Superconformal and Supersymmetric Constraints to Hadron Masses in Light-Front Holographic QCD

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The Proton Mass: at the Heart of Most Visible Matter

ECT*, Trento,

3-7 April 2017

In collaboration with Stan Brodsky, Alexandre Deur, Hans G. Dosch, Cédric Lorce and Raza Sabbir Sufian

- We use a superconformal algebraic structure to construct relativistic light-front (LF) semiclassical bound-state equations which can be embedded in a higher-dimensional classical gravitational theory

- This approach to hadron physics incorporates basic nonperturbative properties which are not apparent from the chiral QCD Lagrangian:

  I. Emergence of a mass scale and confinement out of a classically scale-invariant theory

  II. Occurrence of a zero-mass bound state in the limit of zero quark masses

  III. Universal Regge trajectories for mesons and baryons

  IV. Breaking of chirality in the hadron excitation spectrum

  V. Precise connections between the light meson and nucleon spectra

- Effective theory can be extended to heavy-light hadrons where heavy quark masses break the conformal invariance but the underlying dynamical supersymmetry still holds

- Procedure based on work by de Alfaro, Fubini and Furlan, and Fubini and Rabinovici: a generalized Hamiltonian is constructed as a superposition of superconformal generators which carry different dimensions

- The Hamiltonian remains within the superconformal algebraic structure and leads to unique form of the confinement potential

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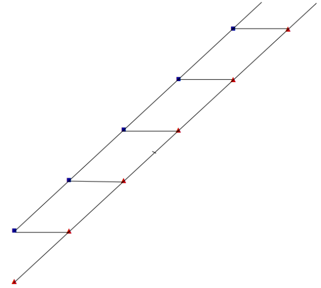


1 Superconformal quantum mechanics

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

- SUSY QM [E. Witten (1981)] contains two fermionic generators Q and $Q^\dagger$ with anticommutation relations

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$

and the Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ which commutes with the fermionic generators

$$[Q, H] = [Q^\dagger, H] = 0$$

closing the graded-Lie $sl(1/1)$ algebra

- Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E, but for a zero eigenvalue we can have the trivial solution $|E = 0\rangle = 0$

- In matrix notation

$$Q = \begin{pmatrix} 0 & q \\ 0 & 0 \end{pmatrix}, \qquad Q^\dagger = \begin{pmatrix} 0 & 0 \\ q^\dagger & 0 \end{pmatrix}, \qquad H = \frac{1}{2}\begin{pmatrix} q\,q^\dagger & 0 \\ 0 & q^\dagger q \end{pmatrix}$$

with

$$q = -\frac{d}{dx} + \frac{f}{x}, \qquad q^\dagger = \frac{d}{dx} + \frac{f}{x}$$

for a conformal theory and f is dimensionless

- Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to the generator of conformal transformations K

$$S = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix}, \qquad S^\dagger = \begin{pmatrix} 0 & 0 \\ x & 0 \end{pmatrix}$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned} \tfrac{1}{2}\{Q, Q^\dagger\} &= H, & \tfrac{1}{2}\{S, S^\dagger\} &= K, \\ \{Q, S^\dagger\} &= f - B + 2iD, & \{Q^\dagger, S\} &= f - B - 2iD \end{aligned}$$

where $B = \frac{1}{2}\sigma_3$, and the generators of translation, dilatation and the special conformal transformation H, D and K

$$\begin{aligned} H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 + 2Bf}{x^2}\right) \\ D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\ K &= \tfrac{1}{2}x^2 \end{aligned}$$

satisfy the conformal algebra

$$[H, D] = iH, \qquad [H, K] = 2iD, \qquad [K, D] = -iK$$

- Following F&R define a fermionic generator R, a linear combination of the generators Q and S

$$R_\lambda = Q + \lambda\, S$$

which generates a new Hamiltonian

$$G_\lambda = \{R_\lambda, R_\lambda^\dagger\}$$

where by construction

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \qquad [R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$$

which also closes under the graded algebra $sl(1/1)$

- The Hamiltonian G_λ is given by

$$G_\lambda = 2H + 2\lambda^2 K + 2\lambda\,(f - \sigma_3)$$

and leads to the eigenvalue equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2}\right)\phi_1 = E\,\phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2\, x^2 + 2\lambda\, f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2}\right)\phi_2 = E\,\phi_2$$

2 Superconformal meson-baryon symmetry

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Meson}} \\ \phi_{\text{Baryon}} \end{pmatrix}$$

- Upon the substitutions (slide 6)

$$\begin{aligned} x &\mapsto \zeta \\ E &\mapsto M^2 \\ \lambda &\mapsto \lambda_B = \lambda_M \\ f &\mapsto L_M - \tfrac{1}{2} = L_B + \tfrac{1}{2} \end{aligned}$$

we find the LF bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M - 1)\right)\phi_{Meson} = M^2\,\phi_{Meson}$$

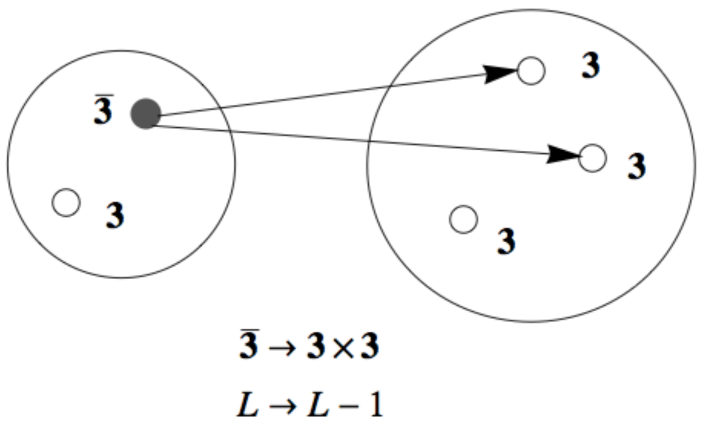
$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_N^2 - 1}{4\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_N + 1)\right)\phi_{Baryon} = M^2\,\phi_{Baryon}$$

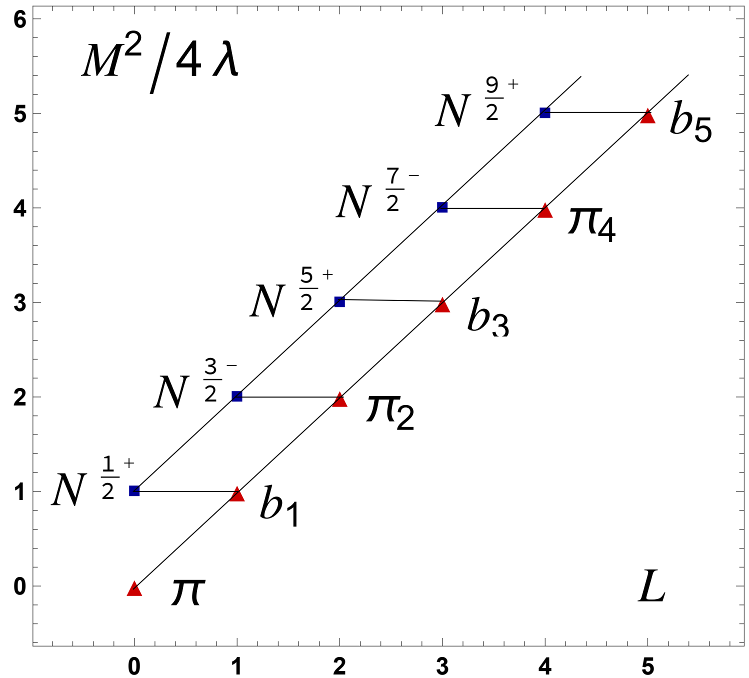
obtained previously from LF holographic QCD (LFHQCD)

S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rep. **584**, 1 (2015)

- ζ is the invariant transverse separation between constituents in LF quantization which is identified with the holographic variable z in AdS classical gravity: $\zeta = z,\ \zeta^2 = x(1-x)b_\perp^2$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $\Rightarrow$ $\boxed{L_M = L_B + 1}$
- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B is the relative angular momentum between the active quark and spectator cluster in the baryon
- Special role of the pion as a unique state of zero energy
  $R^\dagger|M, L\rangle = |B, L-1\rangle, \quad R^\dagger|M, L=0\rangle = 0$
- Emerging dynamical SUSY from SU(3) color
  (Hadronic SUSY introduced by H. Miyazawa (1966))





- Spin-dependent Hamiltonian to describe mesons and baryons with internal spin (chiral limit)

  [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

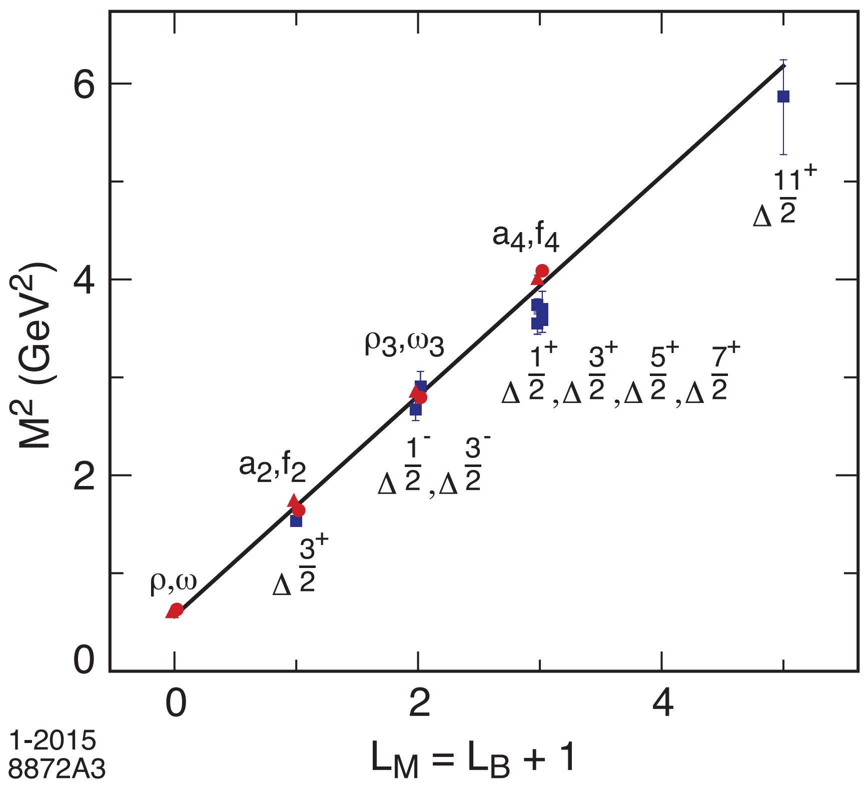
$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S \qquad S = 0, 1$$

- $\text{Mesons}: M^2 = 4\lambda\,(n + L_M) + 2\lambda S$

  $\text{Baryons}: M^2 = 4\lambda\,(n + L_B + 1) + 2\lambda S$

- Superconformal meson-baryon partners

  ($\sqrt{\lambda} = 0.53$ GeV)

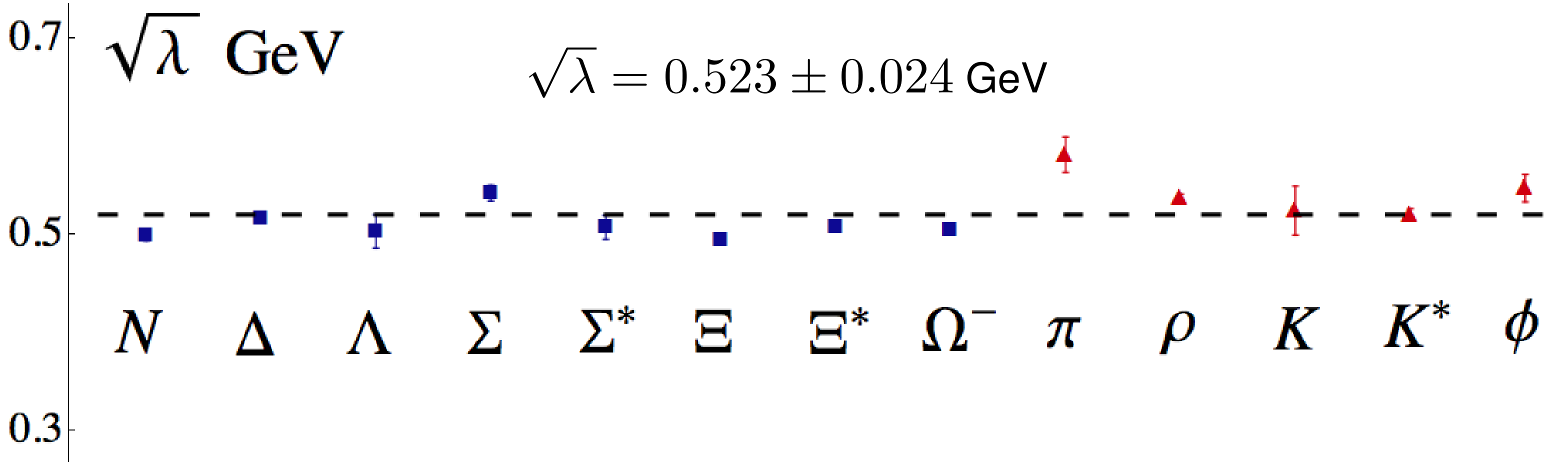


- Quadratic mass correction for light quark masses

$$\Delta M^2[m_1, \cdots, m_n] = \frac{\lambda^2}{F}\frac{\mathrm{d}F}{\mathrm{d}\lambda}$$

with $F[\lambda] = \int_0^1 \cdots \int \mathrm{d}x_1 \cdots \mathrm{d}x_n\, e^{-\frac{1}{\lambda}\left(\sum_{i=1}^n \frac{m_i^2}{x_i}\right)}\delta(\sum_{i=1}^n x_i - 1)$

- How universal is the semiclassical approximation based on superconformal QM and its LF holographic embedding? [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]]



Best fit for hadronic scale $\sqrt{\lambda}$ from the different light hadronic sectors including radial and orbital excitations

- Frame-independent decomposition of the quadratic masses for light hadrons in the chiral limit:

$$M_H^2/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{\text{contribution from 2-dim light-front harmonic oscillator}} \quad + \overbrace{2(L_H + s) + 2\chi}^{\text{contribution from AdS and superconformal algebra}}$$

Here n is the radial excitation number, L_H the LF angular momentum, s is the total spin of the meson or the cluster in the baryon, $\chi = -1$ for mesons and $\chi = +1$ for baryons

3 Supersymmetry across the heavy-light hadronic spectrum

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **92**, 074010 (2015), Phys. Rev. D **95**, 034016 (2017)]

- For light quark masses we apply superconformal dynamics and treat quark masses as perturbations: confinement scale remains universal
- For light quark masses decoupling of transverse degrees of freedom (LF variable ζ) from longitudinal ones (LF longitudinal momentum fraction x) [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]
- Heavy quark mass breaks conformal symmetry but needs not break supersymmetry since it can stem form the dynamics of color confinement: confinement scale depend on the mass of the heavy quark
- Light quarks present in heavy-light hadrons: system still ultrarelativistic and described by LF relativistic bound-state equations
- If separation of kinematic and dynamic variables also persist for heavy-light hadrons, then the holographic embedding constrains specific form of the confinement potential
- Heavy quark effective theory (HQET) determines the dependence of the confinement scale on the heavy quark mass

- SUSY LF bound-state equations for relativistic heavy-light bound states

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + U_M(\zeta)\right)\phi_{Meson} = M^2\,\phi_{Meson}$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_N^2 - 1}{4\zeta^2} + U_M(\zeta)\right)\phi_{Baryon} = M^2\,\phi_{Baryon}$$

where

$$U_{M,B}(\zeta) = V^2(\zeta) \mp V'(\zeta) + \frac{2L_{M,B} \mp 1}{\zeta} V(\zeta)$$

and the superpotential V is a priori unknown

- Embedding in AdS_5 [Phys. Rev. D **95**, 034016 (2017)]

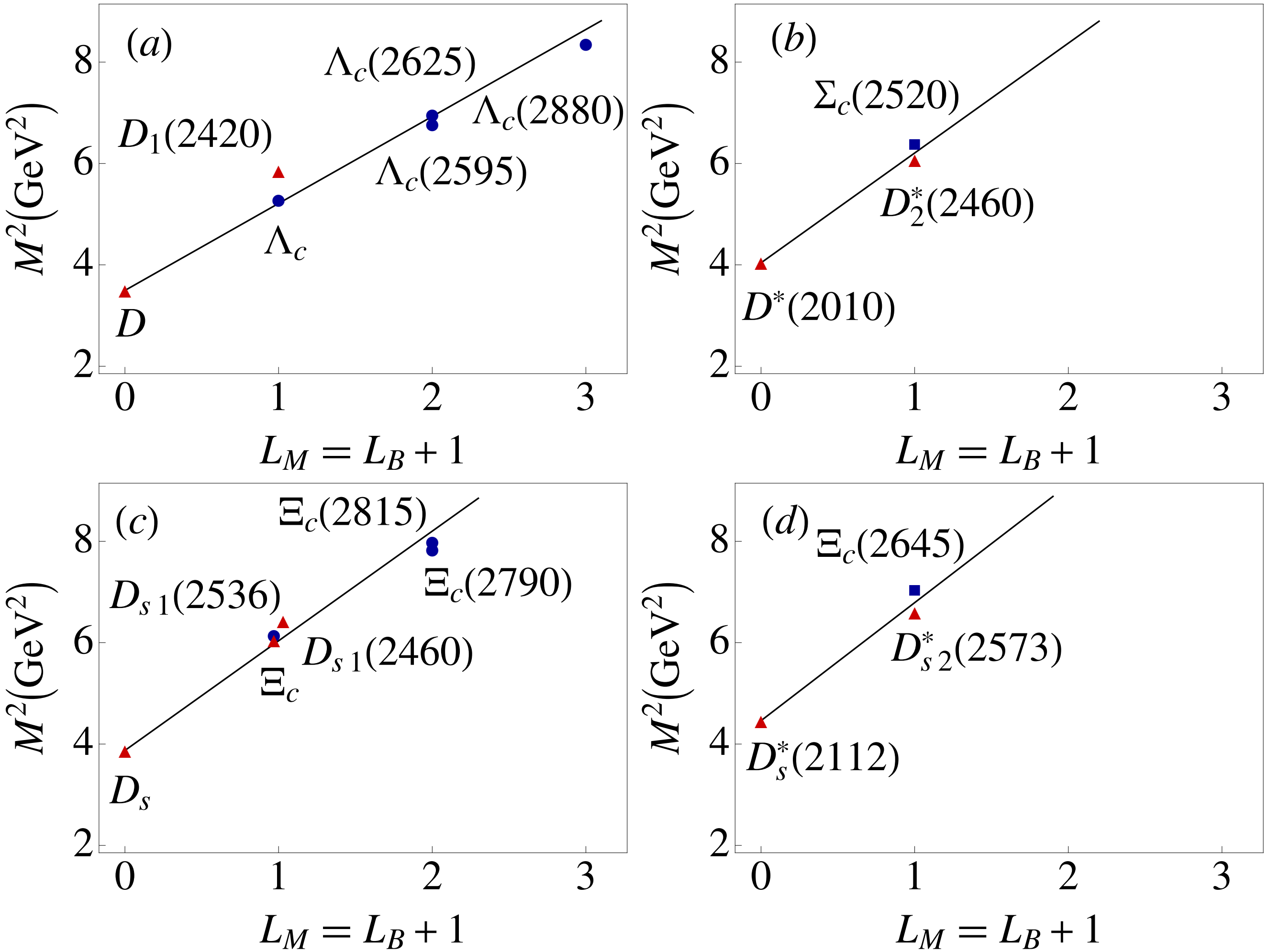
$$V(\zeta) = \frac{1}{2}\left(\lambda\zeta\sigma(\zeta) + \frac{\lambda^2\zeta^2\sigma'(\zeta)}{\lambda^2\zeta^2\sigma(\zeta) + 2(L_M - 1)\lambda}\right)$$

where $\sigma(\zeta)$ is an arbitrary function

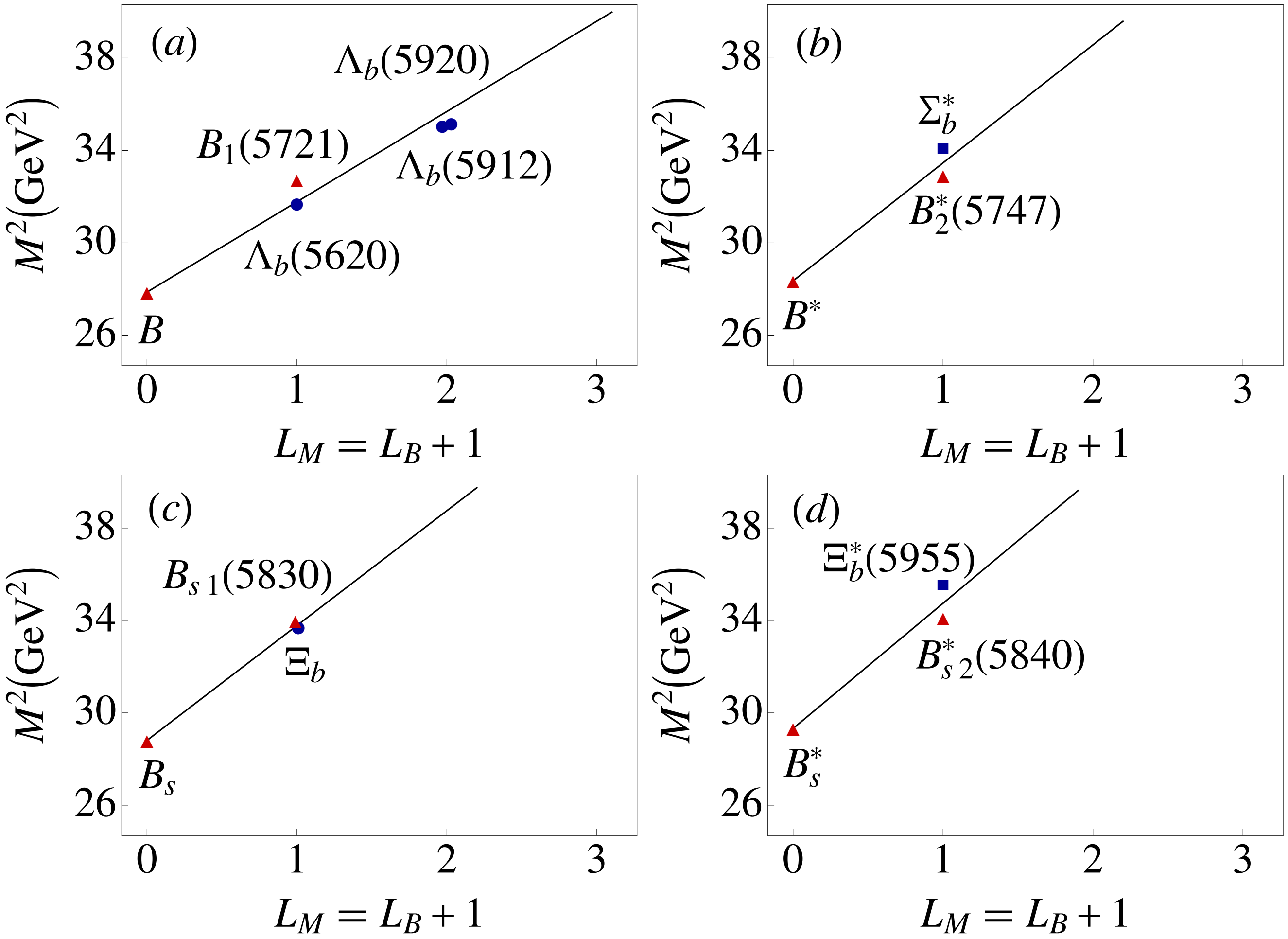
- If embedding is free of kinematical quantities $\sigma'(\zeta) = 0$ and $\sigma = \mathrm{const} \equiv 2A$ with

$$V(\zeta) = \lambda A \zeta$$

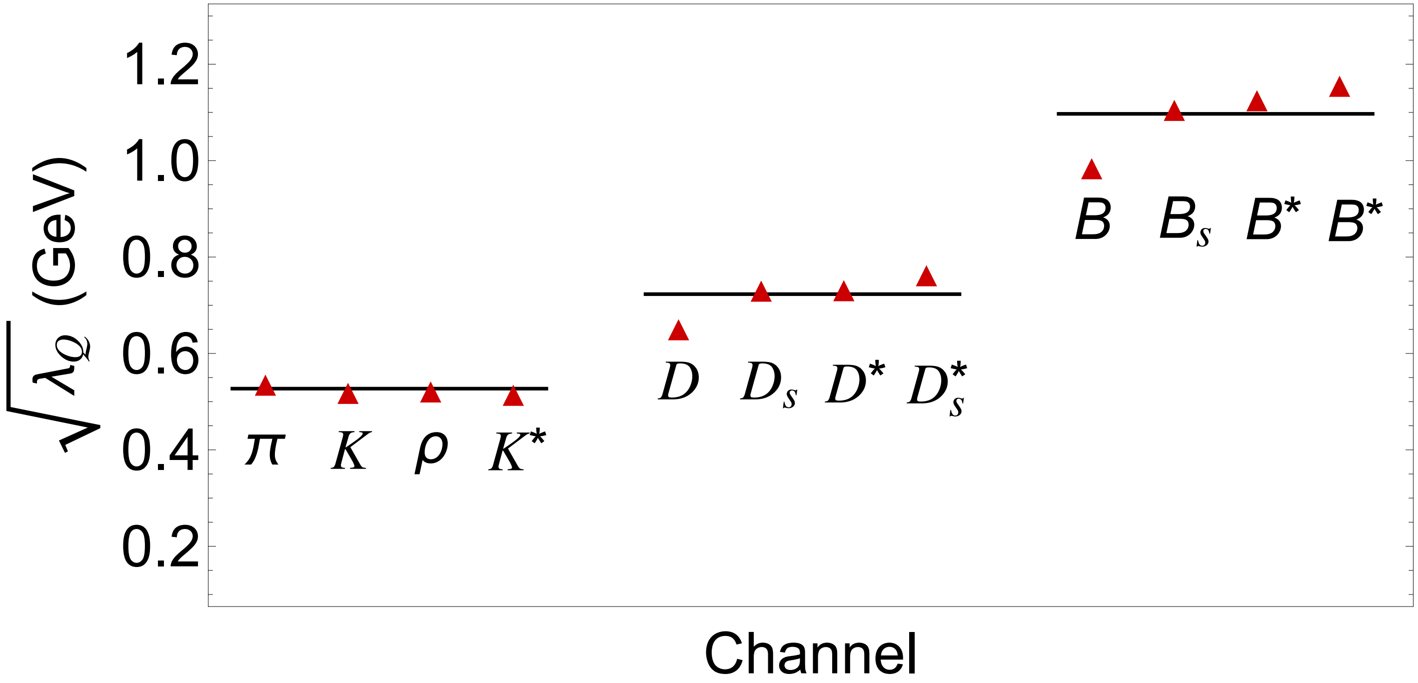
- For strongly broken conformal invariance the potential is still quadratic, $U \sim \zeta^2$, but since A is arbitrary the strength of the potential is not determined



Heavy-light mesons and baryons with one charm quark: $D = q\bar{c}$, $D_s = s\bar{c}$, $\Lambda_c = udc$, $\Sigma_c = qqc$, $\Xi_c = usc$. In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the quark cluster in the baryons



Heavy-light mesons and baryons with one bottom quark: $B = q\bar{b}$, $B_s = s\bar{b}$, $\Lambda_c = udb$, $\Sigma_b = qqb$, $\Xi_c = usb$. In (a) and (c) $s = 0$ and in (b) and (d) $s = 1$, where s is the total quark spin in the mesons or the spin of the diquark cluster in the baryons



Fitted value of $\sqrt{\lambda_Q}$ for different meson-baryon trajectories indicated by lowest meson state on given trajectory

Scale dependence of λ_Q from heavy quark symmetry

- HQET result for heavy mesons M_M: $\sqrt{M_M}\, f_M \to C$
- LFHQCD result for decay constant

$$f_M = \frac{1}{\sqrt{\int_0^1 dx\, e^{-m_Q^2/\lambda(1-x)}}} \frac{\sqrt{2N_C\lambda}}{\pi} \int_0^1 dx \sqrt{x(1-x)}\, e^{-m_Q^2/2\lambda(1-x)}$$

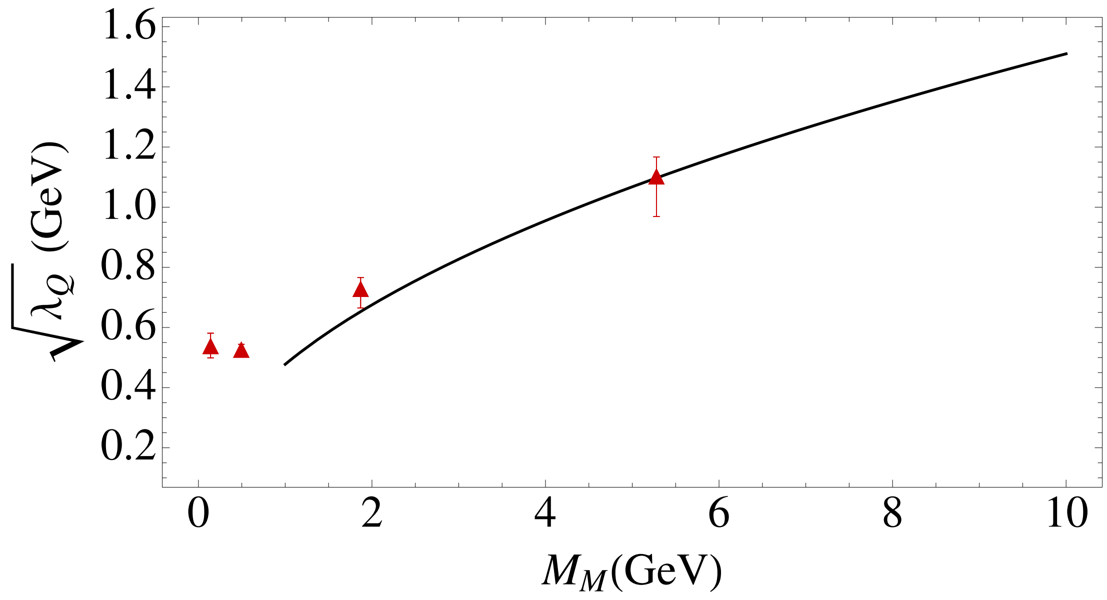
- In the large m_Q limit

$$f_M = \sqrt{\frac{6}{e}} \left(1 + \mathrm{erf}\left(\tfrac{1}{2}\right)\right) \frac{\lambda^{3/2}}{m_Q^2}$$

- From the HQET relation

$$\lambda_Q = \mathrm{C}\, m_Q, \; \dim(C) = [\mathrm{Mass}]$$

- Increase of λ_Q with increasing quark mass is dynamically necessary



4 Infrared behavior of the strong coupling in light-front holographic QCD

[S. J. Brodsky, GdT and A. Deur, PRD **81** (2010) 096010]

[A. Deur, S. J. Brodsky and GdT, PLB **750**, 528 (2015); PLB **757**, 275 (2016), arXiv:1608.04933 [hep-ph]]

- Effective coupling $\alpha_{g_1} = g_1^2/4\pi$ defined from an observable: g_1 scheme from Bjorken sum rule

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx\, g_1^{p-n}(x, Q^2)$$

- Infrared behavior of strong coupling in LFHQCD from Fourier transform of the LF transverse coupling:

$$\alpha_{g_1}^{LFHQCD}(Q^2) = \pi \exp\left(-Q^2/4\lambda\right)$$

- Large Q-dependence of α_s is computed from the pQCD β series:

$$Q^2 \frac{d\alpha_s}{dQ^2} = \beta(Q^2) = -(\beta_0 \alpha_s^2 + \beta_1 \alpha_s^3 + \beta_2 \alpha_s^4 + \cdots)$$

  where coefficients β_i are known up to β_4 in $\overline{MS}$ scheme (five-loops):

- $\alpha_{g_1}^{pQCD}(Q^2)$ expressed as a perturbative expansion in $\alpha_{\overline{MS}}(Q)$:

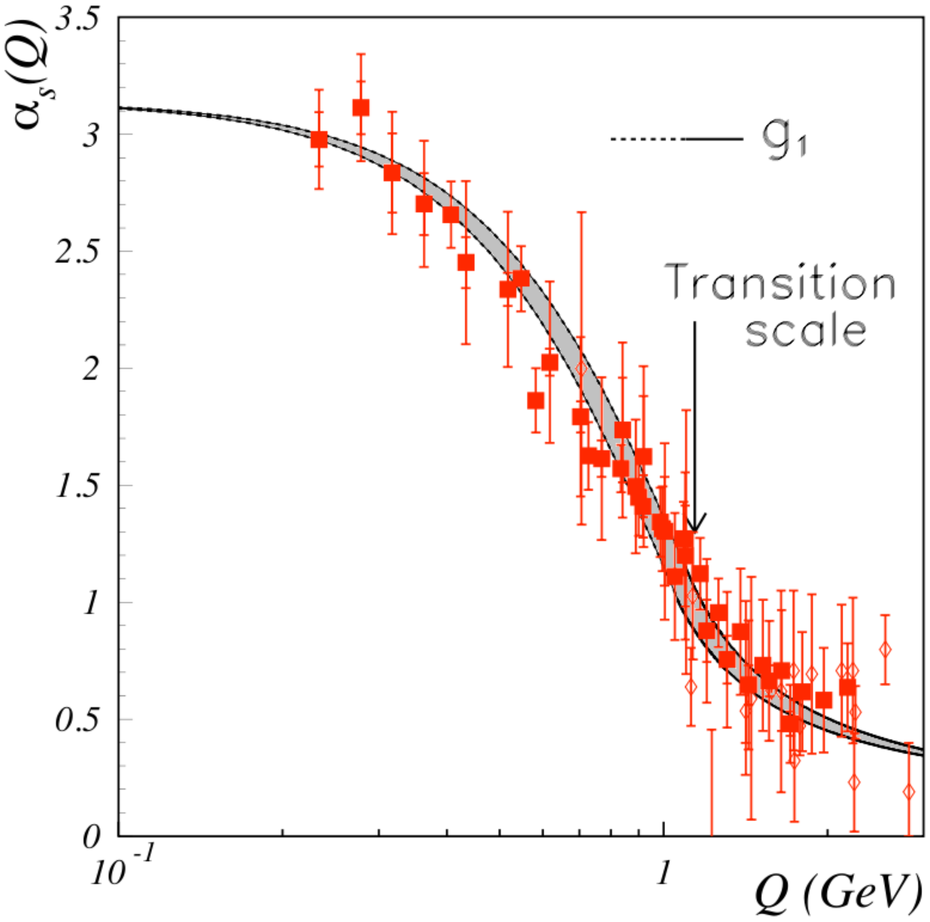
$$\alpha_{g_1}^{pQCD}(Q^2) = \pi\left[\alpha_{\overline{MS}}/\pi + a_1 \left(\alpha_{\overline{MS}}/\pi\right)^2 + a_2 \left(\alpha_{\overline{MS}}/\pi\right)^3 + \cdots\right]$$

  The coefficients a_i are known up to order a_4

- Λ_{QCD} and transition scale Q_0 from matching perturbative and nonperturbative regimes: for $\sqrt{\lambda} = 0.523 \pm 0.024$ GeV

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$$

(World Average: $\Lambda_{\overline{MS}} = 0.332 \pm 0.019$ GeV)
Transition scale: $Q_0^2 \simeq 1$ GeV^2



- Connection between the proton mass $M_p^2 = 4\lambda$ and the fundamental QCD mass scale Λ_{QCD} in any renormalization scheme !

- Nonperturbative β-function from LFHQCD (infrared fixed point $\beta\left(Q^2 \to 0\right) = 0$)

$$\beta(\alpha_s) = Q^2 \frac{d\alpha_s}{dQ^2} = -\pi \frac{Q^2}{4\lambda} e^{-Q^2/4\lambda}$$

- Similar behavior of the IR strong coupling from DSE:

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodriguez-Quintero, arXiv:1612.04835

5 Constraints from the QCD trace anomaly

(In preparation)

- Why the pion is exactly massless in the chiral limit but the proton massive?
- Not a simple problem in QCD where the pion is a bound state of a quark and anti-quark: It requires an exact cancellation of the kinetic and potential energy
- In LFHQCD masslessness of the pion is guaranteed from the strong constraints imposed by super-conformal symmetry
- What is the connection between the universal LFHQCD scale λ and QCD dynamical variables?
- Constraints imposed by the QCD trace anomaly?

$$\Theta^\mu_{\ \mu} = \frac{\beta(\alpha_s)}{2\alpha_s} G^a_{\rho\sigma} G^{a\,\rho\sigma} + (1 + \gamma_m)\, m \overline{\psi} \psi$$

with

$$\beta(\alpha_s) = \mu^2 \frac{d\alpha_s}{d\mu^2}, \quad \gamma_m = \mu \frac{d}{d\mu} \log m$$

- The matrix element of $\Theta^{\mu\nu}$ is

$$\langle P|\Theta^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

- In the chiral limit

$$M^2 = -\frac{1}{4}\frac{\beta(\alpha_s(\mu^2))}{\alpha_s(\mu^2)}\,\langle P|G^2|P\rangle_{\mu^2}$$

evaluated at the renormalization scale $\mu^2 = M^2$

- The pion is a special case since we evaluate the β-function at the scale $\mu^2 = 0$ where it vanishes: The quantum contribution from the anomaly is zero and the pion is massless in the chiral limit

- At the proton scale $-P^2 = \mu^2 = M^2$

$$\lambda = \frac{1}{16}\langle P|G^2|P\rangle_{\mu^2=M_p^2}$$

with $\alpha_s(\mu^2) = \alpha_s(0)e^{-\mu^2/4\lambda}$

- Fundamental connection between LFHQCD scale λ and the QCD matrix element of G^2 in the proton

6 Nucleon form factors in light-front holographic QCD

[R. S. Sufian, G. F. de Teramond, S. J. Brodsky, A. Deur and H. G. Dosch, PRD **91** (2016) 096010]

- LFHQCD leads to analytic expressions for the hadron FFs which incorporates power-law scaling for a given twist τ from hard scattering and vector dominance at low energy

$$F_\tau(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M^2_{\rho_{n=0}}}\right)\left(1 + \frac{Q^2}{M^2_{\rho_{n=1}}}\right)\cdots\left(1 + \frac{Q^2}{M^2_{\rho_{n=\tau-2}}}\right)}$$

  expressed as a product of $\tau - 1$ poles along the vector meson Regge radial trajectory

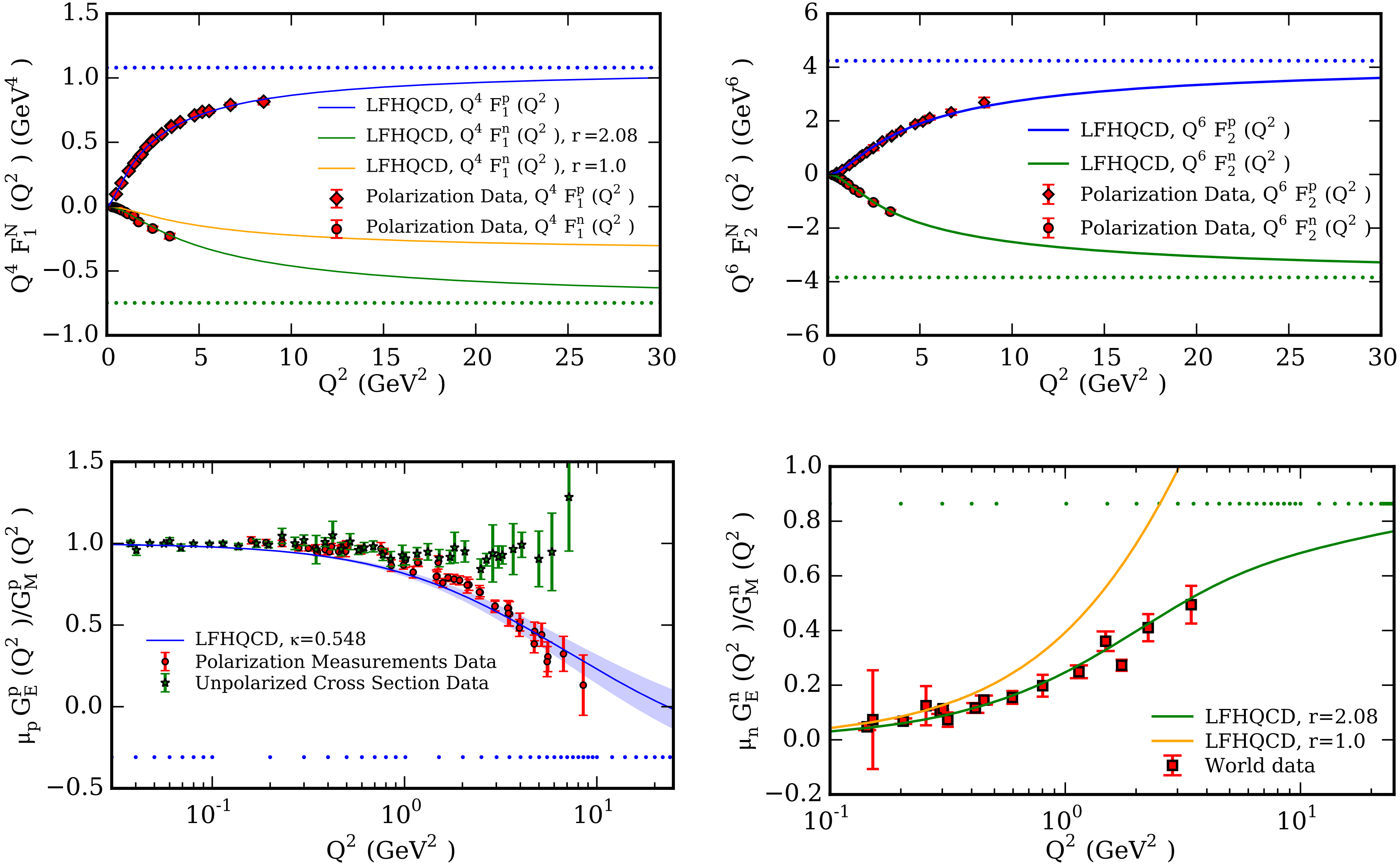
- FF contains a cluster decomposition: hadronic FF factorizes into the $\tau = N - 1$ product of twist-two monopole FFs evaluated at different scales

$$F_i(Q^2) = F_{i=2}\left(Q^2\right)\, F_{i=2}\left(\tfrac{1}{3}Q^2\right)\, \cdots\, F_{i=2}\left(\tfrac{1}{2i-1}Q^2\right)$$

- In the case of a nucleon, for example, the Dirac FF for the twist-3 valence state

$$F_1(Q^2) = F_{i=2}\left(Q^2\right)\, F_{i=2}\left(\tfrac{1}{3}Q^2\right)$$

  is the product of a point-like quark and a diquark-cluster FF consistent with leading-twist scaling, $Q^4 F_1(Q^2) \sim$ const



Comparison of the holographic results with selected world and data and asymptotic predictions

- Free parameters: two parameters for the probabilities of higher Fock states for the Pauli FF and a parameter r for possible SU(6) spin-flavor symmetry breaking effects in the neutron Dirac FF

7 Structure functions in light-front holographic QCD

- Recent progress in computation of nonperturbative structure functions: DAs, GPDs, TMDs . . .
- Require QCD evolution of structure functions from hadronic scale given by LFHQCD to higher scales

T. Gutsche, V. E. Lyubovitskij and I. Schmidt, arXiv:1610.03526

C. Mondal, arXiv:1609.07759

M. C. Traini, arXiv:1608.08410

M. Traini, S. Scopetta, M. Rinaldi and V. Vento, arXiv:1609.07242

T. Maji and D. Chakrabarti, arXiv:1702.04557

M. Rinaldi, arXiv:1703.00348

A. Bacchetta, S. Cotogno and B. Pasquini, arXiv:1703.07669

. . .

Thanks !

Review of LFHQCD, see S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)