Wonders of E_{10} and $K(E_{10})$

Hermann Nicolai

MPI für Gravitationsphysik (AEI), Potsdam

Wonders of Gauge Theory and Supergravity
IHP and LPT-ENS, Paris, 23 - 28 June 2008

(mostly) based on work done in collaboration with:

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Main message of this talk:

Search for unification = search for symmetries

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... and perhaps also for quantum gravity...
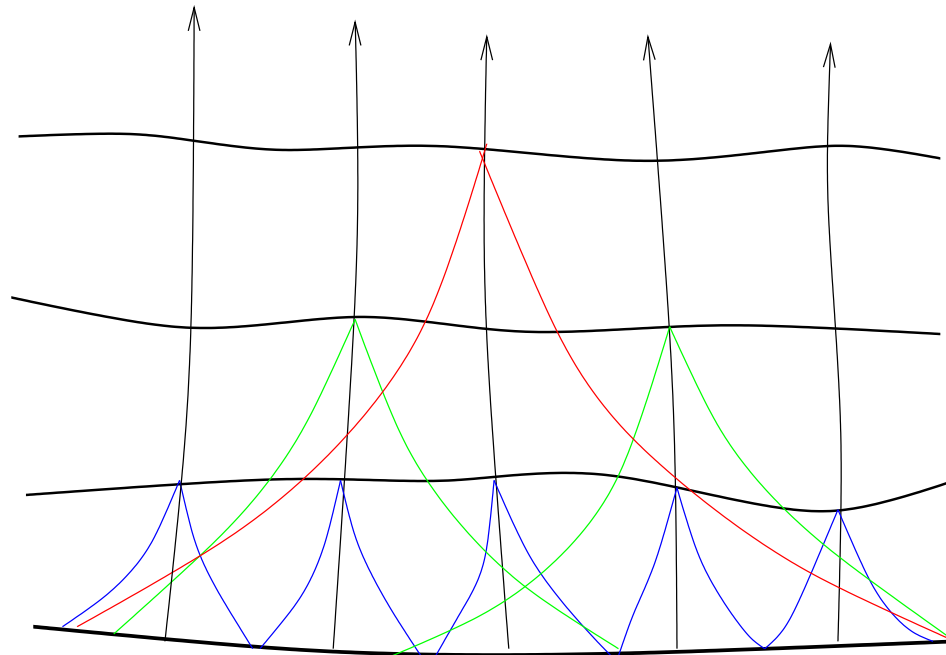
The BKL Paradigm

- Near a spacelike (cosmological) singularity, Einstein equations should simplify $\Rightarrow$ BKL decoupling: $\partial_x \ll \partial_t$?

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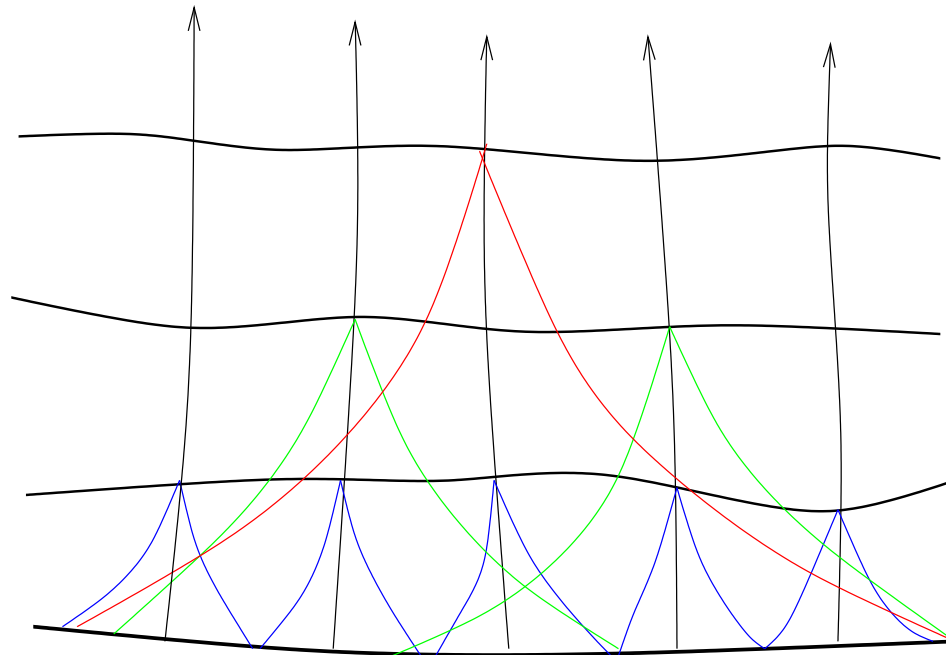
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- Dimensional reduction to one (time) dimension $\rightarrow$ effective dynamics near singularity from gradient expansion? $\rightarrow$ billiards, chaotic oscillations, etc.

Another (old) paradigm

- Cosmological evolution as 'geodesic motion' in the moduli space of 3-geometries [Wheeler, DeWitt,...]:

$$\mathcal{M} \equiv \mathcal{G}^{(3)} = \frac{\{\text{spatial metrics } g_{ij}(\mathbf{x})\}}{\{\text{diffeomorphisms}\}}$$

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- Unification of space-time, matter and gravitation: configuration space $\mathcal{M}$ for quantum gravity should consistently incorporate matter degrees of freedom.

Hidden symmetries

Reduction of SUGRA$_{11}$ to $D = 11-n$ [Cremmer, Julia (1979)]

n	Scalar Coset $E_n/K(E_n)$
1	$GL(1)/\mathbf{1}$
2	$GL(2)/SO(2)$
3	$SL(3) \times SL(2)/U(2)$
4	$SL(5)/SO(5)$
5	$SO(5,5)/SO(5) \times SO(5)$
6	$E_6/USp(4)$
7	$E_7/SU(8)$
8	$E_8/(Spin(16)/\mathbb{Z}_2)$
9	$E_9/K(E_9)$
10	$E_{10}/K(E_{10})$
11	$E_{11}/K(E_{11})$
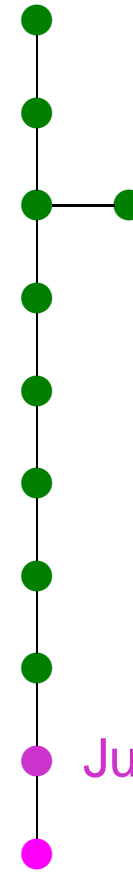
Julia('85), DHN('02)

West (2001)

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Effective dynamics of diagonal metric degrees of freedom is governed by *cosmological billiards* in Weyl chamber of E_{10}!

[Damour, Henneaux, hep-th/0012172; DHN, hep-th/0212256]

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motivates BASIC CONJECTURE: $\mathcal{M} = E_{10}/K(E_{10})$

| Dynamics of supergravity (or some M theoretic extension) | Null geodesic motion on $E_{10}/K(E_{10})$ coset space |

are equivalent! [DHN, hep-th/0207267]

| SUGRA eqs. of motion + canonical constraints | ∞-component geodesic eqn. and coset constraints |

Definition of E_{10}

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E_{10} is the Kac–Moody group with Kac–Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ of rank 10 defined via the Dynkin diagram

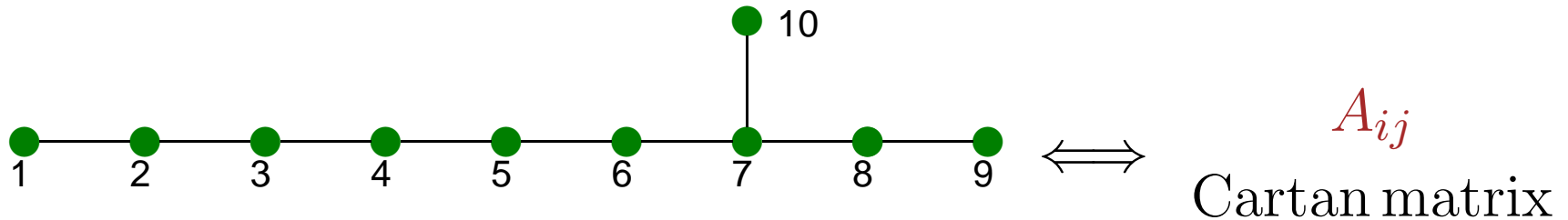
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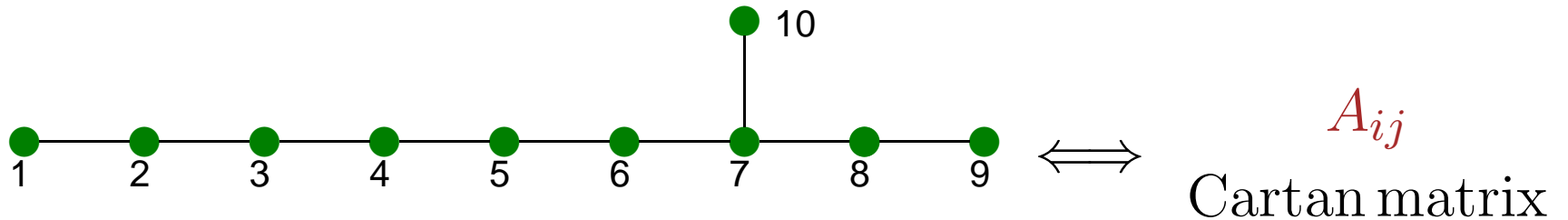
Chevalley–Serre presentation: Generators h_i, e_i, f_i for $i = 1, \ldots, 10$ with relations

$$[h_i, h_j] = 0, \qquad\qquad [e_i, f_j] = \delta_{ij} h_i,$$

$$[h_i, e_j] = A_{ij} e_j, \qquad\qquad [h_i, f_j] = -A_{ij} f_j,$$

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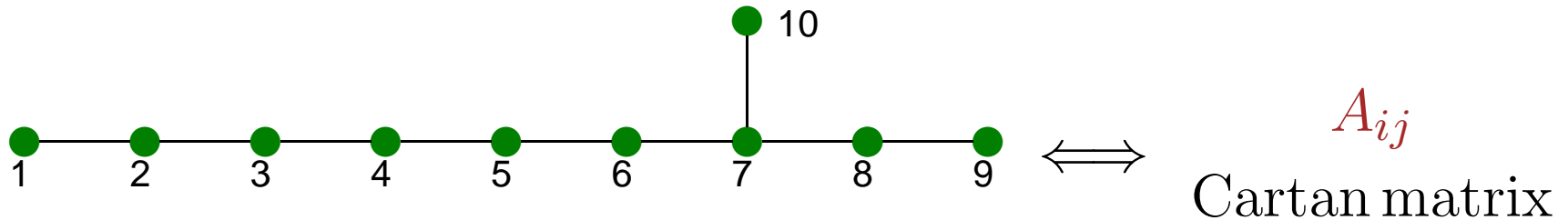
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h_i span Cartan subalgebra $\mathfrak{h}$; e_i and f_i: positive and negative simple root generators

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- Triangular decomposition $\to$ Computability

$$\mathfrak{g} = \mathfrak{e}_{10} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+ \;\; , \quad \text{with } \mathfrak{n}_\pm := \bigoplus_{\alpha \gtrless 0} \mathfrak{g}_\alpha$$

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$$\mathfrak{k}_{10} \equiv K(\mathfrak{e}_{10}) = \left\{ x \in \mathfrak{e}_{10} : \omega(x) \equiv -x^T = x \right\}$$

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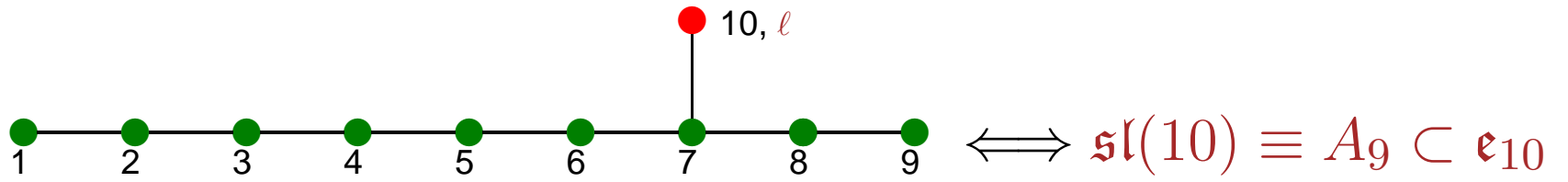$$\mathfrak{k}_{10} \equiv K(\mathfrak{e}_{10}) = \left\{ x \in \mathfrak{e}_{10} : \omega(x) \equiv -x^T = x \right\}$$

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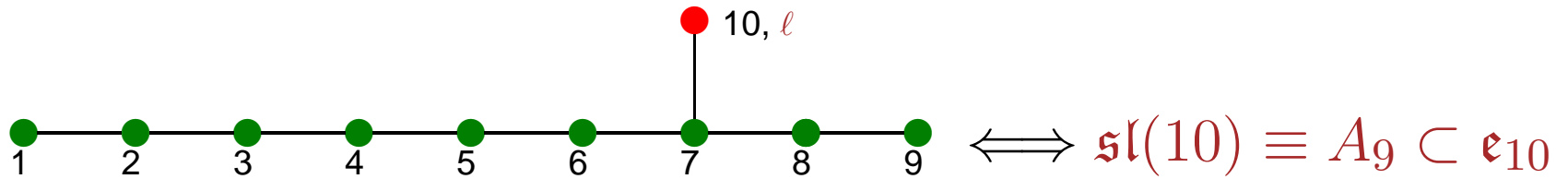However: $\mathfrak{k}_{10}$ is *not* a Kac-Moody algebra [KN, hep-th/0506238]

Level decomposition: $A_9 \subset \mathfrak{e}_{10}$
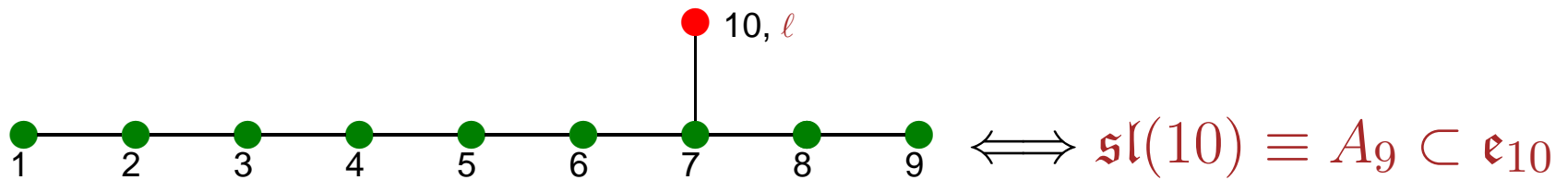
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These are just the representations corresponding to the bosonic fields of $D = 11$ SUGRA and their magnetic duals.

At level $\ell = 3$: dual graviton $h_{a_1...a_8|a_9}$ (with $h_{[a_1...a_8|a_9]} = 0$)

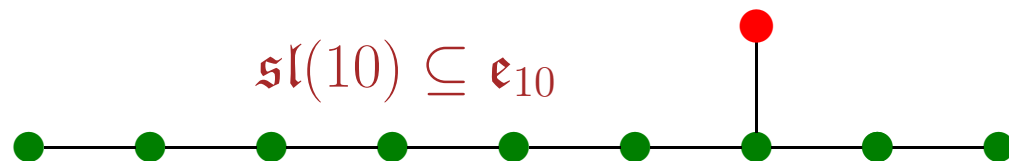[For more representations, see: Fischbacher,N. hep-th/0301017]

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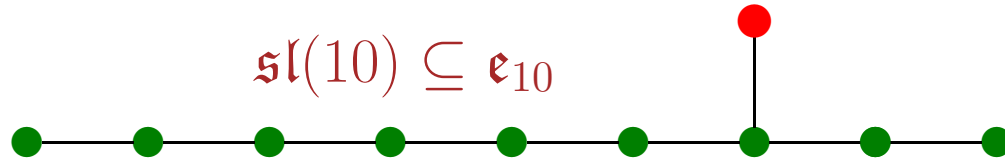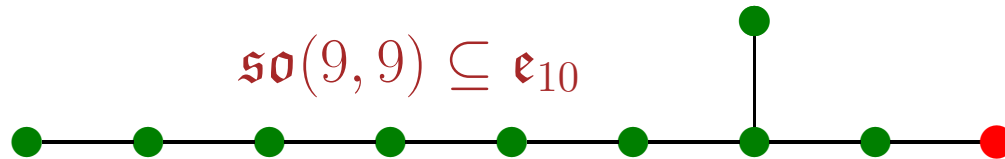The one-dimensional E_{10} σ-model unifies

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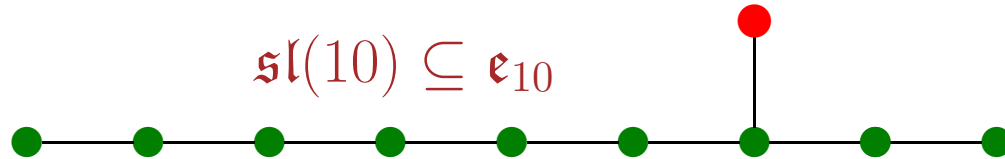$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$

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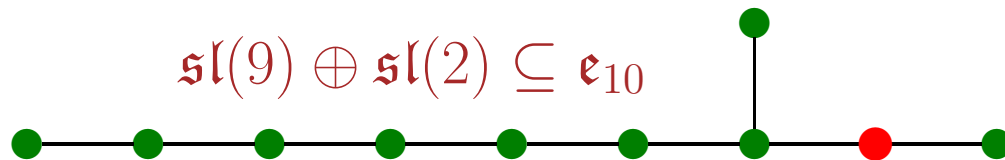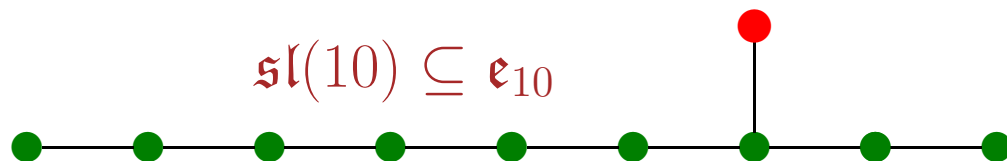$\mathfrak{sl}(10) \subseteq \mathfrak{e}_{10}$

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These are the (maximal) low energy theories of the 'M-theory diagram', now all part of a single model.

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$\Rightarrow$ essentially unique coset Lagrangian ($n(t)=$ lapse)

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Equations of motion: null geodesic on $E_{10}/K(E_{10})$

$$n\partial_t(n^{-1}\mathcal{P}) = [\mathcal{Q}, \mathcal{P}], \quad \langle \mathcal{P} | \mathcal{P} \rangle = 0.$$

Example: $A_9 \subset E_{10}$

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With $\partial_t \mathcal{V} \mathcal{V}^{-1} = \sum_{\ell \geq 0} P^{(\ell)} * E^{(\ell)}$ (schematically) and truncation $P^{(\ell)} = 0$ for $\ell > 3 \Rightarrow$

Equations of motion up to $\ell = 3$ $(a, b = 1, \ldots, 10)$ [DHN; DN, hep-th/0410245]

$$
\begin{aligned}
n \mathcal{D}^{(0)} (n^{-1} P^{(0)}_{ab}) &= -\frac{1}{4} \left(P^{(1)}_{acd} P^{(1)}_{bcd} - \frac{1}{9} \delta_{ab} P^{(1)}_{cde} P^{(1)}_{cde} \right) \\
&\quad - \frac{1}{2 \cdot 5!} \left(P^{(2)}_{ac_1 \ldots c_5} P^{(2)}_{bc_1 \ldots c_5} - \frac{1}{9} \delta_{ab} P^{(2)}_{c_1 \ldots c_6} P^{(2)}_{c_1 \ldots c_6} \right) \\
&\quad + \frac{4}{9!} \left(P^{(3)}_{ac_1 \ldots c_7 | c_8} P^{(3)}_{bc_1 \ldots c_7 | c_8} + \frac{1}{8} P^{(3)}_{c_1 \ldots c_8 | a} P^{(3)}_{c_1 \ldots c_8 | b} \right. \\
&\quad \left. - \frac{1}{8} \delta_{ab} P^{(3)}_{c_1 \ldots c_8 | c_9} P^{(3)}_{c_1 \ldots c_8 | c_9} \right) \\
n \mathcal{D}^{(0)} (n^{-1} P^{(1)}_{abc}) &= -\frac{1}{6} P^{(2)}_{abcdef} P^{(1)}_{def} + \frac{1}{3 \cdot 5!} P^{(3)}_{abcd_1 \ldots d_5 | d_6} P^{(2)}_{d_1 \ldots d_6} \\
n \mathcal{D}^{(0)} (n^{-1} P^{(2)}_{a_1 \ldots a_6}) &= \frac{1}{6} P^{(3)}_{a_1 \ldots a_6 cde} P^{(1)}_{cde} \\
n \mathcal{D}^{(0)} (n^{-1} P^{(3)}_{a_1 \ldots a_8 | a_9}) &= 0 \qquad \text{(with } P^{(3)}_{[a_1 \ldots a_8 | a_9]} = 0).
\end{aligned}
$$

This is a consistent truncation of $E_{10}/K(E_{10})$ coset dynamics: solutions of truncated theory are also solutions of the full theory.

Correspondence with SUGRA$_{11}$

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Bosonic $D = 11$ supergravity equations `[Cremmer, Julia, Scherk 1978]`

$$\mathcal{E}_{AB} \equiv R_{AB} - \frac{1}{3}F_{ACDE}F_B{}^{CDE} + \frac{1}{36}\eta_{AB}F_{CDEF}F^{CDEF} = 0$$

$$\mathcal{M}^{BCD} \equiv D_A F^{ABCD} + \frac{1}{576}\epsilon^{BCDE_1\ldots E_8}F_{E_1\ldots E_4}F_{E_5\ldots E_8} = 0$$

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Consider gauge fixed (*à la* ADM) equations *at some fixed spatial point* $\mathbf{x}_0$:

- keeping all temporal and first order spatial derivatives at $\mathbf{x}_0$

- zero-shift gauge: $E_M{}^A = \left(\begin{array}{c|c} N & 0 \\ \hline 0 & e_m{}^a \end{array}\right)$ and Coulomb gauge: $A_{tmn} = 0$

- Anholonomy coefficients $[\partial_b, \partial_c] = \tilde{\Omega}_{bc|a}\partial_a$ chosen traceless (in some neighborhood of $\mathbf{x}_0$) by exploiting *spatial* Lorentz group, i.e. $\Lambda_{ab} = \Lambda_{ab}(t, \mathbf{x})$ [???]

- Thus the standard ADM procedure leads to usual split into:

  – Dynamical equations: $\mathcal{E}_{ab} = \mathcal{M}_{abc} = D_{[0}F_{bcde]} = R_{[0a\,b]c} = 0$
  – Canonical constraints: $\mathcal{E}_{00} = \mathcal{E}_{0a} = \mathcal{M}_{0ab} = D_{[a}F_{bcde]} = R_{[ab\,c]d} = 0$

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Then with the identifi cation $n = Ne^{-1}$ and (r.h.s. always at fi xed spatial point $\mathbf{x} = \mathbf{x}_0$)

$$\mathcal{D}^{(0)}P_{ab}^{(0)} = R_{ab}^{\text{time derivatives}}$$

$$P_{abc}^{(1)} = NF_{0abc}$$

$$P_{a_1...a_6}^{(2)} = -\frac{1}{4!}N\epsilon_{a_1...a_6 b_1...b_4}F_{b_1...b_4}$$

$$P_{a_1...a_8|a_9}^{(3)} = \frac{3}{2}N\epsilon_{a_1...a_8 bc}\tilde{\Omega}_{bc|a_9}$$

the two sets of dynamical equations coincide! (recall $P_{[a_1...a_8|a_9]}^{(3)} = 0 \Leftrightarrow \tilde{\Omega}_{ab|b} = 0$)

Dynamical equations for mIIA and IIB similarly from level decompositions w.r.t. fi nite dimensional subgroups $D_9 \equiv SO(9,9) \subset E_{10}$ and $A_8 \times A_1 \equiv SL(9) \times SL(2) \subset E_{10}$.

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Conserved E_{10} current $\mathcal{J} = n\mathcal{V}\mathcal{P}\mathcal{V}^{-1}$ $(\equiv$ Noether charge associated with global E_{10}):

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\begin{aligned}
\mathcal{J} &= \frac{1}{9!} J^{m_0|m_1\ldots m_8}_{(-3)} F_{m_0|m_1\ldots m_8} + \frac{1}{6!} J^{m_1\ldots m_6}_{(-2)} F_{m_1\ldots m_6} + \frac{1}{3!} J^{mnp}_{(-1)} F_{mnp} \\
&\quad + J^{n}_{(0)m} K^{m}{}_{n} + \frac{1}{3!} J_{(1)\,mnp} E^{mnp} + \frac{1}{6!} J_{(2)\,m_1\ldots m_6} E^{m_1\ldots m_6} + \ldots
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Consider Sugawara-like ($\propto \mathcal{J} \otimes \mathcal{J}$) expressions   [DKN, hep-th 0709.2691]

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\begin{aligned}
\mathfrak{L}_{(-6)}^{m_1\ldots m_{10};\,n_0|n_1\ldots n_7} &= J_{(-3)}^{n_0|m_1\ldots m_8} J_{(-3)}^{m_9|m_{10}n_1\ldots n_7} \\
\mathfrak{L}_{(-5)}^{m_1\ldots m_{10};n_1\ldots n_5} &= J_{(-2)}^{n_1\ldots n_4 m_1 m_2} J_{(-3)}^{m_3\ldots m_{10}} \\
\mathfrak{L}_{(-4)}^{m_1\ldots m_{10};n_1 n_2} &= \frac{21}{5} J_{(-2)}^{n_1 m_1\ldots m_5} J_{(-2)}^{n_2 m_6\ldots m_{10}} + J_{(-1)}^{n_1 m_1 m_2} J_{(-3)}^{n_2|m_3\ldots m_{10}}
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(with appropriate antisymmetrizations) to re-express canonical constraints:

$$
\begin{aligned}
\mathfrak{L}_{(-6)}^{m_1\ldots m_{10}\,;\,n_0|n_1\ldots n_7} &\propto \epsilon^{m_1\ldots m_{10}} \epsilon^{n_1\ldots n_7 pqr} R_{pq\,rn_0} && \text{Bianchi (I)} \\
\mathfrak{L}_{(-5)}^{m_1\ldots m_{10};n_1\ldots n_5} &\propto \epsilon^{m_1\ldots m_{10}} \epsilon^{n_1\ldots n_5 p_1\ldots p_5} D_{p_1} F_{p_2\ldots p_5} && \text{Bianchi (II)} \\
\mathfrak{L}_{(-4)}^{m_1\ldots m_{10}\,;n_1 n_2} &\propto \epsilon^{m_1\ldots m_{10}} \mathcal{M}^{0 n_1 n_2} && \text{Gauss constraint} \\
\mathfrak{L}_{(-3)}^{m_1\ldots m_9} &\propto \epsilon^{m_1\ldots m_9 n} \mathcal{E}_{0n} && \text{Momentum constraint}
\end{aligned}
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- yields *same information* as maximal supersymmetry:
    - correct supermultiplets of SUGRA$_{11}$ and all $D < 11$ maximal supergravities
    - in particular: self-duality of 5-form field strength in IIB;
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- Fermions correctly described by R symmetry $K(E_{10})$.

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Thank you for your attention