Thomas Schick.

Smooth refinement of cohomology. Idea: combine cohomology and differential forms. Main diagram is a commutative square: $I\colon \hat{H}^*(M) \to H^*(M)$, $R\colon \hat{H}^*(M) \to \Omega^*_c(M)$, $\Omega^*_c(M) \to H^*_{dR}(M)$, and $\mathrm{ch}\colon H^*(M) \to H^*_{dR}(M)$. We also have a natural transformation $a\colon \Omega^{*-1}(M)/\mathrm{im}(d) \to \hat{H}^*(M)$. We want an exact sequence: $H^{*-1}(M) \to \Omega^{*-1}(M)/\mathrm{im}(d) \to \hat{H}^*(M) \to H^*(M) \to 0$. We also want to amend this sequence to a commutative diagram with morphisms $d\colon \Omega^{*-1}(M)/\mathrm{im}(d) \to \Omega^*_c(M)$ and $R\colon \hat{H}(M) \to \Omega^*_c(M)$.

Definition. Given cohomology theory E^*, a smooth refinement $\hat{E}^*$ is a functor $\hat{E}\colon \Pi F \to GRPS$ together with transformations I, R, a. We have to use differential forms with values in $E^*(\bullet) \otimes \mathbf{R}$.

Definition: If E^* is multiplicative, we "say" $\hat{E}^*$ is multiplicative if $\hat{E}$ takes values in graded rings and the transformations are compatible with multiplication, where $a(\omega) \cup x = a(\omega \wedge R(x))$ for all $\omega \in \Omega(M)$ and $x \in \hat{E}(M)$.

Definition: $\hat{E}$ has S^1-integration if there is a natural transformation in M: $\int\colon \hat{E}^*(M \times S^1) \to \hat{E}^{*-1}(M)$ compatible with the integration of formas and such that $\int$ of p^* is zero and conjugation changes the sign.

Homotopy formula: For every $\hat{E}$ and smooth map $h\colon \Pi \times [0,1] \to N$ we have

$$h_1^*(x) - h_0^*(x) = a(\int_{\Pi \times [0,1]/M} h^*(R(x))$$

for all $x \in \hat{E}(M)$.

Theorem (Hopkins-Singer): Additive extensions exist.

It's not evident how to obtain multiplicative structure.

Theorem: Using geometric models multiplicative smooth extensions with S^1-integration are constructed for K-theory (Bunke-Schick, based on local index theory); MU-bordism (Bunke-Schröder-Schick-Wethamp) and from there Landweber exact cohomology theories.

Uniqueness questions: Assume E^* satisfies $E^k(\bullet)$ is torsion for odd k. Then any two smooth extensions with S^1-integration are isomorphic up to unique isomorphism.

If $\hat{E}$ and $\tilde{E}$ are multiplicative the isomorphism is multplicative as well.