Flux Quantization in F-theory and Freed-Witten anomaly

Raffaele Savelli
MPI - Munich

String-Math, Bonn,
July 2012

Based on work with A. Collinucci,
arXiv: 1011.6388, 1203.4542
Motivations
**Motivations**

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.
Global aspects of F-theory configurations are of central importance both phenomenologically and formally. For instance, a key issue is the correct quantization of fluxes.
Global aspects of F-theory configurations are of central importance both phenomenologically and formally.

For instance, a key issue is the correct quantization of fluxes.

This is intimately related to a crucial consistency check, i.e. to the vanishing of Freed-Witten anomalies for 7-branes.
Motivations

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.

- For instance, a key issue is the correct quantization of fluxes.

- This is intimately related to a crucial consistency check, i.e. to the vanishing of **Freed-Witten anomalies** for 7-branes.

- A detailed understanding of fluxes is relevant for problems like **moduli stabilization** and **generation of chiral matter**.
Global aspects of F-theory configurations are of central importance both phenomenologically and formally.

For instance, a key issue is the correct quantization of fluxes.

This is intimately related to a crucial consistency check, i.e. to the vanishing of Freed-Witten anomalies for 7-branes.

A detailed understanding of fluxes is relevant for problems like moduli stabilization and generation of chiral matter.

On a more fundamental level, a generalization of FW anomaly beyond weak coupling would be desirable.
**Motivations**

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.

- For instance, a key issue is the **correct quantization of fluxes**.

- This is intimately related to a crucial consistency check, i.e. to the vanishing of **Freed-Witten anomalies** for 7-branes.

- A detailed understanding of fluxes is relevant for problems like **moduli stabilization** and **generation of chiral matter**.

- On a more fundamental level, a generalization of FW anomaly **beyond weak coupling** would be desirable.

FW anomaly cancellation
Motivations

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.

- For instance, a key issue is the correct quantization of fluxes.

- This is intimately related to a crucial consistency check, i.e. to the vanishing of **Freed-Witten anomalies** for 7-branes.

- A detailed understanding of fluxes is relevant for problems like **moduli stabilization** and generation of **chiral matter**.

- On a more fundamental level, a generalization of FW anomaly beyond weak coupling would be desirable.

---

FW anomaly cancellation prescribes the quantization rules of brane fluxes
Motivations

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.

- For instance, a key issue is the **correct quantization of fluxes**.

- This is intimately related to a crucial consistency check, i.e. to the vanishing of **Freed-Witten anomalies** for 7-branes.

- A detailed understanding of fluxes is relevant for problems like **moduli stabilization** and **generation of chiral matter**.

- On a more fundamental level, a generalization of FW anomaly **beyond weak coupling** would be desirable.

FW anomaly cancellation

prescribes the quantization rules of brane fluxes

constrains the topological type of bulk fluxes
**Motivations**

- **Global aspects** of F-theory configurations are of central importance both phenomenologically and formally.

- For instance, a key issue is the **correct quantization of fluxes**.

- This is intimately related to a crucial consistency check, i.e. to the vanishing of **Freed-Witten anomalies** for 7-branes.

- A detailed understanding of fluxes is relevant for problems like **moduli stabilization** and **generation of chiral matter**.

- On a more fundamental level, a generalization of FW anomaly beyond weak coupling would be desirable.

FW anomaly cancellation

prescribes the quantization rules of brane fluxes  

constraints the topological type of bulk fluxes  

in this talk
Basic set-up
Basic set-up

F-theory on $\mathbb{R}^{1,3} \times CY_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume
Basic set-up

F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

Diagram: T$^2$ $\rightarrow$ $S^1_M \times S^1_T$
Basic set-up

F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

- Reduce $\text{M} \rightarrow \text{IIA}$ on $S^1_M$
- T-dualize IIA $\rightarrow$ IIB on $S^1_T$
- Send $V_{T^2} \rightarrow 0$
F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

Basic set-up

- Reduce M → IIA on $S^1_M$
- T-dualize IIA → IIB on $S^1_T$
- Send $V_{T^2} \rightarrow 0$

Result: IIB string theory on $\mathbb{R}^{1,3} \times B$
with $\tau = C_0 + ie^{-\phi}$ holomorphically varying on B
**Basic set-up**

F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

- Reduce M $\rightarrow$ IIA on $S^1_M$
- T-dualize IIA $\rightarrow$ IIB on $S^1_T$
- Send $V_{T^2} \rightarrow 0$

Result: IIB string theory on $\mathbb{R}^{1,3} \times B$

with $\tau = C_0 + ie^{-\phi}$

holomorphically varying on B

(p,q)7-brane: divisor on which $pS^1_M + qS^1_T$ collapses
F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ “defined as” M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

- Reduce $M \rightarrow$ IIA on $S^1_M$
- T-dualize IIA $\rightarrow$ IIB on $S^1_T$
- Send $V_{T^2} \rightarrow 0$

Result: IIB string theory on $\mathbb{R}^{1,3} \times B$ with $\tau = C_0 + ie^{-\phi}$ holomorphically varying on $B$

(p,q)7-brane: divisor on which $pS^1_M + qS^1_T$ collapses

Collision of 7-branes: singularities of CY$_4$ & non-abelian gauge symmetry
F-theory on $\mathbb{R}^{1,3} \times CY_4$ “defined as” M-theory on elliptic $CY_4$ in the limit of vanishing fiber volume

- Reduce $M \rightarrow IIA$ on $S^1_M$
- T-dualize $IIA \rightarrow IIB$ on $S^1_T$
- Send $V_{T^2} \rightarrow 0$

Result: IIB string theory on $\mathbb{R}^{1,3} \times B$

with $\tau = C_0 + ie^{-\phi}$

holomorphically varying on $B$

Collision of 7-branes: singularities of $CY_4$ & non-abelian gauge symmetry

$(p,q)$7-brane: divisor on which $pS^1_M + qS^1_T$ collapses

Fluxes: $G_4$

- Bulk fluxes

- 7-brane flux
F-theory on $\mathbb{R}^{1,3} \times \text{CY}_4$ "defined as" M-theory on elliptic CY$_4$ in the limit of vanishing fiber volume

- Reduce M $\rightarrow$ IIA on $S^1_M$
- T-dualize IIA $\rightarrow$ IIB on $S^1_T$
- Send $V_{T^2} \rightarrow 0$

Result: IIB string theory on $\mathbb{R}^{1,3} \times B$
with $\tau = C_0 + ie^{-\phi}$
holomorphically varying on B

Collision of 7-branes: singularities of CY$_4$ & non-abelian gauge symmetry

M/F

Fluxes: $G_4$

7-brane flux

bulk fluxes

G$_4$ must have one and only one leg along $T^2$
Purposes
We want to study the aspect of quantization of fluxes
We want to study the aspect of quantization of fluxes

The theory of M2 propagating in CY\_4 may suffer from a global anomaly

$$[G_4] + \frac{c_2(\text{CY}_4)}{2} \in H^4(\text{CY}_4, \mathbb{Z})$$
We want to study the aspect of quantization of fluxes

The theory of M2 propagating in CY\(_4\) may suffer from a global anomaly \(E.\ Witten\ \text{`96}\)

\[ [G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z}) \]

- shift in the quantization of \(G_4\)

- Issue for smooth CY: \(c_2(CY_4)\) has either two or no legs along the fiber!

4D Lorentz in CY-compactifications with odd \(c_2\)!
We want to study the aspect of quantization of fluxes.

The theory of M2 propagating in CY\(_4\) may suffer from a global anomaly, as noted by E. Witten in 1996.

- Shift in the quantization of G\(_4\):
  \[ [G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z}) \]

- Issue for smooth CY: \(c_2(CY_4)\) has either two or no legs along the fiber.

- 4D Lorentz in CY-compactifications with odd \(c_2\)!

Luckily: \(c_2\) is always an even class for smooth elliptic CY\(_4\), as noted by A. Collinucci and R.S. in 2010.
We want to study the aspect of quantization of fluxes.

The theory of M2 propagating in CY$_4$ may suffer from a global anomaly:

$$[G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z})$$

- Issue for smooth CY: $c_2(CY_4)$ has either two or no legs along the fiber!

- CY singularities: Half-quantization arises when the singular locus is non-spin (SU & Sp)
We want to study the aspect of quantization of fluxes

The theory of M2 propagating in CY\(_4\) may suffer from a global anomaly \(E. Witten \ '96\)

\[ [G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z}) \]

\(\rightarrow\) shift in the quantization of \(G_4\)

• Issue for smooth CY: \(c_2(CY_4)\) has either two or no legs along the fiber!

\(\rightarrow\) 4D Lorentz in CY-compactifications with odd \(c_2\)!

Luckily: \(c_2\) is always an even class for smooth elliptic CY\(_4\) \(A. Collinucci, R.S. \ '10\)

• CY singularities: Half-quantization arises when the singular locus is non-spin \((SU \ & Sp)\) \(A. Collinucci, R.S. \ '12\)

Connection to the Freed-Witten anomaly of the corresponding D7-stack \(S\)

\[ [F] - \frac{c_1(S)}{2} \in H^2(S, \mathbb{Z}) \quad \text{with } F \text{ the gauge flux along each Cartan} \]
We want to study the aspect of quantization of fluxes

The theory of M2 propagating in CY$_4$ may suffer from a global anomaly \( E. \text{Witten} \ '96 \)

\[ [G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z}) \]

\( \rightarrow \) shift in the quantization of \( G_4 \)

• Issue for smooth CY: \( c_2(CY_4) \) has either two or no legs along the fiber!

\( \rightarrow \) 4D Lorentz in CY-compactifications with odd \( c_2 \)!

Luckily: \( c_2 \) is always an even class for smooth elliptic CY$_4$ \( A. \text{Collinucci, R.S.} \ '10 \)

• CY singularities: Half-quantization arises when the singular locus is non-spin \( (SU & Sp) \)

Connection to the Freed-Witten anomaly of the corresponding D7-stack \( S \)

\[ [F] - \frac{c_1(S)}{2} \in H^2(S, \mathbb{Z}) \quad \text{with} \ F \ \text{the gauge flux along each Cartan} \]

• We want to find a direct and explicit map:

\( C^{(2)} \in H_2(S, \mathbb{Z}) \)

detecting FW anomaly
We want to study the aspect of quantization of fluxes

The theory of M2 propagating in CY\(_4\) may suffer from a global anomaly

\[ [G_4] + \frac{c_2(CY_4)}{2} \in H^4(CY_4, \mathbb{Z}) \]

- Issue for smooth CY: \(c_2(CY_4)\) has either two or no legs along the fiber!

- CY singularities: Half-quantization arises when the singular locus is non-spin (SU & Sp)

Connection to the Freed-Witten anomaly of the corresponding D7-stack S

\[ [F] - \frac{c_1(S)}{2} \in H^2(S, \mathbb{Z}) \]

We want to find a direct and explicit map:

\[ C^{(2)} \in H_2(S, \mathbb{Z}) \]

detecting FW anomaly

\[ \text{Lift} \]

\[ C^{(4)} \in H_4(CY_4, \mathbb{Z}) \]

detecting M2 anomaly
The SU(2N) case
The SU(2N) case

Resolved fiber over SU(4) locus  →  Affine Dynkin diagram of SU(4)

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \( \{P = Q = 0\} \) due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus \[\xrightarrow{\text{}}\] Affine Dynkin diagram of SU(4)

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \(\{P = Q = 0\}\) due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus  ⇐⇒  Affine Dynkin diagram of SU(4)

\[\rho^2_i\]

We refer to appendix A for the details of the geometry.

One can now easily seek for the detecting 4-cycles in analogy with the analysis done for the $\text{Sp}(N)$ singularities in sec. 3.2, by imposing that:

\[D \equiv P \hat{D} + Q \tilde{D} \quad (4.6)\]

\[a_{2k+2, k}^2 N \equiv P \hat{a}_{2k+2, k} + Q \tilde{a}_{2k+2, k} \quad (4.7)\]

The ansatz makes the gauge symmetry enhance on \(\{P = Q = 0\}\) from $\text{SU}(2N)$ to $\text{SU}(2N+1)$ along the whole matter curve. The enhancement manifest itself as the splitting into two of the node $E_{2N-1} \rightarrow E_1 \cup E_2$. Such transition is shown in fig. 3 for the $N=2$ case.

Figure 3: This shows the transition from the extended Dynkin diagram of $\text{SU}(4)$ (left) to the extended Dynkin diagram of $\text{SU}(5)$ (right) happening along the curve $\{P = Q = 0\}$ due to the singularity enhancement. The fifth D-brane of the $\text{SU}(5)$ stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus \[ \rightarrow \] Affine Dynkin diagram of SU(4)

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \( \{ P = Q = 0 \} \) due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus \[ \rightarrow \] Affine Dynkin diagram of SU(4)

Cartan nodes

D6-brane

Extended node

S\(^1\)-fibration over 2-4 string \[ \rightarrow \] M2

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \{ P = Q = 0 \} due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus ↔ Affine Dynkin diagram of SU(4)

Loops of i-j IIA open strings
The SU(2N) case

Resolved fiber over SU(4) locus \(\rightarrow\) Affine Dynkin diagram of SU(4)

D6-brane

Cartan nodes

Loops of i-j IIA open strings

Extended node

Loops of boundaries on D6\(_i\) and D6\(_j\) \(\rightarrow\) \(C^{(2)}\)

\(E_0\) \(\rightarrow\) \(E_1\) \(\rightarrow\) \(E_3\) \(\rightarrow\) \(E_2\)

\(S^1\)-fibration over 2-4 string \(\rightarrow\) M2

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \(\{P = Q = 0\}\) due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.
The SU(2N) case

Resolved fiber over SU(4) locus ⇐⇒ Affine Dynkin diagram of SU(4)

D6-brane

Cartan nodes

Loops of i-j IIA open strings

Loops of boundaries on D6i and D6j → C^{(2)}

Loops of M2s = En fibered over C^{(2)} → C^{(4)}
The SU(2N) case

Resolved fiber over SU(4) locus \[\leftrightarrow\] Affine Dynkin diagram of SU(4)

Loops of i-j IIA open strings

D6-brane

Cartan nodes

Loops of boundaries on D6\(_i\) and D6\(_j\)

Loops of M2s = En fibered over C\(^{(2)}\)

S\(^1\)-fibration over 2-4 string \[\rightarrow\] M2

Extended node

Figure 3: This shows the transition from the extended Dynkin diagram of SU(4) (left) to the extended Dynkin diagram of SU(5) (right) happening along the curve \(\{P = Q = 0\}\) due to the singularity enhancement. The fifth D-brane of the SU(5) stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.

19
The SU(2N) case

Resolved fiber over SU(4) locus $\rightarrow$ Affine Dynkin diagram of SU(4)

$D_6$-brane

Cartan nodes

Loops of i-j IIA open strings

Loops of boundaries on $D_{6i}$ and $D_{6j}$ $\rightarrow C^{(2)}$

Loops of $M_2$s $= E_n$ fibered over $C^{(2)}$ $\rightarrow C^{(4)}$

However, these 4-cycles are NOT able to detect the $M_2$ anomaly!
The SU(2N) case

Resolved fiber over SU(4) locus \[\rightsquigarrow\] Affine Dynkin diagram of SU(4)

Loops of i-j IIA open strings

Loops of boundaries on D6 \[\downarrow\text{lift}\] \[\rightarrow\] \(C^{(2)}\)

Loops of M2s = En fibered over \(C^{(2)}\) \[\rightarrow\] \(C^{(4)}\)

However, these 4-cycles are NOT able to detect the M2 anomaly!

e.g. \(E_3 \rightarrow C^{(4)}\) \[\downarrow\] \(C^{(2)}\)

\[\int_{C^{(4)}} \frac{C_2}{2} \sim \int_{C^{(2)} \subset D6_3} F|_3 - \int_{C^{(2)} \subset D6_4} F|_4 \quad \text{integer}\]
The SU(2N) case

Resolved fiber over SU(4) locus \[\rightarrow\] Affine Dynkin diagram of SU(4)

Loops of i-j IIA open strings

Loops of boundaries on D6\(_i\) and D6\(_j\) \[\rightarrow\] C\(^{(2)}\)

Loops of M2s = En fibered over C\(^{(2)}\) \[\rightarrow\] C\(^{(4)}\)

However, these 4-cycles are NOT able to detect the M2 anomaly!

e.g. \[E_3 \rightarrow C^{(4)} \rightarrow \int_{C^{(4)}} \frac{C_2}{2} \sim \int_{C^{(2)} \subset D6_3} F|_3 - \int_{C^{(2)} \subset D6_4} F|_4 \quad \text{integer}\]

C\(^{(4)}\) complete-intersection \[\rightarrow\] \[\int_{C^{(4)}} C_2 \quad \text{is even}\]

S.Krause, C.Mayrhofer, T.Weigand `12
Strategy:
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane.

Natural candidate: $O(1)$ invariant D7-brane $W$ with “Whitney Umbrella” shape.
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane.

Natural candidate: O(1) invariant D7-brane \( W \) with “Whitney Umbrella” shape.

Such nodes pop up along the “fundamental-matter” curve \( S \cap W \subset CY_3 \).
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane.

Natural candidate: $O(1)$ invariant D7-brane $W$ with “Whitney Umbrella” shape.

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$.

\[ D \equiv \hat{P} D + \tilde{Q} D \quad (4.6) \]
\[ a_{2N+2} \equiv \hat{a}_{2N+2} + \tilde{a}_{2N+2} \quad (4.7) \]

The ansatz makes the gauge symmetry enhance on $\{P = Q = 0\}$ from $SU(2N)$ to $SU(2N+1)$ along the whole matter curve. The enhancement manifests itself as the splitting into two of the node $E_{2N-1} \rightarrow E_{1}^{(1)} \cup E_{2}^{(2)}$. Such a transition is shown in fig. 3 for the $N=2$ case.

\[ \text{Figure 3: This shows the transition from the extended Dynkin diagram of } SU(4) \text{ (left) to the extended Dynkin diagram of } SU(5) \text{ (right) happening along the curve } \{P = Q = 0\} \text{ due to the singularity enhancement. The fifth D-brane of the } SU(5) \text{ stack is given by the Whitney-type brane. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.} \]

BUT $C^{(2)} \neq S \cap W$ as $W$ is anomaly free!
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane.

Natural candidate: $O(1)$ invariant D7-brane $W$ with “Whitney Umbrella” shape.

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$.

BUT $C^{(2)} \neq S \cap W$ as $W$ is anomaly free!

Constrain $CY_4$ complex structure such that $S \cap W$ is reducible and choose $C^{(2)}$ to be one component.
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane

Natural candidate: O(1) invariant D7-brane $W$ with “Whitney Umbrella” shape

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$

BUT $C^{(2)} \neq S \cap W$ as $W$ is anomaly free!

Constrain $CY_4$ complex structure such that $S \cap W$ is reducible and choose $C^{(2)}$ to be one component

Some integral 4-classes of $CY_4$ acquire holomorphic representatives

Mathematically: $H^{2,2}_H(CY_4) \cap H^4(CY_4, \mathbb{Z}) \neq 0$  

A.Braun, A.Collinucci, R.Valandro `11
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane

Natural candidate: O(1) invariant D7-brane $W$ with “Whitney Umbrella” shape

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$

But $C^{(2)} \neq S \cap W$ as $W$ is anomaly free!

$\Rightarrow$ Constrain CY$_4$ complex structure such that $S \cap W$ is reducible and choose $C^{(2)}$ to be one component

$\Rightarrow$ Some integral 4-classes of CY$_4$ acquire holomorphic representatives

Mathematically: $H^{2,2}_H(CY_4) \cap H^4(CY_4, \mathbb{Z}) \neq 0$ A.Braun, A.Collinucci, R.Valandro '11

They are: $E_3^{(1,2)} \rightarrow C^{(4)}$  $\Rightarrow$ NOT matter surfaces!

$\Rightarrow \int_{C^{(4)}} \frac{c_2}{2} \sim \int_{C^{(2)} \subset D_{63}} F|_3$
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane

Natural candidate: O(1) invariant D7-brane $W$ with “Whitney Umbrella” shape

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$

\[ \text{BUT } C^{(2)} \neq S \cap W \text{ as } W \text{ is anomaly free!} \]

\[ \Rightarrow \text{Constrain } CY_4 \text{ complex structure} \]
\[ \text{such that } S \cap W \text{ is reducible} \]
\[ \text{and choose } C^{(2)} \text{ to be one component} \]

\[ \Rightarrow \text{Some integral 4-classes of } CY_4 \text{ acquire holomorphic representatives} \]

Mathematically: \[ H^{2,2}_H(CY_4) \cap H^4(CY_4, \mathbb{Z}) \neq 0 \]

They are: \[ E_3^{(1,2)} \rightarrow C^{(4)} \]
\[ \downarrow \]
\[ C^{(2)} \]

\[ \Rightarrow \text{NOT matter surfaces!} \]
\[ \Rightarrow \int_{C^{(4)}} \frac{c_2}{2} \sim \int_{C^{(2)} \subset D6} F |_3 \]

General result for $SU(2N) \ N \geq 2$

\[ \int_{C^{(4)}} c_2(CY_4) = \int_{C^{(2)}} 6c_1(B) - (2N - 1)S \]
Strategy: Use a node interpolating between a brane of the stack and a fluxless brane.

Natural candidate: O(1) invariant D7-brane $W$ with “Whitney Umbrella” shape.

Such nodes pop up along the “fundamental-matter” curve $S \cap W \subset CY_3$.

**BUT** $C^{(2)} \neq S \cap W$ as $W$ is anomaly free!

**Constrain** CY$_4$ complex structure such that $S \cap W$ is reducible and choose $C^{(2)}$ to be one component.

**Some integral 4-classes of CY$_4$ acquire holomorphic representatives**

Mathematically: $H^{2,2}_H(CY_4) \cap H^4(CY_4, \mathbb{Z}) \neq 0$

They are: $E_3^{(1,2)} \rightarrow C^{(4)} \rightarrow C^{(2)}$

**NOT** matter surfaces!

$$\int_{C^{(4)}} \frac{c_2}{2} \sim \int_{C^{(2)} \subset D6_3} F|_3$$

General result for $SU(2N)$ $N \geq 2$

$$\int_{C^{(4)}} c_2(CY_4) = \int_{C^{(2)}} 6c_1(B) - (2N - 1)S$$

Same procedure applies for the $Sp(N)$ series.
The SU(2N+1) case
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

Figure 4: This shows the transition from the extended Dynkin diagram of SU(5) (left) to the extended Dynkin diagram of SO(10) (right) happening along the curve \( \{ P = Q = 0 \} \) due to the singularity enhancement. Nodes connected by arrows are identified. The orange nodes are the fibers of the 4-cycles on which it is possible to detect the Freed-Witten anomaly.

This argument suggests that we should constrain the complex structure of the blown-up fourfold, which is given in eq. (A.4), in such a way that the curve \( \{ P = Q = 0 \} \subset B^3 \) is automatically contained in both the D-brane stack and the orientifold plane. Therefore, we impose the following conditions

\[ D \equiv P \hat{D} + Q \tilde{D} \]

Since the polynomial defining the O7-plane is

\[ O7 : h = a_2 + 4a_3 \]

Since the curve \( \{ P = Q = 0 \} \) is a branch of the intersection between the non-abelian stack and the O-plane, we experience on it the gauge symmetry enhancement from SU(2N+1)
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

same strategy, but different fluxless object needed!
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

same strategy, but different fluxless object needed!

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve $S \cap O7 \subset CY_3$
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

same strategy, but different fluxless object needed!

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve $S \cap O7 \subset CY_3$

Again: Constrain CY_4 complex structure such that $S \cap O7$ is reducible
and choose $C^{(2)}$ to be one component
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

\[ \text{Orientifold plane} \]

The new nodes pop up along the “antisymmetric-matter” curve \( S \cap O7 \subset CY_3 \)

Again: Constrain CY4 complex structure such that \( S \cap O7 \) is reducible and choose \( C^{(2)} \) to be one component

Affine Dynkin diagram of SO(10)
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

\[ \text{same strategy, but different fluxless object needed!} \]

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve \( S \cap O7 \subset CY_3 \)

Again: Constrain CY\(_4\) complex structure such that \( S \cap O7 \) is reducible and choose \( C^{(2)} \) to be one component

Affine Dynkin diagram of SO(10)

The new integral, holomorphic 4-cycles are the orange nodes fibered over \( C^{(2)} \)
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

---

The SU(2N+1) case

same strategy, but different fluxless object needed!

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve $S \cap O7 \subset CY_3$

Again: Constrain CY$_4$ complex structure such that $S \cap O7$ is reducible

and choose $C^{(2)}$ to be one component

Affine Dynkin diagram of SO(10)

The new integral, holomorphic 4-cycles are the orange nodes fibered over $C^{(2)}$

Result for SU(2N+1) $N \geq 2$

\[
\int_{C^{(4)}} c_2(CY_4) = \int_{C^{(2)}} S
\]
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

$\rightarrow$ same strategy, but different fluxless object needed!

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve $S \cap O7 \subset CY_3$

Again: **Constrain** CY$_4$ complex structure such that $S \cap O7$ is reducible

and choose C$^{(2)}$ to be one component

Affine Dynkin diagram of SO(10)

The new integral, holomorphic 4-cycles are the orange nodes fibered over C$^{(2)}$

Result for SU(2N+1) $N \geq 2$

$\int_{C^{(4)}} c_2(CY_4) = \int_{C^{(2)}} S$

Interpretation: C$^{(4)}$ lifts loops of closed, non-orientable strings intersecting S in C$^{(2)}$
The SU(2N+1) case

W splits into the 5th brane of the stack and another non-spin surface

\[ \text{same strategy, but different fluxless object needed!} \]

Orientifold plane

The new nodes pop up along the “antisymmetric-matter” curve \( S \cap O7 \subset CY_3 \)

Again: **Constrain** CY\(_4\) complex structure such that \( S \cap O7 \) is reducible

and choose \( C^{(2)} \) to be one component

**Affine Dynkin diagram of SO(10)**

The new integral, holomorphic 4-cycles are the orange nodes fibered over \( C^{(2)} \)

**Result for SU(2N+1) \( N \geq 2 \)**

\[ \int_{C^{(4)}} c_2(CY_4) = \int_{C^{(2)}} S \]

**Interpretation:** \( C^{(4)} \) lifts loops of closed, non-orientable strings intersecting \( S \) in \( C^{(2)} \)

This procedure works also for the SU(2N) series and lends better itself to treating the “U(1)-restricted” cases.
Outlook
For SU(N), the (non)-spin-ness of $N_B S$ decides the quantization of $G_4$

For Sp(N), $T S$ matters

The cases when B is non-spin need clarification
For SU(N), the (non)-spin-ness of $\mathcal{N}_B S$ decides the quantization of $G_4$

For Sp(N), $\mathcal{T} S$ matters

The cases when $B$ is non-spin need clarification

The SU(3) case behaves misteriously... $G_4$ always integral!

What is responsible to cancel 7-brane FW anomalies? Kapustin’s mechanism?

A.Kapustin '99
For SU(N), the (non)-spin-ness of $N_B S$ decides the quantization of $G_4$

For Sp(N), $T S$ matters

The cases when $B$ is non-spin need clarification

The SU(3) case behaves mysteriously... $G_4$ always integral!

What is responsible to cancel 7-brane FW anomalies? Kapustin’s mechanism?

The outlined picture of the lift may be useful for several consistency checks

Make sure that the M2 anomaly leads to well-defined chiral indices

A. Kapustin ’99
For SU(N), the (non)-spin-ness of $N_B S$ decides the quantization of $G_4$

For Sp(N), $\mathcal{T} S$ matters

- The cases when $B$ is non-spin need clarification

The SU(3) case behaves mysteriously... $G_4$ always integral!

What is responsible to cancel 7-brane FW anomalies? Kapustin’s mechanism?

A.Kapustin '99

- The outlined picture of the lift may be useful for several consistency checks

- Make sure that the M2 anomaly leads to well-defined chiral indices

- Sen’s limit of SU(N) F-theory configurations leads to conifold singularities in CY$_3$

R.Donagi, M.Wijnholt '09

- An appropriate treatment of them is crucial for topological matters
For SU(N), the (non)-spin-ness of $\mathcal{N}_B S$ decides the quantization of $G_4$

For Sp(N), $\mathcal{T} S$ matters

The cases when $B$ is non-spin need clarification

The SU(3) case behaves mysteriously... $G_4$ always integral!
What is responsible to cancel 7-brane FW anomalies? Kapustin’s mechanism?

The outlined picture of the lift may be useful for several consistency checks

Make sure that the M2 anomaly leads to well-defined chiral indices

Sen’s limit of SU(N) F-theory configurations leads to conifold singularities in $CY_3$

An appropriate treatment of them is crucial for topological matters

The class of $G_4 | M_5$ must be pure torsion

Analyzing this condition using M/F theory duality may be relevant for the physics of the corresponding type IIB instantons