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Localizing the black M2-M5 intersection

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based on recent work with **K. Siampos**

12xx.xxxx, ``*The M2-M5 ring intersection spins*'', in progress

1206.2935, ``*Entropy of the self-dual string soliton*'', JHEP 1207 (2012) 134

1205.1535, ``*M2-M5 blackfold funnels*'', JHEP 1206 (2012) 175

and older work with **R. Emparan, T. Harmark and N. A. Obers** ➢ **blackfold theory**

1106.4428, ``*Blackfolds in Supergravity and String Theory*'', JHEP 1108 (2011) 154

0912.2352, ``*New Horizons for Black Holes and Branes*'', JHEP 1004 (2010) 046

0910.1601, ``*Essentials of Blackfold Dynamics*'', JHEP 1003 (2010) 063

0902.0427, ``*World-Volume Effective Theory for Higher-Dimensional Black Holes*'', PRL 102 (2009)191301

0708.2181, ``*The Phase Structure of Higher-Dimensional Black Rings and Black Holes*''
+ **M.J. Rodriguez** JHEP 0710 (2007) 110

We learn new things about the fundamentals of string/M-theory by studying the low-energy theories on D-branes and M-branes.

Most notably in M-theory, recent progress has clarified the low-energy QFT on N M2-brane and the $N^{3/2}$ dof that it exhibits.

ABJM '08
Drukker-Marino-Putrov '10

Our understanding of the M5-brane theory is more rudimentary, but efforts to identify the analogous properties of M5-branes, *e.g.* the N^3 scaling of the massless dof, is underway.

Douglas '10
Lambert,Papageorgakis,Schmidt-Sommerfeld '10
Hosomichi-Seong-Terashima '12
Kim-Kim '12
Kallen-Minahan-Nedelin-Zabzine '12
...

It is believed that the M5 theory is a theory of strings.

M2-branes can end on M5-branes

	0	1	2	3	4	5	6	7	8	9	10
M2 :	•	•					•				
M5 :	•	•	•	•	•	•					

just like F-strings can end on D-branes in string theory.

The above intersection is 1/4-BPS (preserves 8 supercharges).

The IR dynamics is controlled by a (1+1)-dim *(4,4)* SCFT that lives in the intersection.

OUR GOAL: to identify this theory, or key features of this theory
e.g. how does the central charge of this CFT scale with N_2 , N_5 ?

We can approach this question in two different ways:

- from a microscopic analysis of the M5/M2 brane physics

- from a supergravity analysis of the corresponding black brane intersection
(*e.g.* near-extremal black brane thermodynamics gives
for M2-branes $S \sim N^{\frac{3}{2}} T^2$
for M5-branes $S \sim N^3 T^5$)

Klebanov-Tseytlin '96

Both approaches are technically complicated
(explains why our understanding of this system is still rather rudimentary).

I will describe progress in the SUGRA approach.

Ultimately we are interested in the development of the microscopic theory that lives at the intersection and its implications for the M5-brane theory.

M5 point of view: the Howe-Lambert-West solution

Descriptions of the intersection are possible either from the M2 or M5-brane point of view.

I will not say much about the M2 point of view, as it will be less relevant for what follows.

From the M5 point of view the string intersection appears as a solitonic solution of the M5 brane worldvolume theory.

The Howe-Lambert-West solution: $N_5=1,\ N_2>0$.

The abelian worldvolume theory on a single M5 brane is known.

It is a theory of a self-dual 3-form field strength and 5 transverse scalars (plus their fermion partners).

Key point: this theory is the leading term in a long-wavelength derivative expansion (analogous to the DBI theory for D-branes).

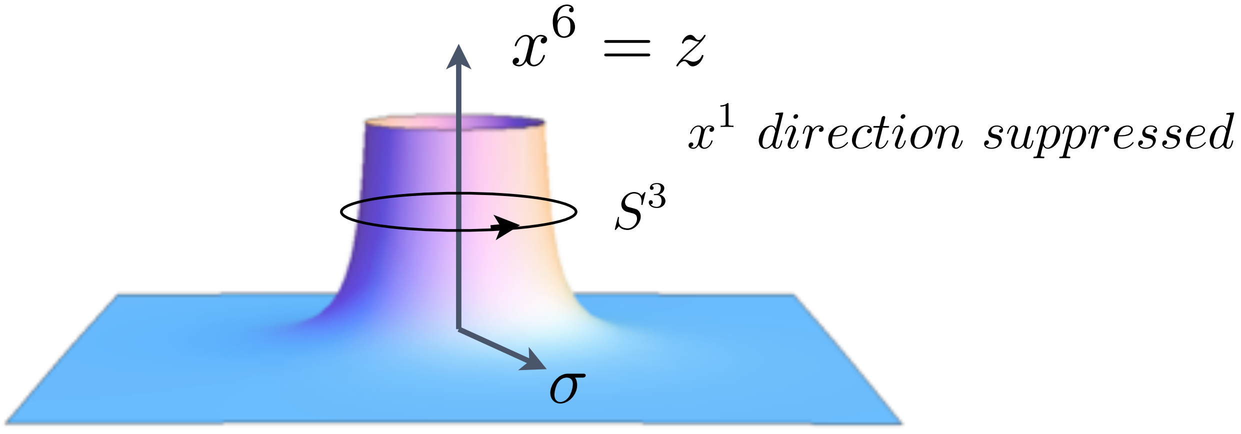
Hold on to this point...

The 1/4-BPS self-dual string soliton solution

(M-theory analog of BIon solution)

$$z(\sigma) = \frac{2Q_{sd}}{\sigma^2}\ , \quad z := x^6$$

$$H_{(3)} = *_6 H_{(3)} = *_4 dz$$



An S^3 spike describes N_2 M2-branes ending on N_5=1 M5-branes.

The solution (and the associated derivative expansion) breaks down at some radius, but a **miracle** happens:

at the tip of the spike one recovers the tension of the orthogonal M2 branes. (We will re-encounter and extend this feature below).

The leading order solution works much better than naively expected.

Technical issue: we do not know the non-abelian M5-brane wv theory. This obstructs a similar analysis at generic N_2, N_5 .

Known supergravity solutions

Supergravity allows us to examine the system in the limit N_2, $N_5 >> 1$.

Brane intersections in supergravity is a subject with a long history and impressive achievements.

Nevertheless, it is technically challenging in many cases to find solutions that describe fully localized intersections.

Finding a solution becomes an even greater challenge as we reduce the amount of supersymmetry, or if we have no supersymmetry at all, e.g. for non-extremal solutions.

In the case of the 1/4-BPS orthogonal M2-M5 intersection

	0	1	2	3	4	5	6	7	8	9	10
M2 :	•	•					•				
M5 :	•	•	•	•	•	•					

we are looking for a solution with SO(1,1) x SO(4) x SO(4) symmetry.

A partially localized solution with metric element

$$ds^2 = H_2^{1/3} H_5^{2/3} \Big[(H_2 H_5)^{-1} (-dt^2 + (dx^1)^2) + H_5^{-1} ((dx^2)^2 + \ldots + (dx^5)^2) + H_2^{-1} (dx^6)^2 + (dx^7)^2 + \cdots + (dx^{10})^2 \Big] \,,$$

$$\nabla^2_{(789(10))} H_5 = 0 \,, \quad \Big(H_5 \nabla^2_{(2345)} + \nabla^2_{(789(10))} \Big) H_2 = 0$$

is known (delocalized along the 6-direction).

Progress towards a fully localized solution has been achieved more recently by Lunin, who reduces the problem to a set of PDEs.

I will now describe a novel treatment of this system in SUGRA that

- works in a long-wavelength DBI-like regime
  (and thus compares more directly with the non-gravity M5 wv description)

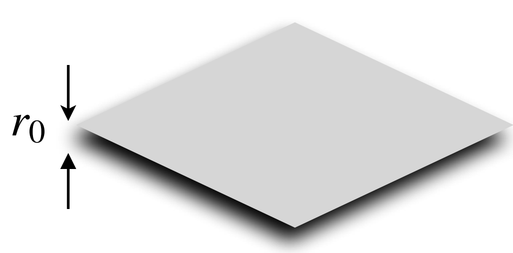
- gives immediate intuitive information, and

- easily provides more complicated (less symmetric) configurations that are well beyond the reach of current exact solution generating techniques.

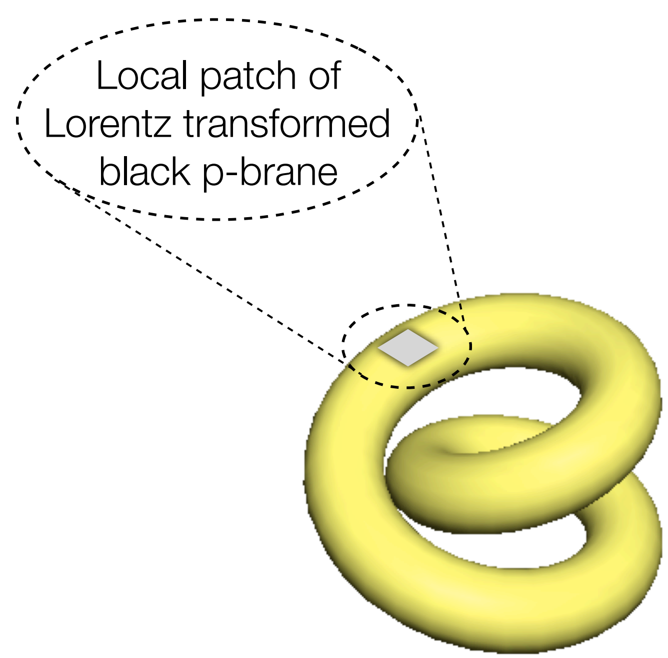
Blackfold theory basics

Blackfolds provide a general effective (long-wavelength) worldvolume description of black brane dynamics

They describe how a black p-brane fluctuates, spins and bends



flat black p-brane



a thin curved blackfold

The **fluid/gravity correspondence** illustrates nicely the general idea.

- a spin-off of the AdS/CFT correspondence

Bhattacharyya-Hubeny-Minwalla-Rangamani '07,...

- it describes temperature and velocity fluctuations of AdS black branes in the long-wavelength approximation $\lambda \gg \frac{1}{T} \sim \frac{L_{AdS}^2}{r_0}$ in terms of a relativistic conformal fluid. ← $\nabla_\mu T^{\mu\nu} = 0$

- the fluid lives on a time-like surface in the asymptotic region of the black hole spacetime (AdS boundary)

- there is a constructive perturbative procedure that maps uniquely the solutions of the fluid equations to regular bulk spacetimes

Blackfolds add co-dimension to the fluid-gravity correspondence

➠ a mix of fluid dynamics + `DBI'.

for neutral AF black branes (in D=n+p+3 dimensions)

$$ds^2_{p-brane} = \left(\eta_{ab} + \frac{r_0^n}{r^n} u_a u_b\right) d\sigma^a d\sigma^b + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega^2_{n+1}$$

- temperature, velocity and worldvolume bending fluctuations in the long-wavelength approximation $\lambda \gg \frac{1}{T} \sim r_0$ in terms of a relativistic non-conformal fluid that lives on a dynamical hypersurface. ← $\nabla_\mu T^{\mu\nu} = 0$

- the fluid lives on a time-like surface in the asymptotic region of the black hole spacetime

- there is a constructive perturbative procedure that maps the solutions of the fluid equations to regular bulk spacetimes (here focus on the leading order of the expansion)

M2-M5 blackfold funnels

We want to describe a spiky deformation of the planar M5 black brane (with dissolved M2 brane charge)

➠ how the planar black M2-M5 bound state deforms

$$ds^2_{11} = (HD)^{-1/3}\Big[-f dt^2 + (dx^1)^2 + (dx^2)^2 + D\left((dx^3)^2 + (dx^4)^2 + (dx^5)^2\right) + H\left(f^{-1}dr^2 + r^2 d\Omega_4^2\right)\Big] ,$$

$$C_3 = -\sin\theta (H^{-1} - 1)\coth\alpha\, dt \wedge dx^1 \wedge dx^2 + \tan\theta D H^{-1} dx^3 \wedge dx^4 \wedge dx^5 ,$$

$$C_6 = \cos\theta D (H^{-1} - 1)\coth\alpha\, dt \wedge dx^1 \wedge \cdots \wedge dx^5 ,$$

$$H = 1 + \frac{r_0^3 \sinh^2\alpha}{r^3} , \quad f = 1 - \frac{r_0^3}{r^3} , \quad D^{-1} = \cos^2\theta + \sin^2\theta H^{-1} .$$

The parameters of the planar M2-M5 bound state are promoted to slowly-varying functions of an effective 5-brane worldvolume

$$r_0(\hat{\sigma}^a)\ ,\ \alpha(\hat{\sigma}^a)\ ,\ \theta(\hat{\sigma}^a)\ ,\ u^a(\hat{\sigma}^a)\ ,\ X^{\perp}(\hat{\sigma}^a)\ ,\ \hat{V}_{(3)}$$

local temperature — *M2, M5 charges* — *local boosts* — *transverse scalars* — *unit 3-volume form controls embedding of M2 charge in 5-brane wv*

Leading order blackfold equations

`DBI' part extrinsic equations

$K_{ab}{}^{i}$ *: extrinsic curvature tensor*

$$K_{ab}{}^{i}T^{ab} = 0$$

`hydrodynamic' part intrinsic equations

$$D_a T^{ab} = 0$$

$$d * J_3 = 0\ ,\quad J_3 = \mathcal{Q}_2 \hat{V}_{(3)}$$

local stress-energy tensor, constitutive relations,...

$$T^{ab} = \mathcal{T}s\left(u_a u_b - \frac{1}{3}\gamma_{ab}\right) - \sum_{q=2,5}\Phi_q \mathcal{Q}_q h^{(q)}_{ab} \qquad .....$$

$SO(1,1) \times SO(4) \times SO(4) \;\; symmetry$

We are looking for a static solution with a single `excited′ transverse scalar

$$x^6 = z(\sigma) \; , \quad \sigma^2 = (x^2)^2 + (x^3)^2 + (x^4)^2 + (x^5)^2$$

For stationary configurations the intrinsic eqs can be solved generically, and we end up with DBI-like eqs for the transverse scalars

For extremal *T=0* configurations we solve the eom of the Dirac action

$$I \simeq \int d\sigma \, \sigma^3 \sqrt{1 + \frac{\kappa^2}{\sigma^6}} \sqrt{1 + z'^2} \; , \quad \kappa = 4\pi \frac{N_2}{N_5} \ell_P^3$$

We recover the extremal (1/4-BPS) 3-sphere spike solution

$$z(\sigma) = 2\pi \frac{N_2}{N_5} \frac{\ell_P^3}{\sigma^2}$$

The blackfold derivative expansion breaks down when the characteristic scale of the solution becomes comparable with the transverse integrated-out characteristic scale

Breakdown scale: $$\sigma_c = \left(\frac{\pi N_5}{\sqrt{2}}\right)^{\frac{1}{3}} \left(1 + \sqrt{1 + \frac{4}{\lambda^2}}\right)^{\frac{1}{6}} \ell_P \ , \quad \frac{1}{\lambda} := \frac{4N_2}{N_5^2}$$

Derivative corrections are controlled by the ratio:

$$\frac{1}{\lambda} = \frac{4N_2}{N_5^2} \ll 1$$

➠ we work in the large-*N* limit

$$N_2,\, N_5 \gg 1 \ , \quad N_2 \ll N_5^2$$

Despite the breakdown of the effective theory the usual **miracle** happens.

The *leading-order* solution reproduces correctly the tension of M2-branes at the tip of the spike (at any λ)

$$\frac{1}{L_t L_{x^1}} \frac{dM}{dz}\bigg|_{\sigma=0} = Q_2 = N_2 T_{M2}$$

(We have also observed this miracle for **non-SUSY extremal** configurations)

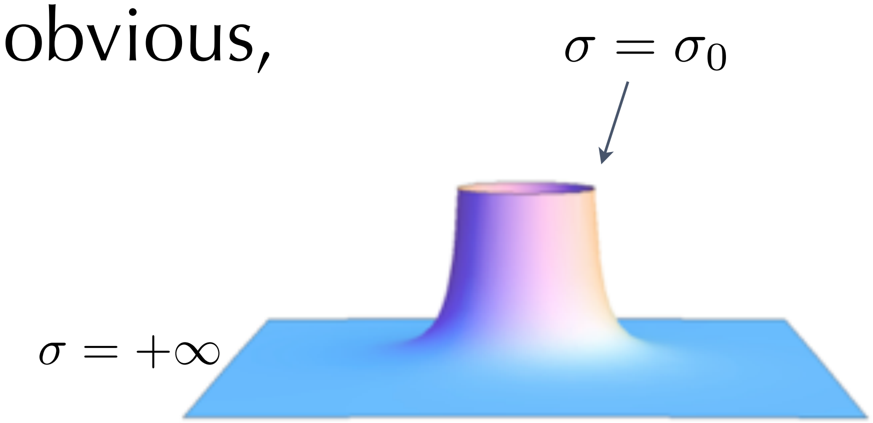
Thermalizing the spike

Spikes at finite temperature can be obtained by solving the eom of the action

$$I \simeq \int d\sigma \sqrt{1+z'^2}\, F(\sigma;\beta)\ , \quad \beta = \frac{3}{4\pi T}$$

$$F(\sigma) = \sigma^3 \left(\frac{1+\frac{\kappa^2}{\sigma^6}}{1+\sqrt{1-\frac{4q_5^2}{\beta^6}\left(1+\frac{\kappa^2}{\sigma^6}\right)}} \right)^{\frac{3}{2}} \left(-2 + \frac{3\beta^6}{2q_5^2} \frac{1+\sqrt{1-\frac{4q_5^2}{\beta^6}\left(1+\frac{\kappa^2}{\sigma^6}\right)}}{1+\frac{\kappa^2}{\sigma^6}} \right) \qquad q_5 = \frac{16\pi G}{3\Omega_{(4)}} Q_5$$

(analogous formula in non-gravity description not obvious, exact non-extremal SUGRA solution also hard)



Boundary conditions:

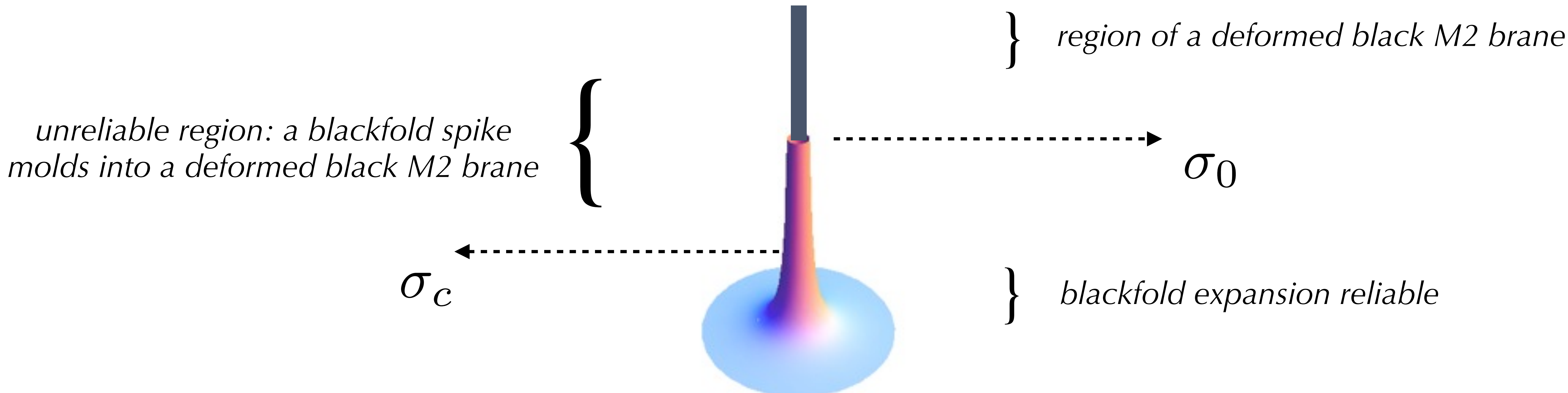
$$\lim_{\sigma\to+\infty} z(\sigma) = 0\ , \quad \lim_{\sigma\to\sigma_0^+} z'(\sigma) = -\infty$$

$\sigma_0 = \sigma_0(T) \ll \sigma_c$ lies in the breakdown region. What determines it?

With these boundary conditions the general solution of the leading order equations is

$$z(\sigma) = \int_{\sigma}^{+\infty} ds \left(\frac{F(s)^2}{F(\sigma_0)^2} - 1 \right)^{-\frac{1}{2}}$$

Black M2 matching conditions and a second set of miracles



The leading order solution `ends' at the tip σ_0 .

Does the extremal matching to M2 at the `tip' extend into the near-extremal regime?

Matching the thermodynamic data of the near-extremal spike with those of the emerging M2 we obtain

$$\left(\frac{1}{L_{x_1}}\frac{dM}{dz}\Big|_{\sigma=\sigma_0^+}\right)_{M2-M5} = \left(\frac{M}{L_{x_1}L_z}\right)_{M2} \ , \quad \left(\frac{1}{L_{x_1}}\frac{dS}{dz}\Big|_{\sigma=\sigma_0^+}\right)_{M2-M5} = \left(\frac{S}{L_{x_1}L_z}\right)_{M2}$$

$$\sigma_0^{(M)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}}\left(c_1^{(M)} + c_2^{(M)}\frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6})\right) \ , \quad c_1^{(S)} \simeq 1.234 \ , \quad c_2^{(M)} \simeq -0.068$$

$$\sigma_0^{(S)} = \frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}}\left(c_1^{(S)} + c_2^{(S)}\frac{q_2^{\frac{1}{2}}}{\beta^3} + \mathcal{O}(\beta^{-6})\right) \ , \quad c_1^{(S)} \simeq 1.189 \ , \quad c_2^{(M)} \simeq 0.052$$

$$\Rightarrow \quad \sigma_0(T) \simeq c_1\frac{q_2^{\frac{1}{4}}}{\beta^{\frac{1}{2}}} \ , \quad c_1 \simeq 1.2 \ , \quad q_2 = \frac{16\pi G}{3\Omega_{(3)}\Omega_{(4)}}Q_2$$

The matching of the leading order coefficients $(c_1^{(M)}, c_1^{(S)})$ within 4% is impressive.

Entropy of thermal spikes

Given a solution of the blackfold equation the formalism provides specific formulae for thermodynamic data. For the entropy in this particular application

$$\cosh\alpha = \frac{\beta^3}{\sqrt{2}q_5}\sqrt{\frac{1+\sqrt{1-\frac{4q_5^2}{\beta^6}(1+\frac{\kappa^2}{\sigma^6})}}{1+\frac{\kappa^2}{\sigma^6}}}$$

$$\frac{S}{L_{x_1}} = \frac{\Omega_{(3)}\Omega_{(4)}\beta^4}{4G}\int_{\sigma_0}^{+\infty} d\sigma\, \sigma^3 \frac{F(\sigma)}{\sqrt{F^2(\sigma)-F^2(\sigma_0)}}\frac{1}{\cosh^3\alpha(\sigma)}$$

Expanding in positive powers of T we wish to identify the $O(T)$ contribution from the (1+1)-dimensional intersection.

The contributions far from the core (M5) and close to the core (M2) are subtracted.

The leading order contribution to S is indeed $O(T)$ (as expected)

$$\frac{S}{L_{x^1}} = \frac{8\sqrt{\pi}\Gamma(\frac{1}{3})\Gamma(\frac{1}{6})}{135c_1^8}\frac{N_2^2}{N_5}T + \mathcal{O}(T^4) \ , \quad c_1 \simeq 1.2$$

Comparing to the Cardy formula for 2-dimensional CFT

$$\frac{S}{L_{x^1}} = \frac{\pi c}{6}T$$

we find an expression for the central charge c

$$c \simeq 0.6\,\frac{N_2^2}{N_5} + \ldots$$

These results indicate that the *d=2 N=(4,4)* SCFT at the intersection has a strong t′ Hooft like expansion

$$N_2 \ , \ N_5 \gg 1 \ , \quad \lambda \sim \frac{N_5^2}{N_2} \gg 1$$

In this limit the leading order contribution to the central charge takes the highly suggestive form

from dimensional analysis !!!

$$c \simeq 0.6 \, \frac{N_2^2}{N_5} + \ldots = 0.04 \, \frac{N_5^{3}}{\lambda^2} + \ldots = 0.3 \, \frac{N_2^{\frac{3}{2}}}{\sqrt{\lambda}} + \ldots$$

- *What is the field theory interpretation of this result?*
  What does it teach us about M2 and M5 brane physics?

- *How does the* $\frac{1}{\lambda}$ *expansion arise in field theory?*

Further work

Much remains to be done:

- The result relied on a set of interesting `miracles'.
  Extra checks are under consideration.

- Explore the implications for field theory.

- Probe more complicated configurations of the intersection.

The M2-M5 ring intersection spins

Search for a closed M2-M5 string intersection in supergravity.

➠ A rotating black M2 cylinder ending on a black M5.

- The configuration preserves less symmetry: SO(1,1) x SO(3) x SO(4)
- The configuration is stationary: the blackfold fluid rotates.
- The black M5 has to be also cylindrical
- The extremal configuration carries a null momentum wave along the intersection.
- Surprisingly, although non-SUSY it exhibits many of the miracles of supersymmetric configurations (e.g. thermo data at the tip of the spike)
- ...