2.3.1   Solitonic brane cobordism via Pontrjagin's Theorem

Remarkably, there is a tight relation between Cohomotopy of spacetime and cobordism classes of submanifolds that behave like branes carrying a corresponding Cohomotopy charge (cf. [SS23-Mf, §2.2] [SS20-Orb, §2.1]):

The **Pontrjagin theorem** [Po38][Ko93, §IX] identifies the unstable n-Cohomotopy of a closed manifold with the cobordism classes of its normally framed submanifolds of co-dimension n.

The **Cohomotopy charge** of a normally framed submanifold (aka *scanning map* or *Pontrjagin-Thom collapse*) is represented by mapping points of the ambient space to their directed distance if inside a tubular neighbourhood, else to ∞.

Conversely, every Cohomotopy class is represented by a smooth map with 0 a regular value, whose preimage is a normally framed submanifold with that Cohomotopy charge.

Under this relation, homotopy of charge maps corresponds to nrml. framed **cobordism** of submnflds.

The cobordism relation exhibits a form of pair creation/annihilation of submanifolds carrying opposite Cohomotopy charges.

When making more ambient dimensions available, the cobordism classes eventually (quickly) exhibit **stabilization** on abelian cobordism cohomology groups. (This might relate *Hypothesis H* to Vafa's *cobordism conjecture* cf. [SS23-Mf, §4]).

This "linearized" Cohomotopy/Cobordism is a **form of K-theory**: algebraic K-theory over the "absolute base field $\mathbb{F}_1$" (cf. [CLS12, Thm. 5.9]).

71