# Limitations of Geometric Engineering: Implications for Model Building


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- Typically, one studies spacetimes of the form $X \times M$, where $X$ is a noncompact geometric space containing the local feature in question, and $M$ is Minkowski space (or sometimes de Sitter or anti de Sitter space).
- My theme today: globalizing the local constructions can be delicate and dangerous.

# ADE singularities

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- These can be studied locally as $X_0 = \mathbb{C}^2/\Gamma$, with their resolutions given by ALE spaces $X$. Much can be learned about the rôle which such singularities play in string vacua by studying $X_0 \times M$ or $X \times M$. The presence of the singularity leads to a nonabelian gauge group in IIA string theory, M-theory and F-theory.

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- However, globalizing these singularities is a different matter. If one asks for a Calabi–Yau 2-fold (i.e., a K3 surface) with such a singularity, then the rank of the gauge group is bounded by 19.
- In this case, it is possible to explicitly describe which gauge groups can occur (cf. DRM, Invent. Math. 75 (1984) 105–121).

- Suppose one wanted to do some kind of statistical analysis of gauge groups in this context. Based on the local analysis, one would likely conclude that groups of type SU($n$) or SO($2n$) occur with probability 1, and the groups of type $E_6$, $E_7$, and $E_8$ occur with probability 0.

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- The global story is much different: because the ranks are bounded, the $E_n$ groups play a significant rôle in the overall statistics, and occur with probability greater than 0.

# 7-branes in F-theory

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- The essential data for an F-theory vacuum is the (complexified) IIB coupling, which is determined from the $j$-invariant of the family of elliptic curves

  $$y^2 = x^3 + px + q,$$

  where $p$ and $q$ are functions on the space $X$ being used to (partially) compactify the IIB string.

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where $p$ and $q$ are functions on the space $X$ being used to (partially) compactify the IIB string.
- The 7-brane wraps the discriminant locus $\{\Delta = 0\}$ of the above equation, where

$$\Delta = 4p^3 + 27q^2.$$

When $X$ is local, we can assume that $p$ and $q$ have no common zeros.

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- At a common zero of $p$ and $q$, $\{\Delta = 0\}$ is itself singular (it has a "cusp" singularity) and the standard analysis of a 7-brane as a submanifold is not correct.
- In fact, there are a number of global issues with 7-branes in F-theory; in six dimensions, many of them can be related to anomaly cancellation in the effective six-dimensional theory (cf. Grassi–DRM, math.AG/0005196).

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# Open strings and D-branes

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- If the D-brane charges do not cancel, leaving a net Ramond–Ramond charge in the vacuum, there is a tadpole anomaly.
- This tadpole is suppressed in local models, since any excess Ramond–Ramond charge can escape to infinity. But it cannot be ignored in global models.
- Still, it has seemed reasonable to study different sources of D-brane charge in a local way, and assemble them later into a global model. This approach has recently been called into question.

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- Recently, in work of Walcher, arXiv:0712.2775 [hep-th], and Cook–Ooguri–Yang, arXiv:0804.1120 [hep-th], it was realized that beyond tree-level, the open topological string theory itself is sensitive to the presence of a net Ramond–Ramond charge, when studied in the context of global models.

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- That is, if one attempts to calculate open topological string amplitudes in a background with net Ramond–Ramond charge (using techniques pioneered by Walcher a few years ago), the answers reveal a new kind of anomaly in the topological string.

- This suggests that the purely local contributions to topological string amplitudes which have been computed in many situations will require unexpected corrections from other sectors of the background. It is not clear at present which conclusions about local physics escaped unchanged.

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- In particular, model building which has relied on a decoupling of the D-brane and orientifold plane sectors of the theory may need to be reexamined.

# Conifold transitions

- One situation in which the relationship of local and global approaches can be made clear is conifold transitions in open string theory, and the relationship to Gopakumar–Vafa large $N$ duality.

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- The local analysis starts with a 3-cycle in a local Calabi–Yau threefold $X$ which is the vanishing cycle for an ordinary double point singularity (when the complex structure parameters are specialized), and the IIA string on $X \times M$ with a D6-brane wrapping the 3-cycle $N$ times. Passing to the double point, and then blowing it up to obtain a 2-sphere, one finds a flux vacuum with $N$ units of Ramond–Ramond flux on the 2-sphere.

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- This transition has a lift to M-theory which was studied by Atiyah–Maldacena–Vafa, hep-th/0011256, Atiyah–Witten, hep-th/0107177, and others. The local M-theory space $Y$ is a neighborhood of $S^3 \times S^3$, and the open string conifold transition is seen as a smooth deformation of M-theory vacua.

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- In the context of the open string model, if we use the coefficients of the homology relation as the $N_i$ on the various 3-cycles, then the net D6-brane charge is zero! Thus, the global requirement from geometry (in order to get a Kähler manifold after the transition) takes care of the tadpole anomaly in string theory as well as the open string anomaly discussed above.

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- How does this lift to M-theory? (cf. DRM, to appear).

- Given a compact Calabi–Yau manifold $X$ and a collection of 3-cycles $Z$, the lift to M-theory should be a G2-manifold $Y$ with a circle action that has fixed points along the 3-cycles $Z \subset Y$.

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- At present, unlike the local case, we do not have existence theorems for such G2-manifolds. However, the topological structure (and hence the classical moduli space) of our postulated G2-manifold can be analyzed. In general, one should consider both fixed points of a circle action, and allow the situation in which the Chern class of the circle bundle over $X - Z$ is non-trivial. (These are the flux vacua.)

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- (These are "extra" when compared with the usual calculation of moduli for the case $X \times S^1$.) These extra parameters allow for the Atiyah et al. explanation of the local conifold transition to be present in the global moduli space. (Of course, we don't know for sure that this happens since we cannot construct the G2-manifolds in question by this method.)

# Lifting to M-theory

- Emboldened by our success for conifold transitions, let us suggest an approach to address global issues for arbitrary IIA vacua with both D-branes and orientifold planes: we propose that liftability to M-theory should be the criterion.

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- The expectation from the past half-decade of string phenomenological studies is that there will be a large number of such vacua, and hence a large number of G2-manifolds which lift them to M-theory.
- Do these all exist? At the moment, the best we can do is to analyze topological restrictions on such manifolds. As indicated above, I've already done this for cases without orientifold planes; work on the more general case is in progress.

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- Statistical conclusions drawn from sampling local models can be misleading.
- Even the local analysis for an open string model, done using topological string theory, can be misleading.
- Global approaches, such as F-theory instead of orientifolds of IIB, or lifing to M-theory instead of orientifolds of IIA, hold much promise for resolving global issues.