SOME PROPERTIES OF O(32) SUPERSTRINGS

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Some properties of the anomaly-free O(32) superstring theory recently discovered by Green and Schwarz are discussed. With proper choice of ground state, the theory leads in four dimensions to an SU(5) theory with any desired number of standard generations (and no exotic or mirror fermions). It predicts axions and stable Nielsen–Olesen vortex lines. It can be consistently compactified only if certain topological conditions are imposed.

Introduction. Superstrings [1], which developed from the Ramond–Neveu–Schwarz spinning string theory, seem very promising as a mathematically consistent approach to quantum gravity which may yield a satisfactory unified theory of all interactions. (The search for a "geometrical" foundation of superstrings was briefly discussed in ref. [2].)

In a stunning development, Green and Schwarz have shown [3] that type I superstrings (unoriented open and closed strings with $N = 1$ supersymmetry) are anomaly free if and only if the Yang–Mills gauge group is O(32). It was already known [4] that anomalies cancel for type II superstrings (closed oriented strings with $N = 2$ supersymmetry), but the immediate phenomenological prospects of the new anomaly free string theory seem much brighter. The present paper will be devoted to a brief study of some aspects of this theory.

Restrictions on compactification. The theory contains a second-rank antisymmetric tensor field $B_{\mu\nu}$ which is crucial in the elegant anomaly cancellation mechanism of ref. [3]. We will use the language of differential forms and not indicate explicitly the indices of antisymmetric tensors. The gauge invariant field strength of B is [5,3]

$$H = \mathrm{d}B - \omega_{3\mathrm{Y}} + \omega_{3\mathrm{L}} , \qquad (1)$$

where $\omega_{3\mathrm{Y}}$ and $\omega_{3\mathrm{L}}$ are the Yang–Mills and Lorentz Chern–Simons three-forms

$$\omega_{3\mathrm{Y}} = \mathrm{tr}(AF - \tfrac{1}{3}A^3) , \quad \omega_{3\mathrm{L}} = \mathrm{tr}(\omega R - \tfrac{1}{3}\omega^3). \qquad (2, 3)$$

Here A is the gauge field, F the Yang–Mills field strength, ω the spin connection, and R the Riemann tensor. H is invariant under gauge transformations of B, $B \to B + \mathrm{d}\lambda$, and also under gauge and Lorentz transformations accompanied by shifts in B [5,3].

Now, in a topologically non-trivial situation, the spin connection, gauge field, and Chern–Simons form are not globally well defined. At best we can cover our manifold with simple open sets $\mathrm{O}^{(i)}$, on each of which these are defined, with suitable relations imposed in intersection regions $\mathrm{O}^{(i)} \cap \mathrm{O}^{(j)}$. However, the gauge invariant field strength H must be globally defined, since (for instance) the energy contains a term H^2.

To see what this implies, note that

$$\mathrm{d}H = -\mathrm{tr}\, F^2 + \mathrm{tr}\, R^2 . \qquad (4)$$

Now, let Q be a closed four-dimensional submanifold in space–time. In general $\int_Q \mathrm{tr}\, R^2$ and $\int_Q \mathrm{tr}\, F^2$ may be non-trivial topological invariants, but (4) implies

$$\int_Q (\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2) = \int_Q \mathrm{d}H = 0 , \qquad (5)$$

where the second equality uses Stokes' theorem and the fact that H is globally defined. Eq. (4) or (5) means that the cohomology class represented by

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$\mathrm{tr}\, R^2 - \mathrm{tr}\, F^2$ is zero. It is a restriction on possible compactifications of the string theory. As we will see, it has a particular significance; it ensures absence of anomalies in four dimensions.

Anomaly cancellation. Suppose that the ten-dimensional world is of the form $\mathrm{M}^4 \times \mathrm{K}$, M^4 being four-dimensional Minkowski space and K a compact six-dimensional manifold. We would like to determine the conditions under which the effective four-dimensional theory is anomaly free. For simplicity, we will assume that the effective four-dimensional gauge group G is a subgroup of O(32) (though we could in a similar spirit consider gauge fields that originate as symmetries of K) and is a semi-simple group such as SU(5) or O(10).

Let H be the subgroup of O(32) that commutes with G. Symmetry breaking from O(32) to G can (in the superstring theory) arise only from expectation values of components of H gauge fields. Under $\mathrm{G} \times \mathrm{H}$, the adjoint representation of O(32) has a decomposition as $\oplus \mathrm{C}_i \otimes \mathrm{L}_i$, where C_i and L_i are certain G and H representations.

Let n_i be the number of left-handed fermion multiplets transforming as C_i under G minus the number transforming as $\overline{\mathrm{C}}_i$. If T is a generic G generator, the G anomaly in the C_i representation is $\mathrm{Tr}_{\mathrm{C}_i} T^3$ ($\mathrm{Tr}_{\mathrm{C}_i}$ and $\mathrm{Tr}_{\mathrm{L}_i}$ will refer, respectively, to traces in the C_i and L_i representations of G and H). The total G anomaly is $\Sigma_i\, n_i\, \mathrm{Tr}_{\mathrm{C}_i} T^3$, and we wish to investigate the conditions under which it vanishes.

The quantity n_i equals the index of the Dirac operator on K for fermions in the L_i representation of K [6,7]. According to the index theorem,

$$n_i = \frac{1}{6 \cdot (4\pi)^3} \int_{\mathrm{K}} (\mathrm{Tr}_{\mathrm{L}_i} F^3 - \tfrac{1}{8} \mathrm{Tr}_{\mathrm{L}_i} F\, \mathrm{Tr}\, R^2)\,, \tag{6}$$

where F is the H field strength. The total anomaly is hence

$$A = \frac{1}{6 \cdot (4\pi)^3} \sum_i \mathrm{Tr}_{\mathrm{C}_i} T^3 \times \mathrm{Tr} \int_{\mathrm{K}} (\mathrm{Tr}_{\mathrm{L}_i} F^3 - \tfrac{1}{8} \mathrm{Tr}_{\mathrm{L}_i} F\, \mathrm{Tr}\, R^2)\,. \tag{7}$$

Now, think of F as a generator of H. Let $\mathrm{Tr}_{\mathrm{adj}}$ and $\mathrm{Tr}_{\mathrm{fund}}$ represent traces in the adjoint and fundamental representations of O(32). Then

$$\sum_i \mathrm{Tr}_{\mathrm{C}_i} T^3\, \mathrm{Tr}_{\mathrm{L}_i} F^3 = \mathrm{Tr}_{\mathrm{adj}}\, T^3 F^3\,.$$

In addition, $\mathrm{Tr}_{\mathrm{adj}}\, T^3F^3$ can be viewed as the term of order $\alpha^3\beta^3$ in $\frac{1}{20}\mathrm{Tr}_{\mathrm{adj}}(\alpha T + \beta F)^6$.

Now the anomaly cancellation mechanism of ref. [3] depends on a peculiar relation in O(32) group theory. One aspect of this is the identity

$$\tfrac{1}{20}\mathrm{Tr}_{\mathrm{adj}}(\alpha T + \beta F)^6 = \tfrac{1}{32}\mathrm{Tr}_{\mathrm{adj}}(\alpha T + \beta F)^4\, \mathrm{Tr}_{\mathrm{fund}}(\alpha T + \beta F)^2\,,$$

for any O(32) generator $\alpha T + \beta F$. The term of order $\alpha^3\beta^3$ is $\frac{1}{8}\mathrm{Tr}_{\mathrm{adj}}\, T^3F\, \mathrm{Tr}_{\mathrm{fund}} F^2$ (here we use the fact that G is semi-simple so $\mathrm{Tr}\, TF = \mathrm{Tr}\, TF^3 = 0$; otherwise we must keep track of more terms, but the eventual conclusion is similar). And we may reexpress $\mathrm{Tr}_{\mathrm{adj}}\, T^3F = \Sigma_i\, \mathrm{Tr}_{\mathrm{C}_i} T^3\, \mathrm{Tr}_{\mathrm{L}_i} F$. The net effect of this is that (7) equals

$$A = \frac{1}{48(4\pi)^3} \sum_i \mathrm{Tr}_{\mathrm{C}_i} T^3 \int_{\mathrm{K}} \mathrm{Tr}_{\mathrm{L}_i} F \times (\mathrm{Tr}_{\mathrm{fund}} F^2 - \mathrm{Tr}\, R^2)\,. \tag{8}$$

Now, we have noted above that the theory in question only makes sense if $\mathrm{Tr}_{\mathrm{fund}} F^2 - \mathrm{Tr}\, R^2 = -\mathrm{d}H$, with some globally defined H. Hence

$$\int_{\mathrm{K}} \mathrm{Tr}_{\mathrm{L}_i} F\, (\mathrm{Tr}_{\mathrm{fund}} F^2 - \mathrm{Tr}\, R^2) = -\int_{\mathrm{K}} (\mathrm{Tr}_{\mathrm{L}_i} F)\, \mathrm{d}H = \int_{\mathrm{K}} (\mathrm{d}\, \mathrm{Tr}_{\mathrm{L}_i} F)\, H = 0\,, \tag{9}$$

where we have integrated by parts and used the Bianchi identity $\mathrm{d}\, \mathrm{Tr}\, F = 0$. Therefore, the topological condition that is needed for the theory to make sense also ensures anomaly freedom in four dimensions.

Models. We will now try to construct realistic models by compactification of the string theory. Without elementary gauge fields, it is probably impossible [6] for a Kaluza–Klein theory to give chiral fermions in four dimensions (this has not been completely proved for Rarita–Schwinger fields). With elementary gauge fields, it is definitely possible to make realistic models. For instance, in ref. [6] it was shown that an O(16) theory with an irreducible fermion representation in ten dimensions can reduce in four dimensions to an

O(10) theory with any even number of ordinary families (and no antifamilies). Similar models have been constructed independently by other authors [8]. We will now see that essentially the same construction can lead for superstrings to an SU(5) ground state with any required number of ordinary generations (and no mirror or exotic fermions).

Some preliminaries will be useful. On any compact oriented two-dimensional manifold M, one can define a U(1) Dirac monopole gauge field $F_{\mu\nu}$ with

$$\int_{\mathrm{M}} \mathrm{d}\Sigma^{\mu\nu} F_{\mu\nu} = 2\pi \,. \tag{10}$$

If a spin 1/2 fermion of charge n interacts with this gauge field, there are $|n|$ zero modes; they have positive or negative chirality for positive or negative n.

For simplicity, the six manifolds we consider will be products $\mathrm{M}_1 \times \mathrm{M}_2 \times \mathrm{M}_3$, where the M_i are compact oriented two manifolds (whose topology will not matter). On each M_i we choose an abelian gauge field $f_{\mu\nu}{}^{(i)}$ such that

$$\int_{\mathrm{M}_i} \mathrm{d}\Sigma^{\mu\nu} f_{\mu\nu}{}^{(j)} = 2\pi\delta_i{}^j \,. \tag{11}$$

If a fermion of unit charge interacts with the gauge field $F_{\mu\nu} = \Sigma_i \, n_i \, f_{\mu\nu}{}^{(i)}$, there are $|n_1 n_2 n_3|$ zero modes, whose chirality equals the sign of $n_1 n_2 n_3$. This may be deduced by separation of variables (reducing to the two-dimensional case we discussed earlier), or by means of a six-dimensional index theorem.

We will construct SU(5) models, though we could simply aim for SU(3) × SU(2) × U(1). Since SU(5) cannot be realized as the isometry group of a six-dimensional manifold, we must regard it as a subgroup of O(32).

The simplest SU(5) embedding in O(32) is the one in which the fundamental representation of O(32) transforms as 5 + $\bar{5}$ singlets. We will consider models based on this embedding [‡1]. With this embedding, the complex representations of SU(5) appearing in the adjoint representation of O(32) are 5, 10, and their conjugates.

[‡1] I thank C.G. Callan, J. Harvey, and L. Yaffe for pointing out an error in a prior attempt with a different embedding, and for helpful comments.

SU(5) so embedded commutes with a U(1) × O(22) subgroup of O(32). Let us denote the U(1) generator as P and normalize it so the fundamental 5 and $\bar{5}$ have $P = \pm 1$.

Let $N_{10-\overline{10}}$ be the number of left-handed fermions transforming in the 10 of SU(5) minus the number transforming as $\overline{10}$; likewise for $N_{5-\bar{5}}$. If $\mathrm{Tr}\, F^2 - \mathrm{Tr}\, R^2 = -\mathrm{d}H$, then anomalies will cancel and $N_{10-\overline{10}} = -N_{5-\bar{5}}$. The number of generations is $N_{10-\overline{10}}$.

Let the U(1) field strength $F_{\mu\nu}$ of P be

$$F_{\mu\nu} = \sum_i p^i f_{\mu\nu}{}^{(i)} \,, \tag{12}$$

where the p^i are constants. At first sight it appears that Dirac quantization requires the p^i to be integers, but that is not so. It is required only that the p^i be integers or half-integers, since Dirac quantization need only be satisfied in the adjoint representation of O(32) (the superstring theory has no charges in the fundamental representation).

The 10 appearing in the adjoint representation of O(32) arises from 5 × 5 so it has $P = 2$ and interacts with $2F_{\mu\nu} = \Sigma_i \, 2p^i f_{\mu\nu}{}^{(i)}$. In view of our previous comments, the number of generations is $N_{10-\overline{10}} = (2p^1)(2p^2)(2p^3) = 8p^1p^2p^3$. With integers or half integers for the p^i, any number of generations can be obtained.

To complete the construction of a model, it is necessary to choose O(22) gauge fields such that $\int_{\mathrm{B}} \mathrm{Tr}\, F^2 = 0$, where B is any four-dimensional closed submanifold of $\mathrm{M}_1 \times \mathrm{M}_2 \times \mathrm{M}_3$. ($\int_{\mathrm{B}} \mathrm{Tr}\, R^2$ is automatically zero for any such submanifold.) The relevant choices of B are $\mathrm{M}_1 \times \mathrm{M}_2$, $\mathrm{M}_2 \times \mathrm{M}_3$, and $\mathrm{M}_3 \times \mathrm{M}_1$.

Many possibilities exist. It is adequate to consider an abelian configuration. O(22) has an eleven-dimensional abelian subgroup O(2) × O(2) × ... × O(2). Denote the field strengths as $F_{\mu\nu}{}^{(a)}$, $a = 1, ..., 11$. We assume

$$F_{\mu\nu}{}^{(a)} = \sum_{j=1}^{3} p^{aj} f_{\mu\nu}{}^{(j)} \,, \tag{13}$$

for some p^{aj}. (Dirac quantization requires that for each a and j, $p^{aj} + p^j$ is an integer.) The condition that $\int \mathrm{Tr}\, F^2 = 0$ on $\mathrm{M}_i \times \mathrm{M}_j$ is that

$$5p^ip^j + \sum_{a=1}^{11} p^{ai}p^{aj} = 0 \,, \tag{14}$$

for each $i \neq j$. It is not difficult to obey these three equations for three dozen unknowns.

One interesting feature of this model is the existence of unbroken symmetries in O(32) commuting with SU(5). One such unbroken symmetry is P; there may be others, depending on the O(22) gauge field (as in the abelian case just considered). Let us discuss what happens to P.

The SU(5) 10's have $P = 2$ and the $\bar{5}$'s have $P = -1$. Others have $P = 0$. Therefore, the fermion representation in four dimensions has a P-SU(5)-SU(5) triangle anomaly. At first sight, this is paradoxical, since SU(5) and the P seem to be unbroken gauge symmetries. The resolution of the paradox is that the P gauge meson is *massive* at tree level. This occurs as follows. Let μ, ν $= 0, ..., 3$ be space–time indices, and let $i, j = 4, ..., 9$ be tangent to $M_1 \times M_2 \times M_3$. Let B be the massless tensor field that plays a key role in anomaly cancellation. If the lagrangian for B were merely $(\mathrm{d}B)^2$, certain components of B, corresponding to the second Betti number of $M_1 \times M_2 \times M_3$, would be massless [6]. Instead the lagrangian for B is $(\mathrm{d}B + \Gamma)^2$, where Γ is the Chern–Simons form. Let A_μ and $F_{\mu\nu}$ be the gauge field and field strength of P. Then $(\mathrm{d}B + \Gamma)^2$ has a term $(\partial_\mu B_{ij} + A_\mu F_{ij})^2$. Since F_{ij} has an expectation value in $M_1 \times M_2 \times M_3$, this reduces in four dimensions to $(\partial_\mu B + A_\mu \langle F \rangle)^2$, when B is the would-be massless scalar. This is a typical Higgs-like lagrangian; B and A_μ combine into a massive vector meson.

However, this mechanism does not disturb *global* conservation of P. In fact, the P-SU(5)-SU(5) anomaly means that P is a Peccei–Quinn symmetry, so that strong *CP* violation will not arise. But realistic low energy breaking of P is hard to achieve.

The special case of our construction in which M_1, M_2, and M_3 are tori may be tractable in the full-fledged string theory.

Axions. In addition to the model-dependent Peccei–Quinn symmetry discussed above, this theory has a model independent mechanism for generating an axion [‡2]. Consider the antisymmetric tensor field $B_{\mu\nu}$ that is crucial in anomaly cancellation. Let us study the mode with $\mu, \nu = 0, ..., 3$ and no dependence on compact coordinates. The equation of motion is $\partial^\mu H_{\mu\nu\alpha} = 0$ with H defined in equation (1). If we define $Y^\mu = \frac{1}{6}\epsilon^{\mu\nu\alpha\beta}H_{\nu\alpha\beta}$, this is $\partial_\mu Y_\nu - \partial_\nu Y_\mu = 0$, so $Y_\mu = M^{-1}\partial_\mu\phi$ for some ϕ (M is an unknown mass chosen so ϕ is canonically normalized in four dimensions).

The "Bianchi identity" $\mathrm{d}H = -\mathrm{Tr}\, F^2 + \mathrm{Tr}\, R^2$ is now equivalent to

$$\Box\phi = -M^{-1}(\mathrm{Tr}\, F_{\mu\nu}\widetilde{F}^{\mu\nu} - \mathrm{Tr}\, R_{\mu\nu}\widetilde{R}^{\mu\nu}) . \qquad (15)$$

This is the standard coupling of an axion, so this theory automatically solves the strong *CP* problem.

In many cases (including the SU(5) models above), the $B\,\mathrm{Tr}\,F^4$ coupling [3] leads in $d = 4$ to a coupling $\epsilon^{\alpha\beta\mu\nu}B_{\alpha\beta}Y_{\mu\nu}$ where $Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$ is the field strength of a U(1) gauge boson Y. In this case (similarly to our discussion of P), B is "eaten", becoming the longitudinal component of Y_μ, and the Y symmetry becomes at low energies a global Peccei–Quinn symmetry.

In a model with several would-be axions, "the" axion is the linear combination coupling to $F\widetilde{F}$.

Vortex lines. Spontaneously broken gauge theories can generate stable vortex lines or strings [9]. The existence of such objects depends on details of the Higgs content. For the O(32) superstring, the Higgs content is fixed; all charged fields are in the adjoint representation. As we will see, the superstring theory with almost any realistic assumptions predicts a variety of vortex lines with different masses.

Choose coordinates so that z is parallel to the hypothetical vortex line, r is the distance from the vortex line, ϕ is the azimuthal angle, and λ^i are coordinates for the Kaluza–Klein space K. At large distances from the vortex line the gauge field must be a pure gauge,

$$A_\mu \xrightarrow{r\to\infty} g^{-1}\partial_\mu g , \qquad (16)$$

where $g(\phi, \lambda^i)$ is a mapping from $S^1 \times K$ into O(32). g must be single-valued in the adjoint representation of O(32) but not necessarily in other representations.

Consider first the "four-dimensional" case in which g depends on ϕ only. The center of O(32) is $Z_2 \times Z_2$, generated by the 2π rotation α (which is -1 in the spinor representation) and an element we will call β which is -1 in the fundamental representation. If α or β is not in the unbroken subgroup H of O(32) then mappings $g(\phi)$ with $g(\phi + 2\pi) = \alpha g(\phi)$ or $g(\phi + 2\pi) =$

[‡2] I thank T. Banks for a question that prompted this observation.

$\beta g(\phi)$ will lead to stable vortex lines. It is rather difficult to make a realistic model in which α and β are in H. For instance, in SU(5) models (like the one above) α and β are not in H and there are stable $Z_2 \times Z_2$ vortex lines.

There may be other vortex lines coming from configurations in which g depends non-trivially on the λ^i. The classification of these is complicated. The simplest comes from $\pi_7(\mathrm{O}(32)) \cong \mathrm{Z}$. Associated with this is an integer valued winding number for maps from any seven-manifold, such as $\mathrm{S}^1 \times \mathrm{K}$, into O(32). Maps $g(\phi, \lambda^i)$ with non-zero winding number will lead to vortex lines unless they can be deformed into maps of $\mathrm{S}^1 \times \mathrm{K}$ into H. Such a deformation is impossible if H is a group such as $\mathrm{SU}(3) \times \mathrm{U}(1)$ whose seventh Betti number is zero. There will therefore be integer valued vortex lines. It is conceivable this can be avoided if the H embedding in O(32) is "twisted" in a way that depends on λ^i.

Parameters. An observation that is not essentially new, but still worth mentioning, is that – as befits a possible unified theory of all interactions – the superstring theory has no adjustable dimensionless parameter. The theory seems to have three parameters, κ, g, and α', of dimension (length)^4, (length)^3, and (length)^2 respectively. But it is known that a relation $\kappa \sim g^2/\alpha'$ is required for consistency of the theory, since a graviton pole appears in the same diagram that has two-gluon exchange. This seems to leave a dimensionless parameter g^4/κ^3. However, the theory contains at tree level a massless scalar ϕ whose expectation value is undetermined. It can be seen from the limiting field tensor [5] or in string theory terms that a shift in this scalar rescales g and α' while keeping κ and g^2/α' fixed. So the value of g^4/κ^3 labels not a one-parameter family of theories but a one-parameter family of vacuum states. If ultimately a non-trivial potential for ϕ is generated, minimizing it will determine g^4/κ^3.

I wish to thank J. Schwarz and M. Green for very valuable discussions.

Note added. As explained in ref. [3], the anomaly cancellation mechanism of that paper works for $\mathrm{E}_8 \times \mathrm{E}_8$ as well as O(32). (This was noted by several physicists including J. Thierry-Mieg and J. Harvey, L. Dixon, and E. Witten.) A consistent theory based on $\mathrm{E}_8 \times \mathrm{E}_8$ is not known at present, but it is interesting to discuss how such a theory would compare to O(32).

Low energy physics can be readily embedded in $\mathrm{E}_8 \times \mathrm{E}_8$. For instance, E_8 has a maximal $\mathrm{E}_6 \times \mathrm{SU}(3)$ subgroup. With suitable abelian configurations of SU(3) gauge fields on $\mathrm{M}_1 \times \mathrm{M}_2 \times \mathrm{M}_3$, one can make realistic E_6 models. For instance, one can pick the diagonal components of the SU(3) magnetic field to be $f_{\mu\nu}{}^{(1)} + f_{\mu\nu}{}^{(2)} + n f_{\mu\nu}{}^{(3)}$; $-f_{\mu\nu}{}^{(2)}$; and $-f_{\mu\nu}{}^{(1)} - n f_{\mu\nu}{}^{(3)}$. (This violates $\mathrm{Tr}\, F^2 = 0$, but one can compensate for that with suitable gauge field expectation values in the second E_8.) This model has n standard E_6 generations (27's).

An attractive feature of such $\mathrm{E}_8 \times \mathrm{E}_8$ models is that there is no analogue of the phenomenologically troublesome P symmetry noted above (and hard to avoid in O(32) models). But the phenomenologically acceptable, model independent axion mechanism noted above for O(32) is still present.

Since $\mathrm{E}_8 \times \mathrm{E}_8$ is simply connected, the "four dimensional" strings of O(32) are absent, but the ones involving higher homotopy groups may still be present.

Since physics as we know it can be embedded in one E_8, it is amusing to speculate that there may be another low energy world based on the second E_8. The two sectors communicate only gravitationally. If the symmetry between the two E_8's is unbroken, it may be that half the stars in the vicinity of the sun are invisible to us, along with half the mass in the galactic disk.

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