Vortices in Type-II Superconductors

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Abstract

Vortices in type-II superconductor arise when the magnetic field starts to penetrate the materials in the form of quantized flux. The vortices interact with each, and can form different phases under the influence of the magnetic field, thermal fluctuations, and the pinning effect of disorder and defects. As the usual theoretical methods towards vortex matter, the London model is briefly introduced while the Ginzburg-Landau models are discussed at length for their capability of describing more interesting phases of the vortices. Some experimental techniques of measuring the vortices are also mentioned at the end of the term essay.

1 Introduction

The history of superconductivity in the 1950s always makes delightful and instructive reading. As is mentioned in A. A. Abrikosov's Nobel lecture[1], in that decade several long-awaited theoretical discoveries were made by extraordinary minds – the famous Ginzburg-Landau (GL) model in 1950 by Vitaly Ginzburg and Lev Landau, the brilliant introduction of type-II superconductors in 1952 and the vortex lattice in 1957 by Alexei Alexeyevich Abrikosov himself, and the celebrated BCS model in 1957 by John Bardeen, Leon Neil Cooper, and John Robert Schrieffer. The details of how these discoveries advanced the research both in and outside of the field of superconductivity require a rather thick book by an expert other than this short term essay by me, and thus can be saved here. But one thing, which is remarkably short but inspiring, is yearning to be mentioned. Without using any microscopic details of the phonon-electron interaction, the phenomenological GL models of conventional superconductors were simply guessed, but they proved even more useful when fitting various experimental data.

Of all these theoretical discoveries, which still have tremendous influence today, personally I want to devote this term essay to the development of theoretical tools on dealing with the vortices in type-II superconductors, because of the freshness (only to me) of the topic, and its relevance to the intriguing high Tc problem.

1.1 Type-I and -II Superconductors

Type-I and -II superconductors mainly differ in their responses to the external magnetic field. The differences generally can be shown by the Ginzburg-Landau theory.

Without magnetic field and excluding the "normal electron" contribution, the GL free energy takes the form of

$$F = \int d^3x \big[\alpha(T)|\Psi|^2 + \frac{\beta(T)}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}|\nabla\Psi|^2\big] \tag{1}$$

where Ψ is the order parameter.

In the presence of the magnetic field, we need to adopt the GL Gibbs energy

$$G = \int d^3x \big[\alpha(T)|\Psi|^2 + \frac{\beta(T)}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}|(\nabla + \frac{ie^*}{\hbar c}\vec{A})\Psi|^2 + \frac{(\vec{B}-\vec{H})^2}{8\pi}\big] \tag{2}$$

where $\vec{H}$ is a constant magnetic field, $(\vec{B}-\vec{H})$ is the magnetization, and $e^* = 2e$.

Minimization with respect to $\vec{A}$ and Ψ leads to the nonlinear Shroedinger equation

$$-\frac{\hbar^2}{2m^*}(\nabla + \frac{ie^*}{\hbar c}\vec{A})^2\Psi + \alpha(T)\Psi + \beta(T)|\Psi|^2\Psi = 0 \tag{3}$$

and the supercurrent equation

$$\frac{c}{4\pi}\nabla \times \vec{B} - \frac{ie^*\hbar}{2m^*}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + \frac{e^{*2}}{cm^*}\vec{A}|\Psi|^2 = 0 \quad (4)$$

Expanding the coefficients to their lowest orders, $\alpha(T) \approx -aT_c(1-t)$ and $\beta(T) \approx \beta = const.$, where $t = T/T_c$.

By looking at the surface energy, and by linearizing the two equations above, some important quantities can be extracted. However, for the sake of simplicity, a complete analysis will not be shown here in order to make space for more interesting physics. I only want to go over the definitions and notations, whose derivation can be found in lots of books and lecture notes, such as M. Tinkham's[2].

- The magnetic penetration depth: $\lambda = \frac{c}{2e^*}\sqrt{m^*\beta/\pi a T_c(1-t)}$
- The correlation length: $\xi = \hbar/\sqrt{2m^* a T_c(1-t)}$
- The ratio of the penetration depth and the correlation length: $\kappa = \frac{cm^*}{\hbar e^*}\sqrt{\beta/2\pi}$
- Type-I superconductor: $0 < \kappa < 1/\sqrt{2}$ (positive surface energy)
- Type-II superconductor: $\kappa > 1/\sqrt{2}$ (negative surface energy)
- Extreme type-II superconductor: $\kappa \gg 1$

Note that the type-II superconductors generally have large dimensionless factor κ, which does not depend on temperature, at least to this order.

1.2 Vortices of Type-II Superconductors and London Model

For a type-II superconductor, there is a lower critical H_{c1} and a upper critical field H_{c2}.

- ($H < H_{c1}$) The superconductor is a perfect diamagnet, exhibiting Meissner effect.
- ($H_{c1} < H < H_{c2}$) The magnetic field penetrates the superconductor in a form of small "tubes" (vortices), each with quantized flux $\Phi_0 = hc/e^*$. When increasing H, the number and density of the vortices will increase.
- ($H > H_{c2}$) The superconductor goes into the normal state.

In principle, the lower and upper critical fields as well as the field profile can be calculated by solving Eq. (3) and Eq. (4). Approximation is generally required becasue of the nonlinearity.

1. As for the upper critical field, it is defined as the boundary of the superconducting phase and the normal phase, hence the order parameter field Ψ is greatly suppressed while magnetic penetration is most prominent. It allows us to make the linear approximation

$$-\frac{\hbar^2}{2m^*}(\nabla + \frac{ie^*}{\hbar c}\vec{A})^2\Psi + \alpha(T)\Psi = 0 \quad (5)$$

Assuming $\vec{H}$ is in $\hat{z}$ direction, choosing $\vec{A} = Bx\hat{y}$ and $\Psi = e^{ik_z z}e^{ik_y y}f(x)$ (because of the translational symmetry), Eq. (5) becomes

$$-f''(x) + (\frac{2\pi B}{\Phi_0})^2(x - \frac{k_y \Phi_0}{2\pi B})f = (\frac{1}{\xi^2} - k_z^2)f \tag{6}$$

This is just the same as the one of a displaced quantum harmonic oscillator. Recall the quantized energy of the oscillator, and assume that $B \approx H$ (almost no magnetization) when $H \approx H_{c2}$, then the largest possible magnetic field, or the upper critical field is given by

$$H_{c2} = \frac{\Phi_0}{2\pi\xi^2} \tag{7}$$

Note that ξ appears to be the radius of the vortex.

2. As for the lower critical field, it is defined as the field when the first vortex is formed. The region close to the lower critical field is where the London model of the vortices is the most reliable. Because the London model is defined with two major assumptions:
(1) Ψ is homogeneous everywhere except at the core of the vortex.
(2) The core of the vortex is so small that the vortex can be essentially taken as a line object.
Assuming $\vec{H}$ is in $\hat{z}$ direction, Eq. (4) should be modified slightly to account for the quantized magnetic field along the vortex line.

$$\frac{4\pi\lambda^2}{c}\nabla \times \vec{J}_s + \vec{B} = \sum_i \hat{z}\Phi_0\delta(\vec{r} - \vec{r}_i) \tag{8}$$

where $\vec{J}_s$ is the supercurrent and $\vec{r}_i$ labels the position of the vortices. Substituting one of the Maxwell equation $\nabla \times \vec{B} = 4\pi\vec{J}_s/c$,

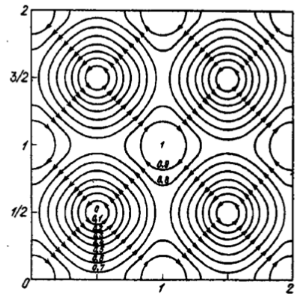
$$\nabla^2\vec{B} - \frac{\vec{B}}{\lambda^2} = -\frac{\Phi_0}{\lambda^2}\hat{z}\sum_i \delta(\vec{r} - \vec{r}_i) \tag{9}$$

whose solution can give the field profile. For a single vortex, by calculating the energy, it can be shown that

$$H_{c1} \approx \frac{\Phi_0}{4\pi\lambda^2} ln\kappa \tag{10}$$

Note that the equation above only works for lare κ.

The London model of the vortices is a very simple one. But it soon becomes invalid when approaching H_{c2}, where Ψ is inhomogeneous and greatly suppressed, and the vortices tend to have overlaps and cannot be linelike. Especially, the London model cannot be used to deal with the more interesting case when the vortices are closely packed into a lattice, which is a common structure when the density of the vortices is high.



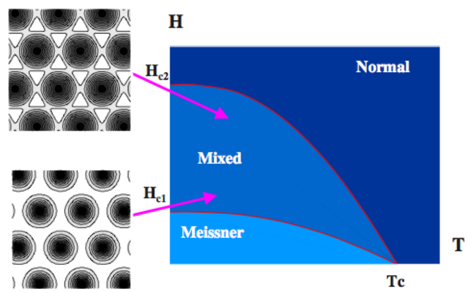


Figure 1: Left: Vortex lattice with constant $|\Psi|$ lines obtained by A. A. Abrikosov. Right: A schematic phase diagram of a type-II superconductor.

The first solution of the vortex lattice of a type-II superconductor was by A. A. Abrikosov[3], where he considered an equation similar to Eq. (6), and got a periodic solution as shown in Fig. 1 above. The graph shown on the right is a schematic phase diagram of the type-II superconductor from Rosenstein and Li, Jan 2010[4], just to show the differences between $H \sim H_{c1}$ and $H \sim H_{c2}$. The Abrikosov vortex lattice as well as the various phases of vortices will be discussed using GL models in Section 2.

2 Ginzburg-Landau Models and Various Phases of Vortices

This section follows the review article of Rosenstein and Li, Jan 2010. Before going into the details of the GL models, I want to mention four things.

1. (When are they valid?) The GL models below are only valid for high κ type-II superconductors in a magnetic field slightly below H_{c2}.

2. (What approximation?) The approximation $\vec{B}(\vec{r}) \approx \vec{H} = const.$ will be made, which is just the opposite of the London model. Linear approximation of the nonlinear Shroedinger equation will be made at first, and later the nonlinear terms can be included as a perturbation on the lowest Landau level states (details will be provided later).

3. (Any thing put in by hand?) During theoretical calculations, the structures of the vortex lattices, no matter square or rhombic, are put in by hand.

4. (What dimension?) Without considering any disorder or fluctuation, a 2-dimensional GL model is sufficient, because of the translational invariance in the direction of the magnetic field, say $\hat{z}$ direction. However, when introducing fluctuation and disorder, the $\hat{z}$ direction terms should be included in case the translational invariance is broken.

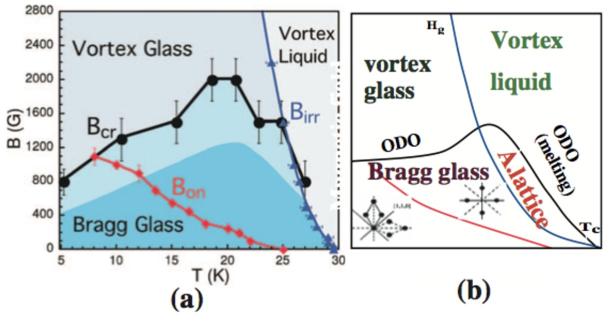


Figure 2: (a)Experimental phase diagram of LaSCO; (b) Theoretical phase diagram advocated by Rosenstein and Li. The red curve is the melting line.

Also, to get an idea of the various magnetic phases of the vortices, an example from Rosenstein and Li, Jan 2010, is shown above. One can get stable Abrikosov latticewhen fluctuation and disorder are both negligible. Increasing fluctuation will generally lead to first order melting towards vortex liquid, while increasing disorder is likely to make it a vortex glass. The Bragg glass, in spite of having multiple metastable states due to the disoder, still maintains long range order, and has Bragg peaks as a solid does.

2.1 Abrikosov Vortex Lattices

$$G = \int dxdy\big[-aT_c(1-t)|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{\hbar^2}{2m^*}|\vec{D}\Psi|^2 + \frac{(\vec{B}-\vec{H})^2}{8\pi}\big] \tag{11}$$

where $t = T/T_c$, $\vec{H} = (0,0,H)$, $\vec{A} = (-By,0,0) \approx (-Hy,0,0)$, and $\vec{D} = \nabla + i(2\pi/\Phi_0)\vec{A}$. Note the Gibbs energy above does not involve

- The magnetic penetration depth is defined as $\lambda = \frac{c}{2e^*}\sqrt{m^*\beta/\pi a T_c}$.
- The correlation length is defined as $\xi = \hbar/\sqrt{2m^* a T_c}$.
- The upper critical field is defined as $H_{c2} = \Phi_0/2\pi\xi^2$.

Note the temperature dependence is dropped from all the definitions. To get a neat expression, we can do rescaling

$$\overline{x} = x/\xi,\ \overline{y} = y/\xi,\ \overline{\Psi} = (\beta/2aT_c)^{\frac{1}{2}}\Psi,\ b = B/H_{c2},\ h = H/H_{c2} \tag{12}$$

$$\overline{G} = \int d\overline{x}d\overline{y}\big[\overline{\Psi}^*\hat{H}\overline{\Psi} - a_H|\overline{\Psi}|^2 + \frac{1}{2}|\overline{\Psi}|^4 + \frac{\kappa^2(\vec{b}-\vec{h})^2}{4}\big] \tag{13}$$

where the linear operator $\hat{H} = -(D_{\overline{x}}^2 + \partial_{\overline{y}}^2 + b)/2$, and $a_H = (1-t-b)/2$. Minimizing the Gibbs energy with respect to $\overline{\Psi}$

$$\hat{H}\overline{\Psi} - a_H\overline{\Psi} + |\overline{\Psi}|^2\overline{\Psi} = 0 \tag{14}$$

and drop the nonlinear term at first (it will be added back later) to get

$$\hat{H}\overline{\Psi} = a_H\overline{\Psi} \tag{15}$$

Assume the wavefunction is

$$\overline{\Psi}(\overline{x}, \overline{y}) = e^{ik_x\overline{x}} f(\overline{y}) \tag{16}$$

This results in the familiar displaced quantum harmonic oscillator equation

$$[-\frac{1}{2}\partial_{\overline{y}}^2 + \frac{b^2}{2}(\overline{y} - k_x/b) - \frac{b}{2}]f = \frac{1-t-b}{2}f$$

which gives quantized ernergies

$$E_N = \frac{1-t-b}{2} = (N + \frac{1}{2})b_N - \frac{1}{2}b_N = Nb_N,\ N = 0, 1, 2, \cdots \tag{17}$$

and corresponding eigenfunctions

$$\phi_{Nk_x} = \pi^{-\frac{1}{4}}\sqrt{\frac{b}{2^N N!}} H_N[b^{\frac{1}{2}}(\overline{y} - k_x/b)]e^{ik_x\overline{x} - \frac{b}{2}(\overline{y} - k_x/b)^2)} \tag{18}$$

where H_N are Hermite polynomials. Recall the definition of the upper crtical field. It can be seen that $N = 0$ (lowest Landau level, short for LLL) gives the largest b at a temperature $t < 1$, or we can say

$$a_H = \frac{1 - t - b_0(t)}{2} = 0 \tag{19}$$

defines the $H_{c2}(T)$ line on the phase diagram.

Now the nonlinear term can be added back as a perturbation. As in the perturbation theory of quantum mechanics, all the eigenstates of $\hat{H}$ should be included as the basis functions. But, since we are considering a magnetic field slightly below H_{c2}, so it is legitimate to only include the $N = 0$ states, or the LLL states as the basis functions. Therefore a variational function can be chosen

$$\overline{\Psi} = \sum_{k_x} C_{k_x}\phi_{k_x}(\vec{r}) \tag{20}$$

where C_{k_x} are variational parameters. However, because the Landau levels are highly degenerate, this variational function is still impractical. Generally, we know that symmetry can be used to reduce the degrees of freedom. At this point, it is the lattice structure that needs to be introduced.

Assuming that the vortices are closely packed into a lattice. And assume that "closely packed" means hexagonal structure (the other structure can be calculated in the same way). Then the two lattice vectors can be denoted as $\vec{d_1} = d(1, 0),\ \vec{d_2} = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

The periodicity in the d_1 or $\vec{x}$ direction means k_x is quantized, i.e. $k_x = 2\pi n/d,\ n = 0, \pm 1, \pm 2, \cdots$. This simplifies the variantional function a little.

$$\overline{\Psi} = \sum_n C_n \phi_n(\vec{r}) \tag{21}$$

It is possible to make further simplification, such as making the two-parameter ansatz $C_{n+2} = C_n$ and $C_1 = iC_0$, which essentially reduces the number of variantional parameters to one. Specific calculations using the variational functions will not be provided here, but it should be mentioned that the nonlinear term lifts the degeneracy of the basis functions.

Now that we know, at least in principle, how to calculate the order parameter field, let's look back at an approximation made earlier to check the consitency.

With the LLL approximation, $\hat{H}\overline{\Psi} = E_0\Psi = 0$, but $a_H = (1 - t - b)/2$ is assumed to be slightly above zero.

$$\overline{G} = \int d\vec{r}\big[-a_H|\overline{\Psi}|^2 + \frac{1}{2}|\overline{\Psi}|^4 + \frac{\kappa^2(\vec{b} - \vec{h})^2}{4}\big] \tag{22}$$

Minimizing with respect to b

$$\kappa^2(h - b(\vec{r})) = |\overline{\Psi}|^2 \tag{23}$$

or

$$b(\vec{r}) = h - \frac{1}{\kappa^2}|\overline{\Psi}|^2 \tag{24}$$

Recall that $\kappa \gg 1$ and $\overline{\Psi}$ is greatly suppressed, so this result is indeed consistent with the approximation that $b \approx h$. Also, it gives the inhomogeneous distribution of $b(\vec{r})$ to the order of κ^{-2}. Fig. 3 shown above is from Rosenstein and Li, Jan 2010, based upon the method mentioned above with hexagonal structure and the two-parameter ansatz. More acuurate results may be obtained by expanding the order paramter field $\overline{\Psi}$ and the magnetic field b in power of κ^{-2}.

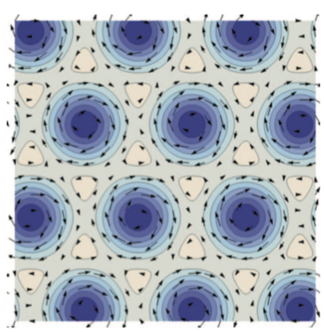


Figure 3: Hexagonal vortex lattice (arrows indicate directions of the superflow)

2.2 Excitations of Vortex Lattices

The excitation of the vortex lattice can also be called "phonons". Since these excitations may break the translational symmetry in $\hat{z}$ direction, it is better to include the $\hat{z}$ direction terms as mentioned above. Still, LLL approximation is assumed. Note that the rescaled $\hat{H}\overline{\Psi} = 0$ in the original units is

$$-\frac{\hbar^2}{2m^*}D^2\Psi = \frac{\hbar e^*}{2m^* c}B\Psi \tag{25}$$

So the we can adopt the following form for the Gibbs energy in the original units.

$$G = \int dxdydz\big[-\frac{\hbar^2}{2m_c^*}|\partial_z\Psi|^2 + aT_c(1-t-b)|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \frac{\kappa^2(\vec{b}-\vec{h})^2}{8\pi}\big] \tag{26}$$

where m_c^* is the effective mass in the $\hat{z}$ idrection. Note there is no derivative term in the trasverse direction. Define the Ginzburg number as

$$Gi = \Big(\frac{e^{*2}\kappa^2\xi T_c\gamma_a}{2\pi c^2\hbar^2}\Big)^2 \tag{27}$$

where $\gamma_a \equiv (m_c^*/m^*)^{1/2}$ is a measure of the anisotropy. Similarly, it is useful to do rescaling

$$x = \frac{\xi}{\sqrt{b}}\overline{x},\ y = \frac{\xi}{\sqrt{b}}\overline{y},\ z = \frac{\xi}{\gamma_a}(\frac{\sqrt{Gi}tb}{4})^{\frac{-1}{3}}\overline{z},\ \Psi^2 = \frac{2aT_c}{\beta}(\frac{\sqrt{Gi}tb}{4})^{\frac{2}{3}}\psi^2 \tag{28}$$

where t and b are the same as before. In the rescaled units, the Boltzman factor becomes

$$g[\psi, b] \equiv \frac{G[\Psi, B]}{T} = \frac{1}{2^{5/2}\pi}f[\psi] + \frac{\kappa^2}{2^{5/2}\pi}(\frac{\sqrt{Gi}tb}{4})^{-4/3}\int d\overline{x}d\overline{y}d\overline{z}\frac{(\vec{b}-\vec{h})^2}{4} \tag{29}$$

$$f[\psi] = \int d\overline{x}d\overline{y}d\overline{z}\big[\frac{1}{2}|\partial_{\overline{z}}\psi|^2 + a_T|\psi|^2 + \frac{1}{2}|\psi|^4\big] \tag{30}$$

$$a_T = -\frac{1-t-b}{2}(\frac{\sqrt{Gi}tb}{4})^{-2/3} \tag{31}$$

Again, it is usefule to invoke the symmetry argument. Based on the structure of the lattice (rhombic, square, ...), it is better to choose some functions $\{\varphi_k(\vec{r})\}$ as the basis functions. Then $\psi(\vec{r})$ can be projected onto this basis.

$$\psi(\vec{r}) = \frac{1}{(2\pi)^{3/2}}\int_k \varphi_k(\vec{r})\psi_k \tag{32}$$

where ψ_k are the new variables. The functional $f[\psi]$ can be rewritten using ψ_k. The first two terms of $f[\psi]$ are quadratic, and last quartic term is regarded as the interaction. As in the case of weakly interacting boson condensates, one can to define

$$\psi_k = v_0(2\pi)^{3/2}\delta_k + \frac{c_k}{\sqrt{2}}(O_k + iA_k) \tag{33}$$

where v_0 represents the condensation part, and O_k and A_k are real fields which are analogous to the optical and acoustic modes in the crystals. Substitute Eq. (33) into the quadratic term of $f[\psi]$. By choosing c_k properly, the quadratic terms can be diagonalized in the forms of $O_k^* O_k$ and $A_k^* A_k$, with the coefficients as the excitation energies, respectively. The acoustic dispersion relation can be shown to be proportional to $|\vec{k}|^4$, which means the "acoustic phonons" are "supersoft".

Also, the effect of the quartic term can be included perturbatively, by summing over Feynman diagrams. From here on, lots of caculation details have to be replaced by hand-waving arguments for simplicity.

2.3 Vortex Liquid

To deal with vortex liquid, one can still start with Eq. (30), but the lack of lattice structure means no basis of functions is particularly favored. There are generally three approaches to calculation of the vortex liquid energy density, with increasing reliability.

1. Treat the quadratic quartic terms as a "free theory", and then include quartic term perturbatively by summing over Feynman diagrams directly.

2. Put a portion of the quadratic term into the perturbation, and treat this portion as a variaional parameter. Then calculate the energy density perturbatively. Finally minimizing the energy density with respect to the variational parameter.

3. Based on Method 2, and use the Borel-Pade resummation technique.

Comparing the vortex liquid energy density with the vortex lattice energy density leads to determination of the melting line, or actually determination of the value of a_T. In this way, it can shown that there is a specific heat jump and a magnetization jump across the melting line. As Li and Rosenstein argues, the theoretical calculations fit very well with the experimental data[5]. The melting of the vortex lattice is believed to be of first order.

2.4 Disorder and Vortex Glass

Vortex glass phase mainly owes its existence to disorder. Impurities and defects generally act attractively to pin the vortex. This random pinning can destroy the vortex lattice, and is most prominent when fluctuation is not too high. The competition between the disorder and fluctuation will severely increase the complexity of the problem, such as in high Tc superconductors.

The usual approach to vortex glass is by adding white noise and using replica trick.

Assuming that the white noise $W(\vec{r})$ is introduced to the coefficient of the quadratic term of the GL free energy (as a random disturbance to the local temperature)

$$F = \int d\vec{r}\Big[\frac{\hbar^2}{2m^*}|\vec{D}\Psi|^2 + \frac{\hbar^2}{2m_c^*}|\partial_z\Psi|^2 + a(T-T_c)(1+W(\vec{r}))|\Psi|^2 + \frac{\beta}{2}|\Psi|^4\Big] \quad (34)$$

So the partition function is

$$Z = \int \mathcal{D}\Psi\mathcal{D}\Psi^* \; exp\Big\{-\frac{1}{k_BT}\big[F[\Psi] + \frac{1-t}{2}\int_r W(\vec{r})|\Psi|^2\big]\Big\} \quad (35)$$

The replica trick comes into play by writing the Helmholtz free energy as

$$\overline{\mathcal{F}} = -k_BT\lim_{n\to 0}\frac{1}{2}\overline{(Z^n-1)} \quad (36)$$

And Z^n can be regarded as a statistical summation of n identical copies of the order parameter field Ψ, thus

$$\overline{Z^n} = \int \mathcal{D}W \; exp[-\frac{1}{2n}\int_r W^2(\vec{r})]Z^n[W] = \int\prod_{a=1}^{n}\{\mathcal{D}\Psi_a\mathcal{D}\Psi_a^*\}e^{-\frac{1}{k_BT}F_n} \quad (37)$$

$$F_n = \sum_a F[\Psi_a] + \frac{1}{2}\gamma(t)\sum_{a,b}\int_r|\Psi_a|^2|\Psi_b|^2 \quad (38)$$

After averaging over the disorder, different replicas get a interacting term. And F_n serves as a starting point for calculating the physical properties of the vortex glass.

Replica symmetry means the invariance of the physical quantities under the permutation of different replicas, or the index a above. And the Bragg glass is believed to spontaneously break the replica symmetry. Rosenstein and Li confirmed that white noise added to the quadratic terms does not spontaneous break the replica symmetry, but white noise added to the quartic term does.

3 Techniques on Measuring Vortices

Here, the usual techniques on measuring vortices are briefly introduced.

3.1 Bitter Decoration

Bitter Decoration provides a indirect illustration of the vortex lattice. It works in a pretty simple way.

1. Spray ferromagnetic powder onto the surface of the superconductor.

2. The tiny ferromagnets are attracted to the locations of the vortex and align themselves in a certain pattern.

This is among the earliest methods to verify the existence of the Abrikosov lattices. But this method cannot be used to extract interesting information about the vortices, and is limited to low magnetic field.

3.2 Scanning Tunneling Spectroscopy

The IV curve obtained by scanning tunneling spectroscopy can be strongly sensitive of vortex core, because of the states confined in the vortex cores. STM can be used under the whole range of the magnetic field, and can detect the change of density of states in the vortex cores.

3.3 μSR Technique

μSR is a indirect method for probing the vortices, and is short for "muon spin rotation". Usually, it uses powdered superconductor. The way it works is as follows.

1. Highly polarized muons are shot onto the surface of the superconductor. The muons' spins will rotate under the influence of the local magnetic of surface.

2. The muons decay and emit positron mostly in the direction of the muon's spins.

3. By measuring the distribution of the positrons, the polarization of the muons' spins in the time domain can be obtained.

4. The inhomogeneous magnetic field of the surface can be extracted, using the details of the London model or the GL models.

The μSR technique can provide information on some physical quantities, such as the penetration depth and correlation length[6], but the information provided is model-dependent.

4 Summary

The study of vortices in type-II superconductors have a long history. As can be seen above, the London model and the GL model are useful in different ranges of the magnetic field, and do a decent job on describing various phases of the vortices. However, since they are all phenomenological theories, neither is capable of providing information on the microscopic details. It is natural that we ask questions, like how does the superconducting gap react to the creation of vortex lattice? How does the creation of vortex lattice influence the pair mechanism and the quasiparticle spectrum near the Fermi surface? These questions are really important. Because there are people who want to use de Haas-Van Alphen technique to study type-II and high Tc superconductors. And there is intriguing observation of paseudogap in the vortex cores of cuprate superconductors. However, these questions, linking the vortices to the microscopic aspects of the superconductors, have not yet had complete answers.

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