Urs Schreiber

New York University, Abu Dhabi

CENTER FOR
QUANTUM &
TOPOLOGICAL
SYSTEMS

Higher Gauge Theory via Differential Nonabelian Cohomology

based in parts on joint work with:

P. Banerjee, D. Fiorenza, G. Giotopoulos, H. Sati

(later this week)

talk at *Physics & Geometry VI*, RBI Zagreb (May 2026)

Monographs:

The Character Map in Nonabelian Cohomology, World Scientific (2023)

Equivariant Principal ∞-Bundles, Cambridge Univ. Press (2026)

Geometric Orbifold Cohomology, CRC Press (2026)

Expositions:

Higher Topos Theory in Physics, Encyclopedia Math. Phys. (2025)

Flux Quantization, Encyclopedia Math. Phys. (2025)

Complete Top. Quantization of Higher Gauge Fields, SciPost Phys. Lect. Notes (2026)

Manuscript:

ncatlab.org/schreiber/show/Zagreb+2026

NB:

There are *two different* approaches to higher gauge theory:

	principal	**cohomological**
specified L_∞-algebra	coefficient of gauge potentials (connections)	coefficient of flux densities (curvatures)
flatness / closure	non-generic	Gauß law / Bianchi identity
examples	heterotic SuGra / B-field on T-folds	type I/II SuGra / 11D SuGra

I started out principal, 20 years ago.
But I think we need cohomological.

Higher Maxwell-type Equations of Motion:

higher fluxes:	$\vec{F} := \left\{ F^{(i)} \in \Omega_{\mathrm{dR}}^{\deg_i}\big(X^{1,d}\big) \right\}_{i \in I}$	
higher Bianchis:	$\mathrm{d}\, F^{(i)} = P^{(i)}\big(\vec{F}\big)$	(polynomials, integrable)
higher duality:	$\star\, F^{(i)} = \mu^{(i)}\big(\vec{F}\big)$	(linear iso)

Examples:

vacuum Maxwell:	$\mathrm{d}F_2 = 0$ $\mathrm{d}G_2 = 0$	$\star F_2 = G_2$
5D MCS/SuGra	$\mathrm{d}\,H_3 = \frac{1}{2}F_2 \wedge F_2$ $\mathrm{d}\,F_2 = 0$	$\star\,F_2 = F_3$
type IIA NS/RR-flux:	$\mathrm{d}\,F_{2\bullet} = H_3 \wedge F_{2\bullet-2}$ $\mathrm{d}\,H_7 = 0$ $\mathrm{d}\,H_3 = 0$	$\star\,F_{2\bullet} = F_{10-2\bullet}$ $\star\,H_3 = H_7$
11D MCS/SuGra	$\mathrm{d}\,G_7 = \frac{1}{2}G_4 \wedge G_4$ $\mathrm{d}\,G_4 = 0$	$\star\,G_4 = G_7$

Prop. The set of germs of solutions on globally hyperbolic spacetime $X^{1,d} \simeq \mathbb{R}^{1,0} \times X^d$ around a Cauchy surface $\{0\} \times X^d$ is

$$\mathrm{Sol} \simeq \left\{ B^{(i)} \in \Omega^{\mathrm{deg}_i}_{\mathrm{dR}}\big(X^d\big) \,\middle|\, \mathrm{d}\, B^{(i)} \underset{\text{Gauß law}}{=} P^{(i)}\big(\vec{B}\big) \right\}$$
$$\simeq \Omega^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big)$$

where $\mathfrak{b} \in L_\infty \mathrm{Alg}_{\mathbb{R}}$ with
$$\mathrm{CE}(\mathfrak{b}) \simeq \mathbb{R}_{\mathrm{d}}\big[\,\vec{b}\,\big] \big/ \big(\mathrm{d}\, b^{(i)} = P^{(i)}(\vec{b})\big)_{i \in I}$$

Hence the *smooth moduli set* of germs of solutions is (boldface for smooth/stacky refinement):

$$\mathbf{Sol} \simeq \mathbf{\Omega}^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big) \in \mathrm{SmthSet} \hookrightarrow \mathrm{SmthGrpd}_\infty$$
$$\mathbf{\Omega}^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big) : U \mapsto \Omega^1_{\mathrm{cl}}\big(U \times X^d; \mathfrak{b}\big)$$

so: $\mathbf{\Omega}^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big) \simeq \mathbf{Map}\big(X^d, \mathbf{\Omega}^1_{\mathrm{cl}}(*; \mathfrak{b})\big)$

Flux Quantization on Phase Space, AHP **26** (2025)

The shape modality

$$\int \;:\; \mathrm{SmthGrpd}_\infty \xrightarrow{\mathrm{Shp}} \mathrm{Grpd}_\infty \xhookrightarrow{\mathrm{Dsc}} \mathrm{SmthGrpd}_\infty$$

where $\mathrm{Shp} \dashv \mathrm{Dsc}$

Differential cohomology in a cohesvive ∞-topos (2013)

Prop. Shape as smooth path ∞-groupoid:

$$\int \mathbf{X} \simeq \varinjlim_n \mathbf{Map}(\boldsymbol{\Delta}^n, \mathbf{X}) \simeq \varinjlim_n \mathbf{X}(\boldsymbol{\Delta}^n)$$

Prop. Smooth Oka principle:

$$X \in \mathrm{SmthMfd} \hookrightarrow \mathrm{SmthGrpd}_\infty \quad \Rightarrow$$

$$\begin{aligned} \int \mathbf{Map}(X, \mathbf{Y}) &\simeq \mathbf{Map}(X, \int \mathbf{Y}) \\ &\simeq \mathbf{Map}(\int X, \int \mathbf{Y}) \end{aligned}$$

(Pavlov et al. 2014-2024, cf. *Equivariant Principal ∞-Bundles* §4.3.2)

Shape of higher flux solutions and Rational homotopy

$$\int \mathbf{\Omega}^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big) \;\simeq\; \mathbf{Map}\big(X^d, \int \mathbf{\Omega}^1_{\mathrm{cl}}(*, \mathfrak{b})\big) \qquad \text{(by smooth Oka)}$$

$$\int \mathbf{\Omega}^1_{\mathrm{cl}}(*; \mathfrak{b}) \;\simeq\; \mathbb{R}\text{-rational space with Whitehead } L_\infty\text{-algebra } \mathfrak{b}$$

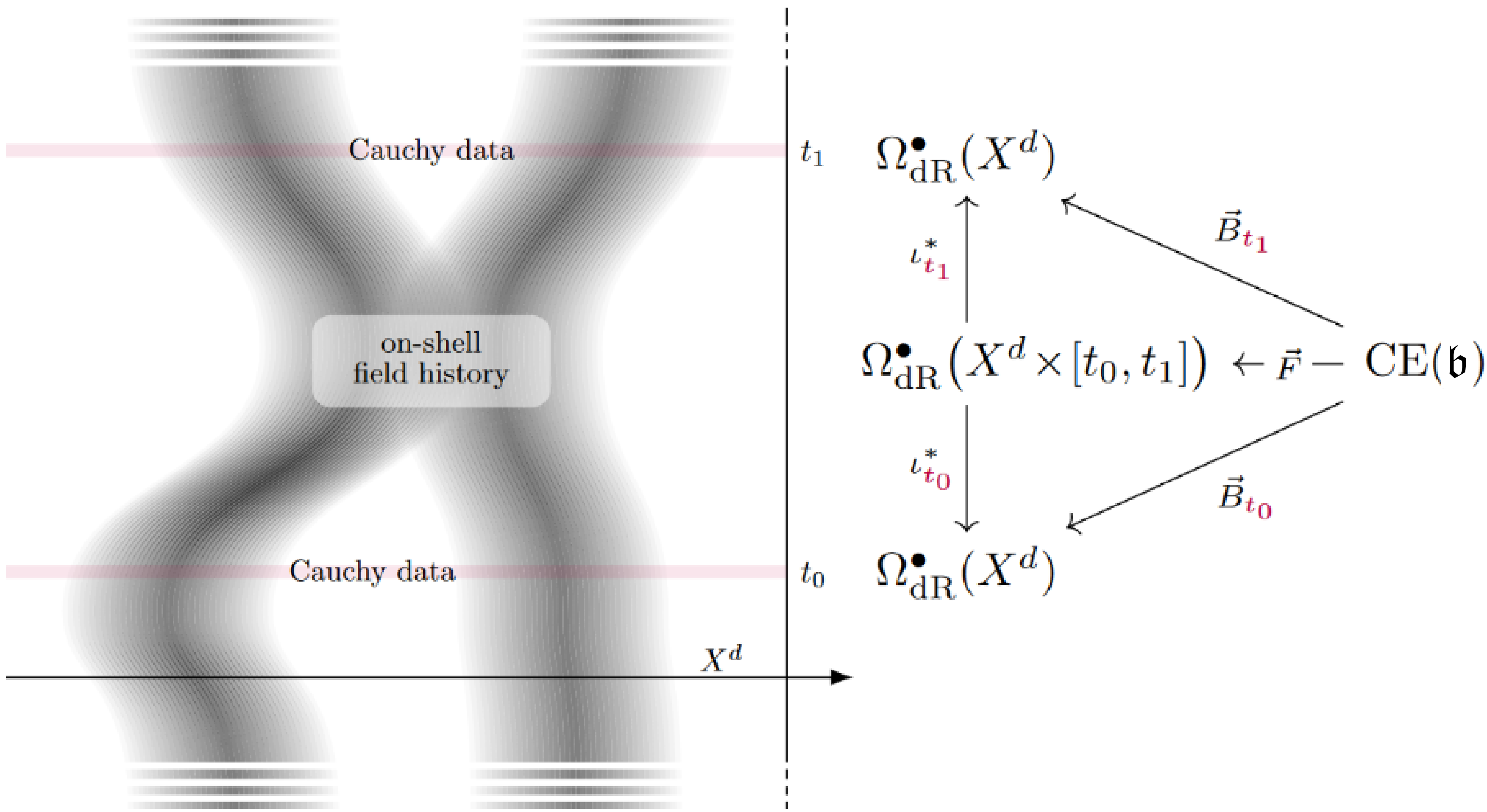
For $\mathcal{B} \in \mathrm{Grpd}_\infty$ (simply connected with fin-dim rational cohomology),
write $\mathfrak{l}\mathcal{B} \in L_\infty\mathrm{Alg}_{\mathbb{R}}$ for the real Whitehead brackets
hence with $\mathrm{CE}(\mathfrak{l}\mathcal{B})$ the minimal Sullivan model of $\mathcal{B}$

then: $\boxed{\int \mathbf{\Omega}^1_{\mathrm{cl}}(*; \mathfrak{l}\mathcal{B}) \;\simeq\; L^{\mathbb{R}}\mathcal{B}}$ is the $\mathbb{R}$-rationalization of $\mathcal{B}$

The Character Map in Nonabelian Cohomology (2023)

Total flux on X^d is in **nonabelian de Rham cohomology**:

$$[\vec{F}] \in H^1_{\mathrm{dR}}\big(X^d; \mathfrak{b}\big) := \Omega^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big)/\mathrm{concordance}$$
$$\simeq \pi_0\Big(\textstyle\int \mathbf{\Omega}^1_{\mathrm{cl}}\big(X^d; \mathfrak{b}\big)\Big)$$



The nonabelian character map.

$$\mathcal{B} \xrightarrow{\ \eta^{\mathbb{R}}\ } L^{\mathbb{R}}\mathcal{B} \xrightarrow{\ \sim\ } \textstyle\int \mathbf{\Omega}^1_{\mathrm{cl}}(\ast, \mathfrak{l}\mathcal{B}) \qquad (\text{composite: } \mathbf{ch})$$

$$\begin{array}{ccc}
\pi_0 \,\mathrm{Map}\big(\textstyle\int X, \mathcal{B}\big) & \xrightarrow{\ \mathbf{ch}_\ast\ } & \pi_0 \,\mathrm{Map}\Big(\textstyle\int X, \textstyle\int \mathbf{\Omega}^1_{\mathrm{cl}}(\ast, \mathfrak{l}\mathcal{B})\Big) \\
\| & & \| \\
H^1(X; \Omega\mathcal{B}) & \xrightarrow{\ \mathrm{ch}\ } & H^1_{\mathrm{dR}}(X; \mathfrak{l}\mathcal{B})
\end{array}$$

nonabelian cohomology | character map | nonabelian de Rham cohomology

The Character Map in Nonabelian Cohomology (2023)

Examples:

de Rham map	$\simeq$	$\mathrm{ch}^{B^n\mathbb{Z}}$
Chern-Dold character	$\simeq$	ch^{E_n} (stage in a spectrum)
Chern-character	$\simeq$	$\mathrm{ch}^{\mathrm{KU}}$
cohomotopy character	$\simeq$	ch^{S^n}

Admissible classifying spaces $\mathcal{B}$ for given Gauß law $\mathfrak{b}$

satisfy: $\boxed{\mathfrak{l}\mathcal{B} \;\simeq\; \mathfrak{b}}$

Examples:

For Maxwell, admissible is $\mathcal{B} \;\equiv\; B^2\mathbb{Z}^2$

for 5D MCS/SuGra, admissible is $\mathcal{B} \;\equiv\; \int S^2$

for NS/RR-flux, admissible is $\mathcal{B} \;\equiv\; (\mathrm{KU}_0 /\!\!/ B\mathrm{U}(1)) \times B^7\mathbb{Z}$

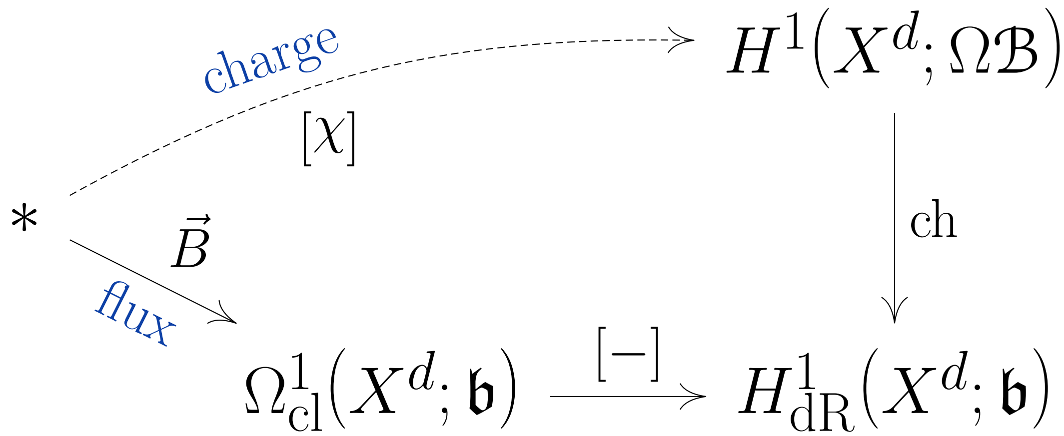
for 11D MCS/SuGra, admissible is $\mathcal{B} \;\equiv\; \int S^4$

But there is in each case an infinity of admissible choices.
This choice of $\mathcal{B}$ is a *global completion* of higher flux theory $\mathfrak{b}$.

Coarse flux quantization:

total flux must be in the image of the character map,

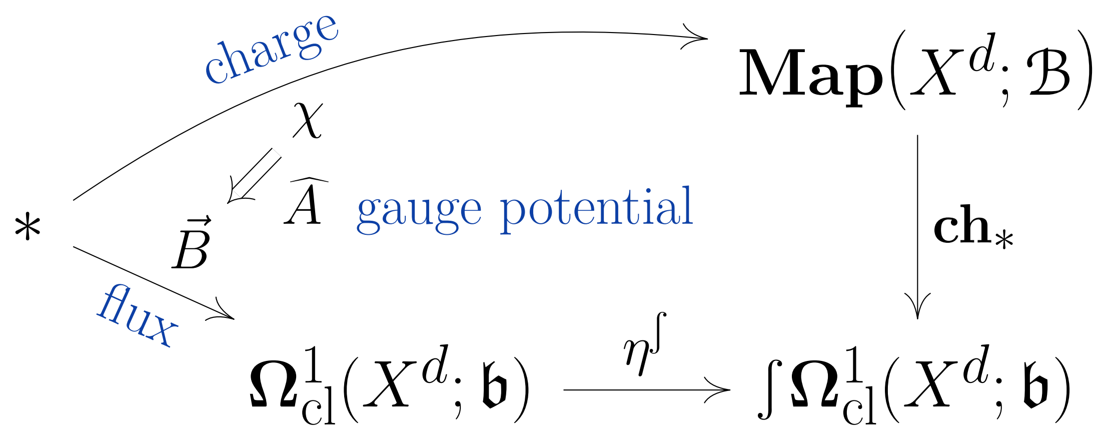
for $\mathcal{B}$ admissible, $\mathfrak{l}\mathcal{B} \simeq \mathfrak{b}$:



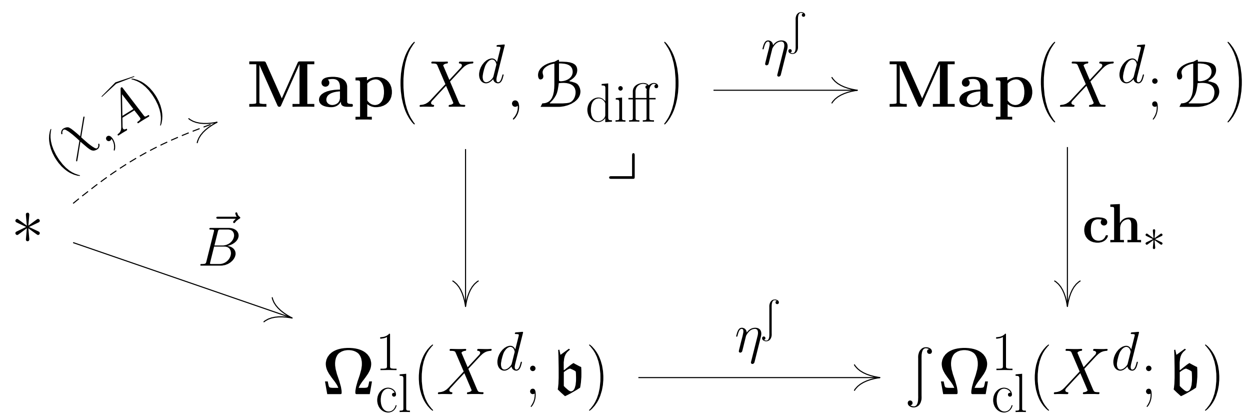
Example: $\mathcal{B} \equiv B^n\mathbb{Z}$: forces n-forms to be integral

NB: Quantizing not just magnetic but also electric fluxes.

Proper flux quantization is local $\Rightarrow$ gauge potentials!



in nonabelian differential cohomology:



$$H^1_{\mathrm{diff}}\big(X^d, \Omega\mathcal{B}\big) \;:=\; \pi_0\big(\flat \mathbf{Map}(X^d, \mathcal{B}_{\mathrm{diff}})\big)$$

Example: Recovering abelian generalized diff. cohomology.

For $\mathcal{B} \equiv E_n$ a stage in a spectrum, this reproduces canonical Hopkins-Singer type differential cohomology, e.g., differential K-theory for $\mathcal{B} \equiv \mathrm{KU}_n$.

Example: Induced gauge potentials in nonabelian case

	on chart	on double-intersections
for $\mathfrak{a} = \mathfrak{l}S^4$	$\mathrm{d}\,C_3 = G_4$ $\mathrm{d}\,C_6 = G_7 - \frac{1}{2}C_3 \wedge G_4$	$\mathrm{d}\,\gamma_2 = C'_3 - C_3$ $\mathrm{d}\,\gamma_5 = C'_6 - C_6 - \frac{1}{2}C'_3 \wedge C_3$
	as expected in 11D SuGra literature	
for $\mathfrak{a} = \mathfrak{l}S^2$	$\mathrm{d}\,A_1 = F_2$ $\mathrm{d}\,B_2 = H_3 - \frac{1}{2}A_1 \wedge F_2$	$\mathrm{d}\,\alpha_0 = A'_1 - A_1$ $\mathrm{d}\,\beta_1 = B'_2 - B_2 - \frac{1}{2}A'_1 \wedge A_1$
	Abelian Chern-Simons term! More in a moment.	

Phase space stack of flux-quantized higher gauge fields:

$$\mathbf{PhsSpc} \equiv \mathbf{Map}\big(X^d; \mathcal{B}_{\mathrm{diff}}\big)$$

Observables: $\mathbf{PhsSpc} \xrightarrow{\;\mathcal{O}\;} \mathbb{C}$

Topological observables: $\mathbf{PhsSpc} \xrightarrow{\;\eta^{\int}\;} \int\mathbf{PhsSpc} \dashrightarrow \mathbb{C}$

Prop: $\int\mathbf{Map}\big(X^d; \mathcal{B}_{\mathrm{diff}}\big) \simeq \mathbf{Map}\big(X^d, \mathcal{B}\big)$

$\Leftrightarrow$ "Topological observables are entirely determined by choice of flux quantization $\mathcal{B}$".

Flux Quantization on Phase Space, AHP **26** (2025)
Quantum Observables of Quantized Fluxes, AHP **26** (2025)

Higher Maxwell-*Chern-Simons* EoMs

More generally, we may pick a morphism $\mathfrak{b}' \xrightarrow{\;\boldsymbol{\prime}\;} \mathfrak{b}$
and consider the further pullback:

$$
\begin{array}{ccccc}
\text{CS constrained} & & \text{phase space} & & \\
\mathbf{Map}\big(X^d, \mathcal{B}_{\mathrm{diff}'}\big) & \rightarrow & \mathbf{Map}\big(X^d, \mathcal{B}_{\mathrm{diff}}\big) & \xrightarrow{\eta^{\int}} & \mathbf{Map}\big(X^d; \mathcal{B}\big) \\
\downarrow & \lrcorner & \downarrow & \lrcorner & \downarrow \mathbf{ch}_* \\
\mathbf{\Omega}^1_{\mathrm{cl}}(X^d; \mathfrak{b}') & \xrightarrow{\;\boldsymbol{\prime}_*\;} & \mathbf{\Omega}^1_{\mathrm{cl}}(X^d; \mathfrak{b}) & \xrightarrow{\;\eta^{\int}\;} & \int \mathbf{\Omega}^1_{\mathrm{cl}}(X^d; \mathfrak{b})
\end{array}
$$

This corresponds to imposing CS-like EoMs, $F^{(i)} = 0$
on those fluxes $F^{(i)}$ not in the image of $\boldsymbol{\prime}_*$.

Example: The Hopf fibration $\boldsymbol{\prime} \equiv \mathfrak{l}h_{\mathbb{C}} \;:\; \mathfrak{l}S^3 \longrightarrow \mathfrak{l}S^2$
implies ordinary abelian CS EoM: $F_2 = 0$

Higher Maxwell EoMs with background fields.

Consider now an inclusion of domains $\begin{array}{c}\Sigma^p \\ \big\downarrow{\scriptstyle \phi} \\ X^d\end{array}$

with "background" flux densities $\vec{B} \in \Omega^1_{\mathrm{cl}}\big(X^d, \mathfrak{b}\big)$ as before,

then higher Maxwell-type theory *relative* to ϕ is

$$\left\{ \vec{B}'^{(j)} \in \Omega^{\deg_j}_{\mathrm{dR}}(\Sigma^p) \,\middle|\, \mathrm{d}B'^{(j)} = P^{(j)}\big(\vec{B}', \phi^\ast \vec{B}\big) \right\}$$

$$\simeq \left\{ \begin{array}{ccc} \Sigma^p & \overset{\vec{B}'}{\dashrightarrow} & \mathbf{\Omega}^1_{\mathrm{cl}}(\ast; \mathfrak{a}) \\ \big\downarrow{\scriptstyle \phi} & & \big\downarrow \\ X^d & \xrightarrow{\vec{B}} & \mathbf{\Omega}^1_{\mathrm{cl}}(\ast; \mathfrak{b}) \end{array} \right\}$$

Relative Whitehead L_∞-algebras

$$
\begin{array}{ccccc}
\text{fibration} & & \text{fibration} & & \text{relative minimal} \\
\text{of spaces} & & \text{of } L_\infty\text{-algs} & & \text{Sullivan model} \\[1em]
\mathcal{A} & & \mathfrak{l}_{\mathcal{B}}\mathcal{A} & & \mathrm{CE}\big(\mathfrak{l}_{\mathcal{B}}\mathcal{A}\big) \\
\Big\downarrow{\scriptstyle \wp} & \longmapsto & \Big\downarrow{\scriptstyle \mathfrak{l}\wp} & \longleftrightarrow & \Big\uparrow{\scriptstyle \mathrm{CE}(\mathfrak{l}\wp)} \\
\mathcal{B} & & \mathfrak{l}\mathcal{B} & & \mathrm{CE}(\mathfrak{l}\mathcal{B})
\end{array}
$$

Examples:

Maxwell with source

$$\begin{array}{ccc} X^d & \overset{G_2}{\dashrightarrow} & \mathbf{\Omega}^1_{\mathrm{cl}}\Big(*;\, \mathfrak{l}_{B^3\mathbb{Z}} EB^2\mathbb{Z}\Big) \\ \Big\downarrow{\scriptstyle \iota} & & \Big\downarrow \\ X^d_{\cup\{\infty\}} & \xrightarrow{\;J_3\;} & \mathbf{\Omega}^1_{\mathrm{cl}}\big(*;\, \mathfrak{l}B^3\mathbb{Z}\big) \end{array}$$

$$\mathrm{d}\, G_2 \;=\; \iota^* J_3$$

$$\mathrm{d}\, J_3 \;=\; 0$$

NS/RR-flux

$$\begin{array}{ccc} X^d & \overset{F_{2\bullet}}{\dashrightarrow} & \mathbf{\Omega}^1_{\mathrm{cl}}\Big(*;\, \mathfrak{l}_{B^3\mathbb{Z}} \mathrm{KU} /\!\!/ B^2\mathbb{Z}\Big) \\ \Big\downarrow{\scriptstyle \iota} & & \Big\downarrow \\ X^d_{\cup\{\infty\}} & \xrightarrow{\;H_3\;} & \mathbf{\Omega}^1_{\mathrm{cl}}\big(*;\, \mathfrak{l}B^3\mathbb{Z}\big) \end{array}$$

$$\mathrm{d}\, F_{2\bullet} \;=\; F_{2\bullet-2} \wedge \iota^* H_3$$

$$\mathrm{d}\, H_3 \;=\; 0$$

flux on M5-brane in 11D

$$\begin{array}{ccc} \Sigma^5 & \overset{H_3}{\dashrightarrow} & \mathbf{\Omega}^1_{\mathrm{cl}}\Big(*;\, \mathfrak{l}_{S^4} S^7\Big) \\ \Big\downarrow{\scriptstyle \phi} & & \Big\downarrow{\scriptstyle \mathfrak{l}(h_{\mathbb{H}})_*} \\ X^{10} & \overset{(G_4, G_7)}{\dashrightarrow} & \mathbf{\Omega}^1_{\mathrm{cl}}\big(*;\, \mathfrak{l}S^4\big) \end{array}$$

$$\mathrm{d}\, H_3 \;=\; \phi^* G_4$$

$$\mathrm{d}\, G_7 \;=\; \tfrac{1}{2} G_4 \wedge G_4$$

$$\mathrm{d}\, G_4 \;=\; 0$$

Twisted relative cohomology.

$$\mathbf{Map}(\phi, \wp) \;:=\; \mathbf{Map}(\Sigma, \mathcal{A}) \underset{\mathrm{Map}(\Sigma, \mathcal{B})}{\times} \mathbf{Map}(X, \mathcal{B})$$

$$= \left\{ \begin{array}{ccc} \Sigma & \dashrightarrow & \mathcal{A} \\ {\scriptstyle \int\phi}\big\downarrow & & \big\downarrow{\scriptstyle \wp} \\ X & \dashrightarrow & \mathcal{B} \end{array} \right\}$$

etc.

Then the $\wp$**-twisted** ϕ**-relative differential cohomology** is:

$$\begin{array}{ccccc} \mathbf{Map}(\phi, \wp_{\mathrm{diff}}) & \longrightarrow & \mathbf{Map}(\phi, \wp_{\mathrm{diff}'}) & \longrightarrow & \mathbf{Map}(\phi, \wp) \\ \big\downarrow & \lrcorner & \big\downarrow & \lrcorner & \big\downarrow{\scriptstyle \mathbf{ch}} \\ \mathbf{\Omega}^1_{\mathrm{cl}}(\phi, \mathfrak{l}\wp') & \xrightarrow{\;'_*\;} & \mathbf{\Omega}^1_{\mathrm{cl}}(\phi, \mathfrak{l}\wp) & \xrightarrow{\;\eta^{\int}\;} & \int \mathbf{\Omega}^1_{\mathrm{cl}}(\phi, \mathfrak{l}\wp) \end{array}$$

Example.

$$\text{Consider}\;\begin{cases} \wp \;\equiv\; h_{\mathbb{C}} \;:\; S^3 \longrightarrow S^2 \\[1em] \boldsymbol{/} \;\equiv\; ʃ(\mathrm{id}, h) \;:\; \begin{array}{ccc} ʃS^3 & \longrightarrow & ʃS^3 \\ \Vert & & \big\downarrow ʃh_{\mathbb{C}} \\ ʃS^3 & \longrightarrow & ʃS^2 \end{array} \end{cases}$$

then:

EoMs	gauge potentials
$F_2 \;=\; 0$	$\mathrm{d}\,\lambda \;=\; \phi^* A_1$
$\mathrm{d}\,H_3 \;=\; F_2 \wedge F_2$	$\mathrm{d}\,A_1 \;=\; 0$

This is as in abelian CS with FJ/WZW boundary!

Claim: Topological observables on phase space stack reproduce in fine detail those of abelian CS with boundary:

- renormalized Wilson loop observables
- Heisenberg observables on torus
- modular functor
- bulk/edge correspondence

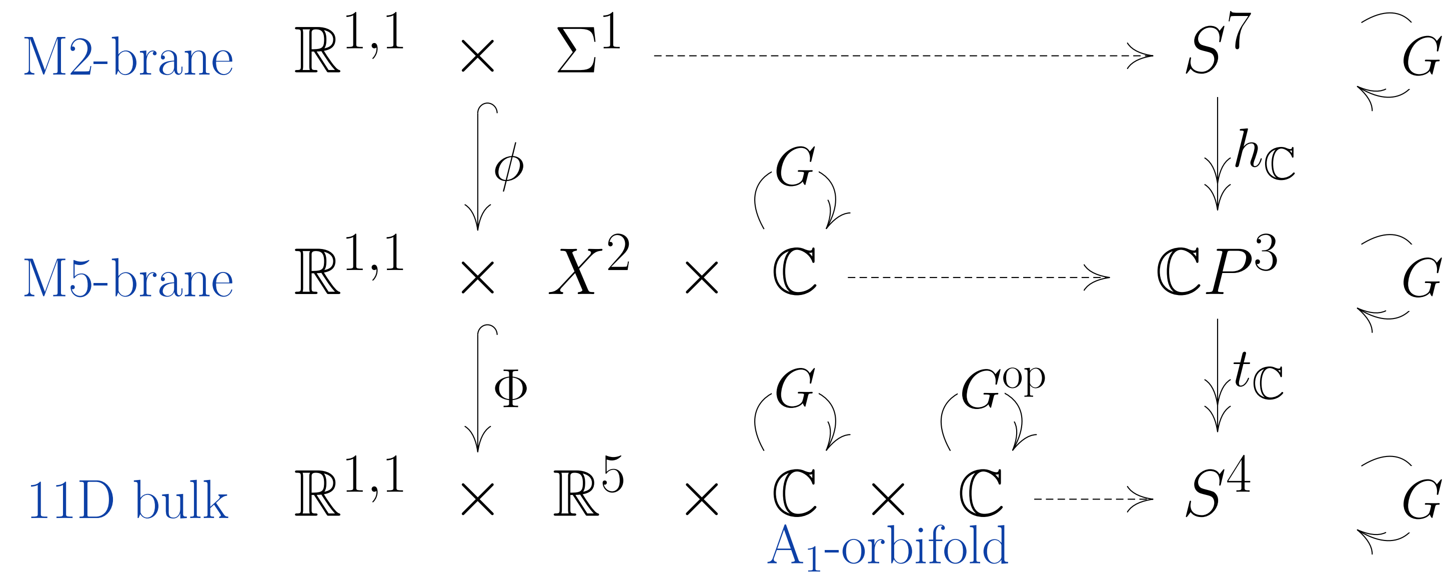
Twisted relative cohomology forms bulk-boundary LESs:

$$
\begin{array}{ccccc}
 & & & & \overset{\text{near bndry unwindings}}{\tilde H^{-2}(\Sigma;\mathcal{A})} \\
 & & \xrightarrow[\text{deep bulk orders unwinding near bndry}]{\partial_1^{\mathrm{uw}}} & & \\
\underset{\text{deep bulk top. orders}}{\tilde H^{-1}(\overline{X};\mathcal{B})} & \underset{\text{among}}{\xrightarrow{\;i_{\mathrm{blk}}\;}} & \underset{\text{total system top. orders}}{\tilde H^{-1}(\phi;\wp)} & \underset{\text{restrict to}}{\xrightarrow{\;r_{\mathrm{bdr}}\;}} & \underset{\text{near bndry top. orders}}{\tilde H^{-1}(\Sigma;\mathcal{A})}
\end{array}
$$

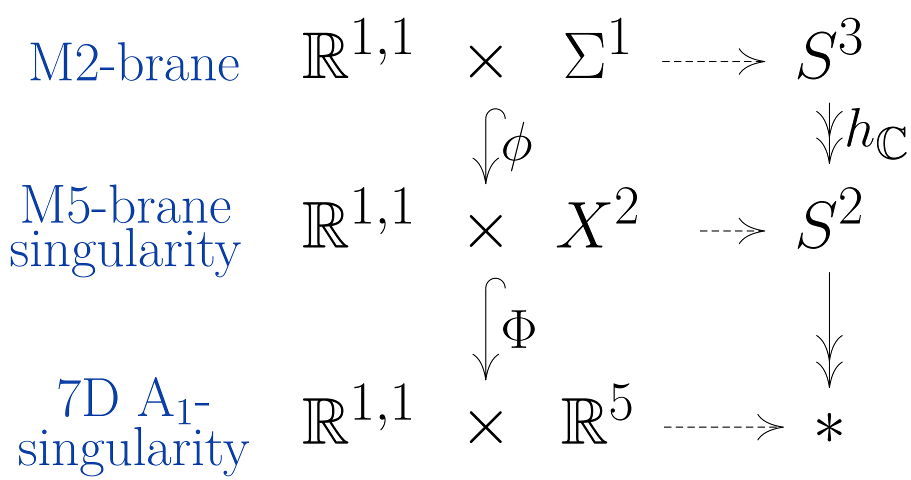
Claim: For the above choice $\wp \equiv h_{\mathbb{C}}$ and $\prime \equiv \mathfrak{l}(\mathrm{id}, h_{\mathbb{C}})$ this reproduces physical expectations for FQH systems ...

ncatlab.org/schreiber/show/BBC

Engineering FQH anyons on M2/M5-branes:



reduces to the orbi-singularity:



and hence to CS/WZW

Flux Quantization on M5-Branes, JHEP 140 (2024)
Anyons on M5-Probes of Seifert 3-Orbifolds, LMP **115** (2025)
Engineering of Anyons on M5-Probes, SciPost Phys. Lec. Notes **107** (2025)
Flux Quantization on M-Strings (2026)

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