dimension of the vectorbundle V_g

$$\dim V_g = \sum_{n=0}^{N-1} |S_{n0}|^{-2(g-1)}. \tag{3.15}$$

It is an amusing exercise to check that the r.h.s. gives integer dimensions for the RCFT's for which the unitary matrix S is known. For example, for the Ising model one recovers the familiar result: $\dim V_g^{\text{Ising}} = 2^{g-1}(2^g + 1) =$ the number of even spin structures on a genus g Riemann surface [2].

4. Some examples

In this section we will illustrate the presented ideas with some concrete examples and in particular we will check our conjecture (3.13). We explicitly work out the $c = 1$ gaussian model, the SU(2) WZW models and the unitary series and we briefly mention some other examples.

The simplest class of RCFT's are the rational gaussian models. They can be described by a free scalar field φ which is compactified on a circle with a rational value for the (radius)^2. They have as symmetry algebra the U(1) current algebra extended with some chiral vertex-operator with conformal spin $\frac{1}{2}N$ and momentum (= U(1) charge) $\sqrt{N}$ (N is an even integer). There are N primary fields $[\phi_p]$ being the vertex-operators with momentum $p/\sqrt{N}$ with $p \in \mathrm{Z}_N$. The fusion rules follow directly from momentum-conservation:

$$\phi_p \times \phi_{p'} = \phi_{p+p'}, \qquad p, p' \in \mathrm{Z}_N. \tag{4.1}$$

For these models the operators $\phi_p(\boldsymbol{c})$ can be defined in a more explicit way using the operator formulation on Riemann surfaces. They can be expressed in terms of the loop-momentum operators introduced in [6]; this is discussed in [7]. Their action on the characters can be calculated by inserting the operator

$$\phi_p(\boldsymbol{c}) = \exp\left(\frac{p}{\sqrt{N}} \oint_{\boldsymbol{c}} \partial\varphi\right) \tag{4.2}$$

into the trace (3.1). (Note that this is not the zero mode of a vertex operator.) This operator measures the momentum flux through the cycle $\boldsymbol{c}$ but at the same time increases the flux through the cycles intersecting $\boldsymbol{c}$. This is reflected in the relation:

$$\phi_p(\boldsymbol{a})\phi_{p'}(\boldsymbol{b}) = \mathrm{e}^{2\pi i p p'/N}\phi_{p'}(\boldsymbol{b})\phi_p(\boldsymbol{a}). \tag{4.3}$$

The operators $\phi_p(\boldsymbol{a})$ and $\phi_p(\boldsymbol{b})$ act on the characters χ_p precisely in the way we