What is String Theory?

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Stony Brook, October 2016

Congratulations to YITP

on the 50th birthday

What is string theory?

String theory

– an attempt to find a unified theory of all the elementary constituents of matter and the forces operating between them.

In reality string theory has much more.

The goal of this talk will be to describe some of what string theory offers us.

1. String theory and the elementary constituents of matter

2. String theory and black holes

3. New frontiers

String theory and the elementary constituents of matter

At present we have a very good understanding of the physics of the elementary constituents of matter and the forces operating between them

– standard model of elementary particle physics

The framework used in describing this model is quantum field theory

– combines the principles of quantum mechanics and special theory of relativity

– elementary constituents are point particles e.g. electron, photon, neutrinos, quarks, $\cdots$

There are few issues that tell us that standard model is not fully correct.

1. A class of elementary particles called neutrinos are required to have zero mass in the standard model, but have been found to have tiny but non-zero mass

2. Most of the matter in the universe is dark matter, but it has no explanation in the standard model

These problems can be solved with suitable 'extensions' of the standard model within the framework of quantum field theory

– but we need experimental input to distinguish between different extensions.

Is the framework of quantum field theory sufficient for a complete description of nature?

Standard model and its extensions based on quantum field theory leave out one important force of nature

GRAVITY

In current experiments, the effect of gravitational force between elementary particles is negligible compared to the other forces

– not observable

But a complete theory of nature must explain all forces, however small.

Gravity

There is a very successful classical theory of gravity known as ‘general theory of relativity’

– discovered by Einstein more than 100 years ago.

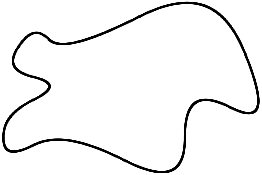
Attempts to make this into a quantum theory run into difficulties

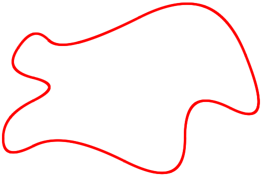
– gives infinite results for experimentally measurable quantities.

String theory resolves this problem in an unexpected fashion. Veneziano, · · ·

— combines the principles of quantum mechanics and special theory of relativity

– takes the elementary constituents of matter as one dimensional objects – strings.



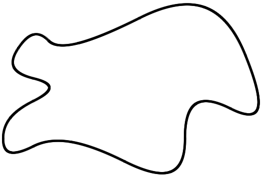


Typical size of a string $\sim 10^{-33}$ cm

This is much smaller than the length scale that can be probed by any present day experiment ($\sim 10^{-17}$ cm.)

Thus to a present day experimentalist the states of the string will appear to be particle like objects.

Different vibrational states of the string appear to us as different elementary particles.



One of the vibrational states of the string has all the properties of a graviton – the mediator of gravitational force.

Yoneya; Scherk, Schwarz

$\Rightarrow$ **string theory automatically contains gravity!**

Furthermore string theory gives finite results for measurable quantities

– a finite quantum theory of gravity.

However string theory is so tightly constrained that we cannot adjust it to suit our needs.

We have to take what string theory gives us.

First of all one finds that there are five consistent string theories collectively known as superstring theories.

They differ from each other in the way the string vibrates.

Furthermore one finds that in each of these five string theories the dimension of space is

9

$\rightarrow$ requires 9 coordinates to describe a point in space instead of the usual 3 coordinates.

This does not describe what we see in nature!

At present there in no intuitive understanding of this number 9, or why there are only five consistent string theories

– emerges after some long calculations.

There are various other versions of string theory, some in other dimensions, but none of them is fully mathematically consistent.

From now on we shall only discuss superstring theories.

This however is not the end of the story.

Not all of the extra dimensions may be large.

e.g. a two dimensional cylinder can be made to look one dimensional by taking the radius of the cylinder to be very small.

Kaluza; Klein

The same idea works in making a 9 dimensional space look 3 dimensional.

Take 6 of the 9 space directions to be small, describing a compact space K.

When the size of K is sufficiently small, the space will appear to be 3 dimensional.

This is known as compactification.

Mathematical consistency puts strong restriction on what kind of spaces K we can use for compactification.

Nevertheless there are many different spaces K that can be used

e.g. Calabi-Yau manifolds Candelas, Horowitz, Strominger, Witten

Thus even if we begin with a specific superstring theory, upon compactification there are more possibilities.

These different possibilities may be regarded as different phases of the underlying superstring theory.

An analogy: The single theory, describing the H_2O molecules and the force between them, has different phases in the form of ice, water and steam.

Similarly a superstring theory has many phases, characterized by many different choices of K.

Just as the environment inside ice, water and steam are very different, similarly the 3-dimensional environment for different choices of the compact space K will be very different.

Even the ‘fundamental constants of nature’ like the number of elementary particles and their masses and charges will appear to be different in different phases.

From what we have described so far, it would seem that there are altogether five consistent superstring theories, each with many different phases.

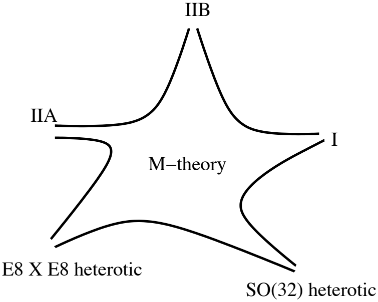
This was our understanding of the subject till the early 1990's.

However subsequent research has shown that the five different superstring theories are not really different, but they all give different descriptions of the same underlying theory. · · · , Witten, · · ·

This theory has been given the name

M-theory

Different phases of M-theory can be schematically represented as different points inside a room with five windows.



The five windows are the five superstring theories.

Through each window we see only a small part of the room.

If there is no overlap between the different parts we see through different windows, then we would not know that we are looking into the same room.

$\rightarrow$ **describes the situation till the early 1990's.**

However once we begin seeing deep enough into the room through each window, we may glimpse some objects through more than one window.

We may then realize that all the windows open into the same room.

$\rightarrow$ **describes the development since mid 1990's.**

For some phases of M-theory the 3-dimensional environment is very similar to the nature that we observe.

$\rightarrow$ has elementary 'particles' and forces similar to what we observe in nature.

Thus besides solving the problem of formulating the quantum theory of gravity, M-theory also offers the possibility of combining this with a theory of elementary particles and their forces.

However finding a phase of M-theory that has exactly the elementary particles we observe in nature remains an open problem.

The phases we have discovered so far probably form only a tip of the iceberg

– many more are waiting to be discovered.

This is one of the most active areas of research known as ‘Superstring phenomenology’.

Hope: Eventually we may find a phase that describes nature that we are familiar with.

One of the major breakthroughs in this field is the discovery of phases of M-theory carrying dark energy

Dark energy was discovered in 1998, but till that date all known phases of M-theory carried zero dark energy

– resolved in 2003

At present there are two competing class of phases carrying dark energy

KKLT Kachru, Kallosh, Linde, Trivedi

Large Volume Scenario Balasubramanian, Berglund, Conlon, Quevedo

This still leaves us with a vexing question

Why does nature prefer one phase over the others?

We do not know the answer to this.

Best explanation offered to date: Multiverse

Bousso, Polchinski

According to this speculation, no single phase of M-theory is special.

Different parts of the universe exist in different phases, and we see a particular phase of M-theory because we happen to live in a particular region.

Analogy: We can have a big reservoir of H_2O molecules with different parts of the reservoir existing in different phases – some part as ice, some as water and some as steam.

For the reservoir of H_2O molecules the system will eventually come to thermal equilibrium, and all parts of the system will be in the same phase.

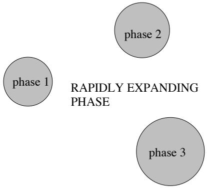
To prevent this we need some driving mechanism, e.g. some heating and cooling systems in different parts of the H_2O reservoir.

What is the mechanism that prevents our universe from coming to thermal equilibrium?

Answer: force of gravity!

Many phases of M-theory have the property that they expand rapidly according to the laws of general theory of relativity.

This rapid expansion separates the different parts in different phases very quickly, preventing the system from coming to thermal equilibrium.



As the universe expands more and more space is created making room for new phases to form.

Since this process continues for infinite time, it is possible that every phase of M-theory will be realized in some region of the universe.

Thus no phase of M-theory gets a special role in the universe as a whole, although in any given region one phase will appear to be special.

M-theory and black holes

The story of M-theory told so far has been geared towards particle physics

But M-theory has another aspect – a quantum theory of gravity.

Earlier attempts to quantize general theory of relativity ran into many obstacles.

Getting infinite results was one such obstacle but there are others.

If M-theory is a quantum theory of gravity then it should be able to address these puzzles.

These problems with quantum theory of gravity are generic problems

– expected to exist in any phase of the theory that contains gravity

Therefore in order to address these puzzles we do not need to first find the phase of the theory that describes us.

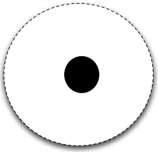
Strategy: Try to solve these problems in phases where you can solve them.

M-theory and black holes

Black holes are objects of very large mass concentrated in a small region.

They are described as classical solutions of the equations of motion of general theory of relativity.

Their gravitational attraction is so large that even light cannot escape a black hole.



A black hole is surrounded by an imaginary surface such that no object inside the surface can ever escape to the outside world.

This surface is called the event horizon.

To an outside observer the event horizon appears completely black since no light comes out of it.

In quantum theory this picture of the black hole gets modified.

A black hole is not completely black, but gives out black body radiation at a definite temperature and carries finite entropy Bekenstein; Hawking

Its entropy is given by the simple formula:

$$S_{BH} = \frac{k_B\, c^3}{4G\hbar} A$$

A: Area of the event horizon c: velocity of light

G: Newton's gravitational constant

k_B: Boltzmann's constant $\hbar$: Planck's constant

Using statistical mechanics one can give an independent definition of entropy.

$$S_{stat} = k_B \log N$$

N: number of quantum states of the black hole

Equality of the two definitions is essential for the consistency of the theory

$$\Rightarrow \frac{k_B\, c^3}{4G\hbar} A = k_B \log N$$

Note: The formula on the left is valid only in the limit of large A.

Consistency of the theory requires

$$\frac{c^3}{4G\hbar} A + \text{corrections} = \log N$$

for every black hole in every phase of the theory.

This would be a remarkable test, since the left hand side is geometric but the right hand side is based on counting.

The main difficulty in testing this is in computing N.

$$\frac{c^3}{4G\hbar} A + \text{corrections} = \log N$$

This issue exists in all phases of M-theory and must be resolved in all the phases, not just in the one that describes nature around us.

Strategy: Choose a convenient phase in which the dynamics of string theory is best understood.

Pick a convenient class of black holes in this phase.

Try to compute N for these black holes and compare with the left hand side.

$$\frac{c^3}{4G\hbar}A + \text{corrections} = \log N$$

It is possible to find convenient classes of phases and convenient class of black holes in these phases for which both sides can be calculated.

In all cases the formula holds in the limit of large A!

Strominger, Vafa; $\cdots$

In many cases one can calculate corrections to the formula proportional to log (A) and the agreement continues to hold!

New Frontier

M-theory and holography

The Bekenstein-Hawking formula hints towards a somewhat intriguing picture of gravity

Unlike in ordinary systems where entropy is proportional to the volume, the entropy of a black hole is proportional to the area.

– suggests that perhaps the degrees of freedom of gravity live on the boundary of space-time

– known as holography ’t Hooft; Susskind

This received a precise formulation in the form of AdS/CFT correspondence

Maldacena

There is a class of phases of M-theory in which space-time has the form AdS $\times$ K.

K: a compact space

AdS: Anti de Sitter space-time whose boundary is an ordinary space-time of the kind in which we live, known as Minkowski space.

Minkowski

Anti-de Sitter

Minkowski

Anti-de Sitter

Conjecture: Such a phase of M-theory is exactly equivalent to a quantum field theory living on the boundary of AdS

The quantum field theory depends on the specific phase we consider.

There is a precise dictionary between what quantities on the M-theory side correspond to what quantities on the field theory side and vice versa

Gubser, Klebanov, Polyakov; Witten

This has been tested in many cases by doing explicit computation on both sides.

– have enhanced our understanding of quantum field theories as well as quantum gravity.

Holography and entanglement

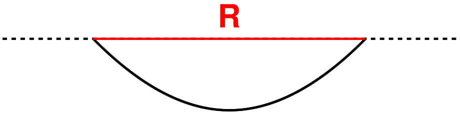
For a quantum field theory in its ground state, we can consider some region R of space and compute the entanglement entropy of the degrees of freedom inside R and outside R

– a measure of to what extent the ground state is not a product of a state in the Hilbert space for R and a state in the Hilbert space outside R.

What computation do we need in gravity for this quantity?

Answer: Area of the minimal surface in AdS bounded by the boundary of the region R in the boundary.

Ryu, Takayanagi



This has led to the suspicion that there is a deep connection between quantum entanglement and classical geometry

There have been many developments but the complete picture is yet to emerge.

Conclusion

M-theory is an exciting subject

We have learned many things about the theory ...

... but probably most of it is yet to be learned