

3-Morphisms in the Degree 2 Degeneracy Maps of Street Nerves

Table of Contents

1	Preliminaries.....	1
2	Extra 2-Unitors	2
3	The 3-Morphisms in the Degree 2 Degeneracy Maps of the Street Nerve.....	6
3.1	$s_0^2(\sigma)$	6
3.2	$s_1^2(\sigma)$	7
3.3	$s_2^2(\sigma)$	8

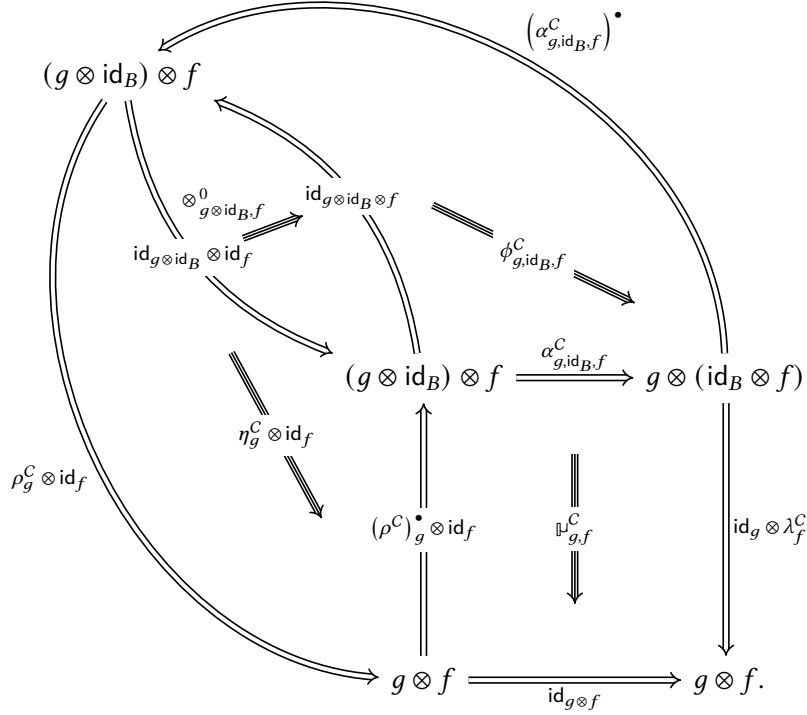
1 Preliminaries

First recall that the components of the left, middle, and right 2-unitors of C are invertible 3-morphisms of the form

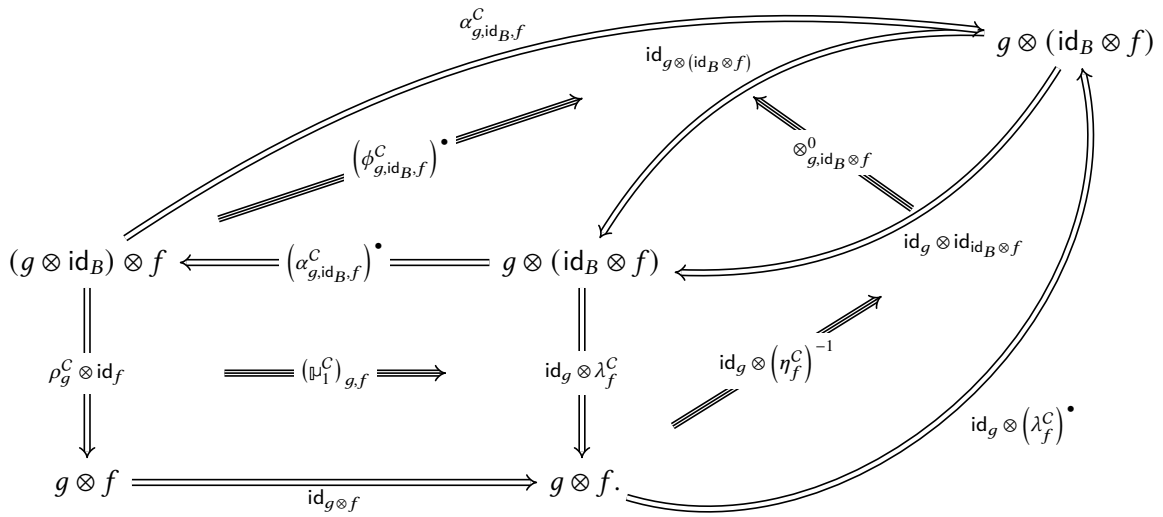
$$\begin{array}{ccc}
 (\text{id}_C \otimes g) \otimes f & \xrightarrow{\lambda_g^C \otimes \text{id}_f} & g \otimes f \\
 \searrow \alpha_{\text{id}_C, g, f}^C & \Downarrow \lambda_{g, f}^C & \nearrow \lambda_{g \otimes f}^C \\
 & \text{id}_C \otimes (g \otimes f) &
 \end{array}
 \qquad
 \begin{array}{ccc}
 (g \otimes \text{id}_B) \otimes f & \xrightarrow{\alpha_{g, \text{id}_B, f}^C} & g \otimes (\text{id}_B \otimes f) \\
 \uparrow (\rho_g^C) \bullet \otimes \text{id}_f & \Downarrow \mu_{g, f}^C & \downarrow \text{id}_g \otimes \lambda_f^C \\
 g \otimes f & \xrightarrow{\text{id}_{g \otimes f}} & g \otimes f
 \end{array}$$

$$\begin{array}{ccc}
 g \otimes f & \xrightarrow{\text{id}_g \otimes (\rho_f^C) \bullet} & g \otimes (f \otimes \text{id}_A) \\
 \searrow (\rho_{g \otimes f}^C) \bullet & \Downarrow \mu_{g, f}^C & \nearrow \alpha_{g, f, \text{id}_A}^C \\
 & (g \otimes f) \otimes \text{id}_A &
 \end{array}$$

1. μ_1^C : The component at f of μ_1^C is given by the following pasting diagram:



2. μ_2^C : The component at f of μ_2^C is given by the following pasting diagram:



3. μ_3^C : The component at f of μ_3^C is given by the pasting of the diagram

$$\begin{array}{ccc}
 (g \otimes \text{id}_B) \otimes f & \xrightarrow{\alpha_{g, \text{id}_B, f}^C} & g \otimes (\text{id}_B \otimes f) \\
 \downarrow \rho_g^C \otimes \text{id}_f & \Uparrow (\mu_2^C)_{g, f} & \uparrow \text{id}_g \otimes (\lambda_f^C)^\bullet \\
 g \otimes f & \xrightarrow{\text{id}_{g \otimes f}} & g \otimes f \\
 \downarrow \text{id}_{g \otimes f} & \Uparrow (\lambda(g \circ f))^{-1} & \downarrow \text{id}_{g \otimes f} \\
 g \otimes f & & g \otimes f
 \end{array}$$

$\text{id}_{g \otimes f} \circ \text{id}_{g \otimes f} \xrightarrow{\text{id}_{g \otimes f}} g \otimes f$

$\text{id}_g \otimes \lambda_f^C$

$((\eta_f^C)^\bullet)^{-1}$

in $\mathbf{Hom}_C[A, C]$, where $\lambda(g \circ f): (\text{id}_{g \otimes f} \circ \text{id}_{g \otimes f}) \Rightarrow \text{id}_{g \otimes f}$ is the invertible 3-morphism defined as the composition

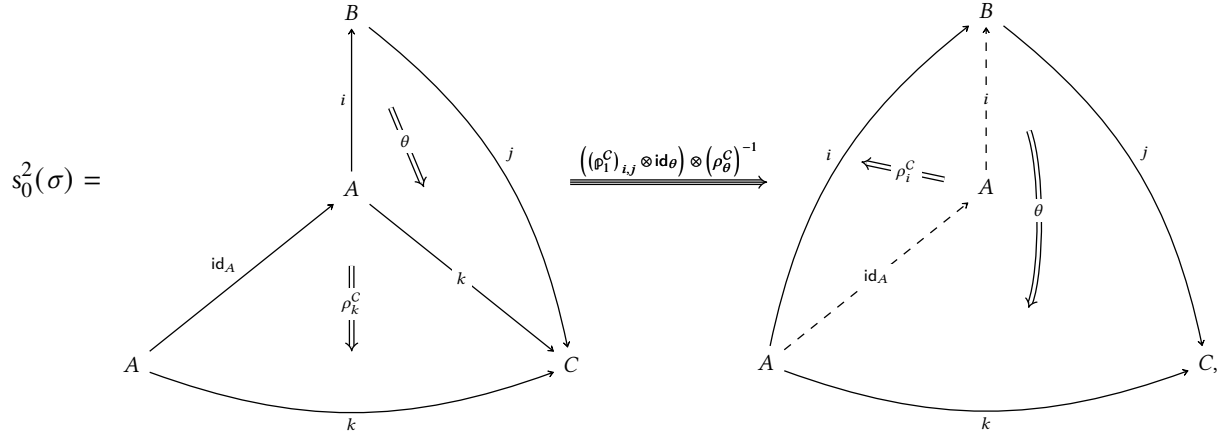
$$\begin{array}{c}
 \text{id}_{g \otimes f} \circ \text{id}_{g \otimes f} \\
 \Downarrow \\
 (\otimes_{g, f}^0)^{-1} * (\otimes_{g, f}^0)^{-1} \\
 \Downarrow \\
 (\text{id}_g \otimes \text{id}_f) \circ (\text{id}_g \otimes \text{id}_f) \\
 \Downarrow \\
 \otimes_{(\text{id}_g \otimes \text{id}_f), (\text{id}_g \otimes \text{id}_f)}^2 \\
 \Downarrow \\
 (\text{id}_g \circ \text{id}_g) \otimes (\text{id}_f \circ \text{id}_f) \\
 \Downarrow \lambda^{\mathbf{Hom}_C(B, C)} \otimes \lambda^{\mathbf{Hom}_C(A, B)} \\
 \text{id}_g \otimes \text{id}_f \\
 \Downarrow \otimes_{g, f}^0 \\
 \text{id}_{g \otimes f}
 \end{array}$$

$\lambda(g \circ f)$

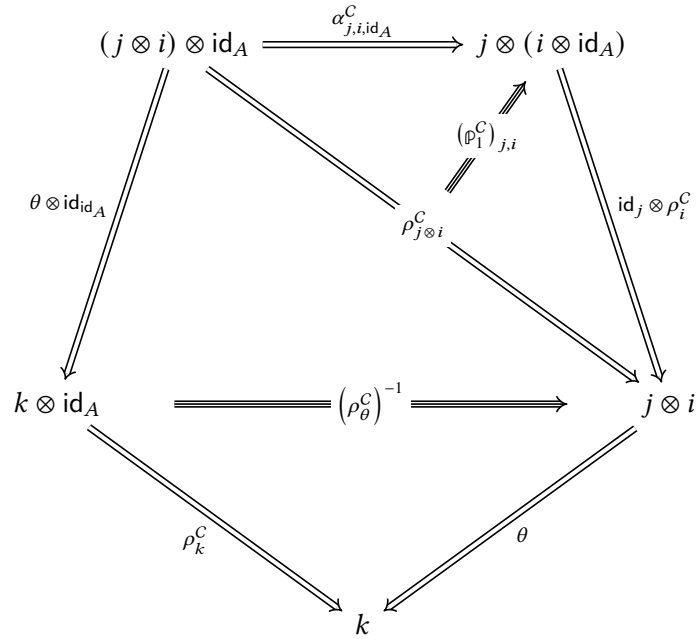
in $\mathbf{Hom}_C(B, C) \times \mathbf{Hom}_C(A, B)$.

3 The 3-Morphisms in the Degree 2 Degeneracy Maps of the Street Nerve

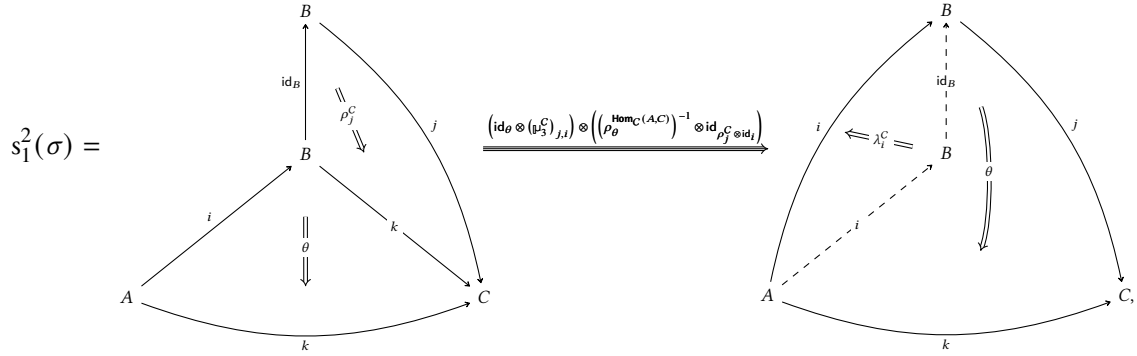
3.1 $s_0^2(\sigma)$



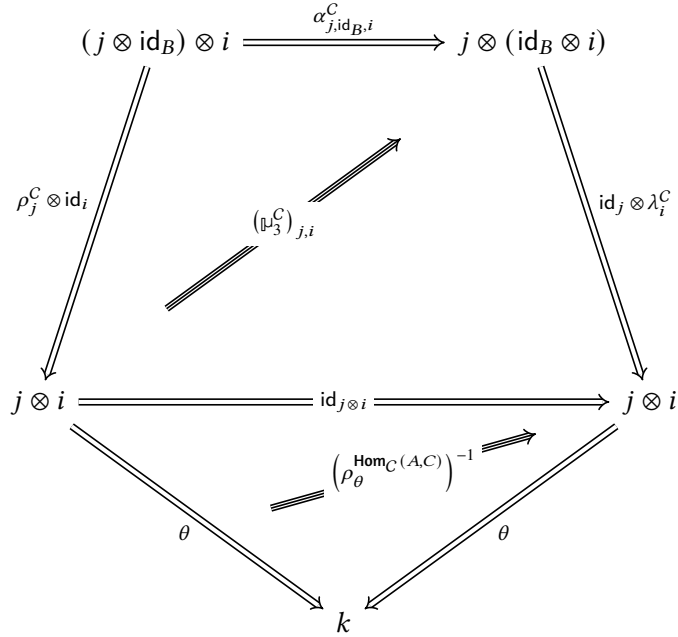
where $\left((\rho_1^C)_{i,j} \otimes \text{id}_\theta \right) \otimes (\rho_\theta^C)^{-1}$ is the pasting of the diagram



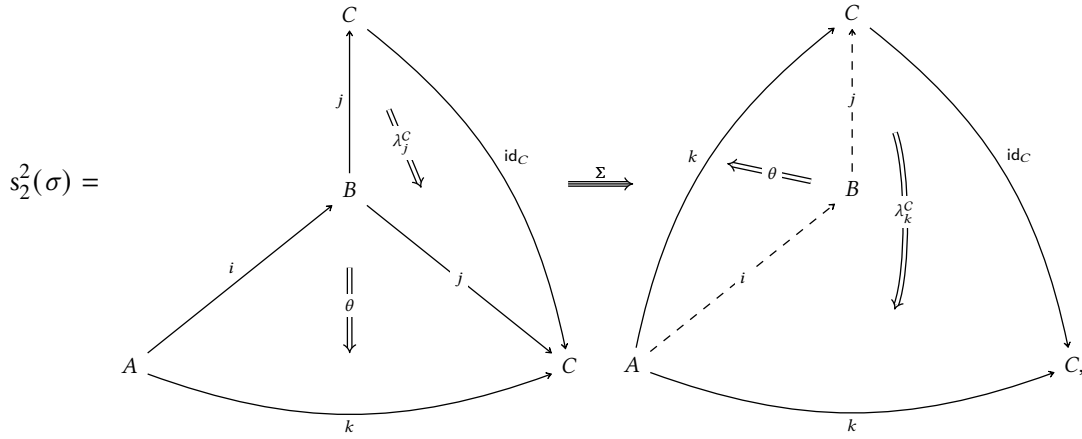
3.2 $s_1^2(\sigma)$



where $(\text{id}_\theta \otimes (\nu_3^C)_{j,i}) \otimes \left((\rho_\theta^{\text{Hom}_C(A,C)})^{-1} \otimes \text{id}_{\rho_j^C \otimes \text{id}_i} \right)$ is the pasting of the diagram



3.3 $s_2^2(\sigma)$



where Σ is the pasting of the diagram

