J. Distler, G. Moore.

\[ \sum^2 \phi \rightarrow X^{10} \]

compact 2-manifold "worldsheet"

smooth manifold "space-time"

2-dim theory \[\rightarrow\] 10-dim theory

Today: given double cover

\[ X_w \xrightarrow{\sigma} X_{10} \]

"space-time with involution"

This will be on orientifold.
Two important cases:

I) trivial (Type I).

II) section $X_w \xrightarrow{\text{fix}} X$ (Type II).

Jacques showed me a formula.

$$i : F \longrightarrow X_w \quad \text{fixed point set}$$

$\text{RR charge} \# \quad \text{in 2d theory [Roraro, same, same?]}

= \pm 2^\# \ i_x \left( \int \frac{L'(F)}{L'(W)} \right)

\text{the normal bundle at } F \to X_w

L' = \prod \frac{x/4u}{\tanh x/4u} \quad \text{the Bott element in K-theory}

looks similar to Hirzebruch L-genus
1. Definition of fields / theory
2. Derive RR charge formula over $\mathbb{Z}_{\left[\frac{1}{2}\right]}$ from 10d.
3. Anomaly cancellation in 2d.

Some background

Differential cohomology

\[ h^*(-; \mathbb{Q}) = H(-; \overline{\frac{h(pt; \mathbb{Q})}{h_{\mathbb{Q}}}}). \]

Will tensor with reals.

\[ h^*(\cdot) \rightarrow H(\cdot; \mathbb{R}) \]

\[ h^*(M) \rightarrow \Omega(M; h_{\mathbb{R}})^* \]

\[ \Omega(M; h_{\mathbb{R}})^* \rightarrow H(M; h_{\mathbb{R}}) \]

Differential cohomology is the "fiber product", i.e. a class is \( a \in h(M) \), \( b \) in \( \Omega(M; h_{\mathbb{R}})^{\text{closed}} \), and a homotopy from \( a \) to \( b \) in \( H(M; h_{\mathbb{R}}) \).
Exact sequences:

\[ \begin{array}{cccccc}
\Rightarrow & h^q(M) & \xrightarrow{\text{curv}} & \Omega(M; h_{\text{ic}})^{\mathbb{Z}} & \xrightarrow{\text{integral forms}} & 0 \\
0 & \Rightarrow & h(M; h_{\text{ic}} \otimes \mathbb{R}/\mathbb{Z})^{q-1} & \Rightarrow & 0
\end{array} \]

\[ 0 \rightarrow \text{forms} \rightarrow \check{h}^q(M) \rightarrow h^q(M) \rightarrow 0 \]

For ordinary diff cohomology: Cheeger + Simons, and Deligne.

Also Hopkins + Singer.

Functorial: given \( h \), get differential cohomology.

\[ h^q(M) = \Pi_0 \left( \text{Map}(M, h^q) \right). \]

so really objects in higher groupoids.

(Need \( \Pi_1, \ldots \))

Also for \( \check{h}^q(M) \).
We will freely use equivariant version, though not entirely worked out.

\[ H^4(M) = \text{Maps}(M, \mathbb{T}) \quad H^4(M) = \pi_0 \text{Maps}(M, \mathbb{T}) \]

\[ H^3(M) = \left\{ \begin{array}{c}
\text{objects of } \mathbb{T} \\
\text{with connection}
\end{array} \right\} \quad H^3(M) = \]

\[ H^3(M) = \text{gerbes, bundle gerbes etc.} \]

In Dirac quantization of monopoles... good example.

These enter in defining topological terms (eg WZW) or gauge theory (Dirac charge quant.)
Twistings of $KR(X_w)$

Twistings could be groupoid approach.

Object in $KR^0(X_w) = \begin{array}{c} E \\ \downarrow \\ X_w \end{array}$

- $\mathbb{Z}_2$-graded complex vector bundle
- twisted
- $^+$ lift of involution

$\sigma^* : E \rightarrow E$

$\sigma^\sigma = \pm 1$.

Special cases:

I) $\sigma$ acts trivially: $K_0^0(X_w)$.

II) $X_w \longrightarrow X \quad K^0(X)$.

May have to

Twistings Pass to a locally equivalent groupoid

$Y_w \longrightarrow Y$

$Y$ is a groupoid

$Y_0 \overset{p_0}{\leftarrow} Y_1 \overset{p_1}{\leftarrow}$
Notation: \[ \phi V = \begin{cases} V, & \phi = 0 \\ \overline{V}, & \phi = 1. \end{cases} \]

**Definition** A twisting of \( KR(X_w) \) is a locally equivalent \( Y_w \to Y \) and a triple \( \tau = (d, L, \theta) \) where:

- \( d : Y_0 \to \mathbb{Z} \) continuous \( \mathbb{Z}_2 \)-graded
- \( L : Y_0 \to Y_1 \) a hermitian line bundle
- \( \theta : \phi L_g \otimes L_f \to L_{gf} \) (\( a \xrightarrow{f} b \xrightarrow{g} c \))

+ Cyclic conditions:
  \( d(b) = d(a) \) for \( (a \xrightarrow{f} b) \).
+ Cyclic data for \( \theta \).
Cohomology group:

For $K(X)$,

$$
\prod_{\tilde{k} \in \mathbb{Z}/2\mathbb{Z}} \tilde{k} \cong \left\{ \mathbb{Z}, \mathbb{Z}/2\mathbb{Z}, 0, \mathbb{Z} \right\}
$$

So without $d$, $\mathbb{Z}_2$-grading, get cohomology class in $3$.

Now also in $0$ and $1$.

For $KO(X, W)$, a Postnikov section of connected $KO$.

$KR(X, W)$; the iso classes are

$$
H^0(X; \mathbb{Z}) \times H^1(X; \mathbb{Z}/2\mathbb{Z}) \times H^3(X; \mathbb{Z}).
$$

as a set.

For differential theory, add connections. Get twisted $B$-fields, etc.

$u \in K^2(\text{pt})$

lifts to $KR^3(\text{pt})$, $4C_1 = 0$

concrete model for $u$ (PTO)
$C^{1/1} = C \oplus \pi C$.

\[\gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

\[\gamma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]

\[\sigma (\overline{\gamma^+}, \overline{\gamma^-}) = (\overline{-\gamma^-}, \overline{\gamma^+}) \]

an odd map which squares to minus the identity.

In particular,

\[\tau \rightarrow \tau \rightarrow \tau + (\tau_1 + \tau_2)\]

i.e. An action of Bolt periodicity on these twistings
Defn An NSAF15 supersymmetric background is

\[ \text{Narow} \quad \text{Schur} \times 2 \]

\[ \Downarrow \text{dilaton data.} \]

(i) \( \mathcal{X} \) a smooth 10d orbifold w/metric and real function

(ii) \( \eta^\text{r} : \mathcal{X}_w \rightarrow \mathcal{X} \) orientifold double cover.

(iii) \( \beta \) a differential twisting of \( \text{KR}(\mathcal{X}_w) \).

\( \text{B-field} \)

(iv) \( K : \mathcal{R}(\beta) \rightarrow \mathcal{C}^{\text{Ko}}(\mathcal{X}^2 - 2) \)

iso of twistings of \( \text{KO}(\mathcal{X}) \)

\[ \left( \begin{array}{c}
\text{a twisted spin structure}
\end{array} \right) \quad \text{class of double cover} \]

\[ \Rightarrow \begin{cases}
\mathcal{W}_1(\mathcal{X}) = tw \\
\mathcal{W}_2(\mathcal{X}) = tw^2 + ow
\end{cases} \]

COME FROM

\[ t = 0 \rightarrow \text{type B} \]

\[ t = 1 \rightarrow \text{type A} \]

There's a Bott shift.

\( \hat{\beta} \rightarrow \hat{\beta} + (\hat{\tau} + 3) \)

\( K \rightarrow (u \hat{\upsilon})^{-1} K \).
A worldsheet $\Sigma$ metric

- with a spin structure

on $\hat{\Sigma} \rightarrow \Sigma$

orientation double cover.

The worldsheet is a map

$\phi: \Sigma \rightarrow X$

plus pullback

$\phi^* W \cong \hat{W}$.

i.e.

\[
\begin{array}{ccc}
\Sigma & \longrightarrow & X_W \\
\downarrow & & \downarrow \\
\Sigma & \longrightarrow & X
\end{array}
\]

spinor fields $\psi, \chi$ on $\hat{\Sigma}$.

\[
e^{-\text{Eff action}} = \left\{ \text{Pfaffian } D_{\Sigma} (\phi^* TX - 2) \right\}
\]

$\times \exp 2\pi i \int_{\Sigma} \phi^* \phi^* + \ldots$
Parameter space $S$.

Section of $L_1$ \( \downarrow \)

1st term in action

Section of $L_2$ \( \downarrow S \)

Second term.

**Theorem:** There is a (canonical) trivialization of $L$.

That's good, because we need a function to integrate.

**Anomaly:**

1. Integrals over fermions \( \ldots \text{eg. } L_1 \text{ above} \) \( \quad \text{(has to do with Pfaffian line bundles)} \)
   \( \quad \ldots \text{geometric index theory of Dirac operators} \)

2. Simultaneous electric and magnetic current
   \( \quad \text{(or self-dual current)} \)

3. Boundaries of topological terms, eg $WZW$, Chern-Simons
(4) **NEW**: exotic orientation \((L_B)\).

Roughly: to integrate \(B\)-field,

\[
\text{(use Schreiber-Waldorf for bosonic case)}
\]

need spin structure.

Insert extra classes: Differential KO of a point is not trivial!

\[
e^{-\text{Eff action}} = \text{pfoffin of } D^*_V (\phi^*TX - i)
\]

\[
\text{exp } 2\pi i \sum \hat{E} \phi^* B
\]

\[
\text{extra classes}
\]