Planar Josephson junction devices with narrow superconducting strips: Topological properties and optimization

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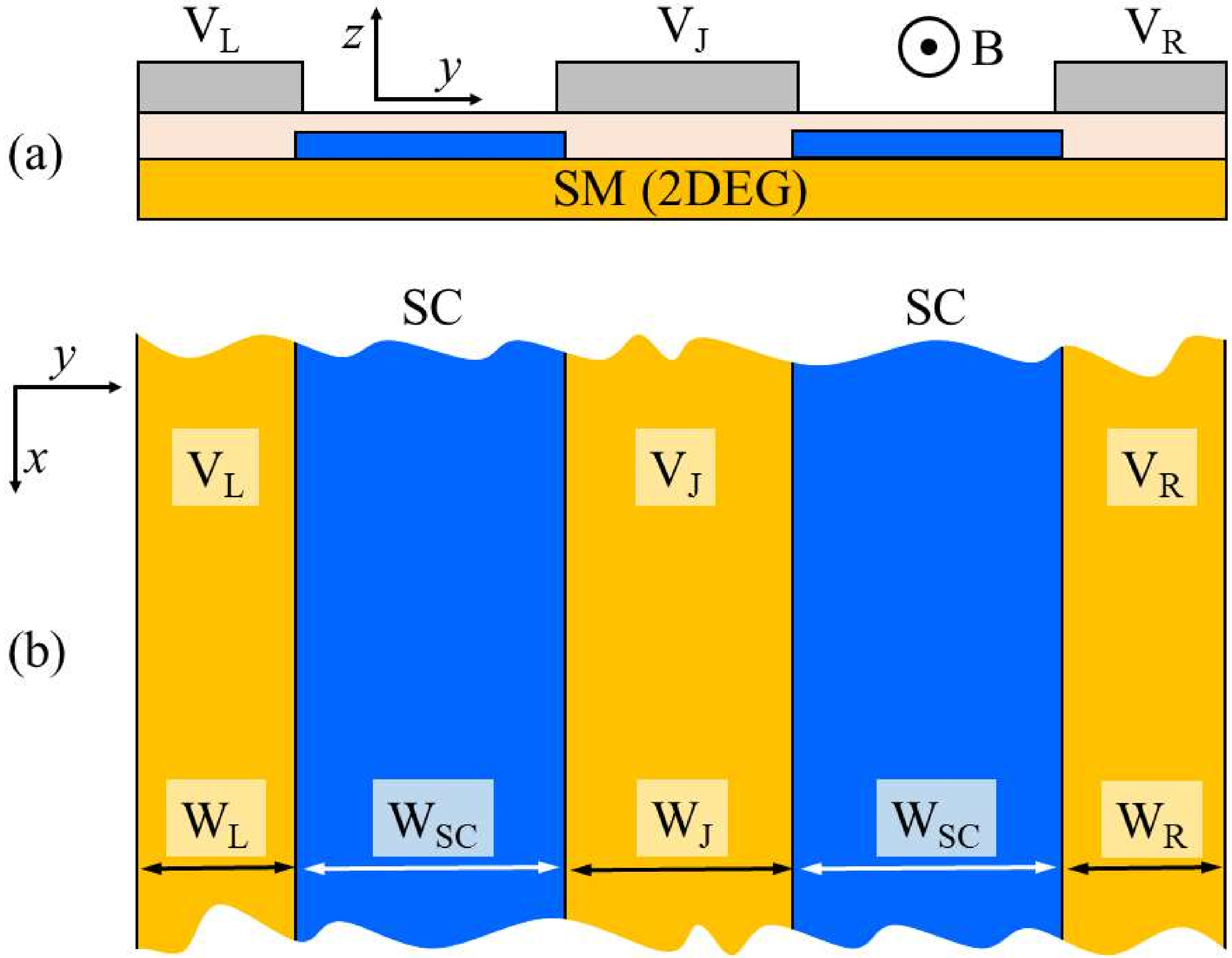
Office of Naval Research
Science & Technology

Hybrid device & modeling



Theoretical approach

$$H = H_{SM} + H_{SC} + H_{SM-SC},$$

Green's function method

$$G_{SM}(\omega,k) = [\omega - \mathcal{H}_{SM}(k) - \Sigma_{SC}(\omega,k)]^{-1}$$

$$\Sigma_{SC}(\omega) = -\frac{\gamma}{\sqrt{\Delta_0^2-\omega^2}}\left(\omega\,\sigma_0\tau_0\mathbb{I}^0_{SC} + \Delta_0\,\sigma_y\tau_y\mathbb{I}^\phi_{SC}\right)$$

with: $[\mathbb{I}^0_{SC}]_{jj} = 1$ $\quad [\mathbb{I}^\phi_{SC}]_{jj} = e^{\pm i\phi/2}$

$$A(\omega,k;j) = -\frac{1}{\pi}\mathrm{Im}\,\mathrm{Tr}[G_{SM}(\omega+i\eta,k)]_{jj},$$

DOS

$$\rho(\omega) = \sum_{k,j} A(\omega,k;j),$$

LDOS

$$\rho(\omega,j) = \sum_{k} A(\omega,k;j).$$

Effective Hamiltonian

$$\sqrt{\Delta_0^2-\omega^2} \approx \Delta_0$$

$$[\mathcal{H}_{eff}(k)]_{jj'} = \begin{cases} [\mathcal{H}_{SC}]_{jj'}; & j,j' \in SM, \\ Z\,[\mathcal{H}_{SC}]_{jj'} - \Delta\,\sigma_y\tau_y\delta_{jj'}; & j,j' \in SC, \\ Z^{\frac{1}{2}}\,[\mathcal{H}_{SC}]_{jj'}; & j(j') \in SM,\ j'(j) \in SC, \end{cases}$$

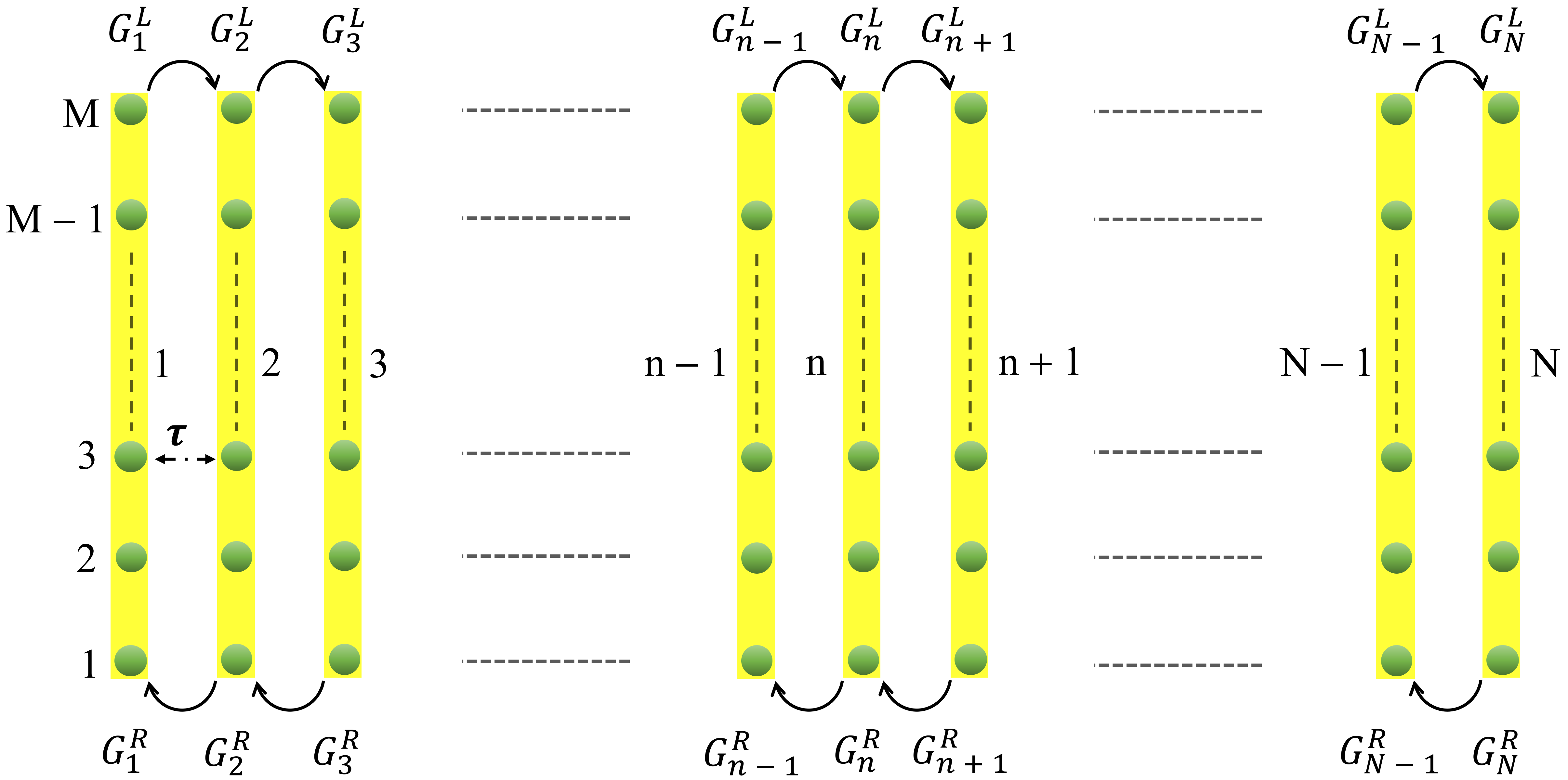
with: $\Delta = \gamma\Delta_0/(\gamma+\Delta_0)e^{\pm i\phi/2}$

$Z = \Delta_0/(\Delta_0+\gamma)$

Z_2 topological invariant:

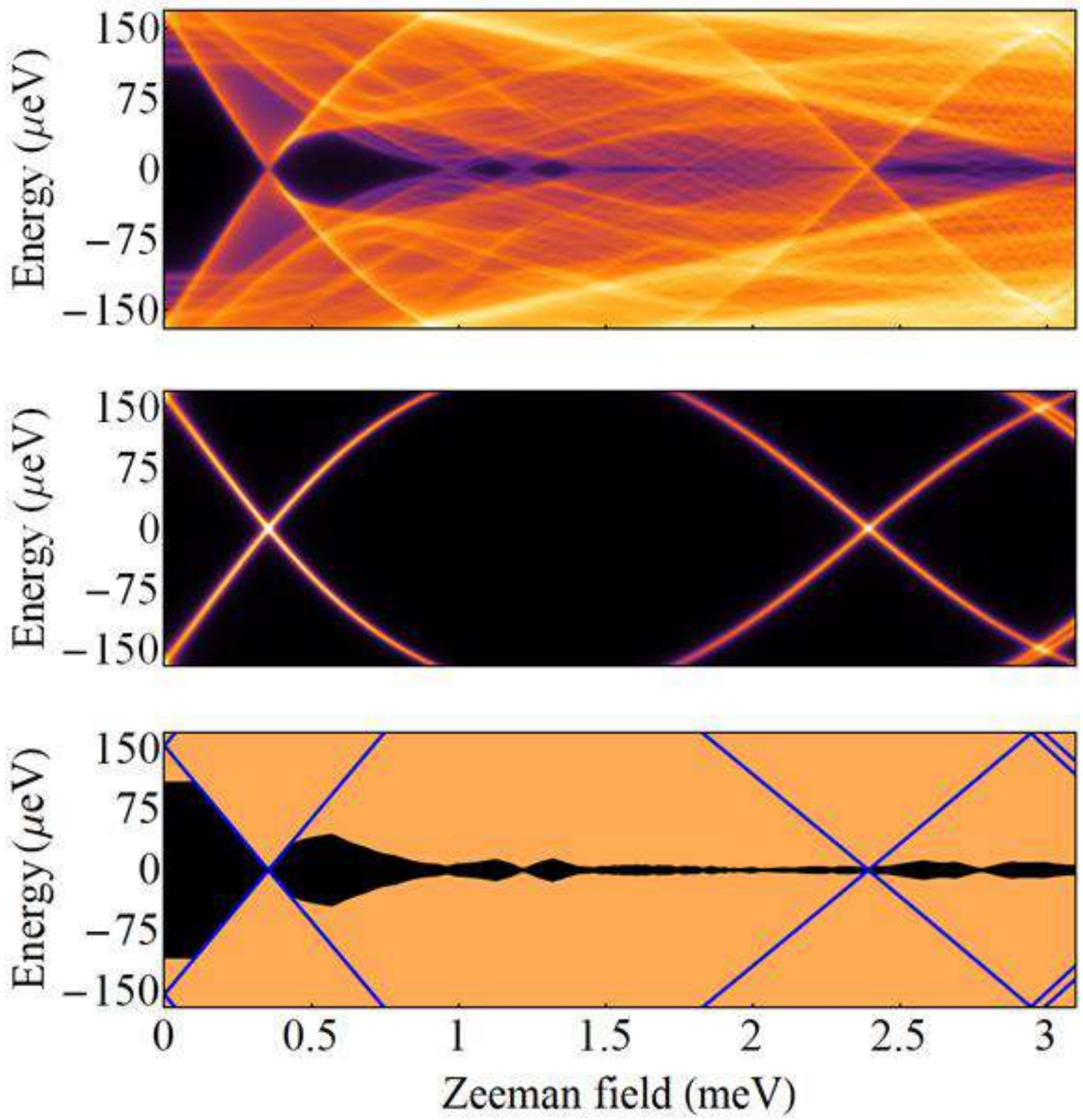
$$\nu = \mathrm{sign}\{\mathrm{Pf}[H_{eff}(0)U_C]\}\ \mathrm{sign}\{\mathrm{Pf}[H_{eff}(\pi)U_C]\} = \pm 1,$$

Recursive Green's function formalism



Low-energy spectral features (comparison)

$\Delta_0 = 0.25$ meV
$\gamma = 0.75$ meV

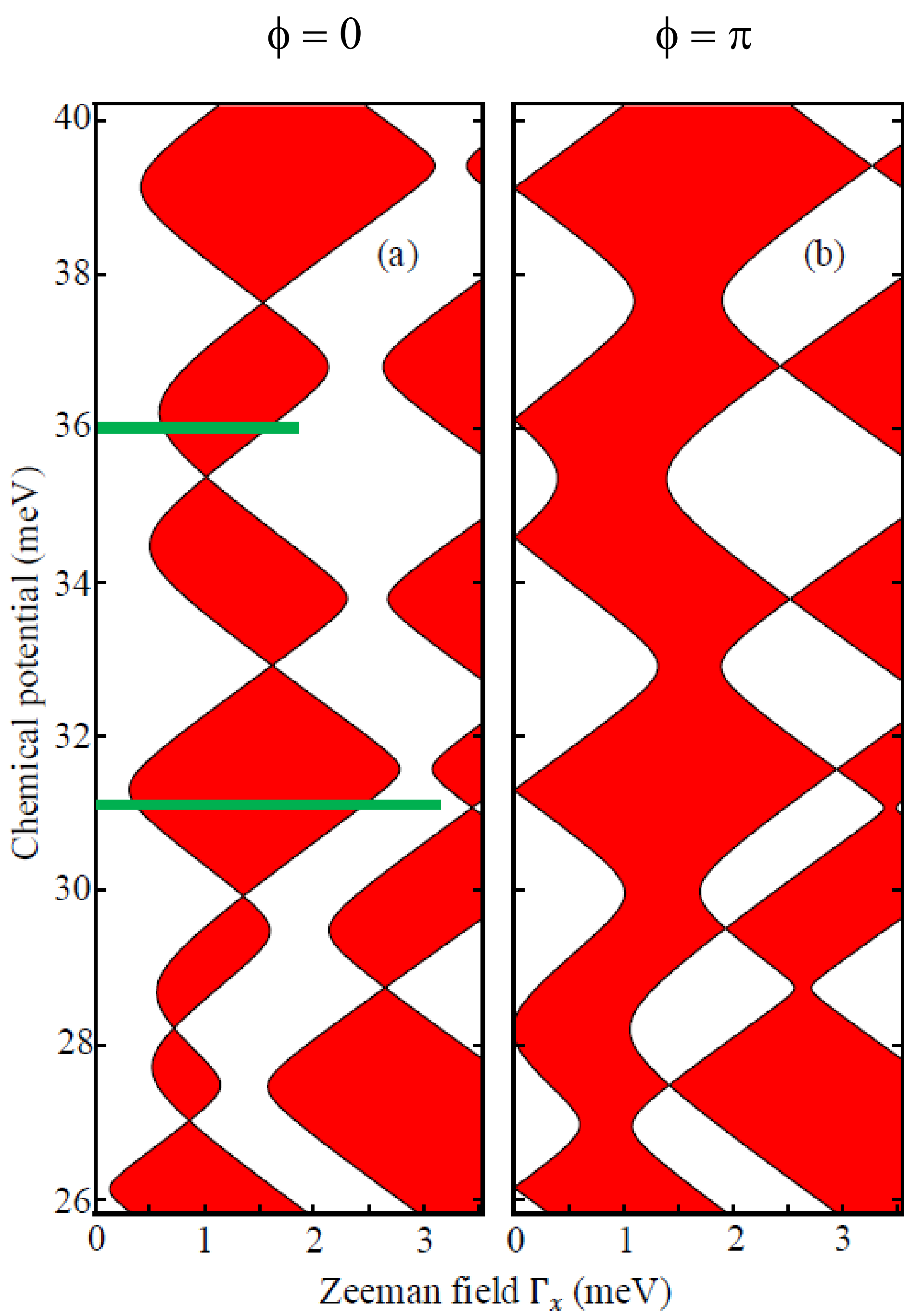


DOS

$A(\omega, k=0)$

$E_{min}(\Gamma)$, $E_{k=0}(\Gamma)$

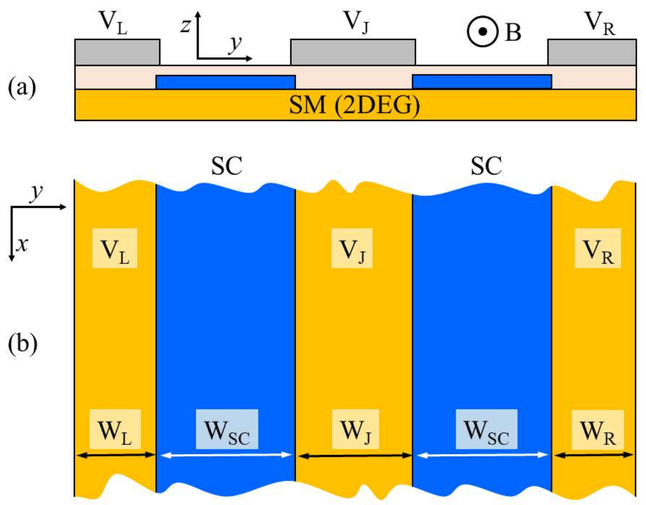
Typical topological phase diagram



$W_{SC} = 150$ nm.

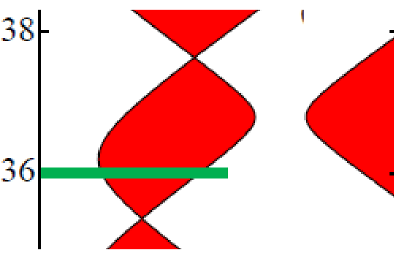
$V_J = 25$ meV

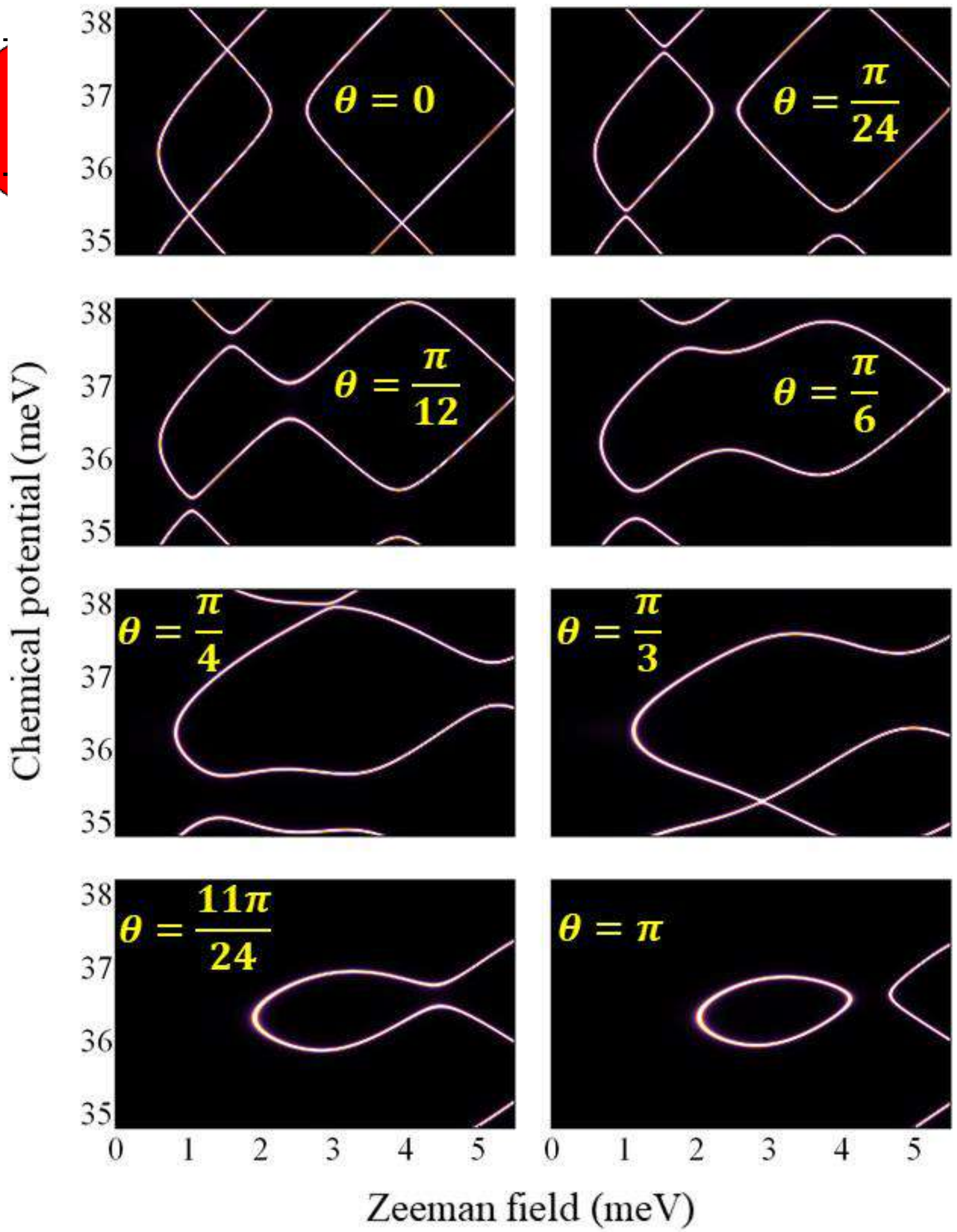
$V_L = V_R = 100$ meV



Effect of rotating the Zeeman field

$\phi = 0$

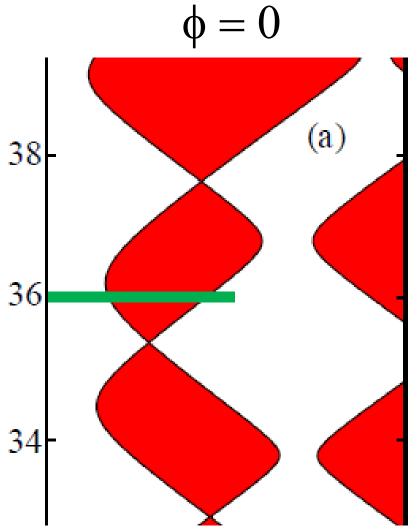


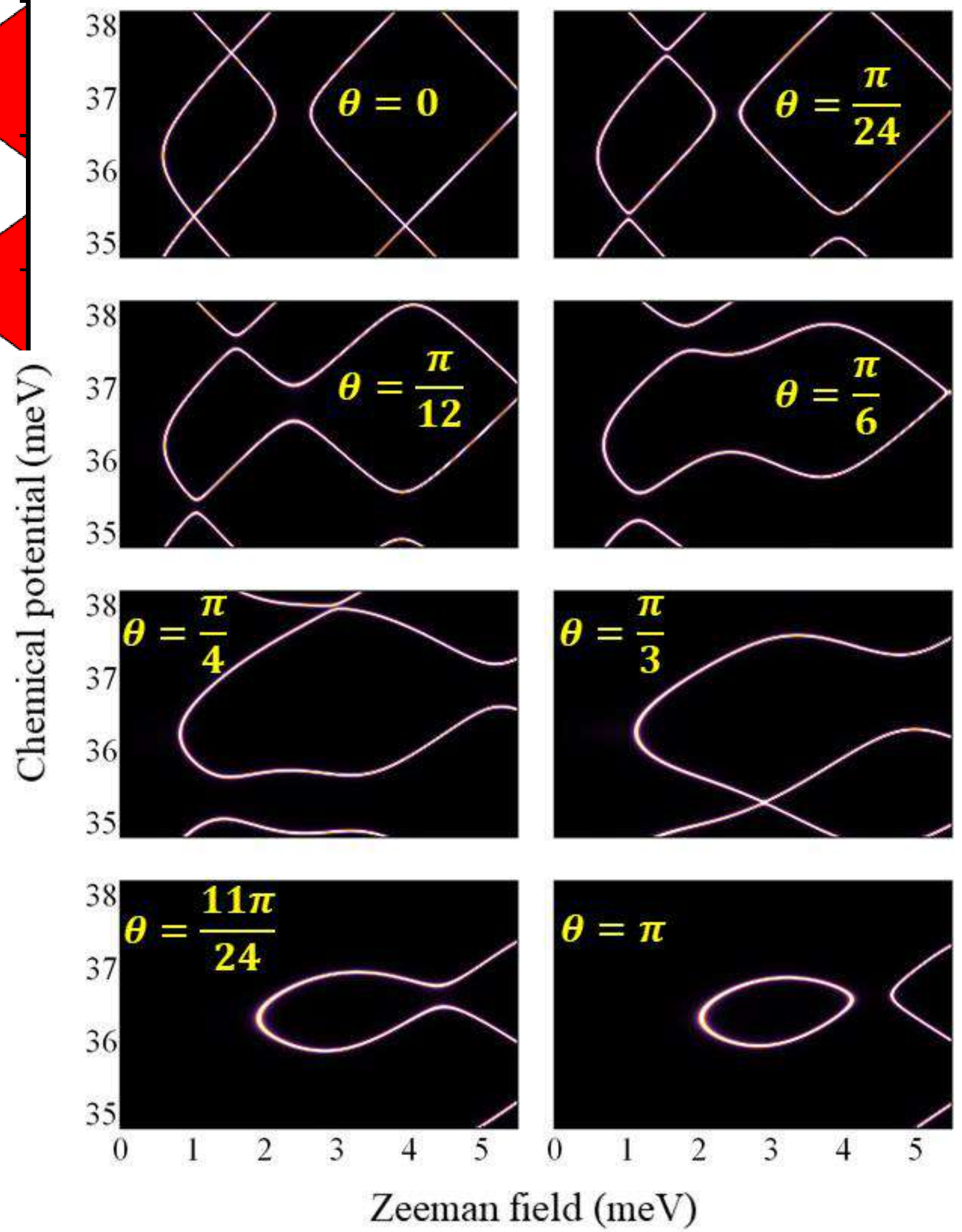


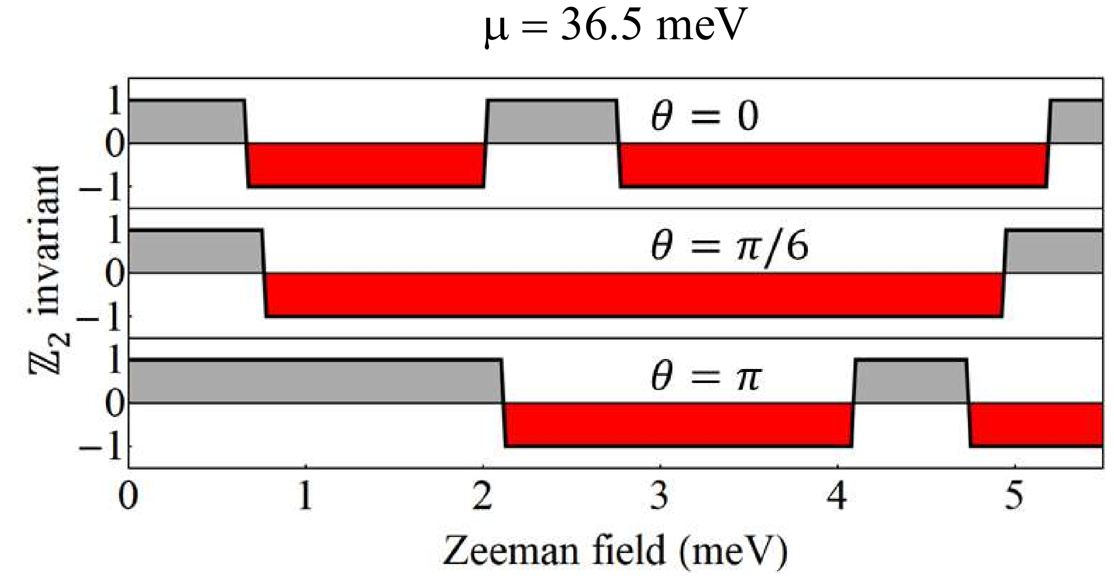
$$\Gamma_x = \Gamma \cos\theta$$

$$\Gamma_y = \Gamma \sin\theta.$$

Effect of rotating the Zeeman field







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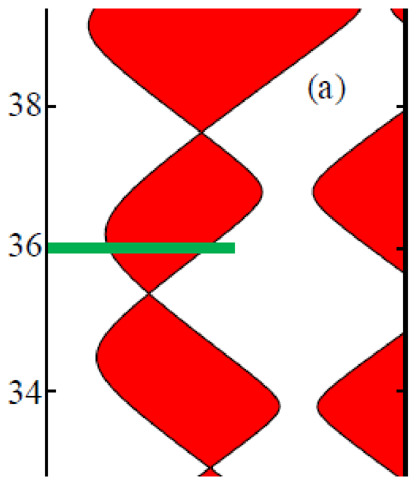
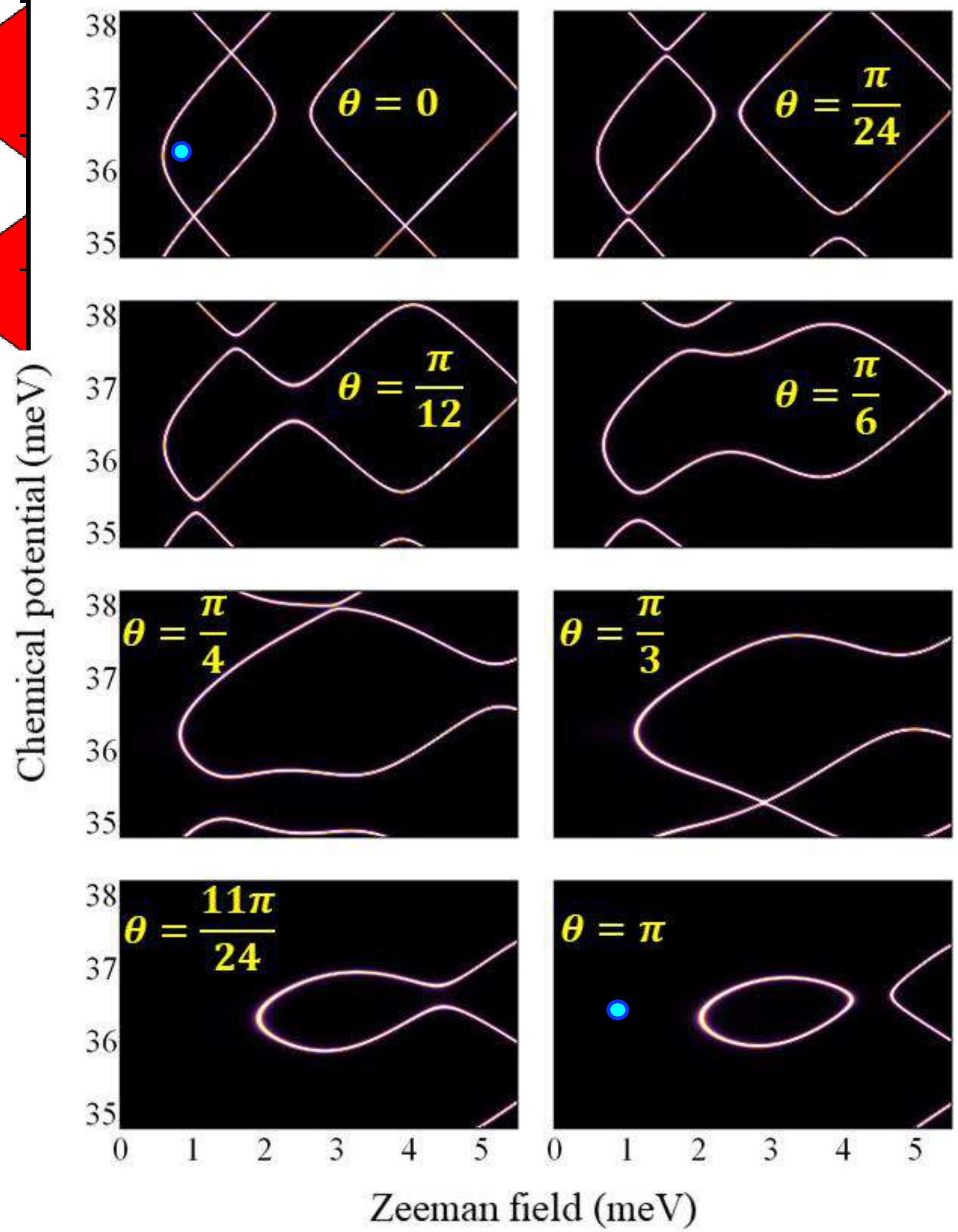
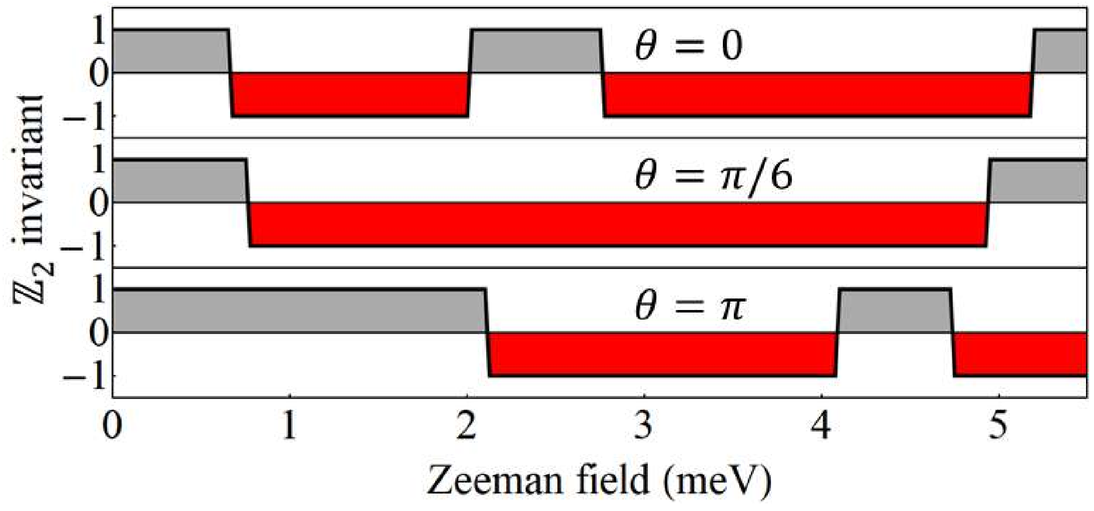
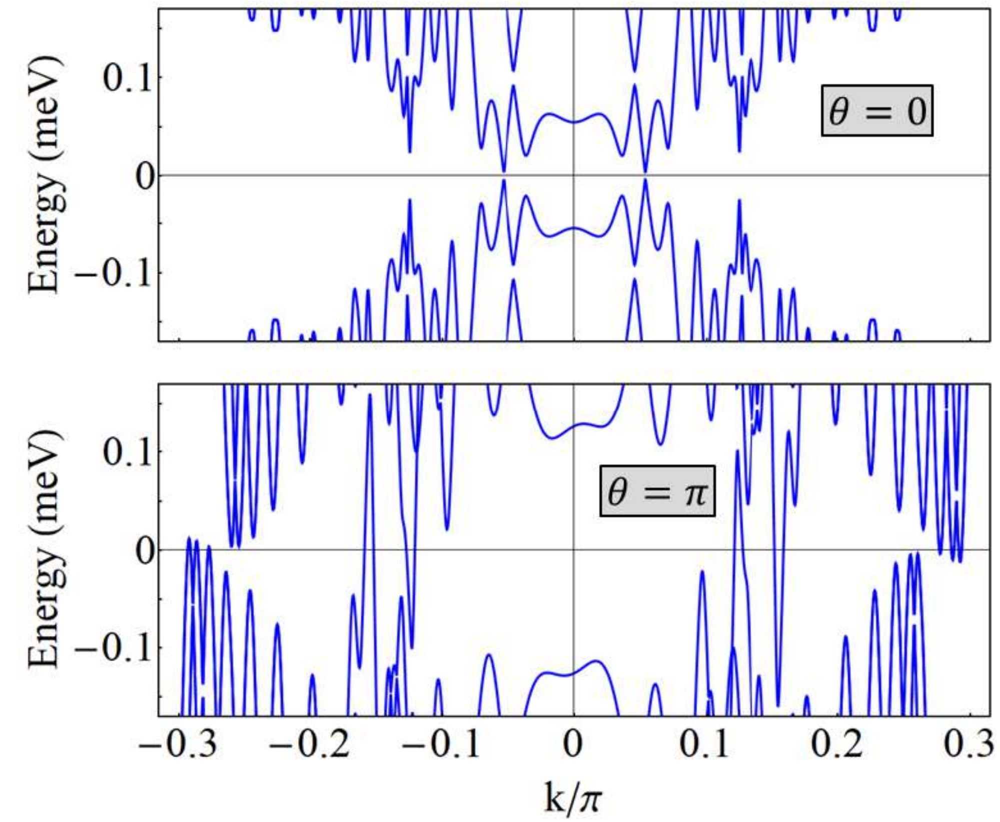




$\mu = 36.5$ meV





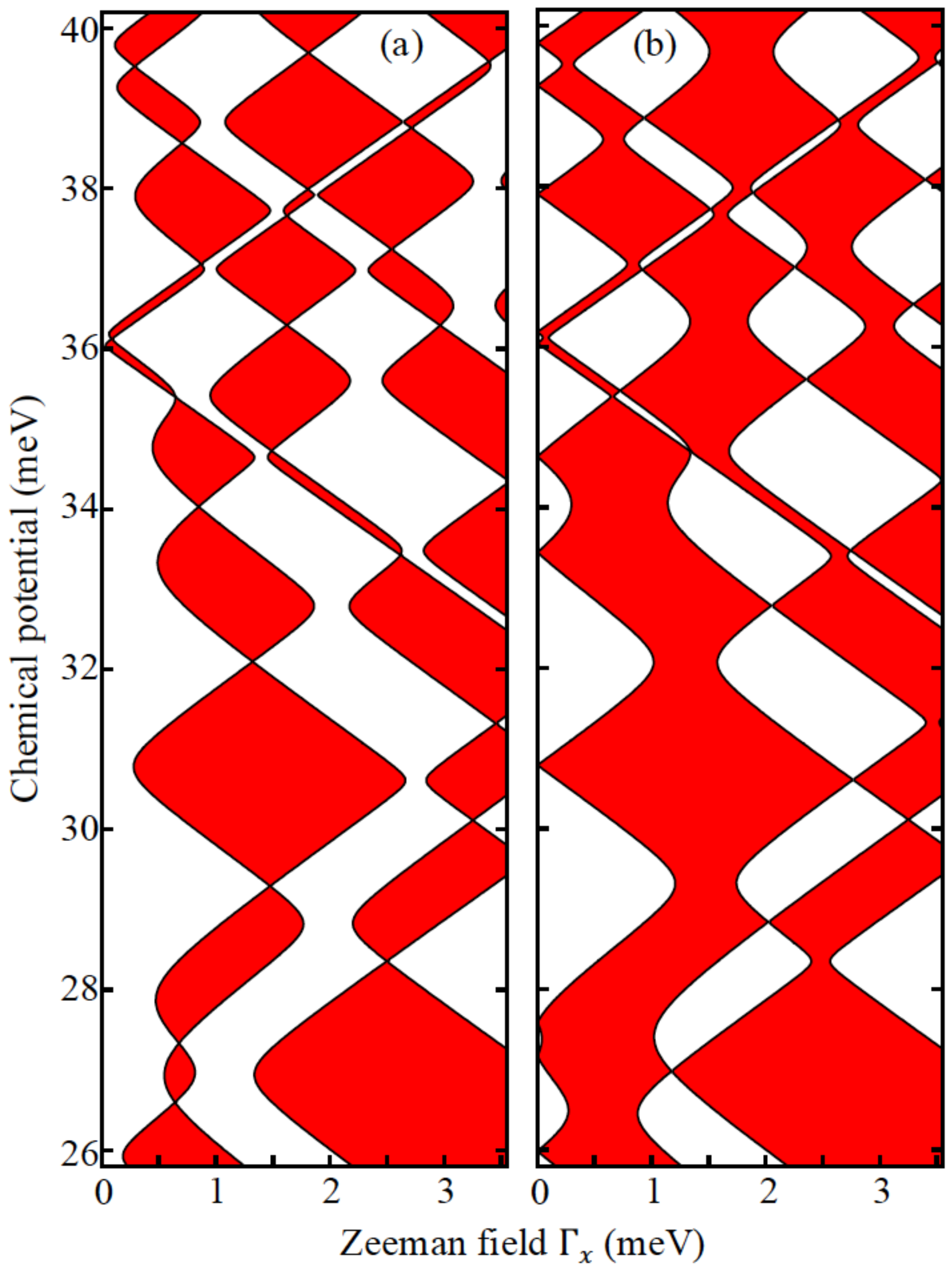
$$\Gamma_x = \Gamma \cos\theta$$

$$\Gamma_y = \Gamma \sin\theta.$$

Structures with undepleted outside SM regions

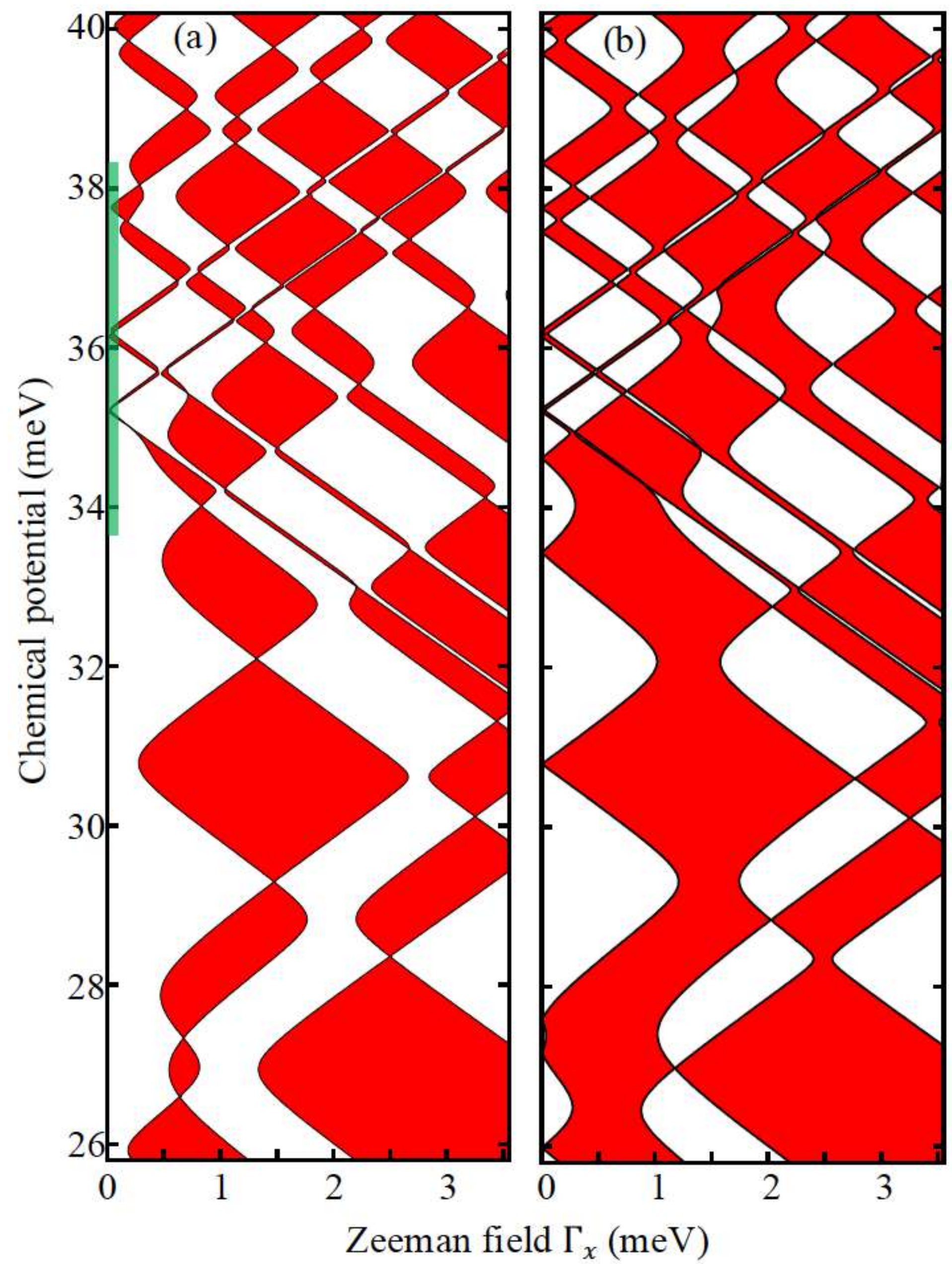
$W_L = W_R = 100$ nm
$V_L = V_R = 35$ meV

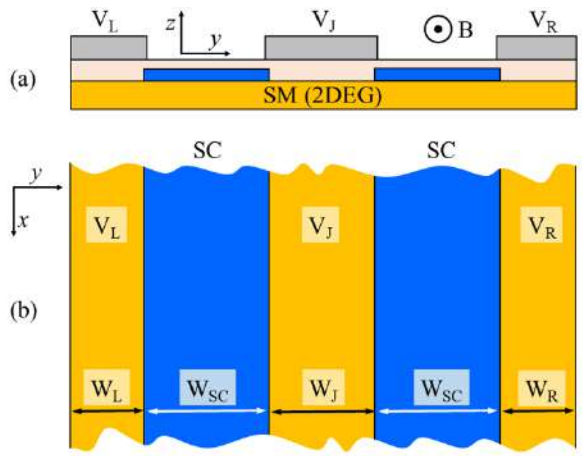
$\phi = 0$ $\phi = \pi$



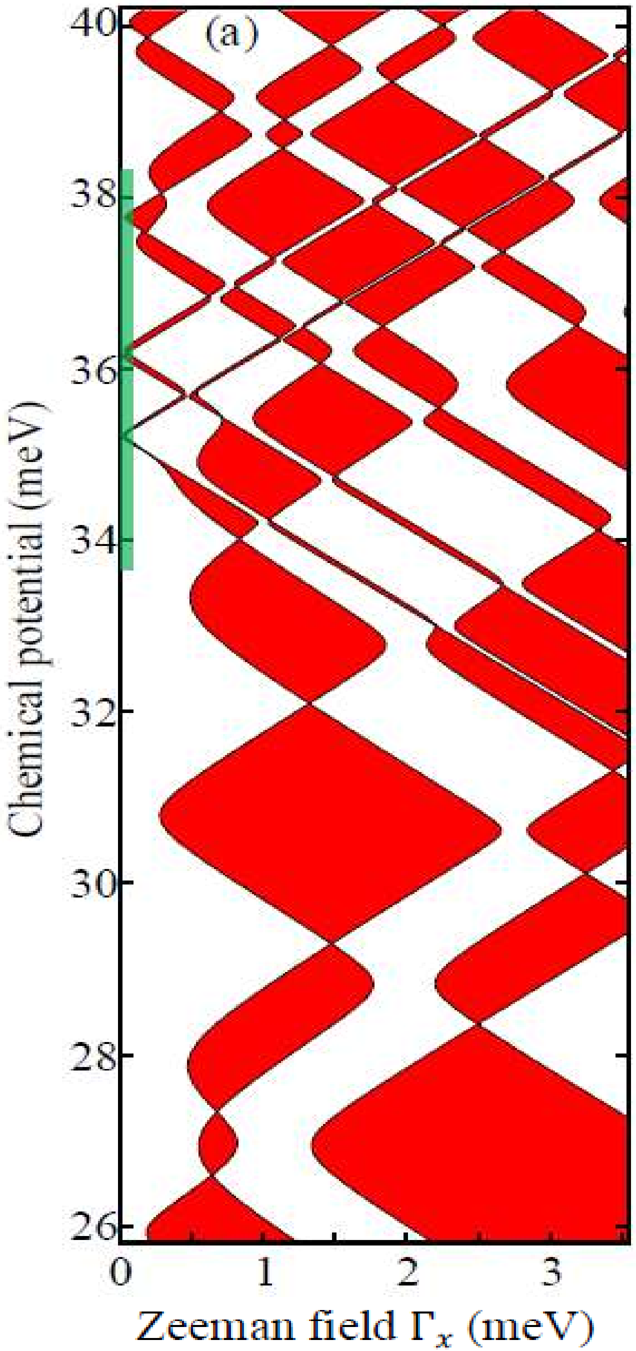
$W_L = W_R = 195$ nm
$V_L = V_R = 35$ meV

$\phi = 0$ $\phi = \pi$





Undepleted external SM regions



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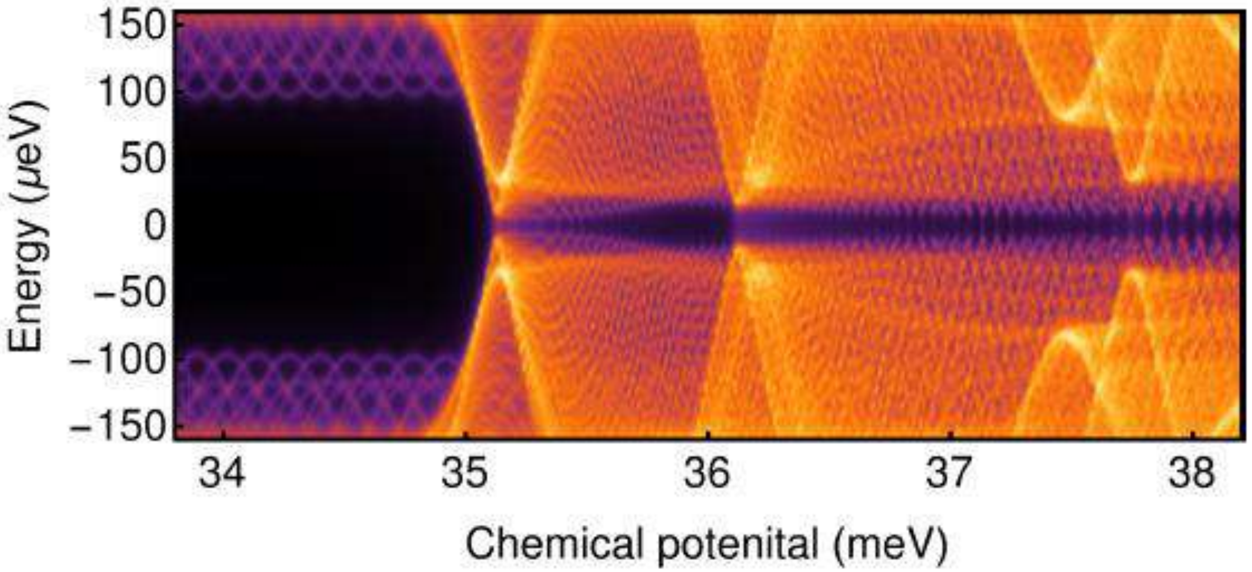
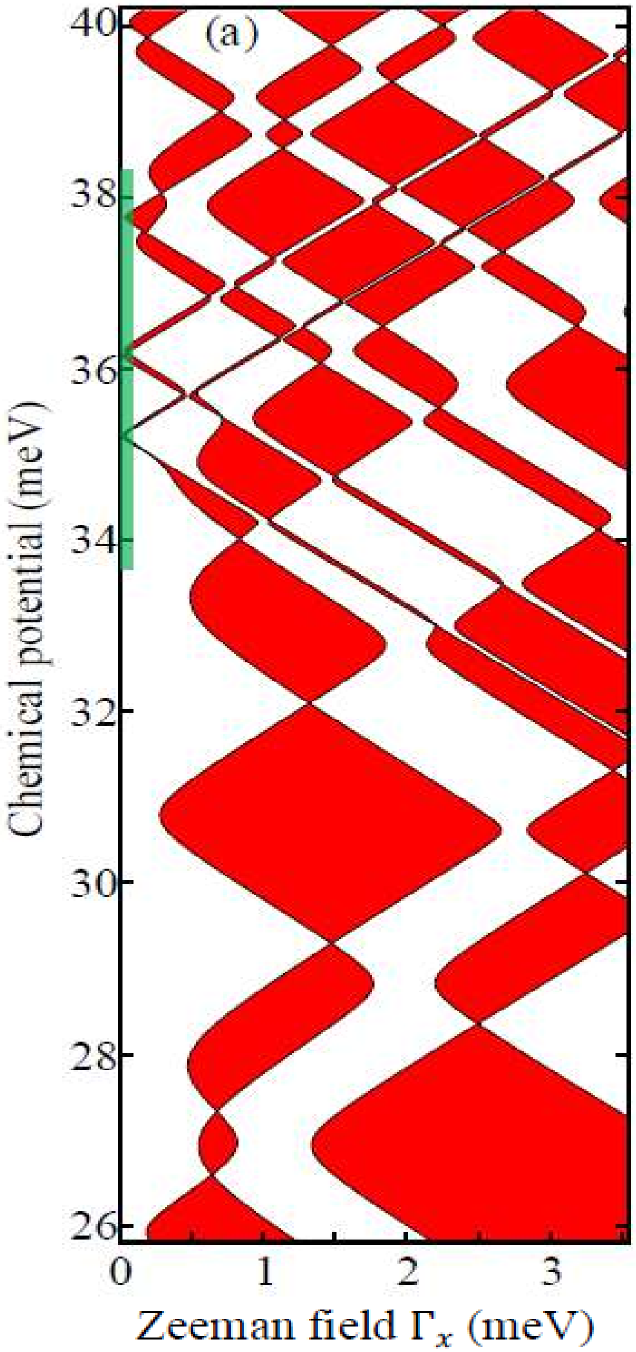
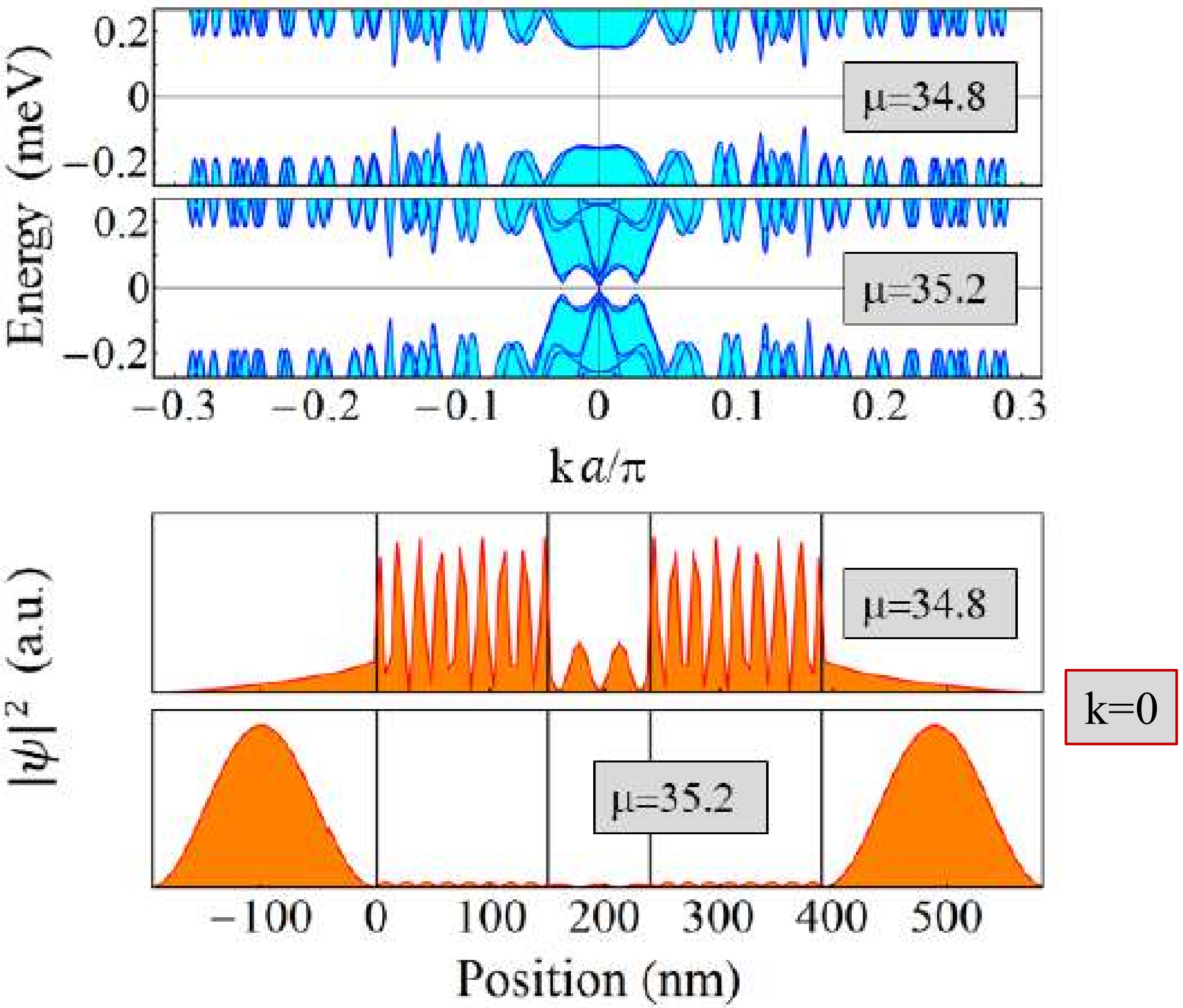


Undepleted external SM regions

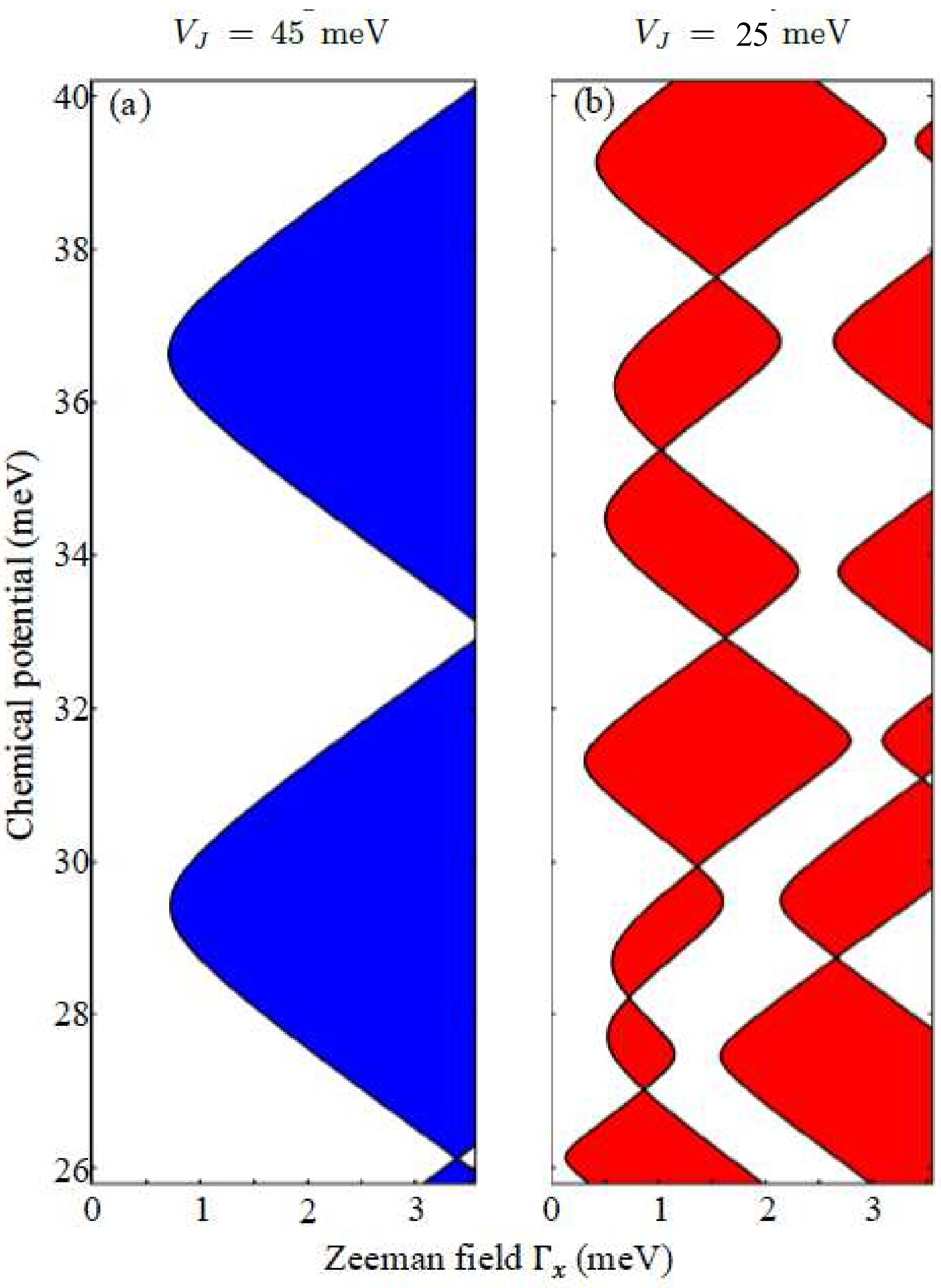




$W_L = W_R = 195$ nm

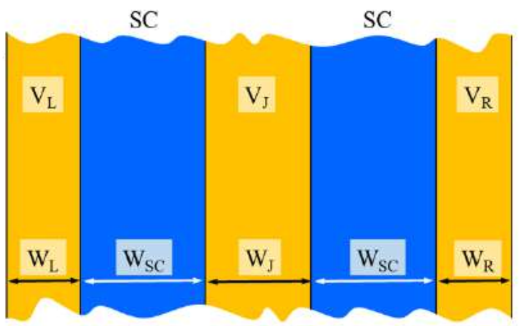
$V_L = V_R = 35$ meV

Depleted junction region & the wire-JJ crossover

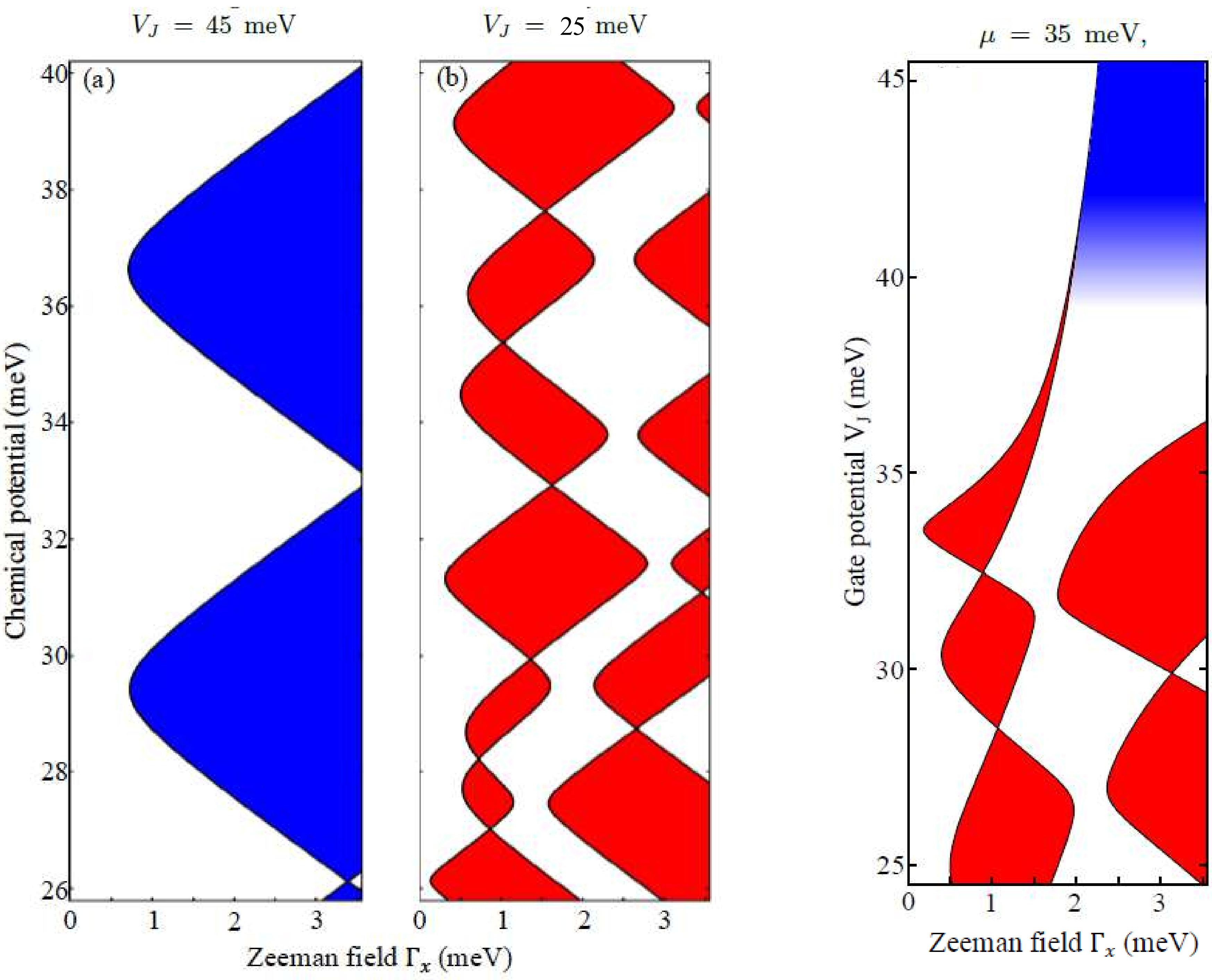


$W_{SC} = 150$ nm.

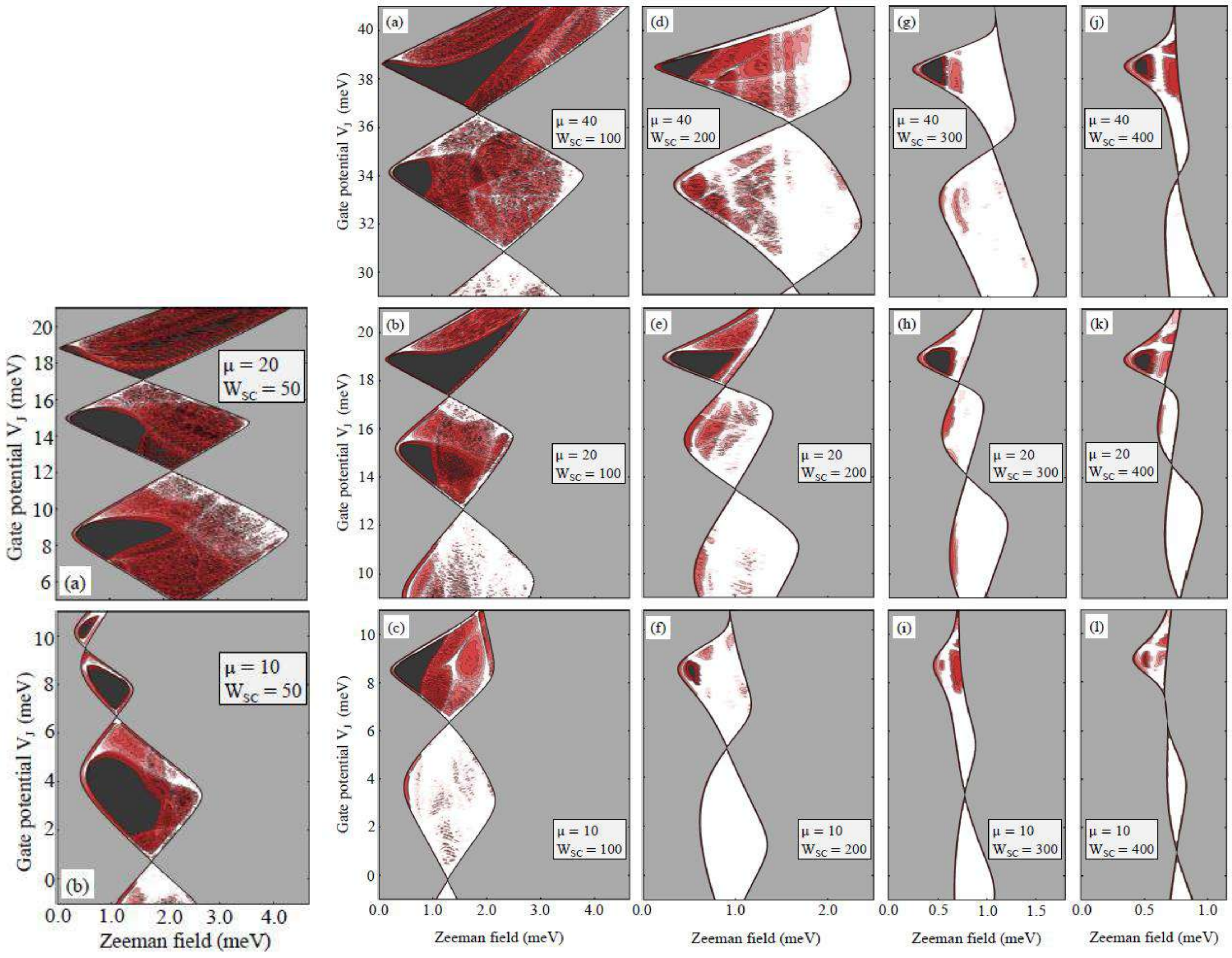
$V_L = V_R = 100$ meV



Depleted junction region & the wire-JJ crossover



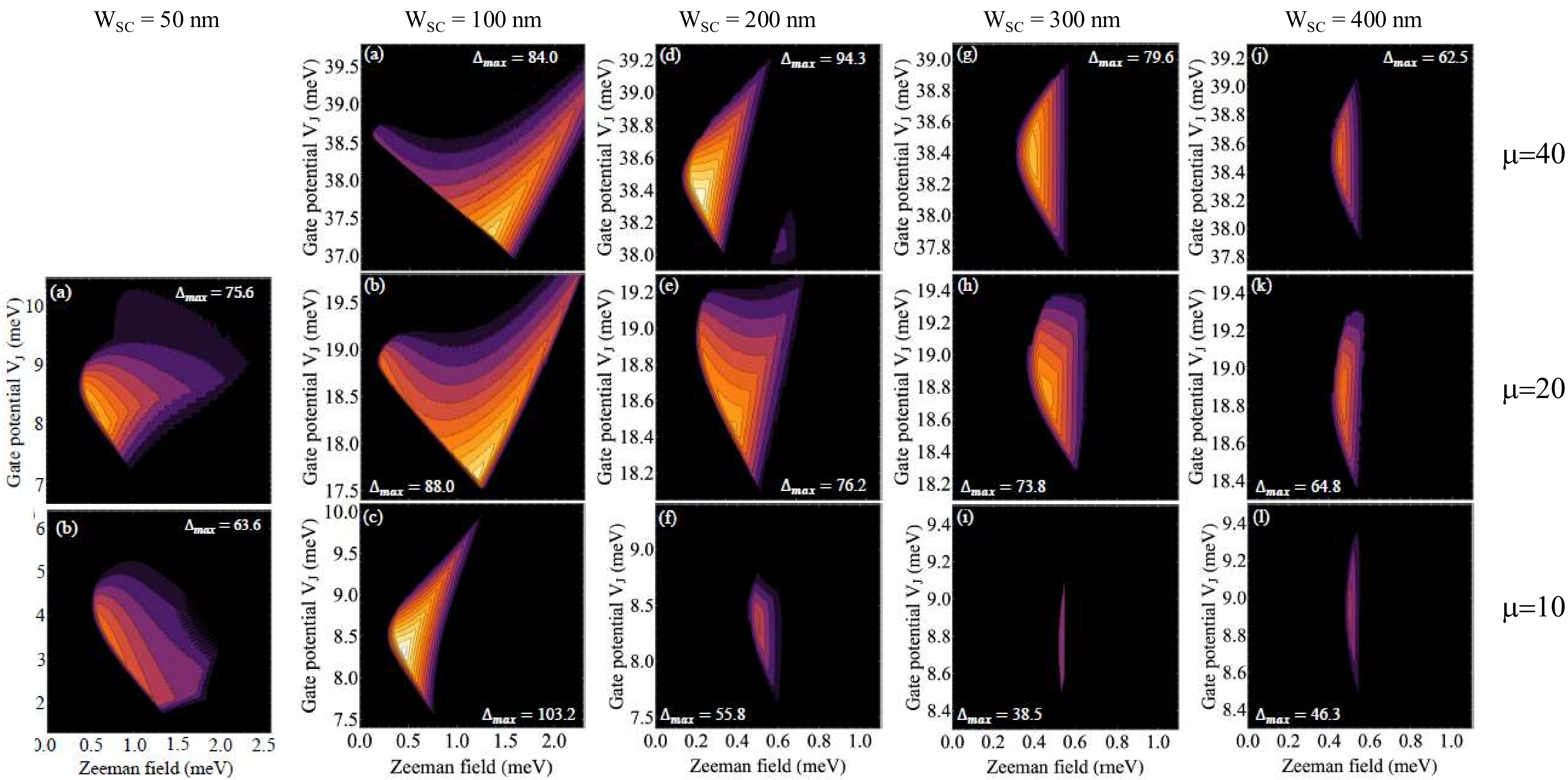
Dependence of the topological gap on the width of the SC films



DOS at $E = 35\ \mu eV$
(ϕ=0)

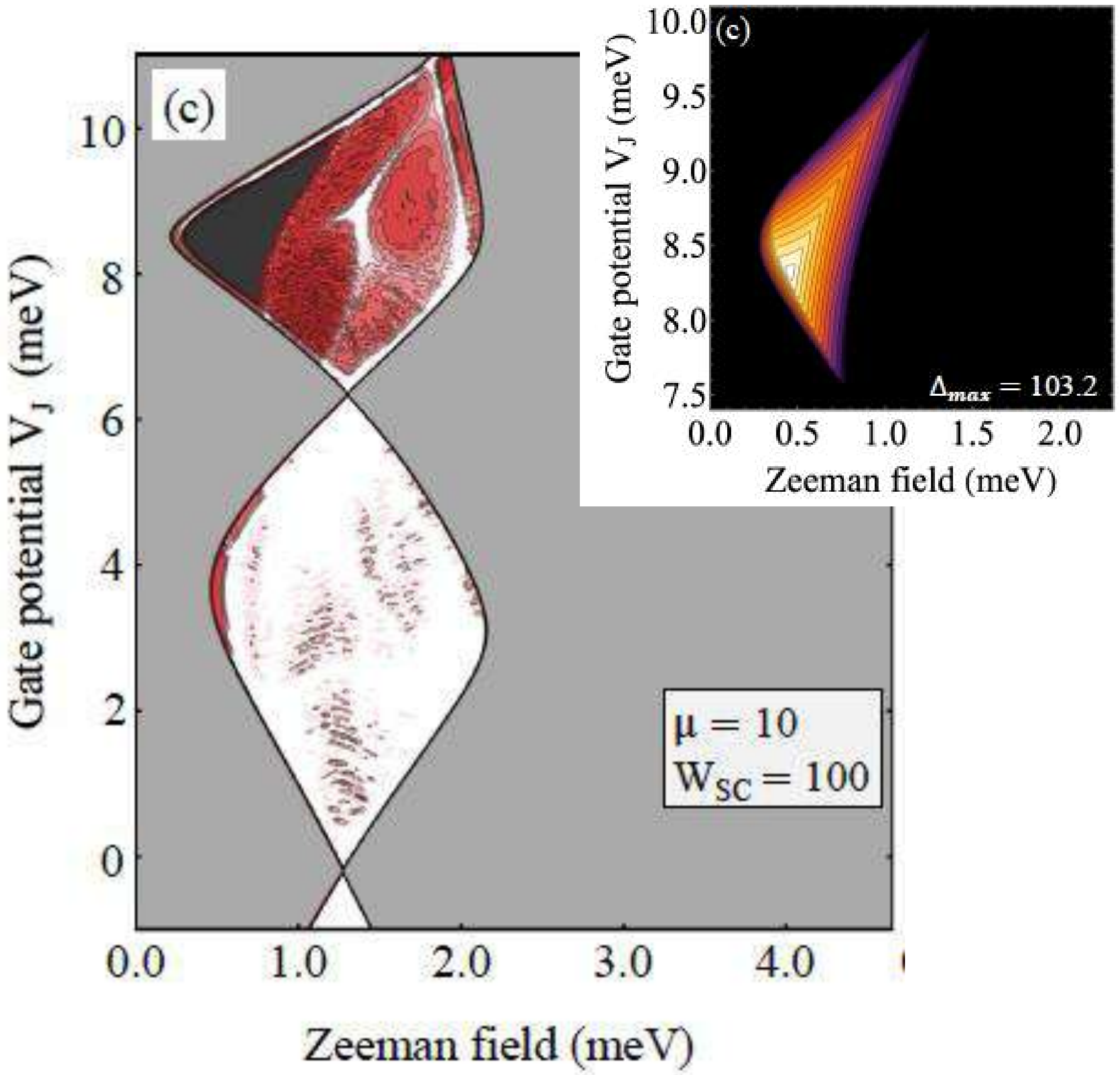
Dependence of the topological gap on the width of the SC films

Topological gap

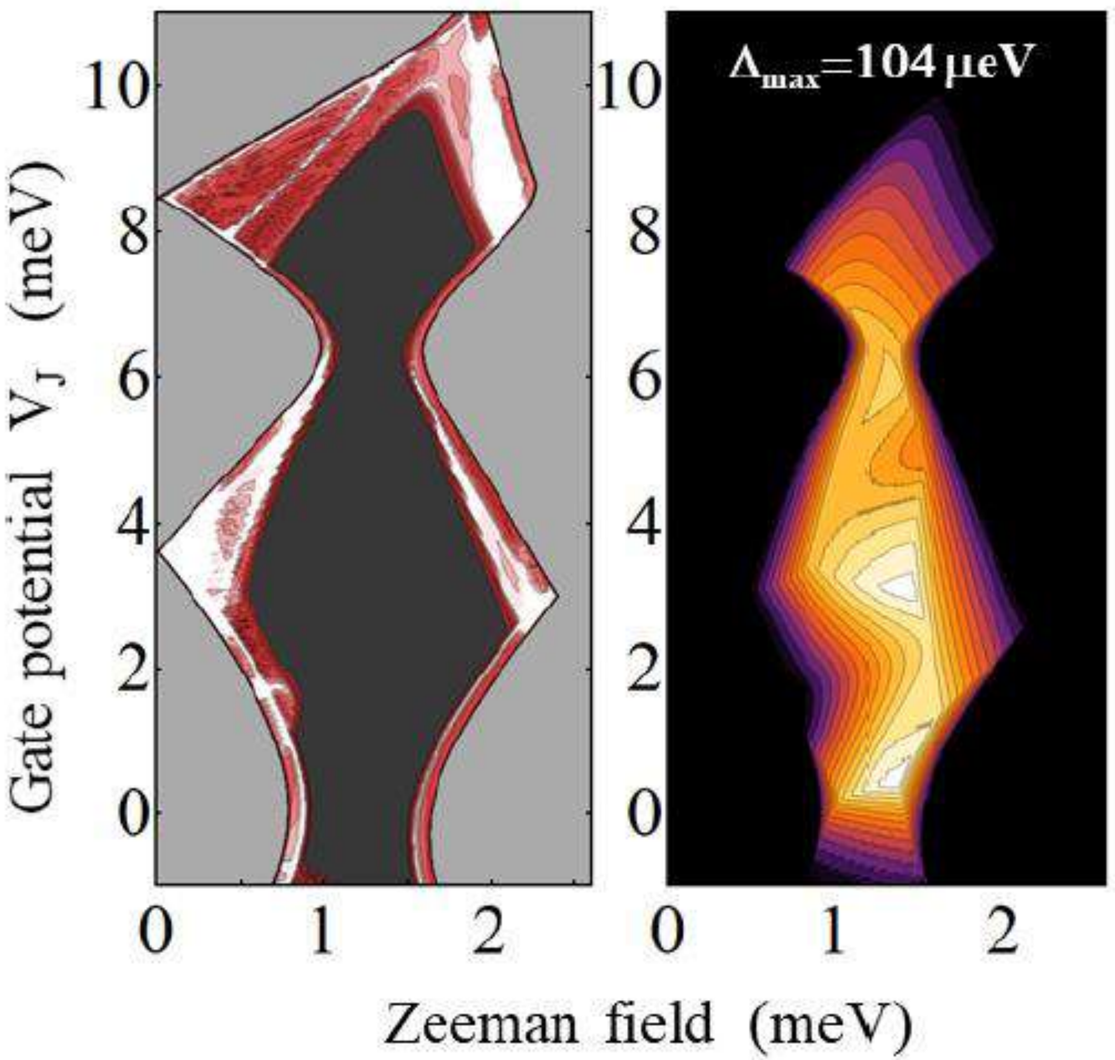


JJ structure with phase difference $\phi=\pi$

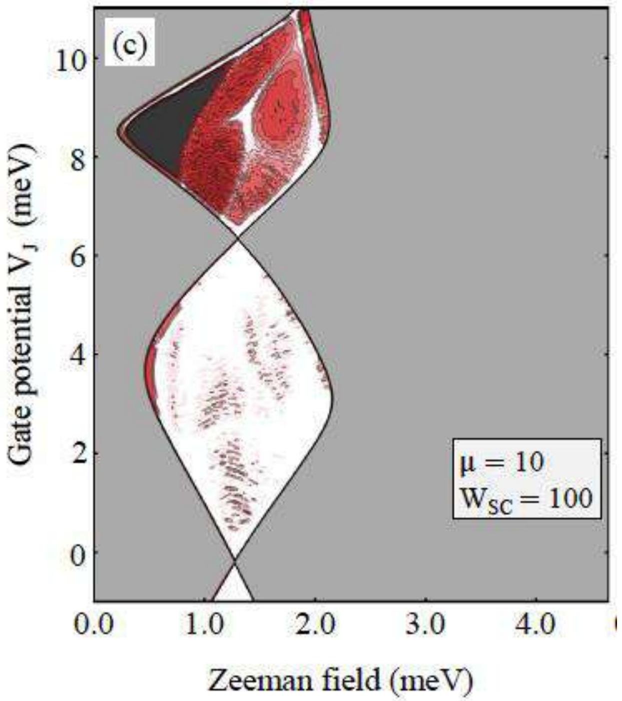
$\phi=0$

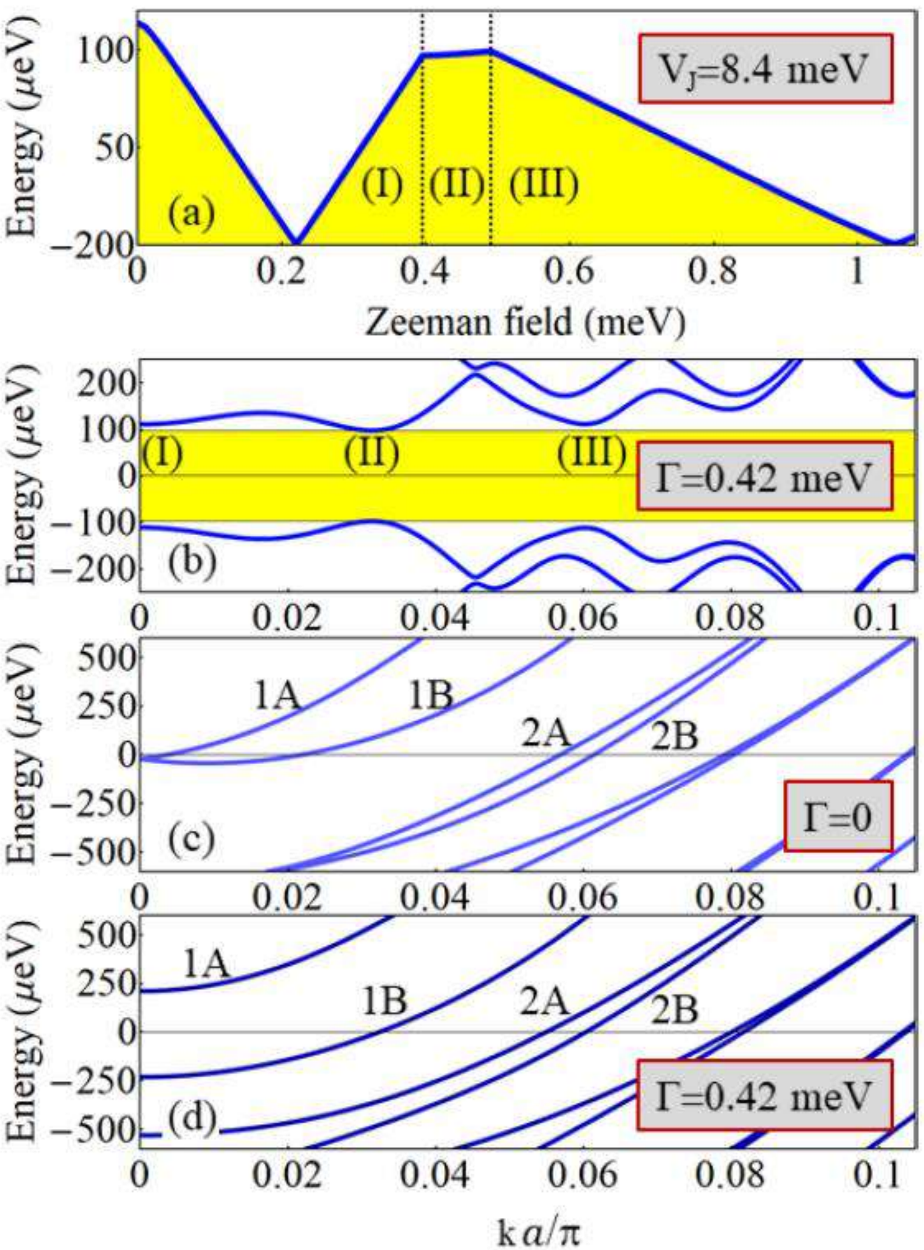


$\phi=\pi$

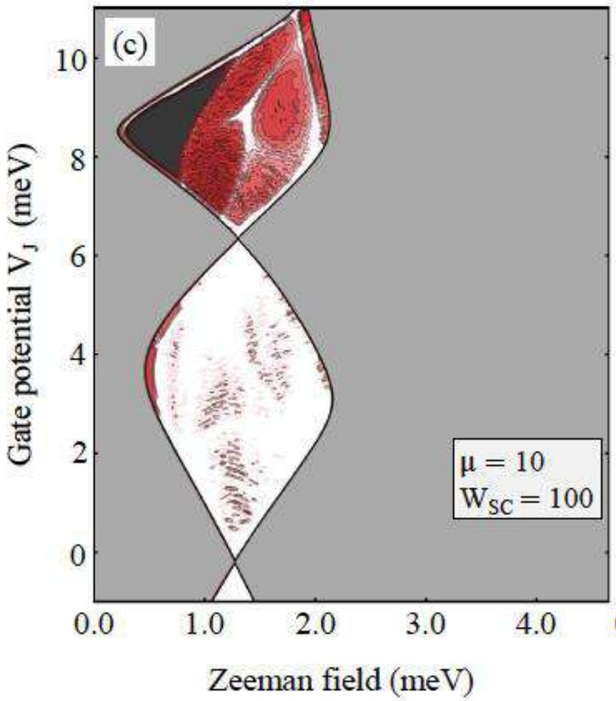


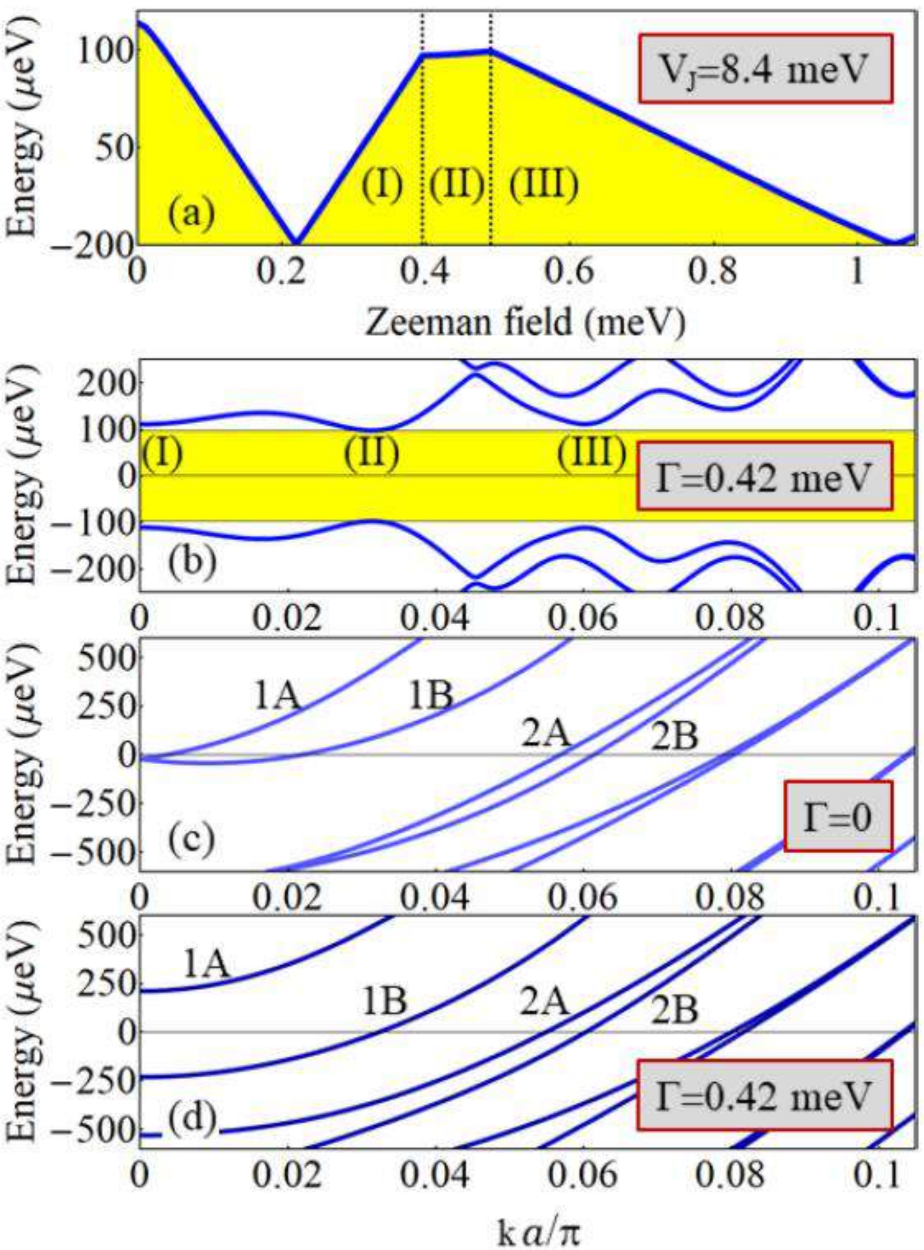
Modes that control the quasiparticle gap

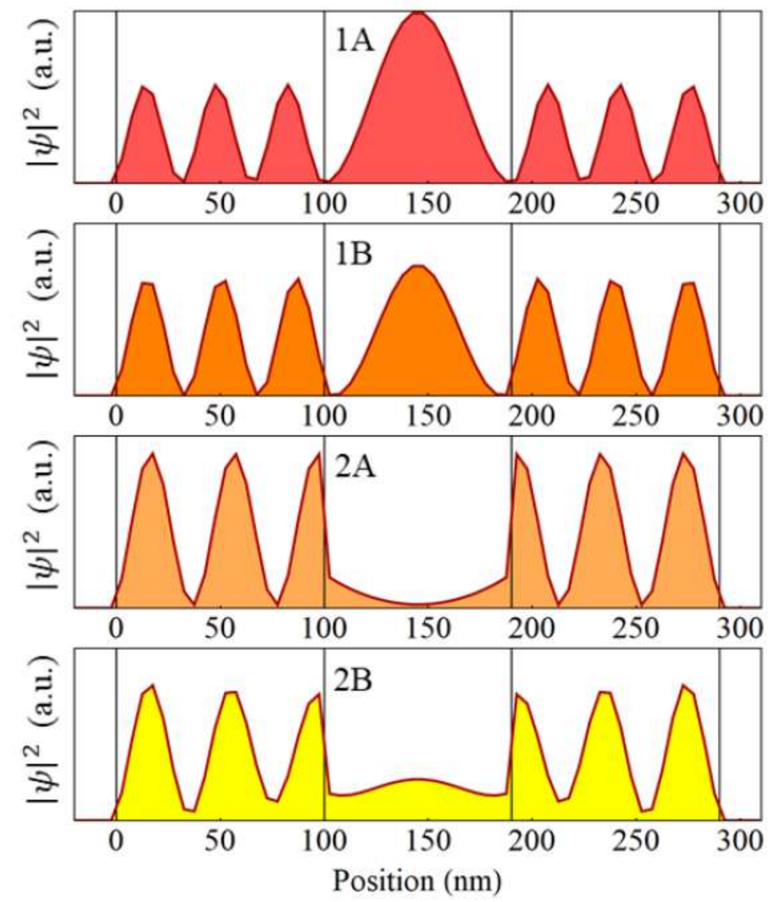




Modes that control the quasiparticle gap

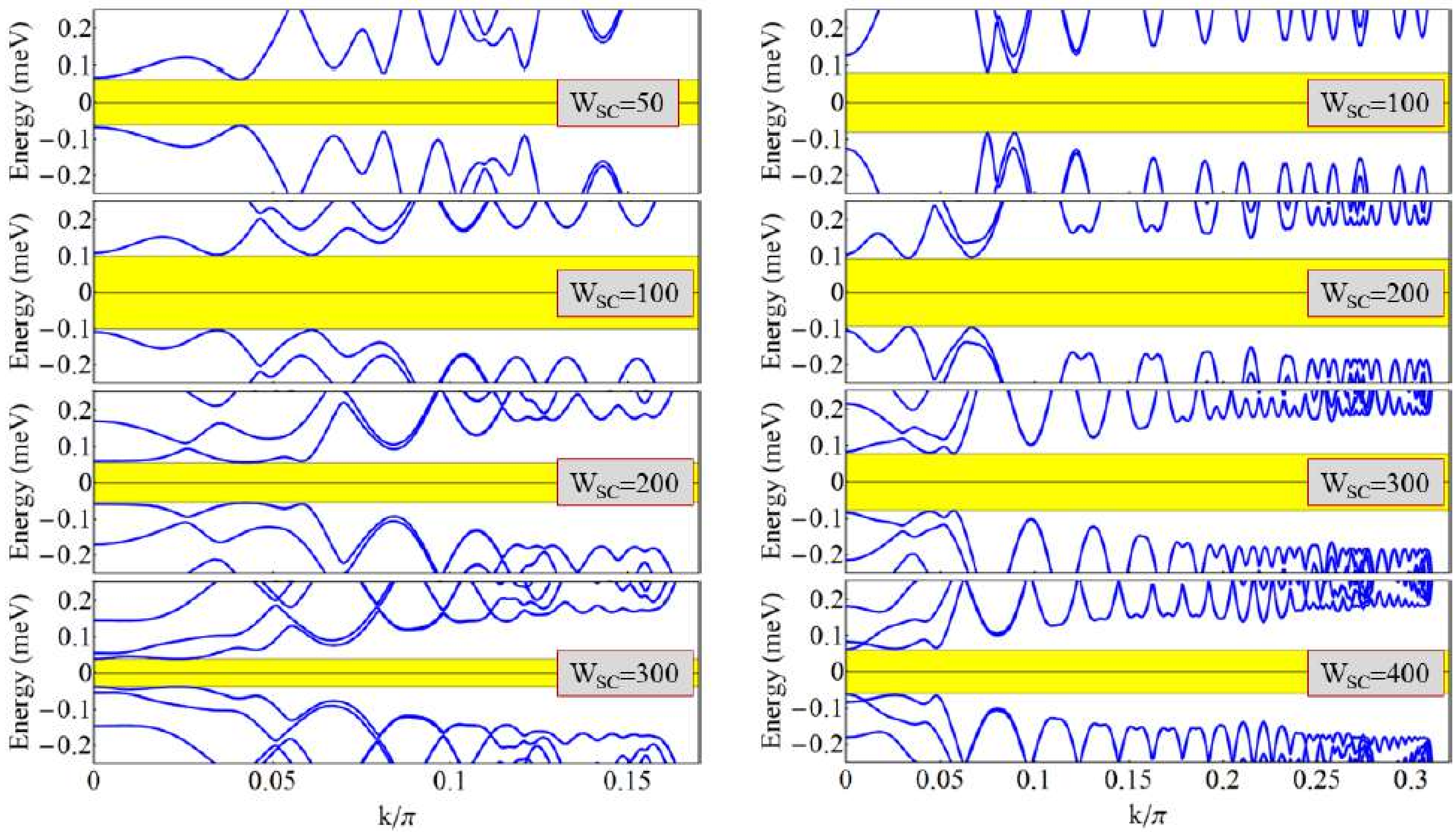




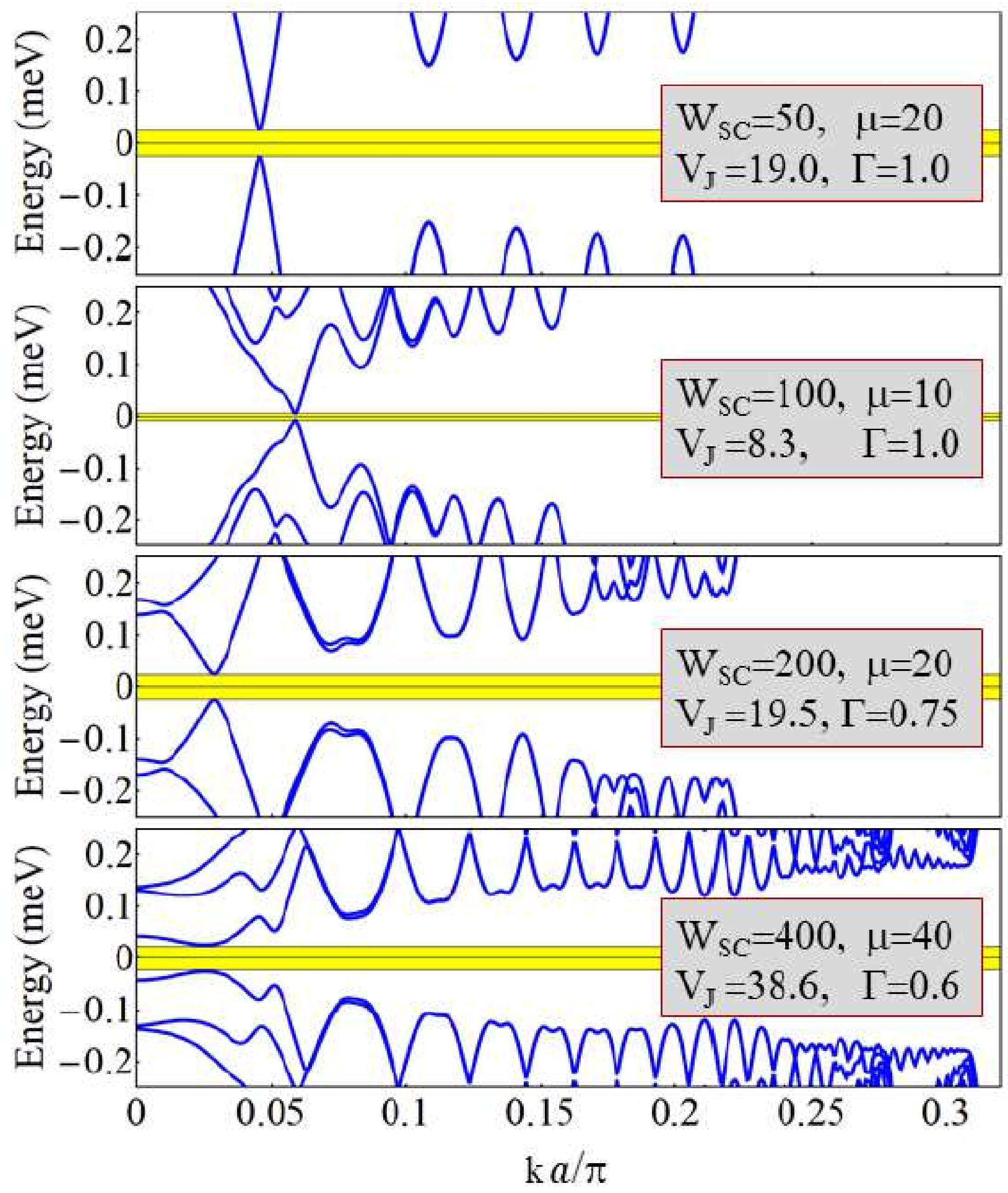


Energy spectra near the maximum topological gap

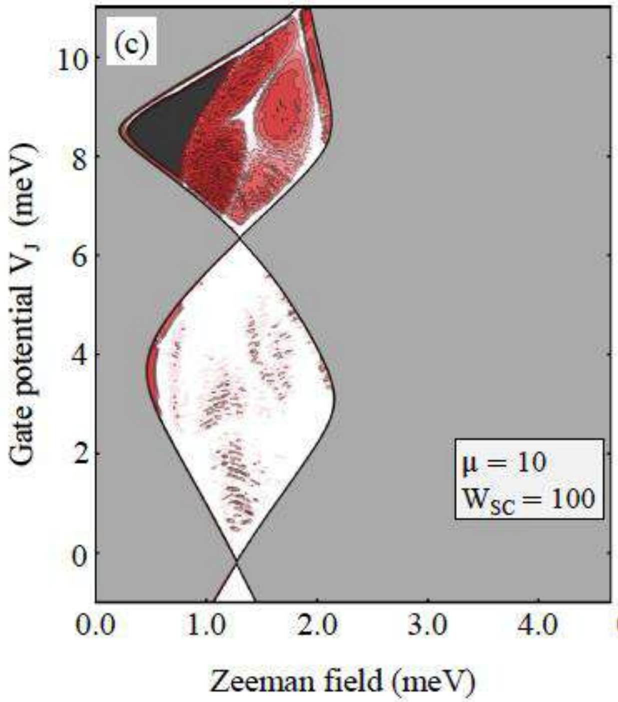
μ=10 meV μ=40 meV

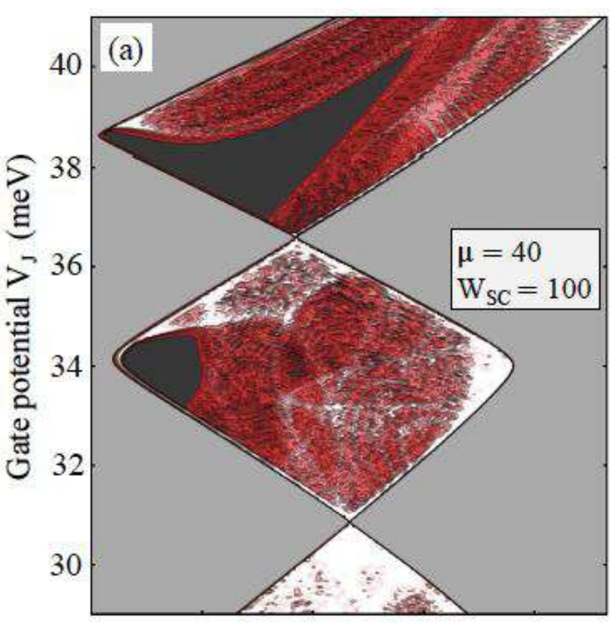


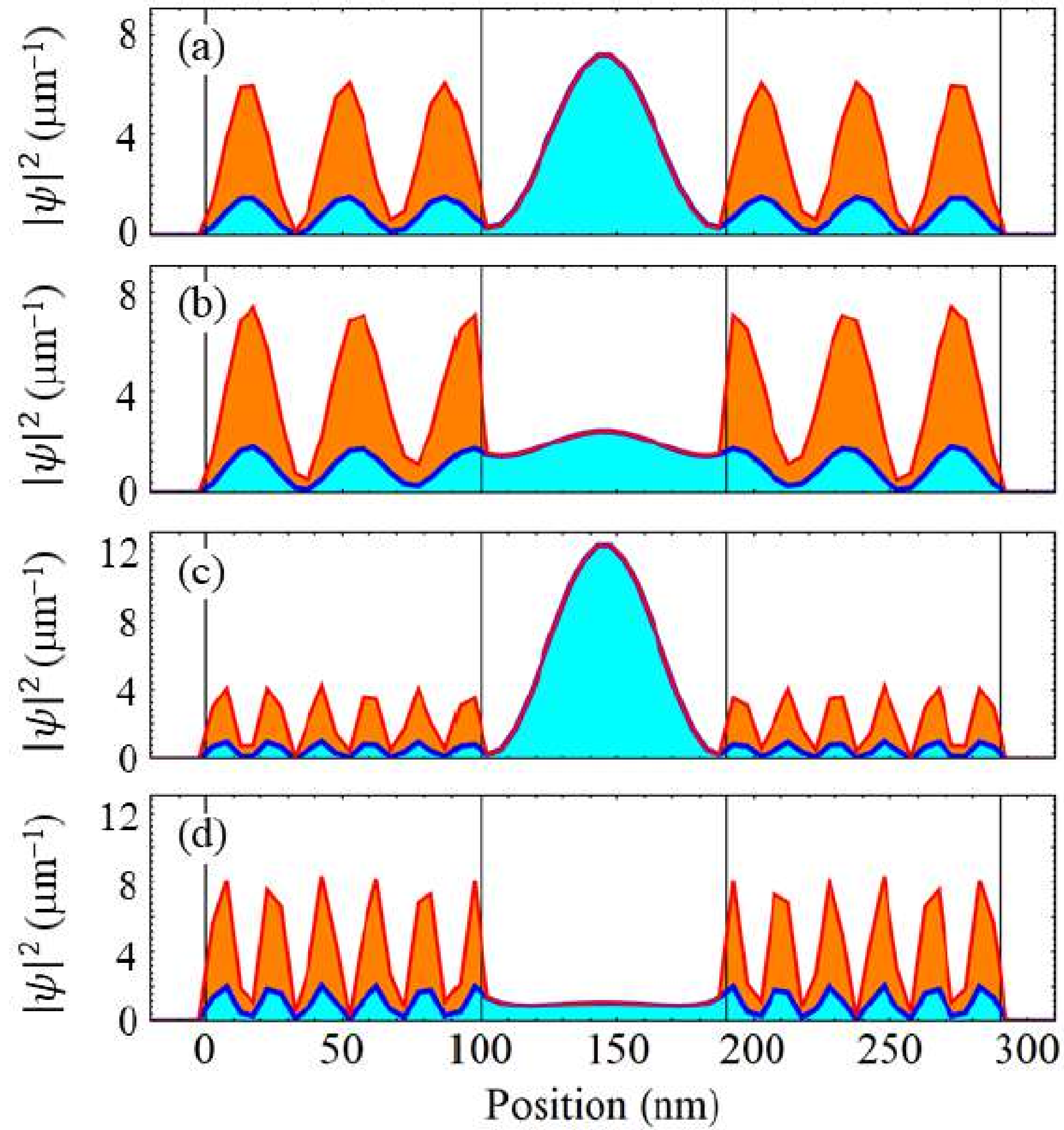
Energy spectra with small topological gaps



Gap-edge modes (near the maximum topological gap)







μ=10 meV

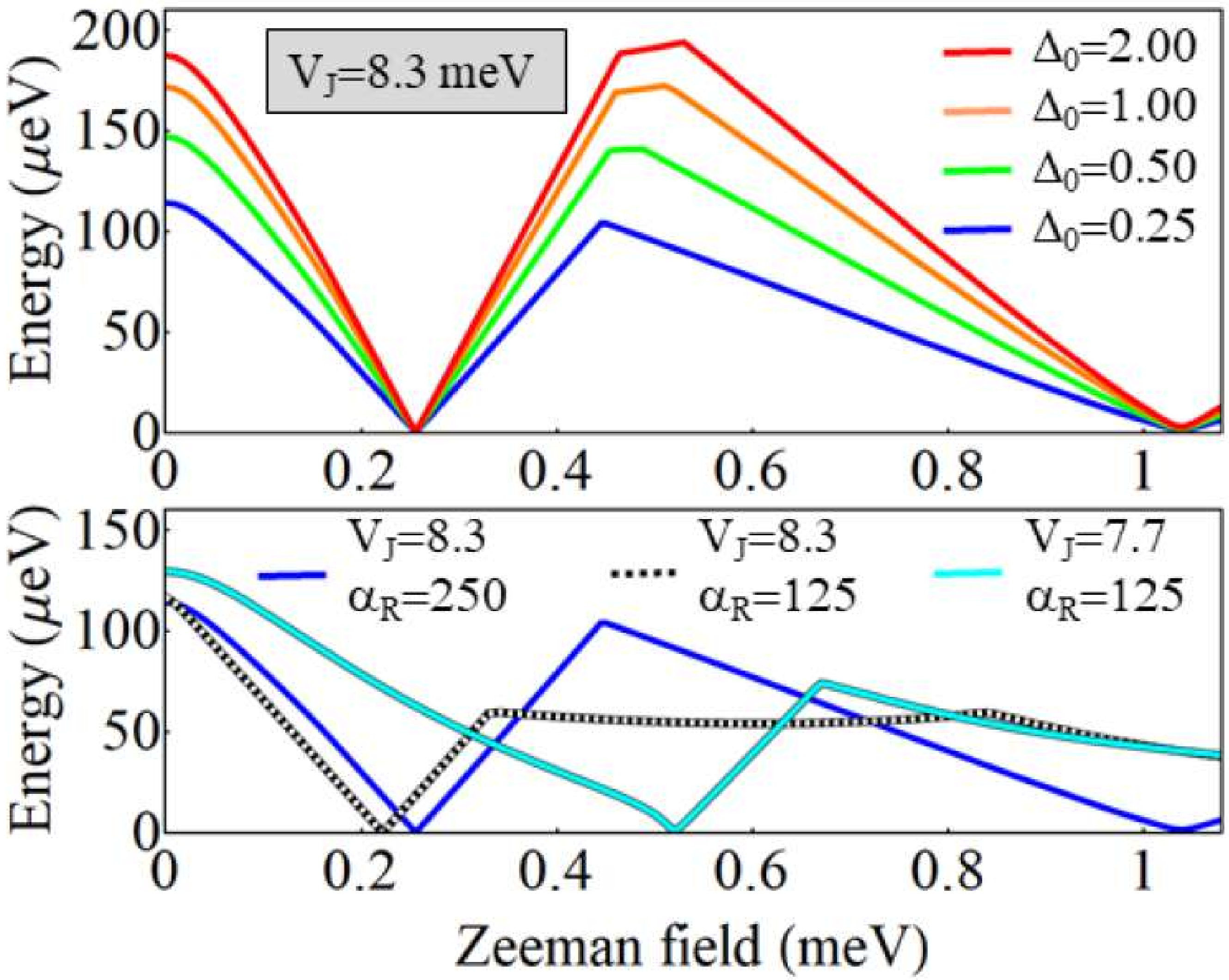
μ=40 meV

Partial conclusion & future tasks

- In planar JJ devices, the topological gap has a nonmonotonic dependence on the width of the SC films, with an optimum width that depends on other system parameters (e.g., μ)

- Determine the dependence of the optimal regime on the parent SC gap and the SOC type & strength
- Investigate disorder effects in JJ structures with narrow superconductors
- Calculate the topological phase diagram (in the presence of disorder)

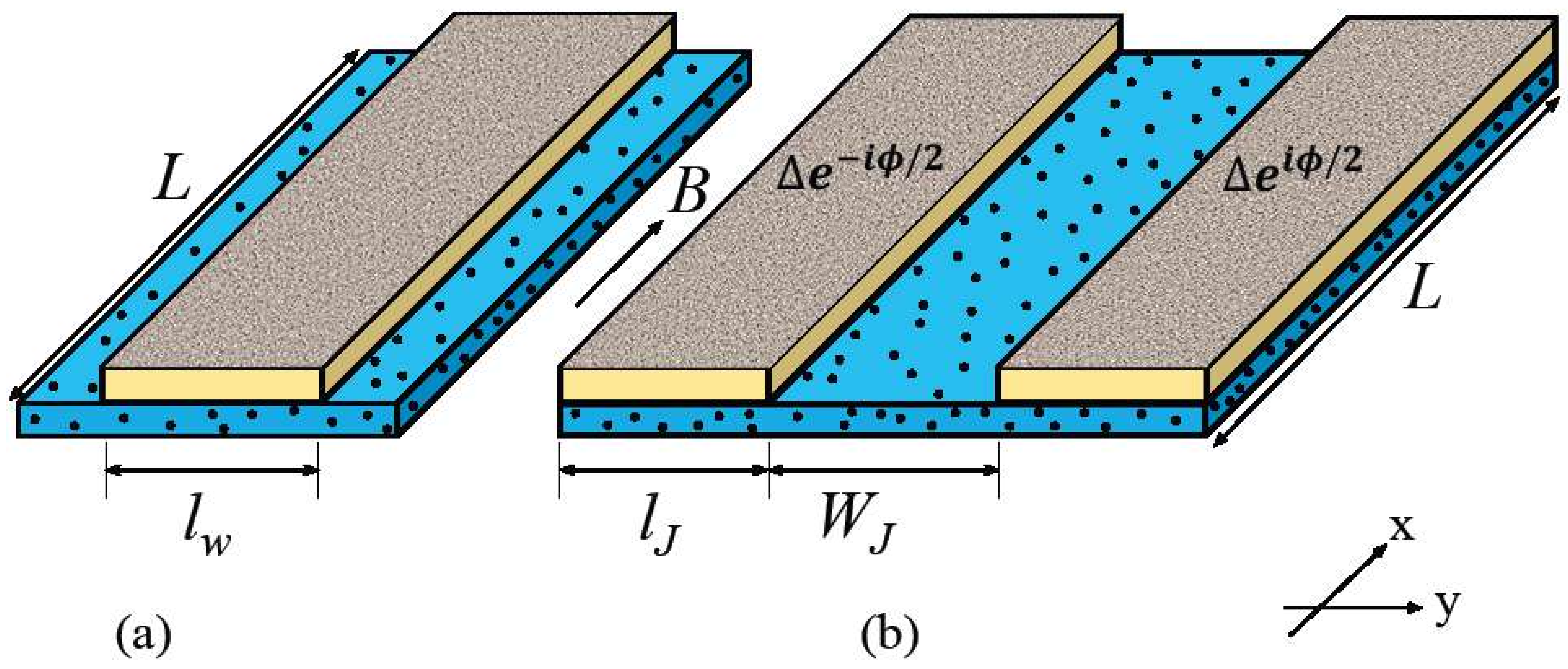
Dependence of the qp gap on the parent SC gap & the Rashba SOC



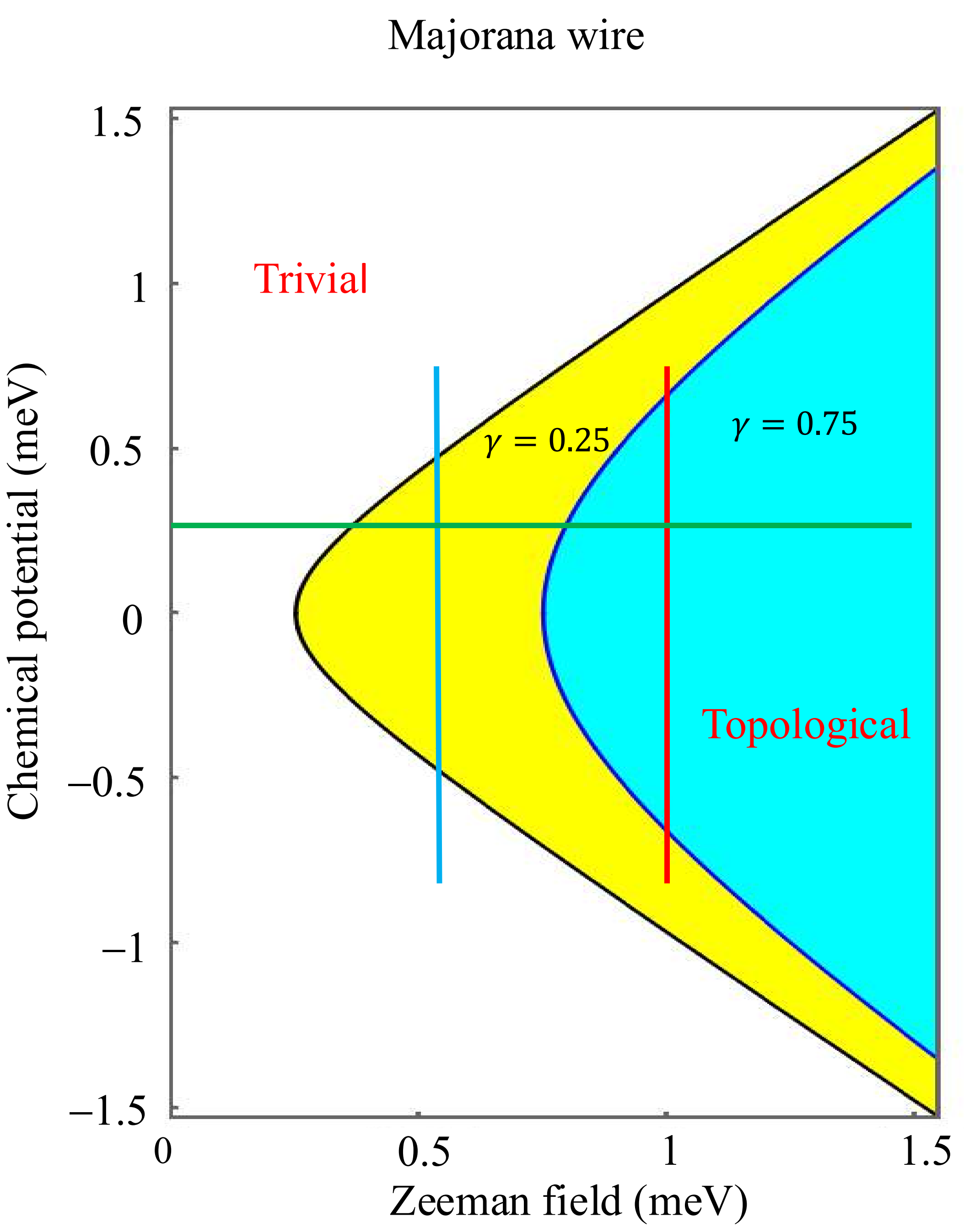
μ = 10 meV
W_{SC} = 100 nm

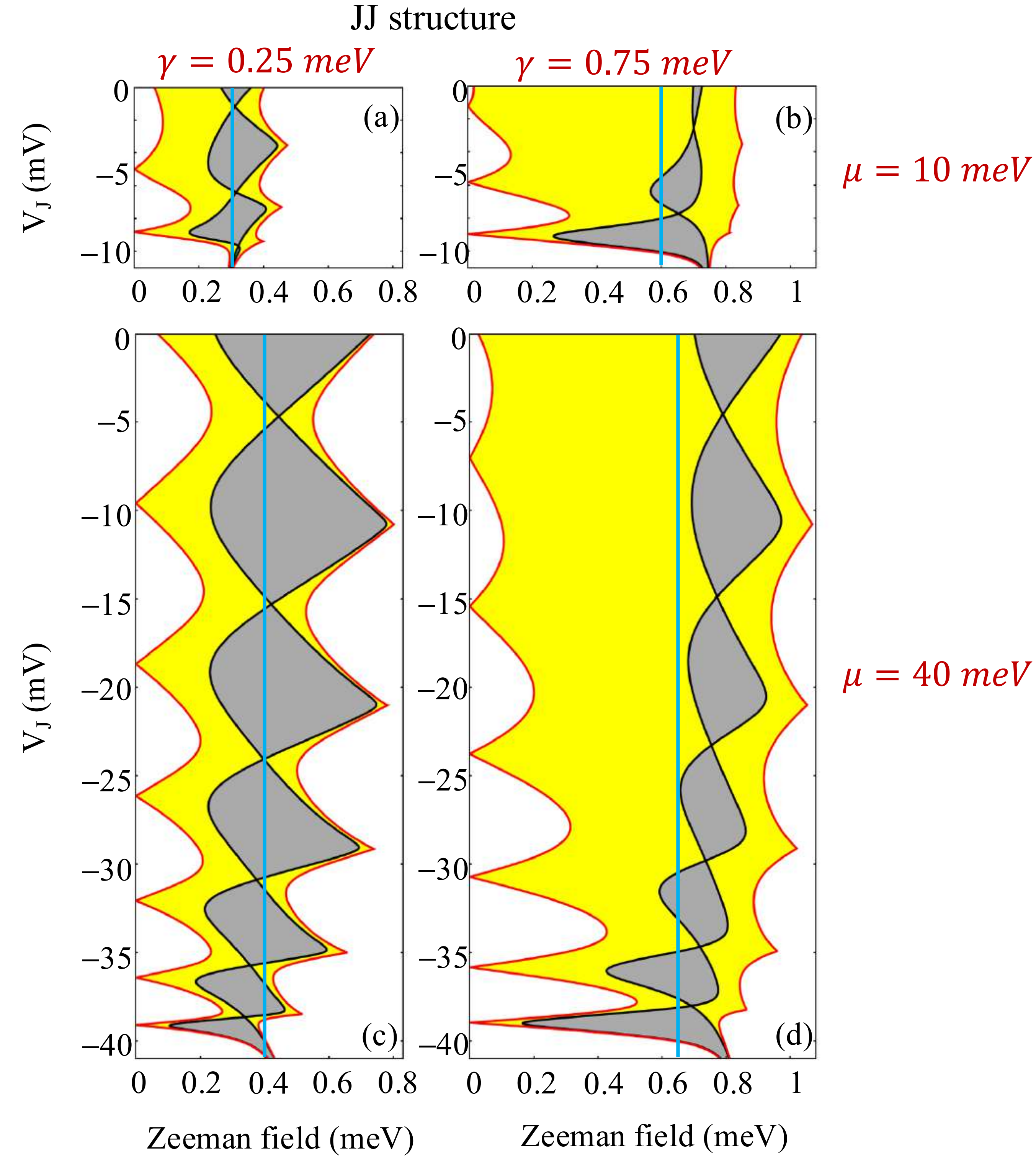
Disorder effects: Majorana wires versus Josephson junctions

$l_w = 100$ nm
$l_J = 800$ nm
$W_J = 100$nm

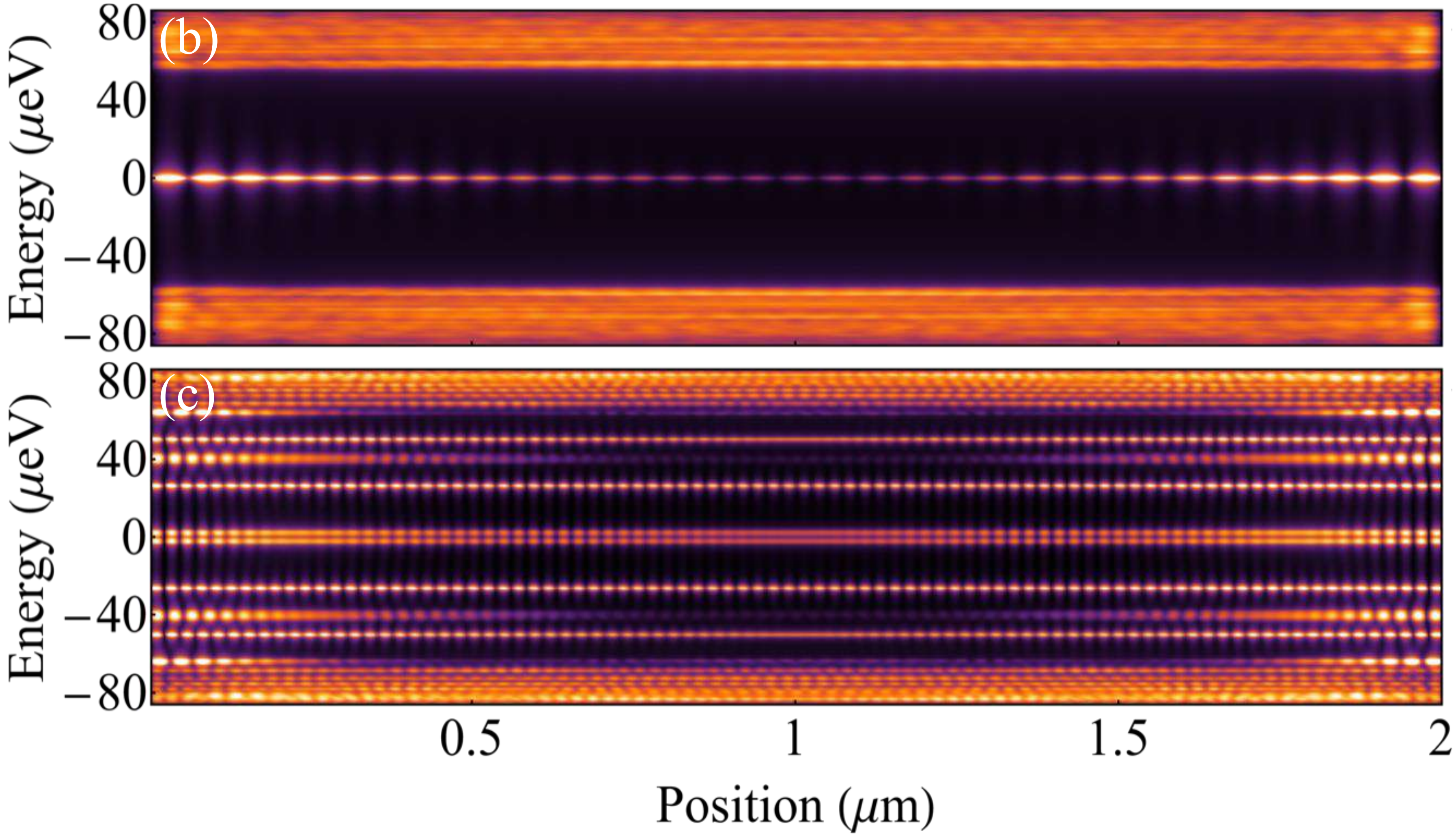


Phase diagrams (clean systems)





Finite size effects (clean finite JJ): LDOS



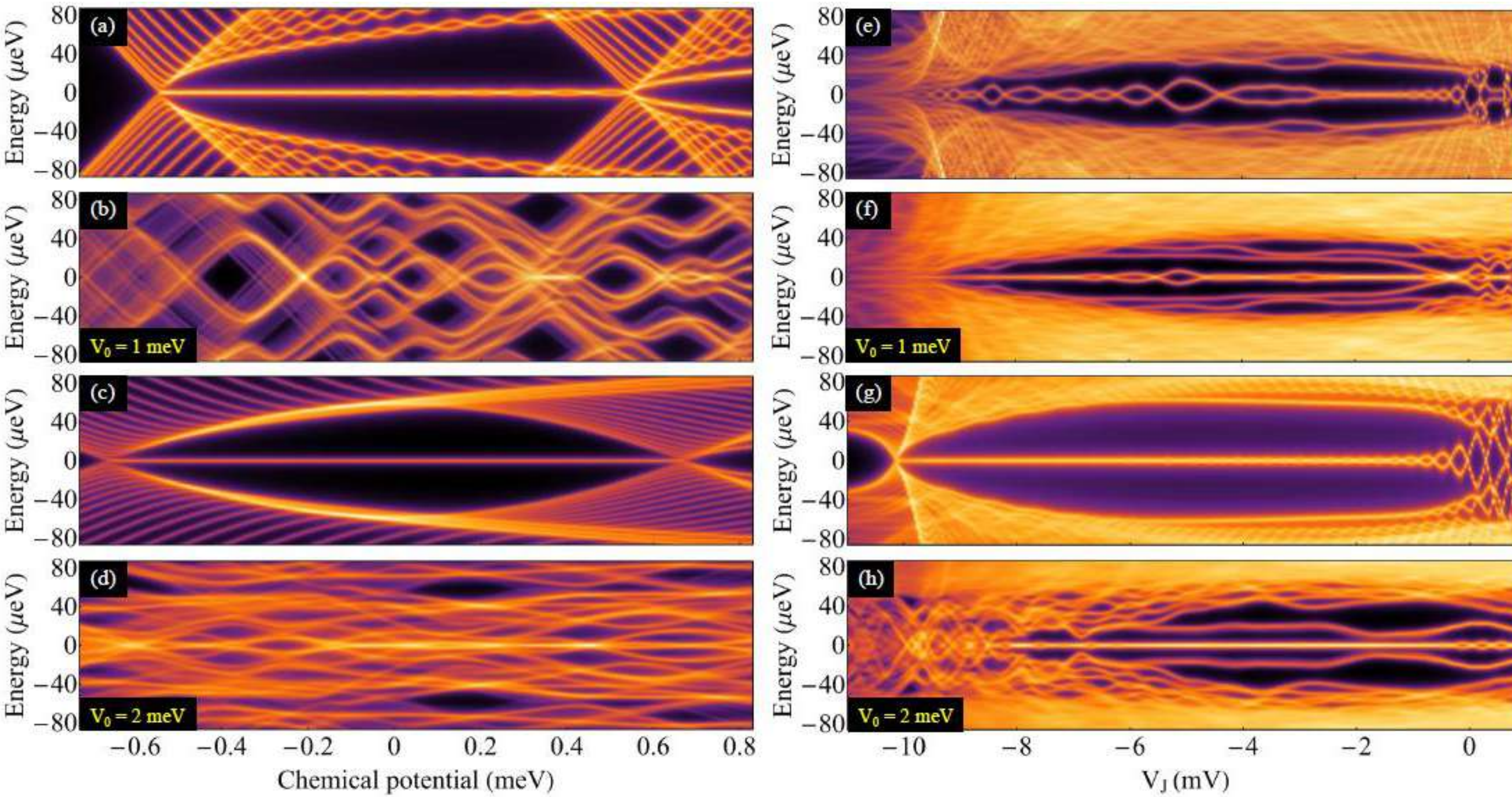
μ = 10 meV
γ = 0.75 meV

μ = 40 meV
γ = 0.75 meV

Systems with SM disorder

Majorana wire

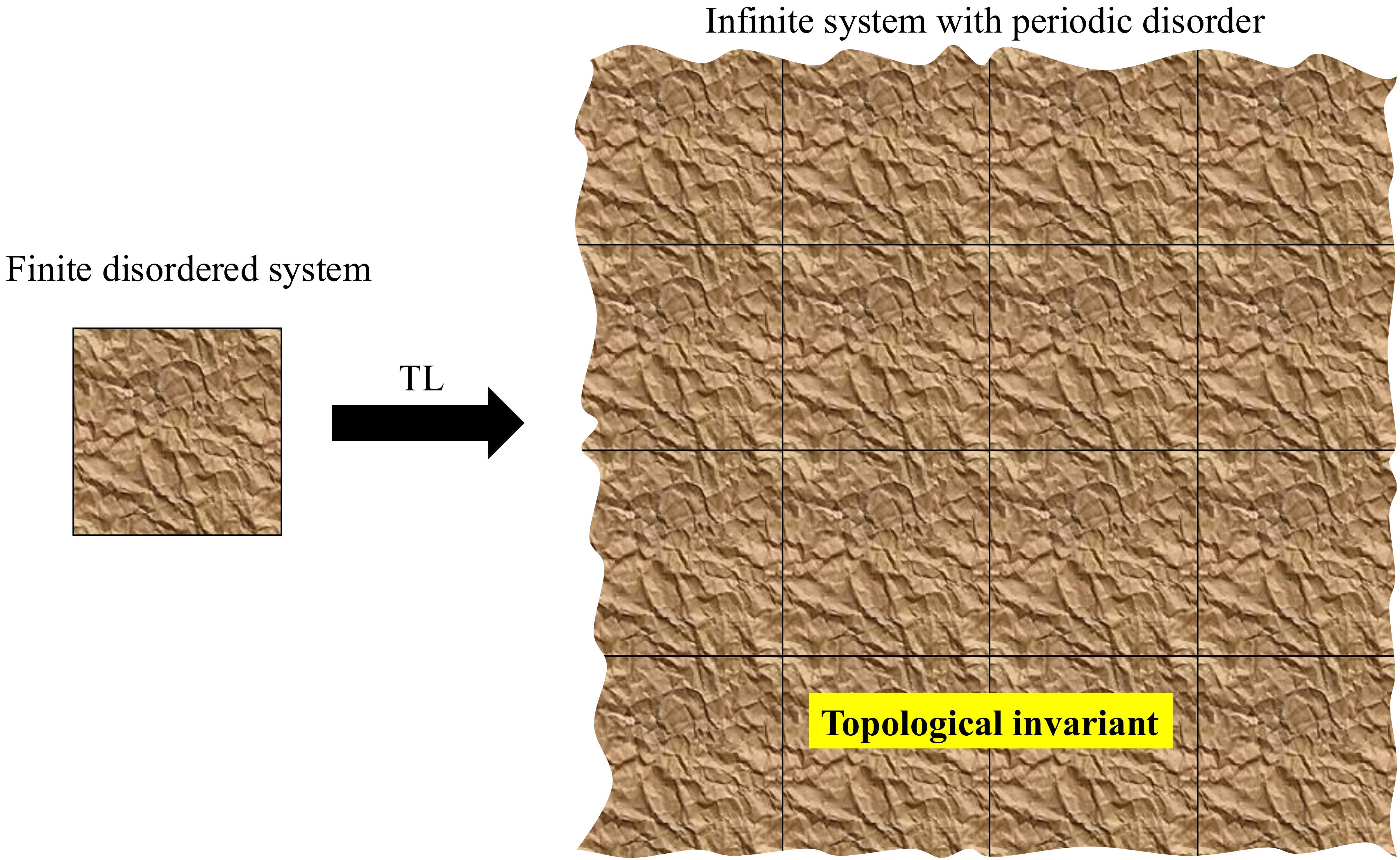
JJ device



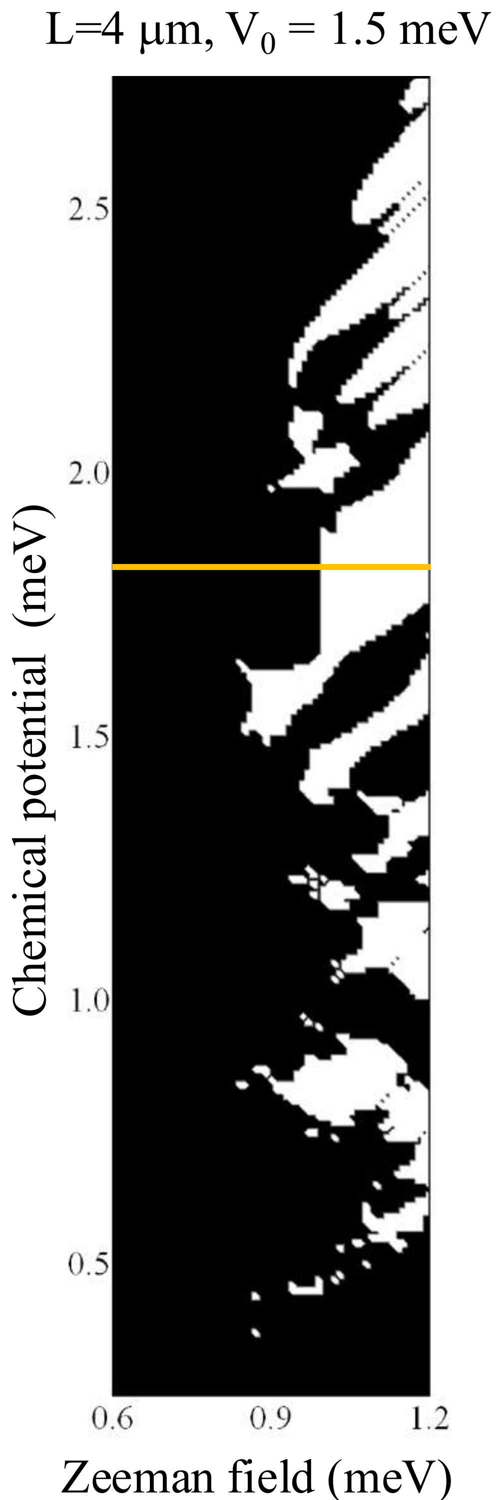
$\mu = 10$
$\gamma = 0.25$

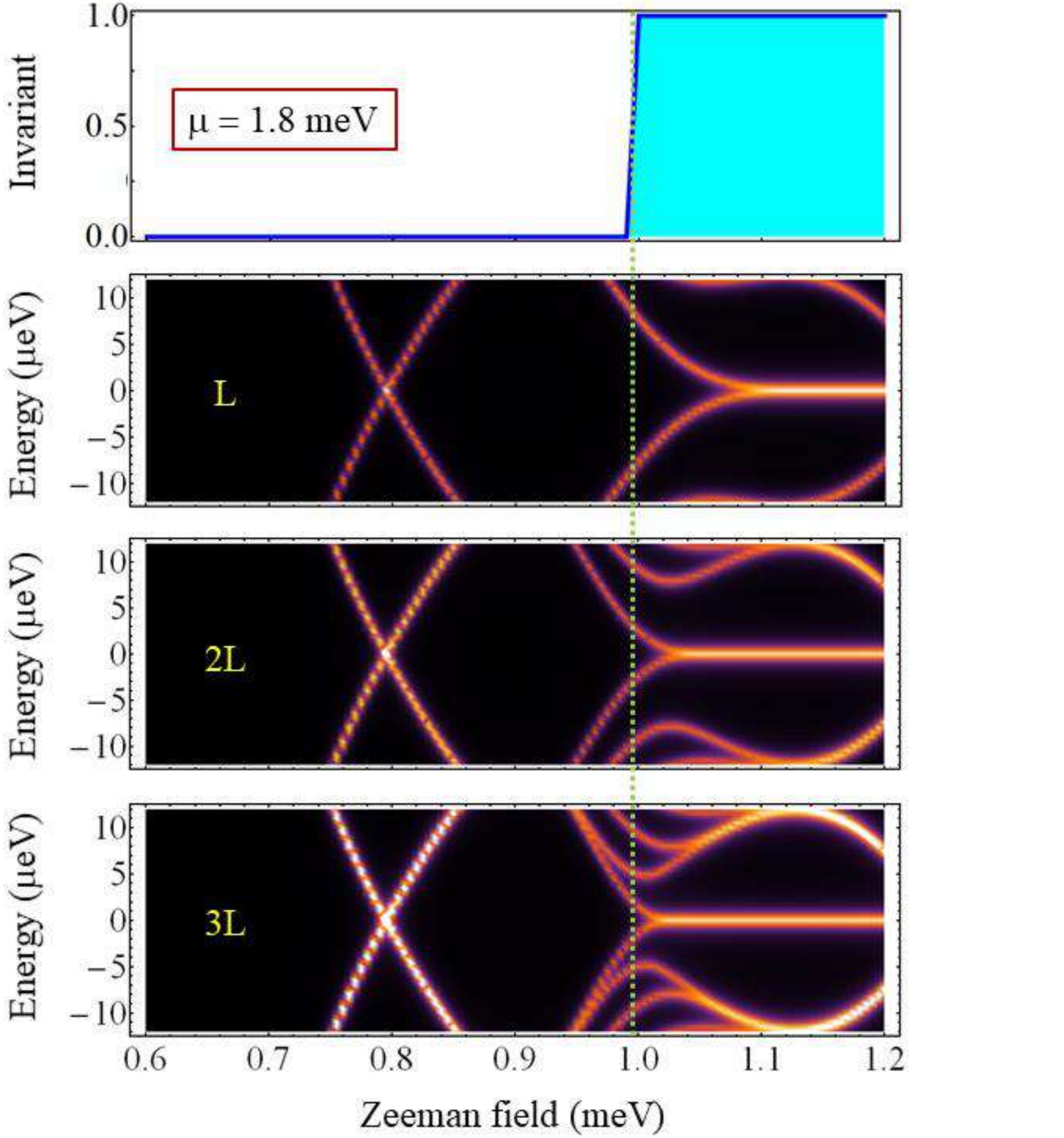
$\mu = 10$
$\gamma = 0.75$

Topological phase diagram: Proposed approach



(Operational) topological phase





Summary & Conclusions

- The quasiparticle gap characterizing JJ structures is typically small; rotating the magnetic field leads to a proliferation of gapless SC phases
- Reducing the potential of the outside SM regions results in a nearly gapless system with a highly "fragmented" topological phase diagram
- Varying the potential in the junction region results in a crossover between a "nanowire regime" and a "Josephson junction regime"
- The topological gap has a nonmonotonic dependence on the width of the SC films, with an optimum width that depends on other system parameters (e.g., μ)
- Investigating the robustness of the topological phase against disorder remains a critical task