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REMEMBERING PROFESSOR MARC KRASNER

It is thirty years since *Marc Krasner*, Professor and Professor emeritus of the University *"Pierre et Marie Curie"* from Paris passed away.

Marc Krasner, a Russian–born, eminent French mathematician of Jewish origin began the life in Odessa on April 9, 1912, and died on May 13, 1985, in Paris.

Marc Krasner received in 1938 his PhD, from *University of Paris – Sorbonne* under *Jacques Hadamard*, with thesis *"Sur la théorie de la ramification des idéaux de corps non–galoisiens des nombres algébriques."* From 1937 to 1960 he was a scientist at CNRS (French National Center for Scientific Research) and from 1960 professor at the *University of Clermont–Ferrand*. In 1965 Krasner was appointed a full professor at the *University "Pierre et Marie Curie"– Paris VI*, from which he retired as professor emeritus, in 1980.

1. THE CHILDHOOD IN ODESSA

Marc Krasner was born to a wealthy Jewish family. His father Ishaac, was an economist and a trade expert, and his mother Sarah came from a well–established bourgeois family. Krasner was only five and a half years old when the October Revolution began, and eight, when the Whites left Odessa.

From childhood Krasner was interested in sciences. He would sneak into the family library and as a seven–year old boy read Pushkin, browsed through history and popular science books and was particularly fond of leafing through the great German Encyclopedia, while envisaging himself as a future scientist.

After completing a seven–year long primary school in Odessa, Marc Krasner continued his education in the two–year professional school of electrical engineering. Although, generally speaking, the circumstances in education were not very favourable at that time, particularly not in 1923, Krasner recalls that in the fifth grade he already felt a powerful call of mathematics and knew that mathematics was all he wanted to do in his life. In his memories from his school days, he fondly remembered his outstanding teacher of mathematics *Adrian Feofanovich Tchivtchinski*, who taught him mathematics in the last three grades of school.

When he was only 11, Krasner got a certain information about *Non–Euclidean Geometry* and, even tried to prove the *Fifth Euclidean postulate.*

Krasner, who had already became very attracted to numbers, decided once to ask his teacher Adrian Feofanovich an interesting question: *"Why, for numeric base, we couldn't take some fraction, say 1/5 ?"* The teacher then laughingly asked him: *"How to express an integer (of course, a real number), in such a system ?"* to which question he didn't know the answer. But this question wasn't meaningless - it contained a little germ of p-adic numbers, specifically the 5-adic ones.

Driven by his love for mathematics and encouraged by Tchivtchinski, his maths teacher, whom he always thought to be his only true teacher and whom he continued to visit after the completion of the seven–year primary school, Krasner continued to read avidly mathematics books. He acquired enviable amount of mathematical knowledge when he was only 14. *We can say, without exaggeration, that he had been a true child prodigy.*

After the Revolution, his parents were impoverished and decided to leave the Soviet Union in 1928. They were lucky to seize the last minute opportunity to leave the country without the departure visas and headed for Paris where they were able to join numerous members of his mother's family already living there.

While on his way to France, and upon the recommendation of his Russian teacher, he took with him two books by *Dimitry Aleksandrovich Gravé's Course of Higher Algebra* and *Egorov's Number Theory* (which he had already read in a scientific library in Odessa), as well as some other books. The first book introduced him to the classical *Galois theory*, which was much later the inspiration for his discovery of *Abstract Galois theory*, while the second book tied him permanently to the number theory.

2. Life and work in France

Having settled in Paris, his family went through financial difficulties and supported themselves in various ways, mainly by selling goods at fairs and shows. His relatives from the mother's side were deeply disappointed when they learnt that he did not want to be neither an engineer, nor a doctor, nor a lawyer, in fact, he only wanted to be a mathematician.

Because Krasner did not have any knowledge of the French language, he was not able to start his studies at the university before 1930. There he demonstrated varying success, because lectures that were offered did not coincide with his already developed interests. Besides, Krasner pointed out that none of his university professors, except maybe Picard, had made a deeper impact on him. He spent time mostly reading books he had brought from Odessa or borrowed first from the Russian library, and then from the scientific and university library in Paris. He also visited Berlin, where, at the University of Humboldt, he attended Issaï Schur's lectures on Algebra.

He could speak some German, since he had briefly studied it at school, as opposed to all the other members of his family whose German was very fluent (owing to the fact that between 1905 and 1910 they moved several times between Vienna and Berlin).

After his arrival in France, Krasner, as a sixteen–year–old boy, continued to be occupied by the famous *Fermat's Last Problem* which he was first informed about in Odessa by his teacher Adrian Feofanovich. Krasner obtained significant results regarding this problem.

Here is what some mathematicians wrote about his first result [13]. Louis J. Mordell wrote to him: *"Your result is very striking"*, and later Paulo Ribenboim in his book wrote: *"a very powerful result ... pointing to the fact that the first case of Fermat's theorem may very well be true"* [20].

Ribenboim's paper *"Krasner versus Fermat"* [21] was also dedicated to Krasner's works on *Fermat's Problem* comparing them with a conquered fortresses that are charging at old warriors who, already tired of their efforts, give up and do not come to the goal. He wrote: *"How much nicer, however, is the young, enthusiastic, fearless, romantic fighter! Coming from Russia to Paris, confident as a talented young man can be, would be, like Rastignac, conquer? ... Not knowing much of what was tried before, Krasner followed a new and original path. His note of 1934 is very ingenious and contains a definite contribution to the first case"* [21].

Encouraged by his first result, Krasner published in subsequent years various other articles on Fermat's Problem. Working on Fermat's Problem, he succeeded, not knowing of their existence, to rediscover the so–called *Abelian equality* and, finally, during 1929, the so–called criterion *Sophie Germain in Wendt's form.*

Those first works by Marc Krasner testified that he was undoubtedly a great mathematical talent. Also, it is those early works by Krasner that attracted the attention of the two important and influential mathematicians of the time: Jacques Hadamard and Helmut Hasse. Krasner will later establish a close working relationship with the former, while the latter, being intrigued and delighted with the results Krasner obtained concerning *Hilbert–Hasse's theory*, decided passing through Paris to meet him in person. After the Second World War, Krasner who was full of bitterness ceased completely his communications with Hasse, because he was a supporter of the German Nazi regime.

Describing Krasner's way of thinking, i.e. his *"mathematical world"* is quite difficult. After *Fermat's Last Theorem*, his intensive and permanent

occupation from 1934 to 1939 were p-adic numbers and p-adic fields, which were the topic of his doctoral thesis published under the title:
- *Sur la théorie de ramification des idéaux de corps non–galoisienne des nombres algébriques*, Memoires de l'Académie Royale de Belgiques, 11 (1938), 1–110.

The results of his Doctoral thesis were already obtained in May 1935. The first draft was written during the summer of the same year, while the final version was brought to light after *Chevalley's remarks* in December 1936, although Krasner did not follow Chevalley's remarks but rather his own new inspiration. The two papers appearing under the same title:
- *La loi de Jordan–Hölder dans les hypergroupes et les suites génératrices des corps de nombres p-adique*, Duke Math. J., 6 (1940), 120–140 and 7 (1940), 121–135),
respectively, were in line with the results of his Thesis.

In both, his Thesis and those two papers, Krasner extended the *Hilbert–Hasse theory of ramification of Galois extensions of p-adic fields to the case of the non–normal finite extensions of p-adic fields.* In order to conveniently formulate his results, Krasner made use of the concept of a hypergroup, formerly introduced by Fédéric Marty. The proof of this lies in *Jordan–Hölder's Theorem* thereby replacing invariance with semi–invariance.

Krasner and Marty met at the Hadamard's Seminar to which Krasner was invited by Hadamard himself after Kranser published several papers focusing on *Fermat's Last Theorem.* Krasner first attended the Seminar in October 1934 and it was sheer coincidence that the co–authors of paper were Krasner and Marty, although this fact is not so obvious and requires clarification. Namely, in the paper in question entitled:
- *Sur une propriété des polynômes de la division de cercle*, C.R. Akad. Sci., 204 (1937), 397–399,
its co–authors were cited as *Marc Kasner and Melle Britt Ranulac.*

The story behind the mentioned co–authorship goes like this. When Paul Levy first used the Seminar to raise the problem of the decomposability of the expression $(x^n-1)(x-1)^{-1}$, Krasner offered the solution to the problem at the subsequent meeting of the Seminar, remarking that he was not very skilled at writing short papers. Marty, then suggested that he, together with Robert Ballieu and Robert Gillis, the two young Belgian mathematicians, act as the co–writers of the note. They chose the pseudonym *Melle Britt Ranulac*, to appear, together with Krasner's name, as the co–authors of the note. Their suggestion was intended to parallel the *"existing author"* who published works under the name of *"Nicolas Bourbaki"* which actually was the pseudonym for a group of mathematicians. In the French Academy of that time such things were not a common thing expected from serious authors, and Emil Picard, while listening Elie Cartan's presentation of the works by *"Nicolas Bourbaki"*, insisted on meeting Nicolas Bourbaki in person.

Krasner's most important paper, before the World War II, also referred to *p-adic numbers* and was entitled:
- *Sur la primitivité des corps p-adique*, Mathematica, Cluj, 13 (1937), 72–191.
In this paper of vast proportion, his attention is focused on primitive extensions of p-adic fields and his research of the conditions for two *Eisenstein equations* to define the same extension. The need to exactly understand the situation led to the study of *an ultramteric on spaces* of polynomials and the discovery of what Krasner called his *"principe fundamental"*, now known as *Krasner's lemma.* In the same paper he gave the arithmetical characterization of primitive extensions of meta–galoisian extensions. This paper contains, in germ, of the calculations which eventually led to Krasner's most important result *explicit formulas for the number of extensions of p-adic field, with given degree and different*, now known as *Krasner's Mass Formula.*

From the list of Krasner's works, before the Second World War, we will mention the work:
- *Sur la présentation exponentionelle dans les corps relativement galoisien de nombres p-adiques*, Acta Arith., 1 (1939), 133–173.
The results presented in this paper were obtained in 1934. In this paper he determined the structure $Z_p(G)$-module E in the case when K/k is "tamely" ramified where G is the Galois's group of extension K of p-adic field k by adjunction p-th roots of 1, and E is the group of units ε of K, such that $\varepsilon \equiv 1\ (modP)$, where P is the maximal ideal of the valuation ring of K. The structure of this module is closely related with *Kummer logarithmic derivatives* and makes it possible to understand the meaning of *Kummer's congruences.* Krasner was led to this by his interest on Fermat's Last Theorem. In this paper there is already a description of the structure of the Galois group of a tamely ramified extension (rediscovered by Jacobson in 1944).

3. The second world war

When Second World War began, life in France was not so terrifying, since the French were not anti–Semitic. But, after the fall of the Petin's Government, the pogroms started and deportations became the destiny for all those who did not change their Jewish names. Krasner's parents were among those who refused to hide their Jewish origin the consequence of which was their deportation to a death camp, in February 1943. The only survivors of the War in his family were Marc Krasner himself and his nine years older sister Frederica.

Marc Krasner was recruited to the French army, where he served, between 1939 and 1940, because, at the time, the French law of obligatory military service applied to both the French and the individuals without the French citizenship but who spent at least 20 years in France. Since this law was first applied in 1936, he never served in the army before. In September 1940 he was freed of military service, but his position was still very dangerous: he was without the citizenship in addition to being a Russian Jew. He stayed in Marseille until November 1942, since the southern zone was occupied by the Germans and afterwards went to Clermont and Grenoble, then occupied by the Italian troops. When these places became too unsafe for him, he moved to the region of Nancy, which had been previously thoroughly "cleansed" and where he could be less conspicuous. In Marseille and Grenoble, Krasner had access to the library and could continue his work. Quite astonishingly, he remained affiliated with the CNRS, throughout all these years.

When the demobilization time came, all the soldiers and officers of his cavalry division, known as the *"Iron Division"*, were awarded the Military Cross. This helped him obtain the identity card which, in turn, helped him retain his status of the scientist at CNRS, which he himself considered a miracle.

4. Some of Krasner's papers after war

The results and ideas of the previous paper *Sur la primitivité des corps p-adique*, as well as the perspective of their generalization, Krasner presented in his paper:

- *Quelques méthodes nouvelles dans la théorie des corps valués complets*, Colloq. Int. CNRS, Paris, (1949), 29–39.

The abridged version of this paper entitled:

- *Généralisation non–abeliennes de la théorie locale des corps des classes*, Proceedings, Vol. II, (1950), 71–76, was the actual text of his half an hour conference lecture at the International Congress of Mathematicians in Cambridge, Mass., 1950.

Krasner's the first more significant paper after the War was published under the title:

- *Certaines propriétés des Series de Taylor d'un ensemble au plus dénombrable de variables dans les corps valués completes et une démonstration structurel de formule de M. Pollaczek*, Bull. Sci. Math., 71 (1947), 123-152 and 180-200.

4.1. **Krasner's Seminar 1953–54 at "Faculté des Sciences", Paris**

The following papers ensued and their titles indicate their contents:

- *Espaces ultramétriques et ultramétroïdes*, 52 pp.
Here is the first detailed paper on the properties of such spaces. Ultrametric spaces had been defined in the note *"Nombre semi–réel et espace ultramétric"* of 1944. Krasner's ultrametroid spaces are now called hyper–ultrametric spaces. This new terminology tends to give the impression that we are in the realm of super–generalizations. Yet, these hyper-ultrametric spaces are the analogue of ultrametric spaces, when the distance takes its values in a totally ordered set having a smallest element 0. These spaces are described as projective limits of discrete spaces (see [22]).
- *Groupes et anneaux valués*, 25 pp.
This is the first publication with the question of the notions of valued and hypervalued groups is considered. Starting from ultrametric spaces Krasner has introduced them in view of a gradual construction of the notion of a valued field. This was first done in his lecture presented at the *Séminaire Châtelet* in 1948, and then in his paper *"Intraduction à la théorie des valuations"*. This paper contains also ultrametroïd rings, valued rings and *"anneaux titrés"*.
- *Corps valués et extensions valués*, 46 pp.
This is an expository paper on valued fields, where Krasner puts the accent on mentioned terminology.
- *Théorie élémentaire des corpoïdes sans torsion, II* exp. 5, 131 pp.
The theory of extensions of corpoïds is similar to the theory of extensions of fields. In some ways the present article resembles Steinitz's book on fields. However the Galois theory of corpoïds had to wait until 1978 to be put into writing by Krasner, for the extensions of finite degree. The original ideas relating to this paper date from the early forties.
- *Prolongement des valuations: Introduction, Idée des differentes méthodes et leur comparaison*, 6 pp.
This paper contains one *"vue d'ensemble"* of the theory of extensions of valuations, in particular the role played by henselian fields, which are characterized as those for which the Newton polygon of every irreducible polynomial is a segment.

4.2. **Some Krasner's papers published in the proceedings of various colloquia**

- *Aproximation des corps valués complets de caractéristique* $p \neq 0$ *pour ceux de caractéristique* 0, Colloque d'Algèbre Supérieur, Bruxelles, (1956), 129–206.
"This paper is concentrated around one idea, which is ultimately a consequence of the so–called Krasner's lemma, or fundamental principle. This novel point of view consists if we like to explain it in a few words – in approximating structures, endowed with an ultrametric, by other structures

which, though not isometric, have invariants ultrametrically close or equal to the ones of the given structure" [22].
- *Nombre des éxtensions d'une degré donné d'un corps p-adique*, Les Tendances Géometriques en Algèbre et Thórie des Nombres, Colloque C.N.R.S., Clermont–Ferrand, 1964, Ed. CNRS, Paris, (1966), 143–169.
- *Anneaux gradués géneraux*, Colloque d'Algèbre Rennes, (1980), 209–308.

4.3. **Some Krasner's papers appeared in lecture notes or preprints**

- *Introduction à la théorie des valuations*, Facultés des Sciences, Université "Pierre et Marie Curie", Paris, 1965–66.
- *Théorie de Galois des corpoïdes commutatifs sans torsion et ses applications à la théorie de la ramifications des extensions algébriques des corps valués*, Preprint, Université "Pierre et Marie Curie", Paris, 1978, 63 pp.
- *Local differents of algébraic finite extensions of valued fields*, Preprint, Université "Pierre et Marie Curie", Paris, 1981, 64 pp.
- *Differents local des extensions algébriques finies des corps valués*, Preprint Université "Pierre et Marie Curie", Paris, 1981, 11 pp.

4.4. **Some papers published in Comtes Rendus de l'Académies des Sciences de Paris**

From 1935 Krasner published about 80 short papers–notes in C.R. Acad. Sci., which contain only an announcement and mostly the sketches of the proofs. Now, we will mention just a few of such papers.
- *Une généralisation de la notion de corps–corpoïde. Un corpoïde remarquable de la théorie des corps valués*, C.R. Acad. Sci. Paris, 219 (1944). 345–347.
"The concept of corpoïde was found in 1941, but it was impossible to publish during the occupation of France, since Krasner was hiding" [22].
- *Nombres semi–réels et espaces ultramétriques*, C.R. Acad. Sci. Paris, 219 (1944). 433–435.
"These notions date back to 1939. The concept of a semi-real number substitutes advantageously the symbols e, u used by Krull in his description of ideals of valuation rings of rank 1. *Dj. Kurepa, in his thesis* [15] (1935) *has indicated the analogue of semi-real numbers for an arbitrary totally ordered set. The notion of an ultrametric space appears here for the first time in this generality"* [22].

In addition, Krasner published with L. Kaloujnine three papers on the wreath product group:
- *Produit complet des groupes de permutations et les problèmes d'extension de groupes I, II, and III*, Acta Sci. Math. Szeged, 13 (1950), 208–230; 14 (1951), 39–66 and 69–82, respectively.

Krasner thought that these papers were the most fundamental in the theory of the wreath product. The wreath product of groups AB is one of the basic constructions in group theory.

The first paper, conceived at the end of 1948 and beginning of 1949, deals with the definitions and pertinent comments. The second paper contains the important theorems of immersion and equivalence. The last paper is devoted to an interpretation of Schreier's systems of factors in the framework of the abstract *wreath product*: this "internal" interpretation is the "dual" of the usual external one.

4.5. **Graded structures in Krasner's sense and their history**

Krasner together with his pupils has worked in domain of graded structures too, and they made a significant contribution to the theory of graded structures including the history of this theory. Now, we will mention the papers from this domain:

- *Le vieux qui est noef*, Rev. Roum. Math. Pures Appl., T. XXVII (1982), 443–472.
- *Anneaux gradués généraux*, Colloque d'algèbre, Rennes, Monograph (1980), 209–308.
- (with M. Vuković), *Structures paragraduées (groupes, anneaux, modules)*, Monograph, Queen's Papers in Pure and Applied Mathematics, Queen's University, No. 77, 1987 Queen's University, Kingston, ON, Canada, 163 pp. and three papers
- (with M. Vuković), *Structures paragraduées (groupes, anneaux, modules) I, II, III*, Proc. Japan Acad. Ser. A, Math. Sci., 62 (1986), No.9, 350–352, and 389–391, and 63 (1987), No.1, 10–12, respectively.

We will start with his historical paper *"Le vieux qui est noef"*. Graduations are old as they are new, claims Krasner, in this work. Really, the notion of graduation is, at least in the light of certain examples, quite an old one. It was introduced explicitly into mathematics during the 18th century when Euler defined the notion of polynomials and real homogeneous functions. But this notion, when applied to the case of a single variable, was already familiar to mathematicians well before Euler (*Diophantes*, some Arab mathematicians, *Viète, Descartes*). The germ of the notion of homogeneity can be seen in early Greek mathematics (the *"multiplication of segments"*) and in some documents from *Babylonian times*.

Other notions of homogeneity and its corresponding grades were introduced after Euler in various algebraic structures. Some of these structures are well known: *weights* of polynomials and functions, *dimension* of geometrical and topological objects, *order* of differential operators, etc.

Later, the more general notion of $\mathbb{Z}$–graded ring (or more generally $\mathbb{Z}_2$–graded, $\mathbb{Z}_3$–graded...) was introduced, mainly in algebraic geometry and topology (*Samuel and Zarisky*).

The first relatively, but not enough, general definition of graded groups limited to the Abelian graded group, was given by Bourbaki. Krasner, leaving aside the unnecessary restrictive hypothesis of commutativity, brings us to *Bourbaki–Krasner's* definition.

In his monograph *"Anneaux gradués généraux"*, Krasner shows that the graded group is characterised by both the underlying abstract group and the homogeneous subset (or even only by the homogeneous subset) and introduces adequate notions of graded rings and modules together with their homogeneous subsets called *"anneïdes"* and *"moduloïdes"*, as well as *"groupoïds"* in the case of groups. The notion of graded homogeneous field, called *"corpoïd"* was also introduced by Marc Krasner, but much earlier (in 1940s).

Marc Krasner together with his pupils developed a homogeneous theory of commutative graded rings from the Noetherian point of view with *Marcel Chadayras* (see [2]), the homogeneous theory of generally non–commutative regular rings, from the Artinian point of view with *Emanuel Halberstadt* (see [11]), and jointly with the author of this text introduced *paragraded structures (groups, rings, modules)*, [14] developing thus the theory which generalizes the corresponding *Bourbaki–Krasner's graded structures* and has in each of the three cases: groups, rings, and modules the property of closure with respect to the direct product and the direct sum in the sense that the support of the homogeneous part of this product is a Cartesian restricted product.

The characterisation axioms of paragraded groups give way to three study methods of these groups which are in principle equivalent: *non–homogeneous, semi–homogeneous, and homogeneous.*

4.6. **Hypergroups, hyperrings and hyperfields**

The hyperstructures are the structures generalizing corresponding algebraic structures. The concept of hyperstructure - algebraic structure H in which addition is not operation, but hyperoperation, i.e. the sum $x + y$ of two elements x, y of H is, in general, not an element, but a subset of H, goes back to Frederic Marty who first introduced hypergroups in 1934 [17]. It is unknown, but almost at the same time, Marc Krasner, independently of Marty, came up with the concept of a hypergroup in the first draft of his Thesis (1935 handed to Chevalley for criticisms [22]). However, he admitted that Marty's concept appeared prior to his and that he also accepted Marty's terminology. Besides Krasner, hypergroups have been studied by numerous authors: S.D. Comer, M. Dresher and O. Ore [6], J.E. Eaton [7], Y. Utumi

[23]. Among others Marc Krasner used hypergroups in his theory of non–normal ramification, W. Prenowitz and J. Jantosciak, in geometry ([19], [12]), P. Corsini for linearly ordered abelian groups [5]. Later, in the second half of the twentieth century, Marc Krasner, in conception with his work on valuation and approximating non–Archimedean fields, introduced the theory of hyperrings (1956) and hyperfields (1982), in his papers:
- *Approximation des corps valués complets de caractéristique* $p \neq 0$ *par ceux de caracteristique* 0, Colloque d'Algèbre Supérieure (Bruxelles, décembre, 1956), CBRM, Bruxelles, Libraire Gauthier Villard, Paris 1957, 129–206, and
- *A class of hyperrings and hyperfields*, Internat. J. Math., 6; Math. Sci., 6 No. 2, (1983), 307–312.

At the beginning the concept of hyperstructures was not well understood nor accepted, even largely criticised and ignored until famous French mathematicians *A. Connes* and *C. Consani* *"started advocating their potential utility in connection with* F_1*-geometry and the Riemann hypothesis"* [1]. Besides, Connes - Consani wrote: *"We showed that the theory of hyperrings, due to M. Krasner, supplies a perfect framework for understanding the algebraic structure of the adèle class space* H_K *of a global field* K*"* [3].

Now, *"a well-known type of a hyperring, called Krasner hyperring"*. Besides, *"Krasner hyperring is essentially hyperring. ... During numerous discussions it was noted that all known hyperrings were obtained via a construction that Krasner had introduced-known as Krasner's construction"* [18].

So, very disputed and negated hyperstructures, with time, have grown as a new separate branch of mathematics having many applications in other areas such as theory of automata and languages, principle physics and artificial intelligence, and possess rich avenues for further research.

We will finish this part with the following: M. Krasner, M. Marshall, A. Connes, C. Consani, M. Baker, and the author came to hyperfields for different reasons, motivated by different mathematical problems, but all of them came to the same conclusion: *"the hyperfields are great and very useful and very underdeveloped in the mathematical literature ..."* ([1]).

4.7. **Mathematical logic and Philosophy of mathematics**

Krasner was great not only in various domains of mathematics, but also in Foundations, Mathematical logic and Philosophy of mathematics. Krasner's interest in the Foundations of mathematics and logic did not come to the fore only in Abstract Galois theory, which he built, but also in his so–called definability. *"Generalizing Évariste Galois ideas, which he considers as the essence of logic"*, Krasner has made a significant contribution to the mathematical logic and philosophy, too. We will mention only a few of his papers from these domains:

- *Théorie de la définition*, J. Math. Pures Appl. Ser. 9, Vol. 36 (1957), 325–357, and Vol. 37 (1958) 53–101.
- *Théorie de la Définition*, J. Symb. Log., 24 (3) (1959), 230–231.
- *La pluralité et l'infini dans la philosophie et les Mathématiques grecques*, 1981, Paris, ENS, 1979, No. 4 , 40 p; IREM Paris–Nord, Collection philosophie mathématiques.
- *La définitionisme*, Ann. Sci. Univ. Clermont–Ferrand 2, Math., (Actes du Colloque de Mathématiques reunie à Clermont à l'occasion du tricentenaire de la mort de Blaise Pascal) T.7, No. 1 (1962), 55–81.

The importance of Krasner's contribution to the Mathematical logic and Philosophy is best witnessed by the fact that even today, thirty years after his death, his mathematical–logical–philosophical thoughts and ideas are not forgotten and are included in the discussions on this subject together with ideas of the most famous thinkers of the last century, such as *Bertrand Russell, Werner Heisenberg*, etc.

So, in the summary of the paper *"Mathematical Logic Between the two World Wars"*, [10] stays: *"The culminating point of a first period was under Alessandro Padoa's and Bertrand Russell's mixed influences in France with Jean Nicodi's and his philosophical essays, and during a second period was Jaques Herbrand's mathematical work blossom. Follows a period of debates among philosophers, mathematicians and physicists, consecrated, totally or in part, to the philosophy of science, sketches of a non–classical logic which leads to set Werner Heisenberg's relations up as principles, and the structuring of mathematical beings*," ... and finally, gives a special place to Marc Krasner which is expressed with: *"The notion of mathematical structure gives rise to two contributions: Marc Krasner generalizes Evariste Galois' ideas attributed to logic and extended them to infinitary languages and Nicolas Bourbaki taking in account the evaluation of mathematics, terms a structure what we call today a model."*

5. Life dedicated to mathematics

What, briefly, can we say about Marc Krasner, his results and works? One would not have complete picture of Marc Krasner if we did not consider his human aspect. During the long term work with Professor Krasner I have gotten to know him as a person who was solid, very correct, an example of intellectual rigor, but also with innate sense of humor, who could listen and truly wanted to help, even raise voice against injustice if it was necessary.

Marc Krasner, born under a lucky star, in the family which could give him all, between the books and toys, chose the books, and replaced the stories by Pushkin's poetry, which later, after a fateful encounter with outstanding teacher Adrian Feofanovich Tchivtchinski, were displaced by mathematics.

Krasner, already at the age of 11 tried to prove the *Fifth postulate of Euclidean geometry.*

Undoubtedly Marc Krasner was not only a great and born mathematician, who dedicated his whole life to mathematics, but also a refined intellectual, a man of wide interests and outstanding general culture and education.

Besides mathematics, his great love was chess. He was an active member of the Potemkin Chess Circle of Levaillouis Perret, whose members were mostly Russians. He often arrived tired from Clermont–Ferrand to play chess for his Circle. His chess game against Antonov 1:0 is remembered, as well as a simultaneous game with Alekhine when his Circle had zero.

In particular we will highlight his great love for travels, natural beauty and historical places. One of his great loves was an artistic photography. With his camera on the shoulder, he toured around the world trying to perpetuate the beauty of Machu Picchu in Peru, the Grand Canyon in America, Norway geysers, frozen iceberg in Greenland Sea, as well as a beauty of Greece - the cradle of European civilisation and culture. Krasner was returning from his trips with hundreds of recorded artistic slides which he showed to his friends.

Almost since his arrival in France in 1928, Krasner was preoccupied with the famous Fermat's Last Theorem – one of the most important open questions in number theory, opened more than three centuries, which many great mathematicians have been trying unsuccessfully to solve. Paolo Ribenboim, one of the greatest living mathematicians in the field of algebraic number theory, devoted one nice paper *"Krasner Versus Fermat"* to Krasner's work about Fermat's Theorem. Together with extension of the Hilbert–Hasse theory of ramification of Galois extensions of p-adic fields, it was his first "very powerful result" [21].

However, Krasner was not only a mathematician who posed and solved thorny problems, but also a founder of many beautiful and important mathematical theories. Thus, he laid the foundations for numerous branches of algebraic and analytic number theory, p-adic analysis and ultrametrics, such as: ultrametric topology introduced in 1944 and ultrametric spaces defined axiomatically which basic properties are described in 1944 (see [16]). *"Definitions of analytic elements in D and quasi–connected sets D, given by Krasner, during the fifties, were the base for construction of a theory of holomorphic functions. ... A famous result on the analytic elements is the so–called Mittag–Leffler theorem, proven by Krasner"* (see [8], [9]). The study of analytic elements is proven to be essential tool for the most works in functional analysis: the p-adic *Nevanlinna theory* and its *Corona problem* in the ultrametric disc. In addition, Krasner introduced hyperrings and hyperfields.

However, Krasner gave significant contributions to valuations theory, field extensions theory and was a founder of abstract Galois's theory, as well as the pioneer in mathematical logic and philosophy of science.

Only the greatest mathematicians are privileged by having mathematical notions named after them. Marc Krasner was one of such with more than 30 notions named after him. Among his numerous great results with vast and lasting impacts are: his discovery of *"principe fundamental"*, now known as *Krasner's lemma, Krasner mass formula* - one of his most important results, *Krasner–Mittag–Leffler theorem, Krasner–Tate algebras* – the algebras of holomorphic functions over non–archimedean fields, *Krasner algebra, Bool–Krasner algebra, Krasner–Serre formula, Krasner–Ore equation, Kaloujnine–Krasner embedding theorem, Bourbaki–Krasner graded group, Krasner hyperring and hyperfield, Graded structures in Krasner's sense: groups, rings, modules* together with their homogeneous subsets called *"groupoïdes," "anneïdes"*, and *"moduloïdes"* as well as their analogous *Krasner–Vuković paragraded structures*, etc.

Professor Krasner's activity as a great mathematician and teacher was extraordinary. During the period 1960–1980 he took numerous different graduate and postgraduate courses at the Faculty of Sciences, *University of Clermont* in Clermont–Ferrand, as well as University *"Pierre et Marie Curie"* in Paris. His lectures were very interesting, delivered with great vigour and eloquence. He also wrote many textbooks for use in teaching. He always tried to convey the structural beauty of mathematics to his readers and listeners. As scientist he was also brilliant, distinguished for many fundamental contributions.

It is hard to say how many doctoral students Krasner had. There isn't a complete list, but certainly he advised over 10 doctoral students from various countries of Europe and Africa. Young mathematicians, from around the world, still work their doctoral theses inspired by Krasner's works and theories introduced by him.

One can freely say that Marc Krasner was one of the best French mathematicians in the field of algebraic number theory, of the last century. For his extraordinary contribution to the development of mathematics he was awarded with the prestigious award the *"Doisteau–Blutet"* by the French Academy of Sciences in 1958, when the award was founded. Besides, he was *"Officier des Palmes de l'Académie des Sciences de Paris"*.

According to the list of Zentral Blatt für Mathematik, Professor Krasner published 147 papers (see [24]).

Marc Krasner presented the results of his studies at numerous international conferences and congresses. He was an invited speaker with a half an hour conference lecture at the *International Congress of Mathematicians in Cambridge*, Massachusetts, 1950.

He was also invited to give the lectures on his results at many universities and scientific institutes, for example in Stockholm, Prague, Moscow State University–Lomonosov, Saint Petersburg, Royal Society London, Oxford, Cambridge, Rome, Budapest, Athens, Belgrade, Sarajevo, etc.

We had the honour and pleasure to greet Professor Krasner in our Department of Mathematics in Sarajevo, three times. We enjoyed his very interesting lectures on the theory of valuations, ultrametrics and general abstract Galois theory and were impressed with the richness of his ideas, as well as the easiness with which he laughingly spoke about very serious mathematical matters.

It should be said, that Krasner, as the immortal Galois, whose theory he extended to the abstract Galois' theory, was not always understood, even controverted because of generality of the theories which he introduced and developed or, rather, because he was often a step ahead of his time. The results, which were controverted, during his life time, found a significant place, not only in different areas of mathematics, but also in other various sciences.

While young generations will know Professor Krasner only through his works, all his friends and students who followed his career would remember him for extraordinary lectures and laughter while he was easily solving difficult problems, for brilliant works and theories that he introduced and created and which now flourished, and are no longer just the poetry of mathematics and mathematical logic, but have found their place in various areas of mathematics and other sciences, even applied ones.

I'll finish my remembering of Professor Krasner with a dedication of famous French mathematicians Alain Connes and Caterina Consani at the beginning of their work: *To the memory of Marc Krasner in recognition of his foresightedness"* [4].

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