Duality and Hadrodynamics†

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1 Introduction

We would like to explore here possible dynamical properties of hadrons suggested by the duality principle of Dolen, Horn and Scmid, and in particular by its mathematical realization due to Veneziano. Our primary concern will be not in the very interesting details of the mathematical properties that the Veneziano model seems to possess in abundance, but rather in guessing at the internal structure and dynamics of hadrons which underlie the Veneziano model, recognizing that the latter is in all likelihood only an approximate and imperfect representation of the former.

On crucial step we take in interpreting the Veneziano model is factorization. Of course this is a trivial problem, at least in principle if not in practice, if one is dealing with a given four-point amplitude. But it becomes a very restrictive condition if one demands that the same set of resonances saturate all $n-$point amplitudes. Perhaps such a condition is unwarranted; in discussing as revolutionary a concept as duality one may have to give up all the conventional notions about resonances, e.g. that the intrinsic properties of a resonance is independent of how it is prepared, and a multiple resonance can be analyzed in terms of the individual resonances, etc. Nevertheless, the fact that a set of resonances have been found to saturate all the $n-$point Veneziano amplitudes of the standard variety is very significant. It suggests that our conventional notions about resonances still make some sense, and the know techniques of field theory can be applied to them.

The condition of factorizability immediately rules out an ad hoc addition of satellite terms to a scattering amplitude because it would in general destroy factorization unless more and more resonances are introduced. If satellite terms do arise, they must do so in a well defined and self-consistent way.

As it stands now, the Veneziano model is still beset with many difficulties. For all its mathematical elegance, its practical successes are few. It would therefore be appropriate to list here its basic predictions as well as difficulties without going into the details.

Basic predictions.

1) Linearly rising trajectories. This is in accordance with observation. Whether the trajectories really keep rising indefinitely or not is of course an open question, but it seems to be a valid simplification to assume that they do.

2) Regularly spaced daughter trajectories, implying a highly degenerate level structure. Again the actual degeneracy may be only approximate, but these

daughters must exist if the model makes sense at all. So far there is some evidence for ϵ (daughter of ρ), but none for ρ' and ϵ' (daughter of f).
3) The factorized Veneziano model implies an even higher degree of degeneracy. At each level many distinguishable states having the same spin exist. The number of states increases with energy as $\sim \exp(cE)$. This situation bears a striking similarity to Hagedorn's model of high energy reactions.

Unsolved practical problems.
1) Baryons. We do no yet have a quantitatively satisfactory picture of meson-baryon and baryon-baryon scattering based on the Veneziano model, in spite of the fact that duality was first discovered in meson-baryon scattering.
2) A general satisfactory unification of the quark model (or its $SU(3)$, $SU(6)$ and chiral $SU(3)$ aspects) with the Veneziano model does not exist yet.
3) No convincing theory of form factors exists.
4)We do not know what the Pomeron is within the framework of the Veneziano model.

We must emphasize, however, that there are numerous attempts and speculations regarding all these problems.

More fundamental difficulties.
1) Unitarity. The original Veneziano model is a zero width approximation. The amplitude wildly oscillates with energy, and only after averaging over an interval does it reproduce the smooth Regge behavior at high energy. If a basic Hamiltonian for the model is given, unitarization might be formally carried out by taking into account higher order processes, although the highly singular nature of the Hamiltonian casts doubt about its meaningfulness.
2) A more serious problem, however, is the existence of ghosts. Here we mean by ghosts unphysical particles having a) negative probability, or b) spacelike momenta (tachyons), or both. The levels of the factorized Veneziano model contain those of four-dimensional harmonic oscillators, where the timelike excitations have an intrinsic negative norm. This does not necessarily mean that the Breit-Wigner residue of a partial wave projection of a given amplitude is not positive. Contributions from many degenerate states can add up to a positive value, but there is no guarantee that this will always happen. Fubini and Veneziano have found a Ward-like identity which accomplishes this cancellation to a certain extent, and Virasoro has extended their result. They rely, however, on very special properties of the Hamiltonian, and it is not clear whether and how these can be preserved in general. Besides, Virasoro's scheme still leaves us with a tachyon ghost ($m^2 = -1$). The killing of

tachyons is easy in special cases like $\pi - \pi$ scattering (where the ρ trajectory has a potential tachyon), but a general prescription in a factorized model seems very complicated, if not impossible.

We have emphasized the ghost problem because this is not peculiar to the Veneziano model alone, but it is rather a common disease afflicting all attempts at a description of hadron states as infinite multiplets. The program of current algebra saturation, as well as the use of infinite-component wave equations, have floundered on the same difficulty. To a certain extent, the two kinds of ghosts seem to be complementary: The use of finite-dimensional Lorentz tensors (as in the factorized dual model) involves negative probabilities, whereas infinite-dimensional unitary representations in general lead to tachyons.

If these ghosts are so difficult to eliminate, why not accept them and look for them? Maybe ghosts of one kind or the other do exist, which would make either T.D.Lee or G.Feinberg happy (or both, plus Sudarshan and others). But the trouble is that the extent to which these ghosts appear in a particular problem seems to depend on one's cleverness and ability to avoid them. How many of the ghosts are "real"? This is the most serious question of principle that haunts us, especially us the theorists.

2 Factorized Veneziano model

As has been shown by various people, the n−point dual amplitude for scalar external particles can be factorized in terms of a set of harmonic oscillators corresponding to an elastic string of finite intrinsic length. In classical theory, the motion of a free mass point can be derived from the action integral

$$I = -m \int d\tau, \quad d\tau^2 = -dx_\mu dx^\mu \quad (\text{metric } (-+++)) \tag{1}$$

Alternatively one may take

$$I' = \frac{1}{2} m \int \left(\frac{dx_\mu}{d\tau} \frac{dx^\mu}{d\tau} - 1 \right) d\tau \tag{2}$$

τ being an independent parameter, and $x^\mu(\tau)$ the dynamical variables. (The constant in the integrand is added for convenience.) This form is more suitable for the transition to quantum theory. Because of the translational invariance under $\tau \to \tau + c$ and $x^\mu \to x^\mu + a^\mu$, both the Hamiltonian and the momenta

$$\begin{aligned} H &= (p_\mu p^\mu + m^2)/2m \\ p^\mu &= m dx^\mu/d\tau \end{aligned} \tag{3}$$

are conserved. By imposing the constraint

$$\frac{dx_\mu}{d\tau}\frac{dx^\mu}{d\tau} = -1 \tag{4}$$

we can normalize the parameter τ, which then becomes the proper time of the mass point. This condition (4) amounts to

$$H = 0. \tag{5}$$

In quantum mechanics, one postulates the commutation relations

$$[x^\mu, p^\nu] = ig^{\mu\nu} \tag{6}$$

and the Schrödinger equation

$$i\frac{\partial}{\partial\tau}\Psi = H\Psi \tag{7}$$

Eq.(5) is to be replaced by

$$H\Psi = 0 \tag{8}$$

which is nothing but the Klein-Gordon equation

$$(p^\mu p_\mu + m^2)\Psi = 0. \tag{9}$$

We see thus the usefulness of the proper-time formalism. If we just integrate Eq.(7), we get

$$\Psi(\tau) = \exp[-i(p^2 + m^2)\tau/2m]\Psi(0) \tag{10}$$

or

$$\begin{aligned} (\Psi(x';\tau), \Psi(x;0)) &= \langle x'|\exp[-i\tau(p^2+m^2)/2m]|x\rangle \\ &= (-im^2/4\pi^2)\exp[imx^2/2\tau - i\tau m/2] \\ &= (-im^2/4\pi^2)\exp[im\tau\{(x/\tau)^2 - 1\}/2] \end{aligned} \tag{11}$$

This is the transition amplitude $x \to x'$ after an elapsed "time" τ. Integrating over τ from 0 to ∞ (we might say it does not matter how much time it has elapsed between the events x and x'), we obtain the Feynman propagator

$$\frac{1}{2m}\int_0^\infty e^{-i\tau(p^2+m^2)/2m} d\tau = -i(p^2 + m^2 - i\epsilon)^{-1}. \tag{12}$$

It is well known that the above results may be interpreted or derived from the path integration method.

After this digression, let us come to the elastic string. We can imagine it to be the limit of a chain of N mass points as $N \to \infty$. Each mass point will trace out a world line, so that in the limit we are dealing with a two-dimensional world sheet. This sheet may be parametrized by two intrinsic coordinates ξ ($0 \le \xi \le \pi$, let us say) and τ ($-\infty < \tau < \infty$), corresponding to spacelike and timelike coordinates. We assume the action integral

$$I = \frac{1}{4\pi}\iint \left(\frac{\partial x_\mu}{\partial \tau}\frac{\partial x^\mu}{\partial \tau} - \frac{\partial x_\mu}{\partial \xi}\frac{\partial x^\mu}{\partial \xi}\right) d\xi d\tau \tag{13}$$

from which follows the equation

$$(\partial^2/\partial\tau^2 - \partial^2/\partial\xi^2)x^\mu = 0 \tag{14}$$

with the boundary condition

$$\partial x^\mu/\partial\xi = 0 \qquad \text{at } \xi = 0,\ \pi. \tag{15}$$

Duality is essentially a result of the symmetry between ξ and τ though it is not yet a perfect symmetry because of the differences in domain and metric. We have chosen the hyperbolic form because only then can one formulate the Hamiltonian principle. Actually it turns out that in computing the scattering amplitudes a switch to an elliptic form through the change $\tau \to -i\eta$ brings out duality more explicitly.

Eq.(13) is invariant under the translations $\tau \to \tau + c$ and $x^\mu \to x^\mu + a^\mu$, which imply the conservation laws

$$\begin{aligned} H &= \int [\pi p_\mu p^\mu + \frac{1}{4\pi}\left(\frac{\partial x_\mu}{\partial \xi}\right)\left(\frac{\partial x^\mu}{\partial \xi}\right)] d\xi = \text{const.} \\ p^\mu &= (1/2\pi)(\partial x^\mu/\partial\tau), \\ P^\mu &= (1/2\pi)\int(\partial x^\mu/\partial\tau)d\xi = \int p^\mu d\xi = \text{const.} \end{aligned} \tag{16}$$

in direct analogy with Eq.(3). A normal mode decomposition of Eqs.(14) and (15) yields

$$\begin{aligned} x^\mu &= x_0^\mu + 2\sum_{n=1}^{\infty} x_n^\mu \cos n\xi \\ p_n^\mu &= \partial x_n^\mu/\partial\tau \ \ (n \neq 0), \ \ p_0^\mu = \frac{1}{2}\partial x_0^\mu/\partial\tau \\ H &= p_{0\mu}p_0^\mu + \frac{1}{2}\sum_{n=1}^{\infty}(p_{\mu n}p_n^\mu + n^2 x_{\mu n}x_0^\mu \\ P^\mu &= p_0^\mu \end{aligned} \tag{17}$$

We may interpret x_0^μ and p_0^μ as the center-of-mass coordinates and momenta.

When the system is quantized, we get the familiar expression

$$\begin{aligned} & H = P_\mu P^\mu + H_0, \ \ H_0 = \sum_{n=1}^{\infty} n a_{\mu n}^\dagger a_n^\mu (+\text{c number}) \\ & x_n^\mu = (a_n^\mu + a_n^{\mu\dagger})/\sqrt{2n} \\ & p_n^\mu = -i(a_n^\mu - a_n^{\mu\dagger})\sqrt{n/2} \ \ (n \neq 0) \\ & [a_n^\mu, a_n^{\nu\dagger}] = g^{\mu\nu}\delta_{nm} \end{aligned} \tag{18}$$

By imposing the subsidiary condition

$$(H - \alpha)\Psi = 0 \tag{19}$$

we can single out an infinite tower of states with the mass spectrum

$$M^2 = H_0 + \alpha = \sum_{\mu,n} nN_{\mu n} + \alpha \tag{20}$$

In order to construct dual scattering amplitudes we introduce an external scalar field φ and postulate

$$H = P_\mu P^\mu + H_0 + g : \varphi(x(\xi = 0)) : \tag{21}$$

We will not discuss how this leads to $n-$point dual amplitudes in the multi-peripheral configuration since it is well known.

We now add several remarks.

1) The condition $0 \leq \xi \leq \pi$ fixes a fundamental scale of length. ξ is here

measured in $(GeV/c)^{-1}$ to fit the trajectory slope of ~ 1. Whether the system is to be interpreted as a rubber string or a rubber band depends on the boundary condition to be imposed. The rubber band, as twice as many modes as the string, but the difference does not show up in a case like Eq.(21) because those modes which have nodes at $\xi = 0$ cannot be excited. Clearly there will be differences if one tries to extend the model, and this will be an important point in constructing a general theory of hadrons.
2) For a point particle Eq.(1) has a purely geometric meaning (as the length of a world line), but Eq.(2) does not since it depends on the scale (gauge) of the unphysical parameter τ. In the case of the string, on the other hand, Eq.(13) is invariant under the scaling $\tau, \xi \to \lambda\tau, \lambda\xi$. This is one aspect of the conformal invariance of two-dimensional Laplace equations, a property which has widely been utilized to study the Veneziano model.

Nonetheless, Eq.(13) is not a purely geometrical quantity. For curiosity, then, let us try to construct a geometric action integral as one does in general relativity. Obviously a natural candidate for it is the surface area of the two-dimensional world sheet; another would involve its Riemann curvature. The sheet is imbedded in the Minkowskian 4-space, so one can parametrize its points as $y^\mu(\xi^0, \xi^1)$, $(\xi^0 \sim \tau, \xi^1 \sim \xi)$. The surface element is a $\sigma-$tensor

$$\begin{aligned} d\sigma^{\mu\nu} &= G^{\mu\nu} d^2\xi, \\ G^{\mu\nu} &= \partial(y^\mu, y^\nu)/\partial(\xi^0, \xi^1) \end{aligned} \tag{22}$$

whereas its line element is

$$\begin{aligned} ds^2 &= g_{\alpha\beta} d\xi^\alpha d\xi^\beta \qquad (\alpha, \beta = 0, 1) \\ g_{\alpha\beta} &= (\partial y_\mu/\partial\xi^\alpha)(\partial y^\mu/\partial\xi^\beta) \end{aligned} \tag{23}$$

A possible action integral would be

$$I = \int |d\sigma_{\mu\nu} d\sigma^{\mu\nu}|^{1/2} = \iint |2\mathrm{det}g|^{1/2} d^2\xi \tag{24}$$

to be compared with the old one (13) which can be written $(y \to x)$

$$I = -\frac{1}{4\pi} \iint g_{\alpha\beta} \overset{\circ}{g}{}^{\alpha\beta} \, d^2\xi, \quad \overset{\circ}{g}{}^{\alpha\beta} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{25}$$

It is obvious that Eq.(24) leads to nonlinear equations. More complicated equations involving curvature would be not only nonlinear, but also have

higher derivatives.
3) It is sometimes useful to consider the "energy-momentum tensor" in the (ξ, τ) space:

$$T_{\alpha\beta} = \frac{1}{2\pi}(g_{\alpha\beta} - \frac{1}{2} \overset{\circ}{g}_{\alpha\beta}\, g_{\gamma\delta}\, \overset{\circ}{g}^{\gamma\delta}) \tag{26}$$

In particular, let us take its space integral over a test function $f(\xi)$,

$$\bar{T}_{\alpha\beta}[f] = \int_0^\pi T_{\alpha\beta}(\xi) f(\xi) d\xi. \tag{27}$$

By virtue of the canonical commutation relations, they generate a commutator algebra

$$\begin{aligned}
&\left[\ (\bar{T}_{00} \pm \bar{T}_{01})[f],\quad (\bar{T}_{00} \pm \bar{T}_{01})[g]\ \right] = -2i(\bar{T}_{00} \pm \bar{T}_{01})[h],\\
&\left[\ (\bar{T}_{00} \pm \bar{T}_{01})[f],\quad (\bar{T}_{00} \mp \bar{T}_{01})[g]\ \right] = 0,\\
&h = f'g - fg'
\end{aligned} \tag{28}$$

as an integral form of the Schwinger conditions. These relations, when applied to the set $f_n = 1 - e^{-2in\xi}$, amount to

$$\begin{aligned}
&\left[\ L_n^\pm,\quad L_m^\pm\ \right] = 2(n-m)L_{n+m}^\pm\\
&\left[\ L_n^\pm,\quad L_m^\mp\ \right] = 0\\
&L_n^\pm = (\bar{T}_{00} \pm \bar{T}_{01})[e^{2in\xi}],\quad L^\pm[f_n] = L_0^\pm - L_{-n}^\pm
\end{aligned} \tag{29}$$

These operators $L^\pm[f_n]$ have been found useful in generating the various gauge operations.
4) As we have mentioned already, the most serous defect of the above formulation is the indefinite metric that appears in defining the covariant commutation relations (18). The mass operator (20), however, acquires as a result the nice property of being positive. The transition from a classical to quantum picture of 4-dimensional harmonic ocsillators is a drastic one. A wave function in coordinate space would behave like $\exp[-c^2(\underline{x}^2 - x_0^2)]$, which explodes in the timelike direction. We could actually insist that it should behave instead like $\exp[-c(\underline{x}^2 + x_0^2)]$. This would amount to interchanging the creation and annihilation operators and using positive metric for the time component. But then the mass operator would not be positive, and moreover

each level would become infinitely degenerate. In group theoretical terms, the former corresponds to non-unitary, and the latter to unitary representations of $U(3,1)$. The former have negative probability ghosts while the latter have negative mass squared ghosts. The Veneziano model seems to prefer the former. Such a choice is necessary to ensure a Regge behavior à la Van Hove, but runs into trouble with form factors. This is a general agony of making the choice, not restricted to Mr.Veneziano alone.

5) We have ignored the problems of the extra scalar excitations which are needed to incorporate the proper trajectory intercepts in the dual channel. This is another rather unphysical aspect of factorization. These extra modes may be taken either as a set of harmonic oscillators in a fifth dimension or as a modification of the propagator. Whether these states have positive metric or not depends on the intercept and the external masses. Actually a sixth dimension would be necessary to take care of two-particle trajectories correctly. We have no illuminating interpretations to offer on this subject.

3 Quarks and the dual model

What we propose here is a program of building a general picture of the structure of hadrons on the basis of the factorized Veneziano model. It has been noted by Harari and Rosner that the duality may be interpreted schematically in terms of quark diagrams, which have a strong predictive power, albeit of qualitative nature. These diagrams are indeed very suggestive. First of all they agree with the foregoing picture that the hadrons form two-dimensional sheets in space-time. Furthermore, they imply that quarks and antiquarks form the boundary lines of the sheets. A meson system, for example, is then a $q-\bar{q}$ molecule bound by an elastic string. We could also imagine a rubber band in which q and $\bar{q}$ are attached to diametrically opposite points. To go on further, we have to make a choice.

On the basis of the duality diagram picture, we will adopt the linear molecule rather than the benzene ring. An advantage of the linear picture is that a linear chain can be broken in two linear chains, thereby accounting for the production mechanism

$$A \longrightarrow B + C \tag{30}$$

In fact the duality diagrams can be interpreted exactly in this way.

To make things a bit more sophisticated, we will present a modified version of the quark model. This is the three-triplet model proposed by Dr. Han and me some time ago. It had the advantage of a) having integral charges, b) naturally accounting for the zero triality of known hadrons, as well as c) for the $SU(6)$ classification of the baryons. In this scheme there are nine fundamental fermions grouped into three $SU(3)$ triplets. We may use the notation T_i^n; $i, n = 1, 2, 3$, where n distinguishes between different triplets. T_i^n behaves like a triplet representation in the ordinary $SU(3)$ space (lower index), and like an antitriplet representation in the new $SU(3)$ space (upper index). These $SU(3)$ spaces are denoted as $SU(3)'$ and $SU(3)''$ respectively. Each space has its own isospin and hypercharge, and the electric charge is the sum of two charge operators $\lambda'_Q + \lambda''_Q$. We call $3\lambda''_Q$ the charm number C. We will also make things a bit more exciting by naming the three different triplets D, N, and A. Their quantum number assignments are given in the table.

	C		Q	
D_i	1	1,	0,	0
N_i	-2	0,	-1,	-1
A_i	1	1,	0,	0

In the lowest approximation, $SU(3)'$ and $SU(3)''$ are separately good symmetries. In particular, all low lying hadron states are assumed to belong to $SU(3)''$ singlets, which turn out to correspond to only zero triality states in $SU(3)'$. (The same is accomplished also by assuming zero charm for hadrons.) Baryons and mesons have the usual pattern TTT and $T\bar{T}$. More precisely,

$$\begin{aligned} &B \sim DNA \\ &M \sim D\bar{D} + N\bar{N} + A\bar{A} \end{aligned} \tag{31}$$

where B is completely antisymmetric in the $SU(3)''$ space in order to be a singlet, and this takes care of the Pauli principle.

The preference of zero triality is thus reduced in this model to the $SU(3)''$ symmetry, which one may attribute to a dynamical property of superstrong interactions having a larger scale of masses ($\gtrsim 1GeV$) than for the strong interactions ($\sim 1GeV$).

The combination of duality and the three-triplet model will then produce the following picture. The hadrons are "molecules" bound by superstrong interactions, with the bond structure

$$\begin{aligned} B &\sim T - T - T \\ M &\sim T - \bar{T} \end{aligned} \tag{32}$$

The bonds must have a saturation property for zero triality. In the original scheme demanding perfect $SU(3)''$ symmetry, the baryon would have to have either a ring structure

$$B \sim \begin{array}{c} D \\ \diagup\;\diagdown \\ N - A \end{array} \tag{33}$$

or a resonating linear structure

$$B \sim D - N - A \quad + \quad \text{permutations.} \tag{34}$$

If $SU(3)''$ symmetry is abandoned and only the neutral charm condition is imposed, we may simply assume

$$B \sim D - N - A \tag{35}$$

In the latter case the charm number is equivalent to valency. But then we lose the distinction between D and A, drifting back to a two-triplet model. Our tentative preference is in Eq.(34) though the other two possibilities should not be ignored. The meson scheme would follow Eq.(31).

What can we do with this model? We have now hadrons endowed with $SU(3)$ and Dirac spin. These are more or less localized at certain points along the string whose function is to carry bulk of the energy and momentum of the system. The triplets (or simply quarks) themselves are massless, or have only small masses. Several remarks are in order.

1) The interaction process (30) is viewed as a creation of pair $T\bar{T}$ at a point where the break occurs. After the cut, each portion subsequently grows into a full grown string, like an earthworm! This would not be possible if the

string is made up of a fixed number of mass points. We must conclude that the number is not only large but also indefinite. Let us examine this situation a little further. Take a string stretched between two fixed points in space with a distance L apart. If the number of mass points is N, the potential energy is

$$V \sim N(L/N)^2 = L^2/N \tag{36}$$

which depends on N. However, if the number of discrete steps in the "time" direction τ also increases with N to sustain duality, the action integral $I \sim NV \sim L^2$ will be independent of N. This is the scale invariance we have discussed. In units of the fictitious time τ, a system lives longer the larger the number N. The actual state of a hadron would be a linear superposition of configurations having different values of N, but their contributions are all proportional to each other.

If this view is accepted, we can define the Hamiltonian (or vertex) responsible for the process (30) as an overlap integral of the three wave functions corresponding to the states A, B, and C. There is a selection rule

$$N_A = N_B + N_C \quad (N_i > 0) \tag{37}$$

and its two cyclic permutations, either one of which must be satisfied. A more explicit expression satisfying (37) would look like

$$\sum_{0<N'<N} C_{N'N} \int \cdots \int \Psi_B^*(x^{(1)}, \cdots, x^{(N')}) \Psi_C^*(x^{(N'+1)}, \cdots, x^{(N)}) \\ \Psi_A(x^{(1)}, \cdots, x^{(N)}) F(x^{(1)}, \cdots, x^{(N)}) \prod d^4x^{(i)} \tag{38}$$

where F is some scalar function. This integral can be appropriately rewritten in terms of the wave functions $\Psi[x(\xi)]$ in the limit N, $N' \to \infty$.
In the actual Veneziano model, the interaction is such that the breaking of a string occurs only at one of the ends, which may be interpreted to mean the limit

$$N \to \infty; \quad N'/N \to 0 \quad \text{or} \quad N'/N \to 1$$

In general, however, there is no reason to impose such a condition. We would still get a dual theory of sorts; an amplitude corresponding to one

configuration will have singularities in the crossed channels too, though there may not be a symmetry between dual channels in individual amplitudes.
2) The triplet or the quark fields introduce extra spins to the system in conformity with the $SU(6)$ type theories, so π and ρ mesons belong to the s wave states of the string. But this also brings in the old headaches of relativistic $SU(6)$ theories as well. How can one eliminate half of the Dirac components in order to avoid parity doubling and ghost states? The difficulty is compounded by the fact that we would like to maintain duality too. A possible scheme based on the Carlitz-Kisslinger type cut mechanism has been developed by Freund et al. We will not discuss it here. Instead, we would like to propose a general formalism which tries to accomplish this in a dynamical way. The basic idea is as follows. Instead of regarding the triplets and the strings as separate entities, let us take a unified picture and replace the string with a chain of $T\bar{T}$ pairs, so that it would look like a polarized medium with opposite charges created at its ends. There will be interactions between neighbors which depend on their Dirac, $SU(3)'$, and $SU(3)''$ spins. To be dual, these interactions must occur in the "time" direction too, thus forming a two-dimensional polarizable medium. These interactions would contribute to the action integral in addition to the kinetic term represented by the string Hamiltonian. Roughly speaking, that would produce a spin and $SU(3)$ dependence of various trajectories. It is conceivable that parity doubling and other problems can be reduced in the same way to ones of dynamical stability. An example of $T\bar{T}$ interaction might be

$$I = g \sum_{(n,n')} (\gamma_\mu^{(n)} \gamma^{\mu(n')} + \text{const}) \tag{39}$$

where $\gamma_\mu^{(n)}$ refers to a triplet sitting on a site n of a two-dimensional lattice, and n' refers to one of its neighbors. We must choose the constant g in such a way that a chain $T\bar{T}T\bar{T}\cdots\bar{T}$ will have the lowest energy (which may be adjusted to zero). To be dual, the same patter must be repeated in the time direction too. We thus end up with a pattern

$$\begin{array}{l} T\bar{T}T\bar{T}\ldots\ldots\ldots\ldots\bar{T} \\ \bar{T}T\bar{T}T\ldots\ldots\ldots\ldots T \\ T\bar{T}T\bar{T}\ldots\ldots\ldots\ldots\bar{T} \\ \bar{T}T\bar{T}T\ldots\ldots\ldots\ldots T \end{array}$$

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In other words, it is a two-dimensional antiferromagnet or ionic crystal! External particles should couple to it like a magnetic field couples to the spin:

$$I' = \sum_n \gamma_\mu^{(n)} \phi^{\mu(n)} \tag{40}$$

The scattering amplitude would then be obtained, following the Feynman principle, from an expression like

$$\mathrm{Tr} \quad \exp[I + I'] \tag{41}$$

the trace being taken with respect to the γ matrices. Eq.(41) is nothing but a partition function! It is not scale invariant, but rather an extensive quantity proportional to the number of constituents (unless $I = 0$). Thus the probability of creating exotic states having many disordered atoms (Large $|I|$) would be severely cut down.

The simplest mathematical model of the above type is the well known Ising model with its glorious Onsager solution. Perhaps we can adopt the Ising model here as a prototype. The transition from a Lagrangian to a Hamiltonian formalism is accomplished by means of the so-called transfer matrix.

3) The world sheet formalism presented here may accommodate Dirac's monopoles, because the monopoles can be described, according to Dirac, in terms of the same kind of world sheets swept out by strings attached to them. If this is the case, the triplets can have magnetic charges. Since all hadrons are magnetically neutral, these charges must add up to zero, which reminds us of the fact that the charm number is also zero for them. Thus we are tempted to identify the charm with the magnetic charge, which would cause strong binding between opposite charges. The two spaces $SU(3)'$ and $SU(3)''$ may be called electric and magnetic $SU(3)$ respectively, corresponding to the 3×3 ways of assigning electric and magnetic quantum numbers.

Such a formalism has been independently proposed by Schwinger from a different motivation. He calls these nine objects dyons. The direct parallelism between dyons and the triplets has been pointed out by Biedenhahn and Han.

Of course there are all sorts of problems associated with monopoles. The most serious and perhaps most intriguing is the large P and T violation one must expect off hand. This could give a natural explanation for CP violation, but the problem is how to suppress it to a degree $\lesssim 10^{-10}$ which is required by neutron electric dipole moment.

At any rate, the close mathematical connection between the dual model and the monopoles lies in the fact that the Maxwell field due to a pair of monopoles is given by

$$F_{\mu\nu}(x) = g\epsilon_{\mu\nu\lambda\rho} \iint \delta^4(x-y) G^{\lambda\rho}(y) d^2\xi \tag{42}$$

where $G_{\mu\nu} d^2\xi = d\sigma_{\mu\nu}$ is the surface element, Eq.(22), of a world sheet spread between the world lines of the pair. Eq.(42) is independent of the choice of the sheet; one gets the correct equations if Eq.(42) is substituted in the Maxwell Lagrangian and the y's are varied, including the end points.

4 Statistical approximation

We will briefly discuss here a high energy approximation to dual amplitudes which was originally based on an intuitive argument, but can also be justified more rigorously. The point is that in a high energy process a very large number of states are available according to the factorized dual theory. In fact the number of states $\rho(s)$ increases like $\exp[c\sqrt{s}]$, which one can easily derive from the Stephan-Boltzmann law for a one-dimensional black body radiation. The only difference is that $s =$ (center-of mass energy)^2 takes the place of the ordinary energy. The constant c is numerically

$$c = 2\pi\sqrt{n/6} \tag{43}$$

where n is the dimensionality (4, or 5, or more) of the oscillator vectors. As has been pointed out by Fubini et al, this happens to give the same number ($c \approx 1/160 MeV^{-1}$ for $n = 6$) as the Hagedorn constant.

The absorptive part of a scattering amplitude consists of a sum over intermediate states with fixed s, i.e. over a microcanonical ensemble:

$$A = \sum_{s_n = s} \langle p' | j(-q') | n \rangle \langle n | j(q) | p \rangle \tag{44}$$

If s is large, one is tempted to replace it with a sum over a canonical ensemble. More precisely

$$\begin{aligned} A &\sim c(\beta)\sum_n \langle p'|j(-q')|n\rangle e^{-\beta s_n}\langle n|j(q)|p\rangle \\ &\equiv c(\beta)\sum_n F_n e^{-\beta s_n} \end{aligned} \tag{45}$$

Here the s_n's are the eigenvalues of the mass operator (20), not the actual $s = (p+q)^2$. This amounts to relaxing the subsidiary condition (19). $c(\beta)$ is a normalization factor. If the sum (45) had a sharp peak around $s_n = s$, it would be a good approximation to Eq.(44) as in the usual statistical mechanics. The parameter β should then be the inverse temperature ($\sim 1/\sqrt{s}$) of the string. Actually, things do not work out that way because F_n does not grow like $\rho(s_n)$ but much more slowly like s^α, which is the Regge behavior. If $\alpha > 0$, still there will be a peak around $s_n = s$ where

$$\alpha/s \approx \beta \tag{46}$$

So we can use this as the definition of β.

In the operator formalism, Eq.(45) can be explicitly evaluated from

$$A = c(\beta)\langle 0|\Gamma' e^{-\beta H_0}\Gamma|0\rangle \tag{47}$$

where Γ and Γ' are appropriate interaction vertices. We find

$$A \sim (1-e^{-\beta})^{-1-\alpha(t)} \sim \beta^{-1-\alpha(t)} \tag{48}$$

which give the correct Regge behavior in view of Eq.(46), if $c(\beta) \sim \beta$. The above idea makes physical sense, perhaps better than the Veneziano model itself, because we apply it to the absorptive parti and smear the resonance peaks as in the discussion of finite energy sum rules. But the real part, when smeared, should also show a similar behavior for reasons of analyticity. Justification of the method depends on the assumption that the absorptive part grows with s ($\alpha > 0$). The formula may still be valid for $\alpha \leq 0$, but that does not follow from the above argument. Assuming the general validity of the procedure, we can also handle the high energy behavior of many particle processes. The main point is that any high energy ("hot") propagator $\sim 1/(s - H_0)$ is replaced by a Boltzmann factor $\sim \beta\exp(-\beta H_0)$

where $\beta \sim 1/s$, which would give a correct answer as far as the $s-$dependence is concerned.

At this point let us indulge in some speculations. The problems are unitarity and the Pomeron, both of which are lacking in the Veneziano model. It is generally assumed that the Pomeron (in the $t-$channel) is equivalent to non-resonant background in the $s-$channel. But the background is after all made up of hadrons, so a unitarized dual theory should naturally contain the Pomeron. Now the unitarization means taking account of dissociation and recombination of resonances among themselves. Wouldn't it be reasonable, then, to consider a grand canonical ensemble of resonances? The main problem is of course how to define the vertex operator in a scattering problem. We would like to suggest the ansatz that instead of Eq.(46), β^{-1} should be the thermodynamic temperature, or

$$\beta \approx c/\sqrt{s} \tag{49}$$

because the distribution of M^2 would be decided by the interaction among the strings in the ensemble, and not by the coupling of these states to the external channels. We would then get the result

$$A \sim s^{\alpha(t)/2} \tag{50}$$

suggesting that the Pomeron trajectory has half the universal slope of resonances. Whether this is a pole or a cut, and what the intercept is, cannot be decided in such a crude picture. It is interesting that the same behavior as Eq.(50) has been obtained by explicitly computing certain higher order diagrams of the dual model.

There is still another possibility for the Pomeron which will be discussed later.

5 Electromagnetic interactions and inelastic $e-p$ scattering

We would like to tackle the problem of inelastic $e-p$ scattering on the basis of our model. Before doing this, we have to make some remarks about the electromagnetic interactions in general. If we just have an elastic spring with a charge distribution along the string but without the quark spin, the problem of setting up the gauge principle is very simple. In the Hamiltonian (16) one makes the replacement

$$p^{\mu}(\xi) \longrightarrow p^{\mu}(\xi) - \rho(\xi)A^{\mu}(x(\xi)) \tag{51}$$

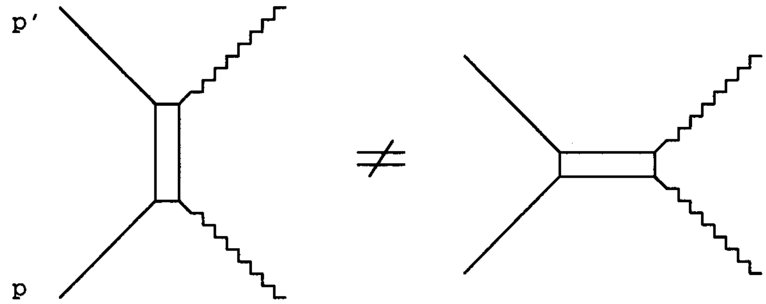
where $\rho(\xi)$ is an arbitrary distribution function. But this is not the most general form. Instead of $x(\xi)$ and $p(\xi)$, we should choose the set $\{x_n, p_n\}$, or any other orthogonal basis, and apply the gauge principle. Because $A(x)$ is nonlinear in x, we get inequivalent results.

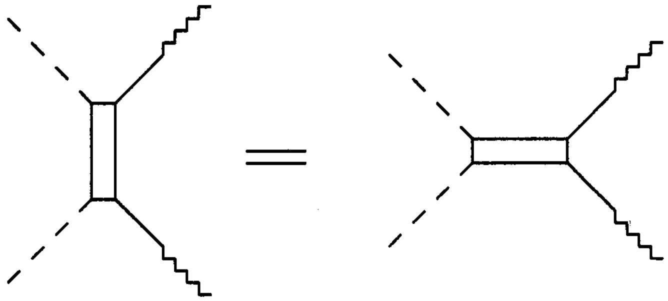
The trouble with this method is that the form factors are all Gaussian, an undesirable characteristic of harmonic oscillators. For a pointlike charge distribution the Gaussian peak is infinitely sharp (in momentum space) since the string has an infinite zero-point length. If we renormalize away the Gaussian factor, on the other hand, what is left is in general a polynomial which is not good either.

Another popular approach to form factors is the spurion method which allows one to obtain pole dominance form factors. A difficulty here is the gauge invariance, or the conservation law, which cannot be automatically guaranteed. At any rate there is a large amount of arbitrariness in either method.

Another serious problem is dealing with external fields in a factorized dual model. It arises from the fact that one needs fifth-dimension oscillators for factorization of a general dual amplitude, but the parameters of the fifth dimension depends on the individual external masses in a non-factorizable way. Thus one cannot maintain duality and factorizability for arbitrary external fields. Full duality must then be abandoned in our model when dealing with electromagnetic interaction. For example, a virtual Compton amplitude should not be dual;

One might argue that in the pole dominance model the lines p and p', instead of the photons, may be treated as external:





but this does not work for many photon cases.

In spite of the unpleasant Gaussian nature of form factors in the first method described above, let us see what will come out if duality is not demanded. The crossed channel singularities in the usual case arises from the singular nature of the vertex operator

$$\Gamma(q) \sim \exp[iq \cdot x(0)] = \exp\left[iq \cdot x_0 + 2iq \cdot \sum_{n=1}^{\infty} x_n\right] \tag{52}$$

In order to blunt the singularity, we assume that the charge is located at a coordinate

$$\bar{x} = \int x(\xi) f(\xi) d\xi = x_0 + 2 \sum_{n=1}^{\infty} x_n f_n \tag{53}$$

such that

$$\sum f_n^2 < \infty \tag{54}$$

According to the gauge principle, the electromagnetic interaction is obtained by the substitution $p_0^\mu \to p_0^\mu - eA^\mu(\bar{x})$, $p_n^\mu \to p_n^\mu - 2ef_n A^\mu(\bar{x})$ in the Hamiltonian. The current operator is then

$$j^\mu = e\left\{\ p_0^\mu + \sum_{n=1}^{\infty} p_n^\mu f_n\ \ ,\ \ e^{iq\cdot\bar{x}}\ \right\}_+ \tag{55}$$

For the ground state (spin zero), this gives a vertex

$$\Gamma_\mu = e(p+p')_\mu \exp\left[-q^2 \sum_{n=1}^{\infty} f_n^2/n\right] \tag{56}$$

Now let us discuss off-diagonal elements $\langle n|j_\mu|0\rangle$ corresponding to inelastic processes. We would like to compare them with the $e-p$ scattering data ignoring the effect of spin. The familiar structure functions W_1 and W_2 should be obtained from

$$\sum_{s_n=n} \langle 0|j_\nu(-q)|n\rangle\langle n|j_\mu(q)|0\rangle \tag{57}$$

where the statistical method could be applied for large s.

There still remains a problem. In the SLAC data, W_1 and W_2 seem to have an s dependence ($W_1 \sim s$, $W_2 \sim 1/s$) which is consistent with the Pomeron picture. On the other hand, Eq.(55) will not give any Regge (or power) behavior. We propose to fix this by including the fifth dimension:

$$\exp[iq\cdot\bar{x}] \longrightarrow \exp[iq\cdot\bar{x} + iq_5 x_5(0)] \tag{58}$$

where q_5 is an appropriate constant. Since $x_5(0)$ is not smeared out, it gives rise to a fixed power behavior $\sim s^\alpha$ where α is constant, corresponding to a flat trajectory. We do not know what all this means, but the Pomeron might be of this nature. At any rate we can evaluate Eq.(57) with the new ansatz, and obtain the result

$$W_1 \sim (s-m^2)^{\alpha_1} \exp\left[-2q^2 \sum_{n=1}^{\infty} \frac{1-e^{-n\beta}}{n} f_n^2\right] \tag{59}$$

and similarly for W_2. Here m^2 is the initial Hadron mass. Actually we have lost gauge invariance from a) addition of the fifth dimension and b) the statistical approximation, so we cannot get the ratio W_1/W_2 exactly. But the main feature of Eq.(59) is that for large s we have

$$\beta \sim c/(s-m^2) \tag{60}$$

so that

$$\begin{aligned}
W_i &\sim (s-m^2)^{\alpha_i} \exp[-2q^2\beta_i \sum f_n^2] \\
&\sim (s-m^2)^{\alpha_i} \exp[-\lambda_i q^2/(s-m^2)] \\
&= (s-m^2)^{\alpha_i} \exp[-\lambda_i/(\omega-1)], \quad i=1,2 \\
\lambda_i &= 2c_i \sum_{n=1}^{\infty} f_n^2, \quad \omega = 1+(s-m^2)/q^2 \ \ (= 2m\nu/q^2)
\end{aligned} \tag{61}$$

Which $\alpha_1 = 1$, $\alpha_2 = -1$, we get the scaling law (for large s)

$$\begin{aligned}
(s-m^2)W_2 &\sim \exp[-\lambda_2/(\omega-1)] \\
\text{or} \qquad 2m\nu W_2 &\sim (\omega/\omega-1)\exp[-\lambda_2/(\omega-1)], \\
(s-m^2)^{-1}W_1 &\sim \exp[-\lambda_1/(\omega-1)]
\end{aligned} \tag{62}$$

The general behavior of the exponential factor in Eq.(62) is: