transverse dimensions is to regard them as fitting the spinor module of $Spin(9)$.

From the theory of cosets, the **16** of $Spin(9)$ describes the tangent space to the symmetric space

$$\frac{F_{4(-52)}}{Spin(9)} \simeq \mathbb{O}P^2,$$

the Cayley-Moufang plane, which is the largest projective octonionic space.

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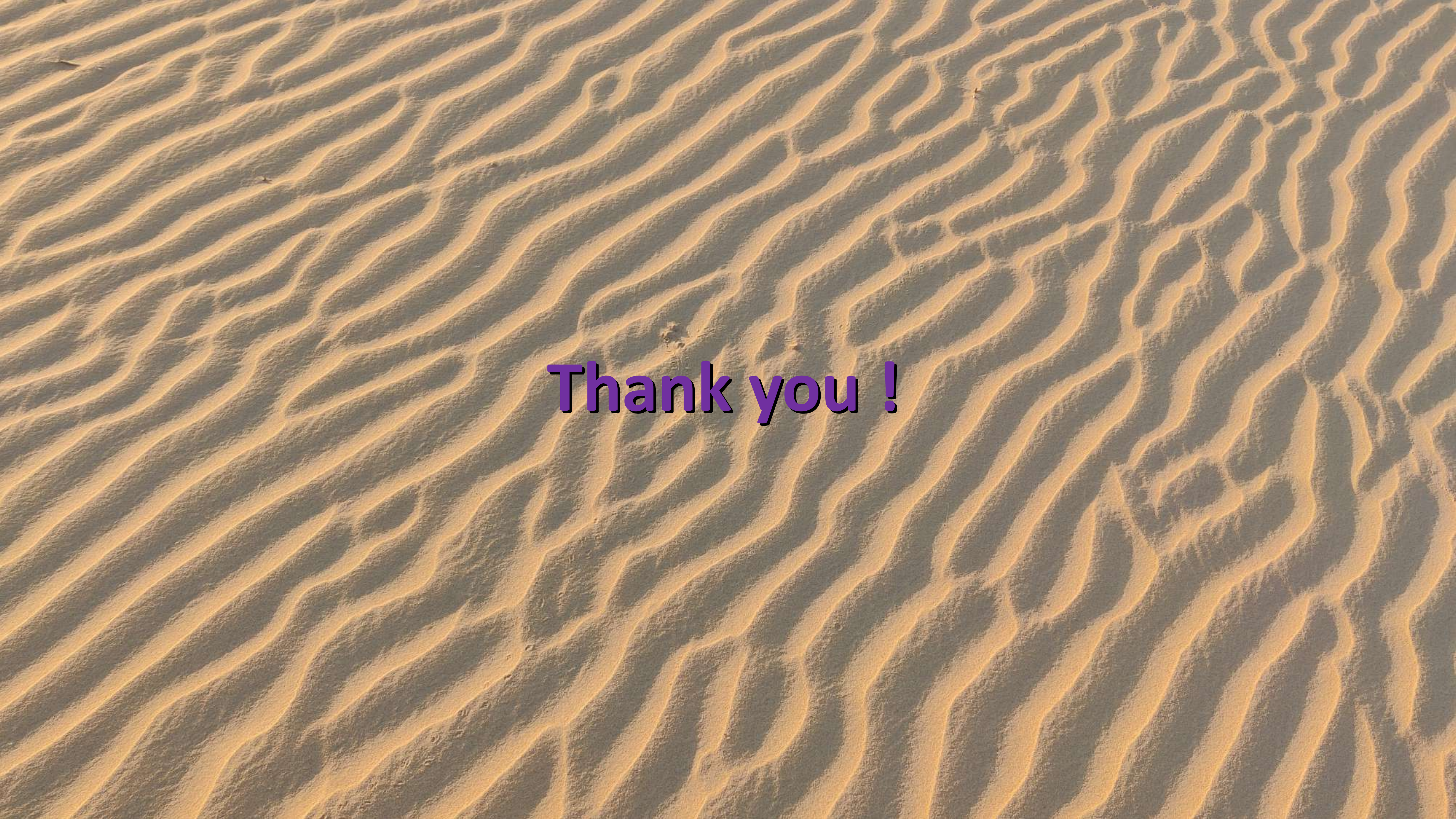
Thus, a **direct consequence of Dynkin's "anomalous" embedding** is the description of the 16 spacial dimensions transverse to the 10-brane in $26+1$ in terms of $\mathbb{O}P^2$.


Thank you !