Algebraic K-theory of finitely presented ring spectra

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The talk introduced a notion of G-Galois extension for commutative S-algebras (or E_∞ ring spectra), and presented the Galois descent problem for algebraic K-theory in this context. With E_n the Lubin-Tate spectrum with $E_{n*} = W(\mathbb{F}_{p^n})[[u_1, \dots, u_{n-1}]][u, u^{-1}]$ and $\bar{E}_n$ a maximal connected pro-Galois extension of E_n, we optimistically conjecture that

$$L_{K(n+1)} K(\bar{E}_n) \simeq E_{n+1}.$$

For $\ell_p = BP\langle 1\rangle_p$ the p-complete Adams summand of connective topological K-theory and L_p its periodic version, C. Ausoni and J. Rognes have computed the mod p and v_1 homotopy of $K(\ell_p)$, the answer being a free $P(v_2)$-module on $4p+4$ generators. This leads to the conjectural formula

$$4p+4 = \sum_{i=1}^{p^2-1} \sum_{n=0}^{\infty} \dim_{\mathbb{F}_p} H^n(\mathrm{Gal}(\bar{L}_p / L_p); \mathbb{F}_{p^2}(i))$$

with $\bar{L}_p = \bar{E}_1$. The talk indicated how algebraic K-theory of topological K-theory is a form of elliptic cohomology. More generally we presented evidence for the following:

Chromatic red-shift problem: Let E be an S-algebra of pure fp-type n. Does $K(E_n)$ have fp-type $n+1$?