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Higher Gauge Theory via Nonabelian Differential Cohomology

and application to bulk/edge correspondence for FQH

via geometric engineering on M2/M5-branes

based in parts on joint work with:

Pinak Banerjee, Domenico Fiorenza, Grigorios Giotopoulos (speaking later today), Hisham Sati

Monographs:

The Character Map in Nonabelian Cohomology, World Scientific (2023)

Equivariant Principal ∞ -Bundles, Cambridge Univ. Press (2026)

Geometric Orbifold Cohomology, CRC Press (2026)

Expositions:

Higher Topos Theory in Physics, Encyclopedia Math. Phys. (2025)

Flux Quantization, Encyclopedia Math. Phys. (2025)

*Complete Top. Quantization
of Higher Gauge Fields*, SciPost Phys. Lect. Notes (2026)

Manuscript:

ncatlab.org/schreiber/show/BBC

NB:

There are *two different* approaches to higher gauge theory:

	principal	cohomological
specified L_∞ -algebra	coefficient of gauge potentials (connections)	coefficient of flux densities (curvatures)
flatness / closure	non-generic	Gauß law / Bianchi identity
examples	heterotic SuGra / B-field on T-folds	type I/II SuGra / 11D SuGra

I started out principal, 20 years ago.
But I think we need cohomological.

Higher Maxwell-type Equations of Motion:

higher fluxes: $\vec{F} := \{F^{(i)} \in \Omega_{\text{dR}}^{\text{deg } i}(X^{1,d})\}_{i \in I}$

higher Bianchis: $d F^{(i)} = P^{(i)}(\vec{F})$ (polynomials, integrable)

higher duality: $\star F^{(i)} = \mu^{(i)}(\vec{F})$ (linear iso)

Examples:

vacuum Maxwell:	$dF_2 = 0$ $dG_2 = 0$	$\star F_2 = G_2$
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5D MCS/SuGra	$dH_3 = \frac{1}{2}F_2 \wedge F_2$ $dF_2 = 0$	$\star F_2 = F_3$
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type IIA NS/RR-flux:	$dF_{2\bullet} = H_3 \wedge F_{2\bullet-2}$ $dH_7 = 0$ $dH_3 = 0$	$\star F_{2\bullet} = F_{10-2\bullet}$ $\star H_3 = H_7$
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11D MCS/SuGra	$dG_7 = \frac{1}{2}G_4 \wedge G_4$ $dG_4 = 0$	$\star G_4 = G_7$
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Prop. The set of germs of solutions on globally hyperbolic spacetime $X^{1,d} \simeq \mathbb{R}^{1,0} \times X^d$ around a Cauchy surface $\{0\} \times X^d$ is

$$\begin{aligned} \text{Sol} &\simeq \left\{ B^{(i)} \in \Omega_{\text{dR}}^{\text{deg } i}(X^d) \mid \underset{\text{Gau\ss law}}{\text{d}} B^{(i)} = P^{(i)}(\vec{B}) \right\} \\ &\simeq \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) \end{aligned}$$

where $\mathfrak{b} \in L_\infty \text{Alg}_{\mathbb{R}}$ with

$$\text{CE}(\mathfrak{b}) \simeq \mathbb{R}_{\text{d}}[\vec{b}] / (\underset{\text{d}}{\text{d}} b^{(i)} = P^{(i)}(\vec{b}))_{i \in I}$$

Hence the *smooth moduli set* of germs of solutions is (boldface for smooth/stacky refinement):

$$\begin{aligned} \mathbf{Sol} &\simeq \mathbf{\Omega}_{\text{cl}}^1(X^d; \mathfrak{b}) \in \text{SmthSet} \hookrightarrow \text{SmthGrpd}_\infty \\ &\mathbf{\Omega}_{\text{cl}}^1(X^d; \mathfrak{b}) : U \mapsto \mathbf{\Omega}_{\text{cl}}^1(U \times X^d; \mathfrak{b}) \end{aligned}$$

$$\text{so: } \mathbf{\Omega}_{\text{cl}}^1(X^d; \mathfrak{b}) \simeq \mathbf{Map}(X^d, \mathbf{\Omega}_{\text{cl}}^1(*; \mathfrak{b}))$$

The shape modality

$$\int : \text{SmthGrpd}_\infty \xrightarrow{\text{Shp}} \text{Grpd}_\infty \xleftarrow{\text{Dsc}} \text{SmthGrpd}_\infty$$

where $\text{Shp} \dashv \text{Dsc}$

Differential cohomology in a cohesvive ∞ -topos (2013)

Prop. Shape as smooth path ∞ -groupoid:

$$\int \mathbf{X} \simeq \varinjlim_n \mathbf{Map}(\Delta^n, \mathbf{X}) \simeq \varinjlim_n \mathbf{X}(\Delta^n)$$

Prop. Smooth Oka principle:

$$X \in \text{SmthMfd} \hookrightarrow \text{SmthGrpd}_\infty \quad \Rightarrow$$

$$\begin{aligned} \int \mathbf{Map}(X, \mathbf{Y}) &\simeq \mathbf{Map}(X, \int \mathbf{Y}) \\ &\simeq \mathbf{Map}(\int X, \int \mathbf{Y}) \end{aligned}$$

(Pavlov et al. 2014-2024, cf. *Equivariant Principal ∞ -Bundles* §4.3.2)

Shape of higher flux solutions and Rational homotopy

$$\int \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) \simeq \mathbf{Map}(X^d, \int \Omega_{\text{cl}}^1(*, \mathfrak{b})) \quad (\text{by smooth Oka})$$

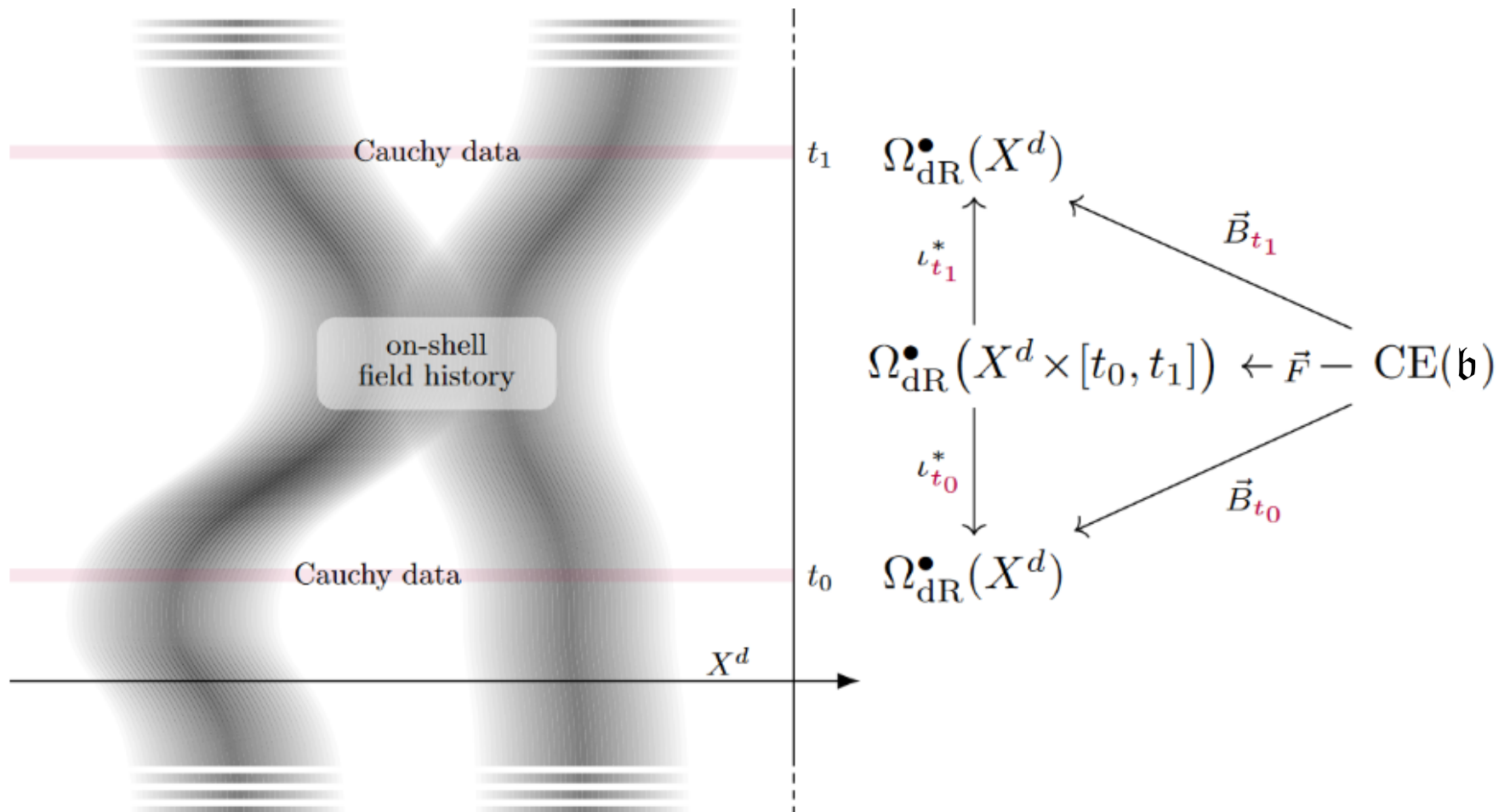
$$\int \Omega_{\text{cl}}^1(*; \mathfrak{b}) \simeq \mathbb{R}\text{-rational space with Whitehead } L_\infty\text{-algebra } \mathfrak{b}$$

For $\mathcal{B} \in \text{Grpd}_\infty$ (simply connected with fin-dim rational cohomology),
write $\mathfrak{B} \in L_\infty\text{Alg}_\mathbb{R}$ for the real Whitehead brackets
hence with $\text{CE}(\mathfrak{B})$ the minimal Sullivan model of \mathcal{B}

then: $\boxed{\int \Omega_{\text{cl}}^1(*; \mathfrak{B}) \simeq L^\mathbb{R}\mathcal{B}}$ is the \mathbb{R} -rationalization of \mathcal{B}

Total flux on X^d is in **nonabelian de Rham cohomology**:

$$[\vec{F}] \in H_{\text{dR}}^1(X^d; \mathfrak{b}) := \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) / \text{concordance} \\ \simeq \pi_0(\int \Omega_{\text{cl}}^1(X^d; \mathfrak{b}))$$



The nonabelian character map.

$$\begin{array}{ccc}
 & \text{ch} & \\
 & \longmapsto & \\
 \mathcal{B} & \xrightarrow{\eta^{\mathbb{R}}} & L^{\mathbb{R}}\mathcal{B} \xrightarrow{\sim} \int \Omega_{\text{cl}}^1(*, \mathcal{B}) \\
 \\
 \pi_0 \text{Map}(\int X, \mathcal{B}) & \xrightarrow{\text{ch}_*} & \pi_0 \text{Map}(\int X, \int \Omega_{\text{cl}}^1(*, \mathcal{B})) \\
 \Downarrow & & \Downarrow \\
 H^1(X; \Omega\mathcal{B}) & \xrightarrow{\text{ch}} & H_{\text{dR}}^1(X; \mathcal{B}) \\
 \text{nonabelian} & \text{character map} & \text{nonabelian} \\
 \text{cohomology} & & \text{de Rham} \\
 & & \text{cohomology}
 \end{array}$$

Examples:

de Rham map \simeq $\text{ch}^{B^n\mathbb{Z}}$

Chern-Dold character \simeq ch^{E_n} (stage in a spectrum)

Chern-character \simeq ch^{KU}

cohomotopy character \simeq ch^{S^n}

Admissible classifying spaces \mathcal{B} for given Gauß law \mathfrak{b}

satisfy: $\boxed{\mathcal{B} \simeq \mathfrak{b}}$

Examples:

For Maxwell, admissible is $\mathcal{B} \equiv B^2\mathbb{Z}^2$

for 5D MCS/SuGra, admissible is $\mathcal{B} \equiv \int S^2$

for NS/RR-flux, admissible is $\mathcal{B} \equiv (KU_0//BU(1)) \times B^7\mathbb{Z}$

for 11D MCS/SuGra, admissible is $\mathcal{B} \equiv \int S^4$

But there is in each case an infinity of admissible choices.

This choice of \mathcal{B} is a *global completion* of higher flux theory \mathfrak{b} .

Coarse flux quantization:

total flux must be in the image of the character map,

for \mathcal{B} admissible, $\mathfrak{l}\mathcal{B} \simeq \mathfrak{b}$:

$$\begin{array}{ccc}
 * & \xrightarrow[\text{charge}]{[\chi]} & H^1(X^d; \Omega\mathcal{B}) \\
 & \searrow_{\text{flux}}^{\vec{B}} & \downarrow \text{ch} \\
 & \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) & \xrightarrow{[-]} H_{\text{dR}}^1(X^d; \mathfrak{b})
 \end{array}$$

Example: $\mathcal{B} \equiv B^n\mathbb{Z}$: forces n -forms to be integral

NB: Quantizing not just magnetic but also electric fluxes.

Proper flux quantization is local \Rightarrow gauge potentials!

$$\begin{array}{ccc}
 * & \xrightarrow{\text{charge}} & \mathbf{Map}(X^d; \mathcal{B}) \\
 & \searrow \vec{B} & \downarrow \mathbf{ch}_* \\
 & \xrightarrow{\text{flux}} & \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) \xrightarrow{\eta^f} \int \Omega_{\text{cl}}^1(X^d; \mathfrak{b})
 \end{array}$$

χ
 \hat{A} gauge potential

in nonabelian differential cohomology:

$$\begin{array}{ccc}
 * & \xrightarrow{(\chi, \hat{A})} & \mathbf{Map}(X^d, \mathcal{B}_{\text{diff}}) \xrightarrow{\eta^f} \mathbf{Map}(X^d; \mathcal{B}) \\
 & \searrow \vec{B} & \downarrow \lrcorner \\
 & \xrightarrow{\text{flux}} & \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) \xrightarrow{\eta^f} \int \Omega_{\text{cl}}^1(X^d; \mathfrak{b})
 \end{array}$$

$\downarrow \mathbf{ch}_*$

$$H_{\text{diff}}^1(X^d, \Omega\mathcal{B}) := \pi_0(\mathfrak{b}\mathbf{Map}(X^d, \mathcal{B}_{\text{diff}}))$$

Example: Recovering abelian generalized diff. cohomology.

For $\mathcal{B} \equiv E_n$ a stage in a spectrum, this reproduces canonical Hopkins-Singer type differential cohomology, e.g., differential K-theory for $\mathcal{B} \equiv \text{KU}_n$.

Example: Induced gauge potentials in nonabelian case

	on chart	on double-intersections
for $\mathfrak{a} = \mathfrak{so}(4)$	$d C_3 = G_4$	$d \gamma_2 = C'_3 - C_3$
	$d C_6 = G_7 - \frac{1}{2} C_3 \wedge G_4$	$d \gamma_5 = C'_6 - C_6 - \frac{1}{2} C'_3 \wedge C_3$
	as expected in 11D SuGra literature	
for $\mathfrak{a} = \mathfrak{so}(3)$	$d A_1 = F_2$	$d \alpha_0 = A'_1 - A_1$
	$d B_2 = H_3 - \frac{1}{2} A_1 \wedge F_2$	$d \beta_1 = B'_2 - B_2 - \frac{1}{2} A'_1 \wedge A_1$

Abelian Chern-Simons term! More in a moment.

Phase space stack of flux-quantized higher gauge fields:

$$\mathbf{PhsSpc} \equiv \mathbf{Map}(X^d; \mathcal{B}_{\text{diff}})$$

Observables: $\mathbf{PhsSpc} \xrightarrow{\mathcal{O}} \mathbb{C}$

Topological observables: $\mathbf{PhsSpc} \xrightarrow{\eta^{\int}} \int \mathbf{PhsSpc} \dashrightarrow \mathbb{C}$

Prop: $\int \mathbf{Map}(X^d; \mathcal{B}_{\text{diff}}) \simeq \mathbf{Map}(X^d, \mathcal{B})$

\Leftrightarrow “Topological observables are entirely determined by choice of flux quantization \mathcal{B} ”.

Flux Quantization on Phase Space, AHP 26 (2025)

Quantum Observables of Quantized Fluxes, AHP 26 (2025)

Higher Maxwell-*Chern-Simons* EoMs

More generally, we may pick a morphism $\mathfrak{b}' \xrightarrow{\iota} \mathfrak{b}$ and consider the further pullback:

$$\begin{array}{ccccc}
 \text{CS constrained} & & \text{phase space} & & \\
 \mathbf{Map}(X^d, \mathcal{B}_{\text{diff}'}) & \rightarrow & \mathbf{Map}(X^d, \mathcal{B}_{\text{diff}}) & \xrightarrow{\eta^f} & \mathbf{Map}(X^d; \mathcal{B}) \\
 \downarrow & \lrcorner & \downarrow & \lrcorner & \downarrow \mathbf{ch}_* \\
 \Omega_{\text{cl}}^1(X^d; \mathfrak{b}') & \xrightarrow{\iota_*} & \Omega_{\text{cl}}^1(X^d; \mathfrak{b}) & \xrightarrow{\eta^f} & \int \Omega_{\text{cl}}^1(X^d; \mathfrak{b})
 \end{array}$$

This corresponds to imposing CS-like EoMs, $F^{(i)} = 0$ on those fluxes $F^{(i)}$ not in the image of ι_* .

Example: The Hopf fibration $\iota \equiv \mathfrak{h}_{\mathbb{C}} : \mathfrak{S}^3 \longrightarrow \mathfrak{S}^2$
implies ordinary abelian CS EoM: $F_2 = 0$

Higher Maxwell EoMs with background fields.

Consider now an inclusion of domains

$$\begin{array}{ccc} & \Sigma^p & \\ & \downarrow \phi & \\ & X^d & \end{array}$$

with “background” flux densities $\vec{B} \in \Omega_{\text{cl}}^1(X^d, \mathfrak{b})$ as before,

then higher Maxwell-type theory *relative* to ϕ is

$$\left\{ \vec{B}'^{(j)} \in \Omega_{\text{dR}}^{\text{deg } j}(\Sigma^p) \mid dB'^{(j)} = P^{(j)}(\vec{B}', \phi^* \vec{B}) \right\}$$

$$\simeq \left\{ \begin{array}{ccc} \Sigma^p & \xrightarrow{\vec{B}'} & \Omega_{\text{cl}}^1(*; \mathfrak{a}) \\ \downarrow \phi & & \downarrow \\ X^d & \xrightarrow{\vec{B}} & \Omega_{\text{cl}}^1(*; \mathfrak{b}) \end{array} \right\}$$

Relative Whitehead L_∞ -algebras

fibration
of spaces

fibration
of L_∞ -algs

relative minimal
Sullivan model

$$\begin{array}{ccccc}
 \mathcal{A} & & \mathfrak{l}_{\mathcal{B}}\mathcal{A} & & \mathrm{CE}(\mathfrak{l}_{\mathcal{B}}\mathcal{A}) \\
 \downarrow \wr_{\varphi} & \dashrightarrow & \downarrow \wr_{\varphi} & \dashleftarrow & \uparrow \mathrm{CE}(\wr_{\varphi}) \\
 \mathcal{B} & & \mathfrak{l}\mathcal{B} & & \mathrm{CE}(\mathfrak{l}\mathcal{B})
 \end{array}$$

Examples:

Maxwell
with source

$$\begin{array}{ccc}
 X^d & \xrightarrow{G_2} & \Omega_{\text{cl}}^1(*; \mathfrak{l}_{B^3\mathbb{Z}} EB^2\mathbb{Z}) \\
 \downarrow \iota & & \downarrow \\
 X_{\cup\{\infty\}}^d & \xrightarrow{J_3} & \Omega_{\text{cl}}^1(*; \mathfrak{l}B^3\mathbb{Z})
 \end{array}
 \quad \begin{array}{l}
 dG_2 = \iota^* J_3 \\
 dJ_3 = 0
 \end{array}$$

NS/RR-flux

$$\begin{array}{ccc}
 X^d & \xrightarrow{F_{2\bullet}} & \Omega_{\text{cl}}^1(*; \mathfrak{l}_{B^3\mathbb{Z}} \text{KU} // B^2\mathbb{Z}) \\
 \downarrow \iota & & \downarrow \\
 X_{\cup\{\infty\}}^d & \xrightarrow{H_3} & \Omega_{\text{cl}}^1(*; \mathfrak{l}B^3\mathbb{Z})
 \end{array}
 \quad \begin{array}{l}
 dF_{2\bullet} = F_{2\bullet-2} \wedge \iota^* H_3 \\
 dH_3 = 0
 \end{array}$$

flux on
M5-brane
in 11D

$$\begin{array}{ccc}
 \Sigma^5 & \xrightarrow{H_3} & \Omega_{\text{cl}}^1(*; \mathfrak{l}_{S^4} S^7) \\
 \downarrow \phi & & \downarrow \mathfrak{l}(h_{\mathbb{H}})_* \\
 X^{10} & \xrightarrow{(G_4, G_7)} & \Omega_{\text{cl}}^1(*; \mathfrak{l}S^4)
 \end{array}
 \quad \begin{array}{l}
 dH_3 = \phi^* G_4 \\
 dG_7 = \frac{1}{2} G_4 \wedge G_4 \\
 dG_4 = 0
 \end{array}$$

Twisted relative cohomology.

$$\mathbf{Map}(\phi, \wp) := \mathbf{Map}(\Sigma, \mathcal{A}) \times_{\mathbf{Map}(\Sigma, \mathcal{B})} \mathbf{Map}(X, \mathcal{B})$$

$$= \left\{ \begin{array}{ccc} \Sigma & \dashrightarrow & \mathcal{A} \\ \downarrow \phi & & \downarrow \wp \\ X & \dashrightarrow & \mathcal{B} \end{array} \right\}$$

etc.

Then the \wp -twisted ϕ -relative differential cohomology is:

$$\begin{array}{ccccc} \mathbf{Map}(\phi, \wp_{\text{diff}}) & \longrightarrow & \mathbf{Map}(\phi, \wp_{\text{diff}}') & \longrightarrow & \mathbf{Map}(\phi, \wp) \\ \downarrow & \lrcorner & \downarrow & \lrcorner & \downarrow \mathbf{ch} \\ \Omega_{\text{cl}}^1(\phi, \wp') & \xrightarrow{!_*} & \Omega_{\text{cl}}^1(\phi, \wp) & \xrightarrow{\eta^{\int}} & \int \Omega_{\text{cl}}^1(\phi, \wp) \end{array}$$

Example.

$$\text{Consider } \left\{ \begin{array}{l} \wp \equiv h_{\mathbb{C}} : S^3 \rightarrow S^2 \\ \prime \equiv \mathfrak{l}(\text{id}, h) : \begin{array}{ccc} \mathfrak{l}S^3 & \rightarrow & \mathfrak{l}S^3 \\ \parallel & & \downarrow \mathfrak{l}h_{\mathbb{C}} \\ \mathfrak{l}S^3 & \rightarrow & \mathfrak{l}S^2 \end{array} \end{array} \right.$$

$$\begin{array}{ccc} \text{then:} & \begin{array}{c} \text{EoMs} \\ F_2 = 0 \\ dH_3 = F_2 \wedge F_2 \end{array} & \begin{array}{c} \text{gauge} \\ \text{potentials} \\ d\lambda = \phi^* A_1 \\ dA_1 = 0 \end{array} \end{array}$$

This is as in abelian CS with FJ/WZW boundary!

Claim: Topological observables on phase space stack reproduce in fine detail those of abelian CS with boundary:

- renormalized Wilson loop observables
- Heisenberg observables on torus
- modular functor
- bulk/edge correspondence

Engineering FQH anyons on M2/M5-branes:

$$\begin{array}{rcccl}
 \text{M2-brane} & \mathbb{R}^{1,1} \times \Sigma^1 & \dashrightarrow & S^7 & \curvearrowright G \\
 & \downarrow \phi & & \downarrow h_{\mathbb{C}} & \curvearrowright G \\
 \text{M5-brane} & \mathbb{R}^{1,1} \times X^2 \times \mathbb{C} & \dashrightarrow & \mathbb{C}P^3 & \curvearrowright G \\
 & \downarrow \Phi & & \downarrow t_{\mathbb{C}} & \curvearrowright G \\
 \text{11D bulk} & \mathbb{R}^{1,1} \times \mathbb{R}^5 \times \underbrace{\mathbb{C} \times \mathbb{C}}_{A_1\text{-orbifold}} & \dashrightarrow & S^4 & \curvearrowright G
 \end{array}$$

reduces to the orbi-singularity:

$$\begin{array}{rcccl}
 \text{M2-brane} & \mathbb{R}^{1,1} \times \Sigma^1 & \dashrightarrow & S^3 & \\
 & \downarrow \phi & & \downarrow h_{\mathbb{C}} & \\
 \text{M5-brane singularity} & \mathbb{R}^{1,1} \times X^2 & \dashrightarrow & S^2 & \text{and hence to CS/WZW} \\
 & \downarrow \Phi & & \downarrow & \\
 \text{7D } A_1\text{-singularity} & \mathbb{R}^{1,1} \times \mathbb{R}^5 & \dashrightarrow & * &
 \end{array}$$

Flux Quantization on M5-Branes, JHEP 140 (2024)

Anyons on M5-Probes of Seifert 3-Orbifolds, LMP **115** (2025)

Engineering of Anyons on M5-Probes, SciPost Phys. Lec. Notes **107** (2025)

Flux Quantization on M-Strings (2026)

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