

I have read the nlab entry on ‘geometric realization of simplicial topological spaces’ (specifically the entry which was last updated on January 11 2011). I believe that all of the mathematics is correct. There are however, a couple of things which I think the article should say, but it does not say. I will give some suggestions below that I think should or could be incorporated into the article, and suggest some references which might also be included.

1. After it is explained that the geometric realization of a simplicial space extends to define a functor $|-|: \mathbf{Top}^{\Delta^{\text{op}}} \rightarrow \mathbf{Top}$, I think it should be noted that $|-|$ has a right adjoint $S: \mathbf{Top} \rightarrow \mathbf{Top}^{\Delta^{\text{op}}}$ which sends a topological space T to the simplicial space $S(T)$ whose space of n -simplices is the space $\mathbf{Hom}(\Delta^n, T)$ of singular n -simplices. One might even note later in the entry that $|S(T)| \rightarrow T$ is a weak homotopy equivalence for any topological space T (see Proposition 3.1 of Seymour ‘*Kan fibrations in the category of simplicial spaces*’). There is actually an interesting discussion on this last point at mathoverflow — see the question here.
2. I’m not quite sure what the philosophy of the nlab is with such matters, but I thought after defining geometric realization via the coend formula, that the article could have mentioned that this formula reduces to the description as a quotient of $\coprod_{n=0}^{\infty} X_n \times \Delta^n$, especially as this is the description which is used in May’s book and in Segal’s original paper. As far as I am aware — which does not count for much — it seems that this paper ‘*Classifying spaces and spectral sequences*’ was the first place where the geometric realization of a simplicial space was defined, although it was implicit in earlier descriptions of classifying spaces. Further in this direction it seems that Mac Lane’s article ‘*The Milgram bar construction as a tensor product of functors*’ SLNM Vol. 168 was one of the first places where it was recognized at least implicitly that geometric realization was an example of a coend.
3. The proposition which states that there is a homeomorphism between the geometric realization of the diagonal simplicial set of the bisimplicial set $Sing(X)$ and the geometric realization of the simplicial space $[n] \mapsto |Sing(X_n)|$ has a generalization: if X is a bisimplicial *space* then there is a homeomorphism between the geometric realization of the diagonal simplicial space, the geometric realization of the simplicial space $[m] \mapsto X_{n,m}$ and the geometric realization of the simplicial space $[n] \mapsto X_{n,m}$. A reference for this is the proof of the Lemma on page 94 of Quillen’s ‘*Higher Algebraic K-theory I*’ (this lemma is attributed to Tornehave by Quillen). An alternative reference is Theorem 1 of the Appendix of Bracho ‘*Haefliger structures and linear homotopy*’.
4. In the subsection Nice simplicial topological space, I think the Proposition which identifies proper simplicial spaces with Reedy cofibrant ones should come immediately after the definition of proper simplicial space. In fact I think that it should be explicitly stated that the latching object in this case is the union of the degeneracies.
5. In the subsection Models for the homotopy colimit I was unclear exactly what was being proven: does the proof just refer to the last statement of the proposition? If so this might be indicated as it is a little confusing. Incidentally a proof that the fat realization coincides with the homotopy colimit is sketched in Dugger’s notes ‘*A primer on homotopy colimits*’ available on his website — see 17.4, and Example 18.2. I think that Dugger’s notes are an excellent resource for this topic and could be included amongst the references.
6. One last comment on this subsection, I would be a little uncomfortable in saying that Segal sketches a proof (see Lemma A.5 of his ‘*Categories and cohomology theories*’ that $||X|| \rightarrow |X|$ is a weak homotopy equivalence when X is good, I think that his proof (while terse) is complete. Also, it is interesting to note that his proof of Lemma A.5 implicitly contains the statement that every good simplicial space is automatically proper.
7. I’m not sure about the intended scope of the entry, but it might be nice to include the example of the geometric realization of the nerve of a topological group G as a model for BG .