Defects, SymTFTs, and (-1)-form symmetry from M-theory

Based on [2411.19683] with Marwan Najjar and Yi-Nan Wang

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Tsinghua University, Beijing

Outline

- ▷ Act I: Crash course on (-1)-form symmetries.

  based on [2403.03119] w/ R.J. Szabo

  - A closer look on higher form symmetries.
  - Tensions and resolutions with gerbes $\Longrightarrow$ (-1)-form symmetries.

- ▷ Act II: Defects and symmetries from M-theory.

  based on [2411.19683] w/ M. Najjar & Y.N. Wang + previous work [2112.02092]

  - Geometric engineering in 2 minutes.
  - Differential cohomology.
  - Classification of defects and symmetries.

- ▷ Finale: Geometric engineering of (-1)-form symmetries.

  based on [2411.19683] w/ M. Najjar & Y.N. Wang

  - Examples: 5d and 4d.
  - Applications.

Disclaimers

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- Questions are welcome at any time.

Act I: (-1)-form symmetries

Symmetries

Textbook symmetries are *group* actions on *local* operators.

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Generalized symmetries,

- relax 'local' $\Longrightarrow$ higher form symmetries;

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Textbook symmetries are categorical actions on *local* operators.

Generalized symmetries,

- relax 'local' $\Longrightarrow$ higher form symmetries;
- relax 'group' $\Longrightarrow$ non-invertible symmetries;

and combinations of the two.

Symmetries

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and combinations of the two.

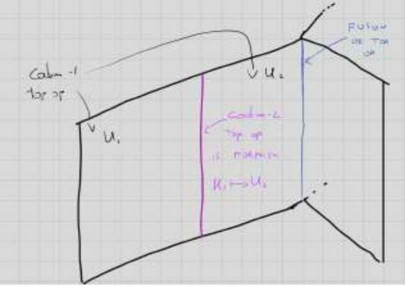
Modern definition/paradigm:

Symmetries of QFT are topological operators in it.

Categorical symmetries

Topological operators of d-dim QFT $\xrightarrow{\text{expectation}}$ $(d-1)$-category.

p-form symmetry $\Longleftrightarrow$ codim-$(p+1)$ top. op. $\Longleftrightarrow$ p-morphism.



Accounts for topological operator of codimension $1, \ldots, d \Longrightarrow p$-form symmetries,

$$0 \leq p \leq (d-1)$$

Gauging symmetries

Symmetries can be gauged.

- Gauge 0-form in $d = 2 \implies$ dual 0-form; *[Vafa '89]*

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- Gauge 0-form in $d = 2 \Longrightarrow$ dual 0-form; *[Vafa '89]*
- Gauge p-form in $d \Longrightarrow$ dual $(d - p - 2)$-form; *[GKSW '14, Tachikawa '17]*
- Gauge p-form in $d \Longrightarrow$ dual category. *[Bhardwaj-Tachikawa '17, Chang-Lin-Shao-Wang-Yin '18]*

Lower form symmetries?

Is the **lower form** symmetry a thing?

- **No!** No ($p \leq -1$)-objects to act on.

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Is the **lower form** symmetry a thing?

- **No!** No ($p \leq -1$)-objects to act on.
- **Yes!** (-1)-form symmetry is generated by top. op. filling connected components of spacetime.
- **Yes!** Gauging is 'reversible' $\Longrightarrow$ gauge ($d-1$), must obtain a dual (-1)-form symmetry. *[Sharpe '19]*

  «Gauging cannot be undone» S.H. Shao. Here I am talking about invertible symmetries only.

Many issues:

- What is a (-1)-form gauge transformation?
- What is a gauge field for (-1)-form symmetry?

Lower form symmetries

Many issues:

- What is a (-1)-form gauge transformation?
- What is a gauge field for (-1)-form symmetry?

Solution: *[LS-Szabo [2403.03119]]*

Invertible p-form symmetries are p-gerbes.

(-1)-form symmetries from gerbes

Invertible p-form symmetries are p-**gerbes** with connection.

$$\boxed{p\text{-form symmetry on } M \Longrightarrow \text{characteristic class} \in H^{p+2}(M,\mathbb{Z}).}$$

$\Longrightarrow$ Characteristic class $\exists$ for $p \geq -2$;

$\Longrightarrow$ Gauge fields $\exists$ for $p \geq -1$.

(-1)-form Avenues

Open problem: cast (-1)-form symmetries in categorical language

- Category of top. op. of SymTFT will include (-1)-form symmetry generators;
- Non-invertible (-1)-form symmetry. (example in *[LS-Szabo [2403.03119]]*)

Rest of this talk: Show presence of (-1)-form symmetries from M-theory compactifications.

Act II: Defects and SymTFTs from M-theory

M-theory

M-theory is defined on 11d manifold M_{11}.

- In this talk: $M_{11} = M_d \times X_{11-d}$, with

$$\begin{cases} d = 7 & X_4 \text{ is CY2} \\ d = 5 & X_6 \text{ is CY3} \\ d = 4 & X_7 \text{ is G2} \\ d = 3 & X_8 \text{ is CY4} \end{cases}$$

  or circle compactifications thereof.

- Dynamical 3-form field C_3 with curvature G_4 *locally $G_4 = dC_3$*
- Lagrangian

$$-\underbrace{G_4 \wedge G_7}_{\text{analogue of YM}} - \frac{1}{3!}\underbrace{C_3 \wedge G_4 \wedge G_4}_{\text{analogue of CS}}.$$

Geometric engineering in 2 minutes (1/2)

$\{\beta^a\}$ curve classes generating $H_2(X_{11-d}, \mathbb{Z}) \Longrightarrow$ harmonic 2-forms $\{\omega_a\}$,

$$\int_{\beta^a} \omega_b = \delta^a_b.$$

Expand C_3 in this basis *(sum over repeated indices understood)*

$$C_3 = A^a_1 \wedge \omega_a + \cdots$$

$\Longrightarrow U(1)$ Gauge fields A^a_1 propagating in M_d.

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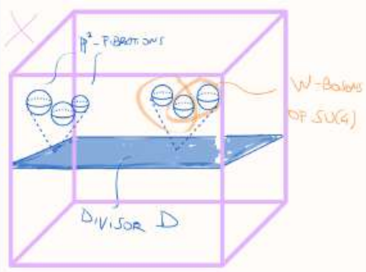
$$C_3 = A^a_1 \wedge \omega_a + \cdots$$

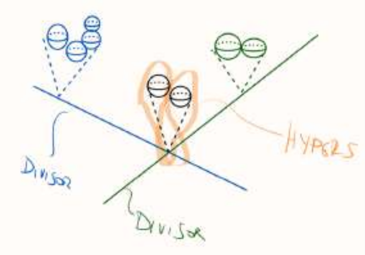
$\Longrightarrow$ $U(1)$ Gauge fields A^a_1 propagating in M_d.

- If X_{11-d} **compact**, all symmetries are gauged.
- If X_{11-d} **non-compact**, $\mathrm{PD}(\omega_a)$ may be compact or not
  - $\mathrm{PD}(\omega_a)$ **compact divisor** $\Longrightarrow$ A^a_1 dynamical gauge field of Cartan of total gauge algebra;
  - $\mathrm{PD}(\omega_a)$ **non-compact divisor** $\Longrightarrow$ A^a_1 background gauge field of Cartan of total symmetry algebra.

BPS states from wrapping M2-branes on curves in X

($d = 5$, X CY3 for exposition)





Differential cohomology for M-theory

What about torsion classes $\mathrm{Tor} H_2(X_{11-d}, \mathbb{Z})$?

Long known that appropriate formalism for M-theory is (a generalized) **differential cohomology** *[Hopkins-Singer '02, many others including Freed-Moore-Segal, (Sati-Schreiber)^n, (Fiorenza-Sati-Schreiber)^m, . . .]*

Differential cohomology in 5 minutes (1/5)

Def. Differential character of degree-p on M is group homomorphism

$$\chi_p : \{(p-1)\text{-cycles in } M\} \longrightarrow U(1)$$

subject to some conditions.

Def. Group of all degree-p differential characters on M is Cheeger–Simons group $\check{H}^p(M)$.

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Key advantage: deal with free and torsion at once.

Differential cohomology in 5 minutes (2/5)

$\check{H}^p(M)$ is Abelian group with properties:

- There exist a **field strength** map

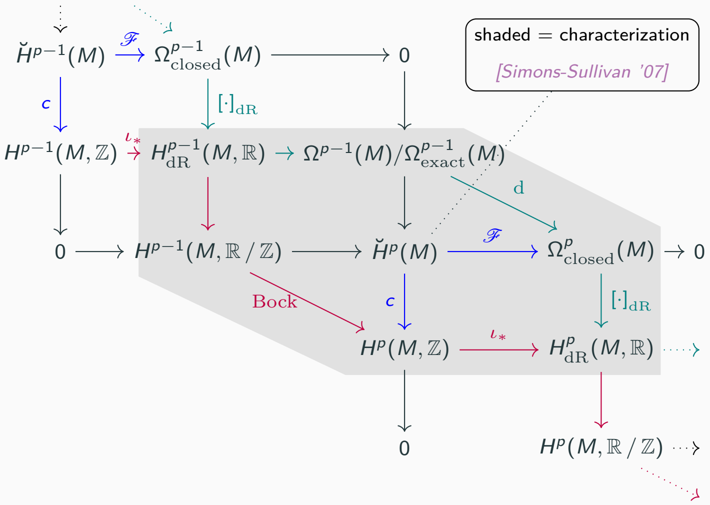
$$\mathscr{F} \ : \ \check{H}^p(M) \longrightarrow \Omega^p_{\mathbb{Z}}(M)$$

  and a **characteristic class** map

$$c \ : \ \check{H}^p(M) \longrightarrow H^p(M,\mathbb{Z}).$$

- $\chi_p(\partial\Sigma) = \exp\int_\Sigma F$ if $\mathscr{F}(\chi_p) = F$.
- $\pi_0\check{H}^p(M) = H^p(M,\mathbb{Z})$ and $\pi_1\check{H}^p(M) = H^{p-1}(M,\mathbb{Z})/\mathrm{Tor}H^{p-1}(M,\mathbb{Z})$.

Differential cohomology in 5 minutes (3/5)



Differential cohomology in 5 minutes (4/5)

Geometry Topology Seminar $\Longrightarrow$ study gerbes

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Geometry Topology & Physics Seminar	$\Longrightarrow$	study gerbes with connection

Differential cohomology in 5 minutes (4/5)

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Differential character generalizes **holonomy** of gauge field:

- If A_{p-1} globally defined gauge field,

$$\chi_p(\Sigma) = \exp \oint_\Sigma A_{p-1}.$$

- If $\Sigma = \partial\Sigma'$, by def.

$$\chi_p(\Sigma) = \exp \oint_\Sigma A_{p-1}.$$

Free + torsion $\Longrightarrow$ provide gauge fields for $U(1)$ **and discrete** symm.

Differential cohomology in 5 minutes (5/5)

$\exists$ **internal product** $\star$ inducing graded ring structure

$$\star \ : \ \breve{H}^p(M) \otimes \breve{H}^q(M) \longrightarrow \breve{H}^{p+q}(M).$$

It descends to $\wedge$ product in $\Omega^\bullet(M)$ and to cup product $\smile$ in $H^\bullet(M, \mathbb{Z})$,

$$\star \xrightarrow{\ \mathscr{F}\ } \wedge \qquad\qquad \star \xrightarrow{\ c\ } \smile \ .$$

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Fibration $M \hookrightarrow \mathcal{M} \twoheadrightarrow B$, M smooth of $\dim M = d$.
Integration map:

$$\int^{\breve{H}}_{\mathcal{M}/B} \ : \ \breve{H}^p(\mathcal{M}) \longrightarrow \breve{H}^{p-d}(B).$$

In particular, using $\breve{H}^1(\mathrm{pt}) \cong \mathbb{R}\,/\,\mathbb{Z}$, we have **integral**

$$\int^{\breve{H}}_{M} \ : \ \breve{H}^{d+1}(M) \longrightarrow \mathbb{R}\,/\,\mathbb{Z}\,.$$

Notation: χ_p with $\mathscr{F}(\chi_p) = F_p$ denoted $\breve{F}_p$.

Properties of product and integral:

- $\breve{F}_p \star \breve{G}_q = (-1)^{pq} \breve{G}_q \star \breve{F}_p$.
- When $\breve{F}_p$ is topologically trivial, $\mathscr{F}(\breve{F}_p \star \breve{G}_q) = \mathrm{d}\left(A_{p-1} \wedge \frac{G_q}{2\pi} \right)$.
- The integral operation defines a perfect pairing

$$\int_M^{\breve{H}} \ : \ \breve{H}^p(M) \times \breve{H}^{d+1-p}(M) \longrightarrow \mathbb{R}/\mathbb{Z},$$

$$(\breve{F}_p, \breve{G}_{d+1-p}) \ \mapsto \ \int_M^{\breve{H}} \breve{F}_p \star \breve{G}_{d+1-p}.$$

Chern–Simons term in differential cohomology

Notation: χ_p with $\mathscr{F}(\chi_p) = F_p$ denoted $\breve{F}_p$.

3d Chern–Simons action $\frac{k}{4\pi}\int_{M_3} A \wedge F$, but A is not globally defined
$\Longrightarrow$ Replace by $\frac{k}{4\pi}\int_{M_3}^{\breve{H}} \breve{F} \star \breve{F}$.

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Chern–Simons action for 3-form field C_3 becomes

$$\frac{1}{3!}\int_{M_{11}}^{\breve{H}} \underbrace{\breve{G}_4 \star \breve{G}_4 \star \breve{G}_4}_{\in \breve{H}^{12}(M_{11})} .$$

(locally $dC_3 = G_4$)

Includes both $U(1)$ **and discrete** Chern–Simons terms.

Geometric engineering of defects and SymTFTs

Key insight: *[Apruzzi Bonetti García-Etxebarria Hosseini Schafer-Nameki '21]*

differential cohomology + **geometric engineering**
$\Longrightarrow$ classify defects and symmetries of QFT_d on M_d.

Geometric engineering of defects and SymTFTs

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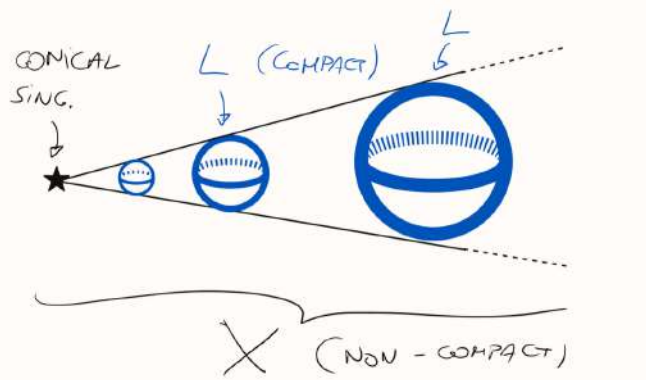
$\Longrightarrow$ classify defects and symmetries of QFT_d on M_d.

- Idea: SymTFT for discrete symmetries from differential cohomology
  [Apruzzi Bonetti García-Etxebarria Hosseini Schafer-Nameki '21]
  [García-Etxebarria Hosseini '24] [+ related work by many others, refs in the paper]
- New ingredients *[Najjar-LS-Wang 2411.19683]*
  - Unify **continuous & finite** symmetries;
  - Attention to (-1)-**form** symmetries.

SymTFTs from Geometric engineering (1/3)

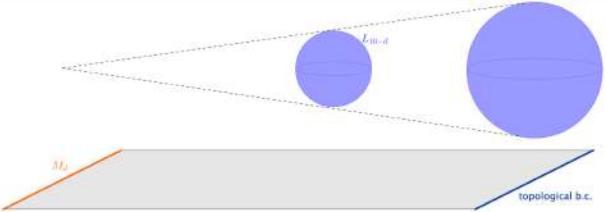
Assume $X_{11-d} = \text{Cone}(L_{10-d})$, where

$$L_{10-d} := \text{link of the singularity } X_{11-d}$$



- **differential cohomology:** Replace $G_4 \mapsto \breve{G}_4$;
- **Geometric engineering:** Expand $\breve{G}_4$ in differential cohomology of L_{10-d};

$\Longrightarrow$ Obtain $(d+1)$-dim theory called **SymTFT**, coupled to physical QFT_d at the 'tip-of-the-cone' boundary.



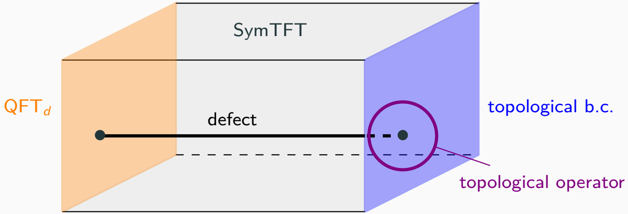
SymTFTs from Geometric engineering (3/3)

$\{v_k^i\}$= basis of Free$H^k(L,\mathbb{Z})$; $\{t_k^\alpha\}$= basis of Tor$H^k(L,\mathbb{Z})$,

$$\breve{G}_4 = \sum_{k=0}^{4}\sum_i \breve{F}_{4-k,i} \star \breve{v}_k^i \ + \ \sum_{k=0}^{4}\sum_\alpha \breve{B}_{4-k,\alpha} \star \breve{t}_k^\alpha$$

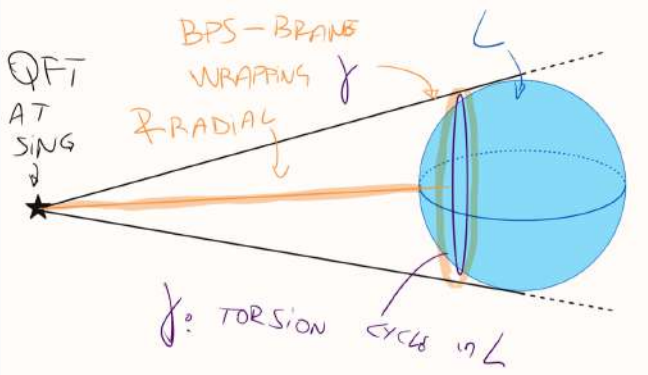
- Reduce along free cycle $\Longrightarrow$ $F_{4-k,i}$ curvature of $(2-k)$-form $U(1)$;
- Reduce along torsion cycle $\Longrightarrow$ $B_{4-k,\alpha}$ discrete gauge field of $(3-k)$-form $\mathbb{Z}_{n_\alpha}$;

all are **global symmetries** of QFT$_d$, fields live in SymTFT$_{d+1}$.



Defects from M-branes

Wrap M2 or M5 on $\gamma \in \mathrm{Tor} H_\bullet(L_{10-d}, \mathbb{Z})$ and stretch in radial direction
$\Longrightarrow$ stretch in radial direction of SymTFT and pierces through physical QFT_d
$\Longrightarrow$ Engineer defect charged under **finite** symmetry.



Topological operators from M-branes (1/3)

$\int_{L_{10-d}}^{\breve{H}}$ induces two (perfect) pairings:

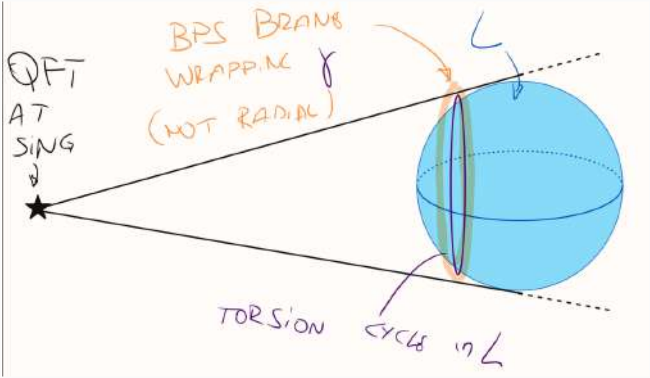
$$\mathrm{Tor}H^k \times \mathrm{Tor}H^{11-d-k} \longrightarrow \mathbb{R}/\mathbb{Z} \xrightarrow{\text{Poincaré dual}} \text{linking pairing}$$

$$\mathrm{Free}H^k \times \mathrm{Free}H^{10-d-k} \longrightarrow \mathbb{Z} \xrightarrow{\text{Poincaré dual}} \text{intersection pairing}$$

$H_\bullet$	charged defect	symmetry op.
Tor	M2	M5
Tor	M5	M2
Free	M2	P_7-flux ($\neq G_7$-flux)
Free	M5	P_4-flux ($= G_4$-flux)

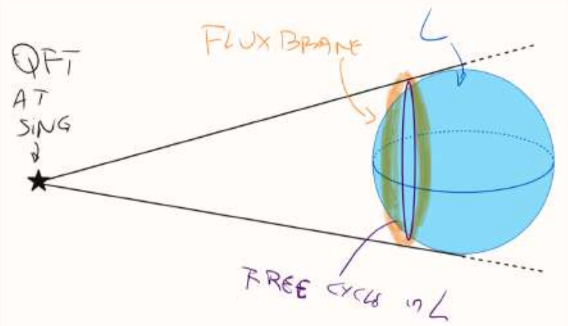
$\Longrightarrow$ M-branes on **Tor** cycles, fluxbranes on **Free** cycles

Wrap M2 or M5 on $\gamma \in \mathrm{Tor} H_\bullet(L_{10-d}, \mathbb{Z}) \Longrightarrow$ live in SymTFT parallel to physical QFT_d and to topological boundary conditions
$\Longrightarrow$ Engineer topological operator generating **finite** symmetry.



Topological operators from M-branes (3/3)

Fluxbrane on $\gamma \in \mathrm{Free}H_\bullet(L_{10-d},\mathbb{Z}) \Longrightarrow$ live in SymTFT parallel to physical QFT$_d$ and to topological boundary conditions
$\Longrightarrow$ Engineer topological operator generating $U(1)$ symmetry.



Fluxbranes, M5 and differential cohomology cf. Fiorenza-Sati-Schreiber '12, '19 etc

«Fluxbranes are unstable». But they are topological from SymTFT perspective.

Finale: Classification of (-1)-form symmetries from M-theory

Applications to 5d and 4d

Applications of the formalism:

- X_6 CY3 $\Longrightarrow$ defects and SymTFTs of 5d SQFTs;
- X_7 G2 manifold $\Longrightarrow$ defects and SymTFTs of 4d SQFTs.

CY3 and 5d SCFTs

X_6: generic CY3 singularity. L_5=link of $X_6 \Longrightarrow$ 5d $\mathcal{N} = 1$ SCFT.

Classify defects from branes:

	M2		M5	
$\mathrm{Tor}H_1(L_5,\mathbb{Z}) \times [0,\infty)$	Wilson line	♢	Domain wall	♠
$\mathrm{Tor}H_2(L_5,\mathbb{Z}) \times [0,\infty)$	Local operator	∘	3d defect	△
$\mathrm{Tor}H_3(L_5,\mathbb{Z}) \times [0,\infty)$	—	♣	Magnetic string	♡
$\mathrm{Tor}H_1(L_5,\mathbb{Z})$	2-form sym. generator	♡	(-1)-form sym. generator	♣
$\mathrm{Tor}H_2(L_5,\mathbb{Z})$	3-form sym. generator	△	0-form sym. generator	∘
$\mathrm{Tor}H_3(L_5,\mathbb{Z})$	4-form sym. generator	♠	1-form sym. generator	♢

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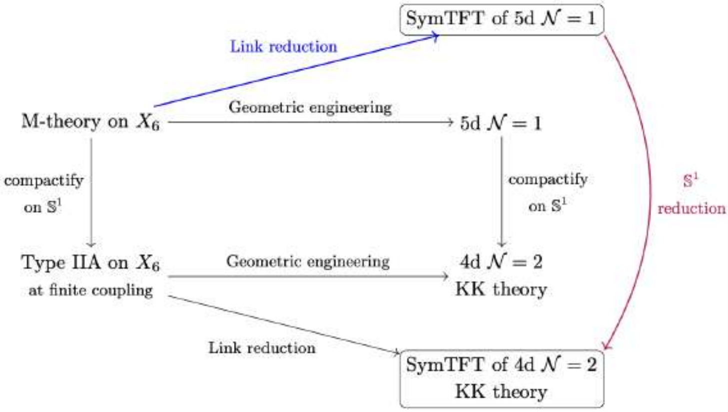
SymTFT derivation in differential cohomology from M-theory:

- Find background fields for all the symmetries;
- Find all mixed **anomalies**.
- Find (-1)-form symmetry background fields.

G2 and 4d N=2 SQFTs

$X_7 = X_6 \times \mathbb{S}^1$: obtain 4d $\mathcal{N} = 2$ KK theory.

SymTFT from circle reduction:



G2 and 4d N=1 SQFTs

X_7: a manifold with G2 holonomy and link $L_6 = (\mathbb{S}^3/\mathbb{Z}_{pN}) \times (\mathbb{S}^3/\mathbb{Z}_p)$
$\Longrightarrow$ obtain 4d $\mathcal{N}=1$ SQFT.

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$\mathrm{Tor} H_3(L_6, \mathbb{Z}) \times [0,\infty)$	—	♣	Surface defect	△
$\mathrm{Tor} H_4(L_6, \mathbb{Z}) \times [0,\infty)$			't Hooft line	♡
$\mathrm{Tor} H_1(L_6, \mathbb{Z})$	1-form sym. generator	♡		
$\mathrm{Tor} H_2(L_6, \mathbb{Z})$	2-form sym. generator	△	(-1)-form sym. generator	♣
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$\mathrm{Tor}H_4(L_6,\mathbb{Z})$			1-form sym. generator	♢

SymTFT derivation in differential cohomology from M-theory:

- Find background fields and mixed **anomalies** for all the symmetries;
- Find **continuous + finite** (-1)-form symmetry background fields.
- Find **modified instanton sum** *[Tanizaki-Unsal '19]*

(-1)-form symmetries in 5d

Fact: We have found discrete (-1)-form symmetries in 5d from M-theory geometric engineering.

Open problem: Understand them better with more canonical methods.

- Lagrangian description?
- Dynamical implications?

(-1)-form symmetries in 5d

Fact: We have found discrete (-1)-form symmetries in 5d from M-theory geometric engineering.

Open problem: Understand them better with more canonical methods.

- Lagrangian description?
- Dynamical implications?

Open problem: More analysis in lower dimension and lower supersymmetry.

SymTFT for continuous symmetries

⚠ $\exists$ Three proposals for SymTFT of **continuous** symmetries

$\begin{cases} \text{Proposal 1} \\ \text{Proposal 2} \end{cases}$

Proposal 1 *[Brennan-Sun 2401.06128]*

Proposal 2 *[Antinucci-Benini 2401.10165]*

Proposal 3 *[Apruzzi-Bedogna-Dondi 2402.14813]*

Proposal 4 *[Bonetti-Minasian-DelZotto 2402.12347]*

Differ in electric/magnetic coupling:

- Proposal 1 & 2: $F \wedge h$ with h gauge field for $\mathbb{R}$-bundle;
- Proposal 3: $F \wedge H$ with H curvature.

SymTFT for continuous symmetries

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$\begin{cases} \text{Proposal 1} \\ \text{Proposal 2} \end{cases}$ Proposal 1 *[Brennan-Sun 2401.06128]*, Proposal 2 *[Antinucci-Benini 2401.10165]*

Proposal 3 *[Apruzzi-Bedogna-Dondi 2402.14813]*

Differ in electric/magnetic coupling:

- Proposal 1 & 2: analogy with discrete symmetry BF-term;
- Proposal 3: derived from **geometric engineering** in M-theory *[Najjar-LS-Wang 2411.19683]*.

SymTFT for continuous symmetries

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- Proposal 1 *[Brennan-Sun 2401.06128]*
- Proposal 2 *[Antinucci-Benini 2401.10165]*

Proposal 3 *[Apruzzi-Bedogna-Dondi 2402.14813]*

Differ in electric/magnetic coupling:

- Proposal 1 & 2: derived from symmetry descent in **Type IIB** *[Gagliano-García-Extebarria 2411.15126]*
- Proposal 3: derived from geometric engineering in **M-theory** *[Najjar-LS-Wang 2411.19683]*.

Open problem: settle this puzzle or reconcile approaches.

Conclusions

Summary

Formalism for
(-1)-form symmetries

Formalism for
SymTFT from M-theory

Classification of (-1)-form symmetries
from M-theory

Formalism for (−1)-form symmetries → Classification of (−1)-form symmetries from M-theory

Formalism for SymTFT from M-theory → Classification of (−1)-form symmetries from M-theory

Classification of (−1)-form symmetries from M-theory

This talk:

Reality:

Conclusions

- QFT: we gain a lot of mileage treating (-1)-form symmetries as actual symmetries;
- M-theory: (-1)-form symmetries are derived on same footing as any other symmetry.

Differential cohomology + geometric engineering on $L =$ link of X
$\Longrightarrow$ we obtained

- ▷ Full SymTFT and classification of symmetries, **both discrete and continuous**;
- ▷ Detailed study of (-1)-form symmetries, **both discrete and continuous**, in 5d and 4d.

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Thank you for your attention!