## 1.1 Branes imprinted on flux

The concept of *branes* (see [IU12, §6][Fr13, §7][HSS19, §2]) is the core aspect of the historical re-thinking of string theory that came to be known as the "second superstring revolution" [Schw96], in that it is the key for the non-perturbative completion of the theory [Du00]: The "M" in "M-theory" originates [HW96, p. 2] as a "non-committal" abbreviation for *membrane*.

Conversely this means that the precise meaning of "brane" has been almost as elusive as that of "M-theory" itself. Or rather: There is a range of specialized meanings of the term, some versions of which do have precise definitions, but it has remained unclear how exactly any and all of these notions are aspects of a unified concept of "branes".

**Our strategy for formalizing the concept of branes** is by careful examination of the **nature of the flux** they source.

(1.) We recall here (§1.1) the notion of **branes as** higher-dimensional generalizations of *poles* and hence as **sources of flux**.
(2.) We **express this notion in algebro-topological terms** (§1.2), from which we motivate *Hypothesis H* (§1.3).
(3.) We observe a natural notion of **light-cone quantization** by passage to Pontrjagin homology algebras (§2.2).
(4.) All inspection of brane physics in §2 proceeds by mathematical unravelling of this algebro-topological stucture.

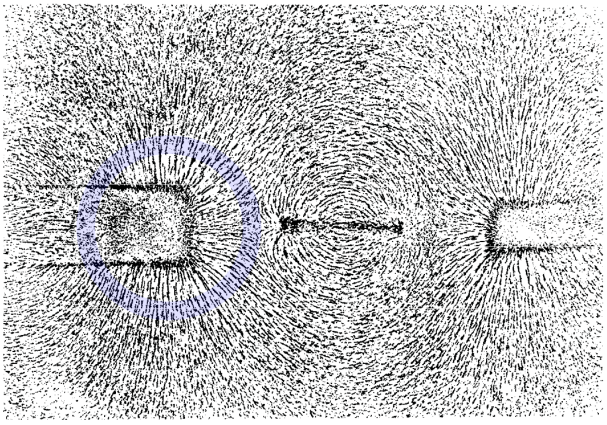
### 1.1.1 Branes as concentrations of flux

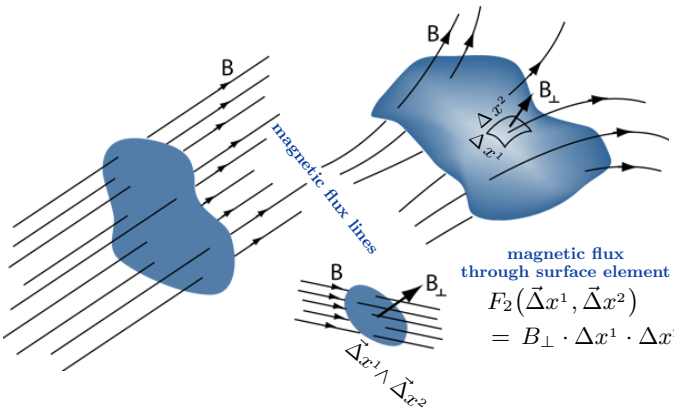
To get ground under our feet, it is expedient — our ambitious goal nonwithstanding — to start with elementary reflections on *flux lines* (*flux densities*) sourced by charged *poles* as originally conceived by Faraday in the 19th century, and as more generally sourced by higher dimensional charged *branes*, like the charged *membranes* already considered by [Dirac1962]. While most of these objects (famously including magnetic mono-poles) are notorious for remaining hypothetical entities not currently seen in experiment, we highlight (p. 7) the example of *vortex strings* in superconductors which have been observed in detail and which — whether one likes to refer to them as "1-branes" or not — do constitute an example of the general notion of classical branes in question.[a] This highlights the relevance of the distinction between singular and solitonic branes (and their difference in dimension) which may not to have received due attention before, cf. §2.3.3.

[a] [Polyakov 12, p. 1] regrets not to have understood vortex lines as strings. See also the emphasis on vortex *worldsheets* in [Beekman & Zaanen 2011].

| **Field flux.** | |
|---|---|
| $X^D \in$ Mfds | spacetime manifold |
| $\Omega^r_{\mathrm{dR}}(X^D) \in$ Sets | differential $r$-forms |
| $F^{(i)} \in \Omega^{\deg_i}_{\mathrm{dR}}(X^D)$ | flux density |
| $\star : \Omega^r_{\mathrm{dR}}(X^D) \to \Omega^{D-r}_{\mathrm{dR}}(X^D)$ | Hodge star (§1.1.2) |

| **Classical Example: Electromagnetic flux** | |
|---|---|
| $X^3 = \mathbb{R}^{3,1}$ | Minkowski spacetime |
| $\Omega^1_{\mathrm{dR}}(\mathbb{R}^{3,1}) = \{A_i\,\mathrm{d}x^i + \phi\,\mathrm{d}t\}$ | vector potentials |
| $F_2 : \Omega^2_{\mathrm{dR}}(\mathbb{R}^{3,1})$ | Faraday tensor |
| $= \star(E_{ij}\,\mathrm{d}x^i \wedge \mathrm{d}x^j)$ | electric flux density |
| $+ B_{ij}\,\mathrm{d}x^i \wedge \mathrm{d}x^j$ | magnetic flux density |






**Magnetic flux lines.** On the left: Iron filings in the magnetic field around magnetic poles (from *Faraday's diary of experimental investigation*, entry of 11th Dec 1851, reproduced by Martin 2009). The light circle around one of the poles is our addition, for emphasis. On the right: Magnetic flux(-line) density as a differential 2-form (adapted from hyperphysics.phy-astr.gsu.edu/hbase/magnetic/fluxmg.html).

**Solitonic vs. Singular branes.** Imprinted on the flux density may be *two kinds* of branes, to be called:[7]

(1.) **singular branes** (*black branes*) reflected in **diverging flux density** at **singular loci** in spacetime,

(2.) **solitonic branes** reflected in *localized* but **finite flux density**, namely **vanishing at infinity**.

This distinction is often not made quite clear in the literature, but it is crucial for the analysis of brane effects (§2). Therefore we first highlight the issue in the familar case of electromagnetism: