

Transcript Excerpt: Holographic Confinement

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Abstract

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This document contains an edited transcript of the section of the lecture focusing on gauge/gravity duality, the large N expansion, and holographic confinement via the Wilson loop, spanning from [00:37:49] to [00:53:34]. Minor punctuation and mathematical formatting have been added to the auto-generated transcript for readability.

[00:37:49] Now let's recall our two questions. So the first one was: do we think the $1/N$ expansion is useful quantitatively in the real world? Second question is: do we believe that 't Hooft was right, and have we made progress in understanding his conjecture? So we're now going to move on to the second question.

[00:38:11] Well, we have far more confidence in gauge/string duality today than 50 years ago, speaking personally, because AdS/CFT duality—that is, duality between gravity in asymptotically anti-de Sitter space and conformal field theory—has given concrete examples of duality between a 4D gauge theory and a string theory with, as 't Hooft predicted, $1/N$ as the string coupling constant. The most basic example is duality between $\mathcal{N} = 4$ super Yang-Mills in four dimensions and Type IIB superstring theory on $AdS_5 \times S^5$.

[00:38:49] Once we have this basic example, all kinds of other examples follow. For one thing, we can make a relevant deformation of the $\mathcal{N} = 4$ theory, and this will be dual to some sort of deformation of the string theory. So that will give us more gauge theories that are dual to string theories. For example, Polchinski and Strassler used this approach to find a dual string theory description of a certain gauge theory sometimes called the $\mathcal{N} = 1^*$ theory with $\mathcal{N} = 1$ supersymmetry. The dual string theory made confinement manifest precisely when it was expected in the field theory.

[00:39:30] There are many other examples. You don't have to start with $\mathcal{N} = 4$, that's just a basic example. Many other examples of these dualities are known and many of them can be perturbed to give other confining models...

[00:40:06] ...I won't explain the details of the model, but I will explain why such a model can naturally explain confinement or predict confinement. But before I do that, I want to say that personally I think the setup really implies that 't Hooft was right that pure $SU(N)$ gauge theory is dual to a string theory, because it's an example of a theory that can be reached by relevant deformation of the $\mathcal{N} = 4$ theory, and I do believe that the corresponding perturbation of the string theory will give whatever 't Hooft was talking about.

[00:40:37] But there's a big catch. The only problem—but it's a problem that you could drive a herd of elephants through—is that to get the pure gauge theory, we need to make a relevant

deformation with a large deformation parameter. Or we could make a relevant deformation and make the energies very small. And we don't know at all how to take that limit in the string theory. In fact, the string theory we want is probably quite unlike any that we actually know as of now. The reason to think so is that we've got, in QCD or in pure gauge theory, we have asymptotic freedom governing the short-distance behavior. We have weak coupling and asymptotic freedom, hard parton behavior, as David explained yesterday, at short distances, and we don't know how to make a string theory that does that. So I feel that this setup basically implies that 't Hooft was right, but there's a trick involved that we really fundamentally don't understand.

[00:41:43] Anyway, what I want to do is to explain... why such models can naturally lead to confinement. We have to remember that AdS/CFT duality is holographic. The gauge theory lives on the conformal boundary of the spacetime on which the strings propagate. So here I've written the metric of AdS_5 . You can think of t and \vec{x} as the usual coordinates of Minkowski space. z is the fifth coordinate that is introduced as this holographic dual of ordinary physics on the boundary. The conformal boundary where the gauge theory lives is at $z = 0$. The space is at z non-negative, or really positive. The infrared region is the region where z goes to infinity. To be precise, to make a state of very low energy in the boundary theory, its dual description has to extend to very large z . I guess roughly it's because for large z , this red-shifting factor $1/z^2$ becomes very large... or very small, very large in the denominator. Anyway, the gauge theory lives at $z = 0$, and in the dual description, the infrared region is at very large z , very far from the boundary.

[00:43:20] Now suppose we're given a Wilson operator W in the fundamental representation. In other words, an operator that describes an external quark by which we want to probe the system. And I'll put it in the boundary...

[00:44:00] Now Wilson taught us a criterion for confinement in terms of the Wilson operator. Let A be the area of a minimal surface in the boundary that has for its boundary this red circle... Let A be the area of the blue disk, but I'm calling it A_{boundary} to emphasize that that's the area measured in the boundary. According to Wilson, confinement means that the expectation value of the Wilson operator decays exponentially with a constant times the area for large area.

[00:44:45] So we want to decide whether the AdS/CFT duality predicts such a result. Well, Maldacena and Rey explained how to compute the expectation of W in the large N limit in AdS/CFT duality. You view the Wilson loop at infinity as the boundary of an elementary string worldsheet, which I'll call S ...

[00:46:12] The path integral over S computes the expectation value of the Wilson operator in the large N limit. Now... in the large N limit, you just pick for S the bounding surface of minimal area, except this is minimal bulk area, so I've called it \mathcal{A} , which is area sub bulk...

[00:47:10] ...According to Maldacena and Rey, the expectation value for the Wilson operator in the large N limit is the exponential of minus $1/\alpha'$ (where α' is the string tension) times the area \mathcal{A} , the area measured in the bulk.

[00:47:31] So we'll have confinement, which says that the expectation value of W is the exponential of minus a constant times the boundary area, if and only if the bulk area is a constant times the boundary area for large A . In other words, we make the circle bigger and bigger, so the boundary area gets bigger and S gets bigger and bigger. If the area of S , the surface S , grows in proportion to the area measured on the boundary, that will give us confinement.

[00:48:00] Now that might make you think that gauge/string duality always implies confinement, because how can we avoid \mathcal{A} , the bulk area, being proportional to the boundary area? Well, the way we avoid it is that when A becomes large, the surface S can bend farther and farther into the bulk, and that means it reaches the region at very large z where the metric is

small. And if this surface lives mostly at very large z , the area will not be too big, and \mathcal{A} can avoid being proportional to straight A .

[00:48:49] In fact, if the bulk is really AdS_5 , the dual gauge theory on the boundary is conformally invariant, so it definitely doesn't have an area law. It's gapless and conformally invariant and not confining.

[00:49:05] However, suppose we turn on a relevant perturbation that causes the theory to develop a mass gap. Well, since very large z was the infrared region which you have to probe if you're going to see massless or very low energy states, if there's a gap in the dual description, that will cut off the bulk spacetime at some $z = z_0$, eliminating the vanishing of $1/z^2$ as z goes to infinity. Then there's a limit to how much the bulk area can be reduced by bending in the bulk, and it's natural you would have confinement.

[00:49:45] Here's a picture of what happens in such a case... The bulk is cut off at some $z = z_0$. It's not necessarily cut off sharply as I've drawn, it could be cut off in some smooth or fuzzy way... So there's a limit to how far the bulk surface can go looking for large z . So the optimal area of the bulk surface... it will go toward the cutoff and then stay there where $1/z^2$ is as small as possible, and then come back. And the distance it has to propagate near the cutoff is just the same as the distance in the boundary between these two points. So now we're going to get in such a case... the bulk area to be proportional to the boundary area when both of them are big. So this leads to the bulk area being a multiple of the boundary area, giving confinement.

[00:51:41] Now this description might make you think that confinement is an inevitable consequence of the mass gap, but that's false. It's possible to have a gapped gauge theory that's not confining. How does that happen? In this picture, the alternative is that the elementary string may condense, which is a fancy way to say that it might be capable of ending in the infrared region.

[00:52:11] So then the picture would be schematically like this. Now the surface... has two components, each of which is a string that goes from the boundary to the cutoff, and then at the cutoff there's some kind of physics that makes it possible for the string to condense or end. And now the bulk length is just the length of these two segments, and it's completely independent of this distance. So if you go back to the full picture, you'll see that in this case the Wilson operator will have a perimeter law rather than an area law...

[00:53:17] In modern language, the infrared physics might violate the one-form symmetry related to the existence of the string. Then the string will be capable of propagating into the region where that physics prevails and ending there.