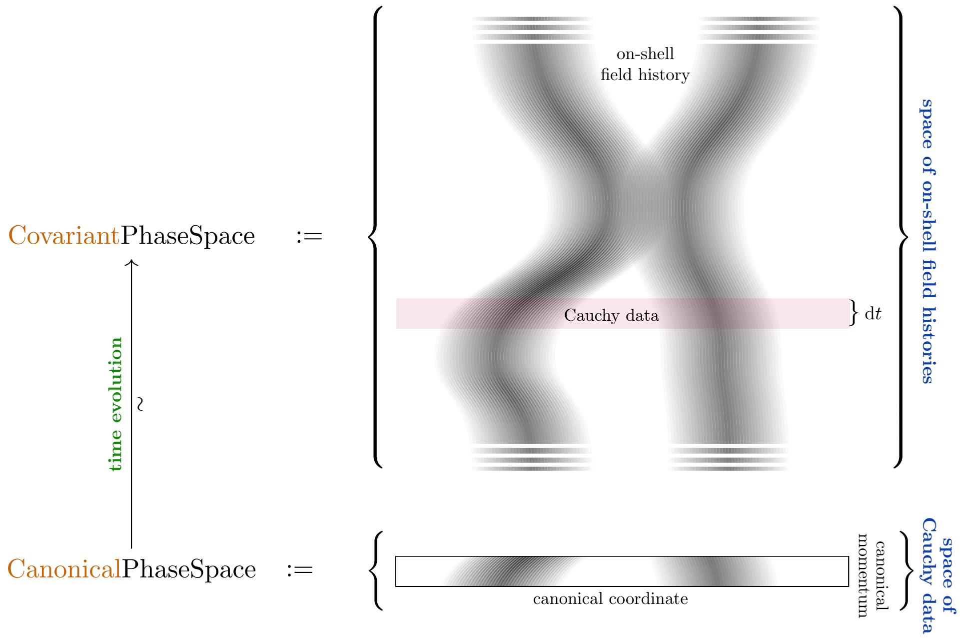
The phase space. Abstractly, the *phase space* of a field theory is nothing but the space of all those field histories that satisfy the given equations of motion (the "on-shell" field histories). Phrased this way, this is sometimes called the **covariant phase space** ([Wi86, p. 314][ČW87][HT92, §17.1]; see [Kh14][GiS23] for rigorous discussion) to emphasize that no choice of foliation of spacetime by Cauchy surfaces has been or needs to be made.

The more traditional **canonical phase space** is instead a parameterization of the covariant phase space by initial value data on a given Cauchy surface, after choosing a foliation of spacetime by spatial hypersurfaces (cf. [SS23-FQ, p. 5]). This choice breaks the "manifest covariance" of the covariant phase space. Nevertheless, if a Cauchy surface exists at all (hence on globally hyperbolic spacetimes), then both these phase spaces are equivalent, by definition, the equivalence being the map that generates from initial value data the essentially unique on-shell field history that evolves from it (possibly up to gauge transformation).



Solution space of on-shell flux densities. At this point in our discussion, we do not yet know what the full field content of our field theories really is – this will be implied by a choice of flux quantization in §1.2 – we only know the corresponding flux densities. To remember this, we shall call the space of flux densities solving their equations of motion the *solution space*, and we are after its incarnation as a *canonical solution space* of initial value data on a Cauchy surface. But the canonical phase will simply consist of all flux-quantized gauge potentials compatible with these flux solutions (cf. p. 21).

higher Maxwell-type equations of motion in duality-symmetric form

$$\begin{array}{|c|}\hline \text{Bianchi identities} \\ \mathrm{d}\,\vec F \;=\; \vec P(\vec F) \\[4pt] \star F \;=\; \vec\mu(\vec F) \\ \text{self-duality} \\ \hline\end{array}\qquad \begin{array}{|l|}\hline \underset{\text{flux species}}{I \,\in\, \mathrm{Set}}\,, \;\; \underset{\text{flux degrees}}{\big(\deg_i \,\in\, \mathbb{N}_{\geq 1}\big)_{i \in I}}\,, \;\; \underset{\text{flux densities}}{\vec F \,\equiv\, \Big(F^{(i)} \,\in\, \Omega^{\deg_i}_{\mathrm{dR}}\big(X^D\big)\Big)_{i \in I}} \\[6pt] \underset{\text{flux self-sourcing}}{\vec P \text{ graded-symm. polynomial}}\;, \quad \underset{\text{vacuum permittivity}}{\vec\mu \text{ invertible matrix}} \\ \hline\end{array} \tag{15}$$

Proposition 1.1 ([SS23-FQ]). *On a globally hyperbolic spacetime $X^D \simeq \mathbb{R}^{0,1} \times X^d$, the solution space to higher Maxwell-equations of motion brought into the duality-symmetric form* (15) *is isomorphic to the solution of the Bianchi identities on any Cauchy surface $\iota : X^d \hookrightarrow X^D$, then to be called the higher Gauß law:*

$$\begin{aligned}
\underset{\substack{\text{space of flux densities}\\ \text{on spacetime, solving}\\ \text{the equations of motion}}}{\mathrm{SolSpace}} \;\equiv\;& \left\{ \underset{\text{electromagnetic flux densities on spacetime}}{\vec F \,\equiv\, \Big(F^{(i)} \,\in\, \Omega^{\deg_i}_{\mathrm{dR}}\big(X^D\big)\Big)_{i\in I}} \;\middle|\; \begin{array}{c} \text{Bianchi identities} \\ \mathrm{d}\,\vec F \;=\; \vec P(\vec F) \\ \star F \;=\; \vec\mu(\vec F) \\ \text{self-duality}\end{array} \right\} \;\; \text{covariant form} \\
\underset{\iota^*}{\simeq}\;& \left\{ \underset{\text{magnetic flux densities on Cauchy surface}}{\vec B \,\equiv\, \Big(B^{(i)} \,\in\, \Omega^{\deg_i}_{\mathrm{dR}}\big(X^d\big)\Big)_{i\in I}} \;\middle|\; \underset{\text{Gauß law}}{\mathrm{d}\,\vec B \;=\; \vec P(\vec B)} \right\} \;\; \text{canonical form}
\end{aligned} \tag{16}$$

Gravity "decouples" on canonical phase space. As above, the inverse isomorphism (16) is given by time evolution of initial value data. Notice that the background metric (the background field of gravity) enters *only* in determining the nature of this isomorphism ι^*, but does not affect the nature of the initial value data (of the canonical phase space) as such (cf. [SS23-FQ]).

It is this "decoupling" *on the canonical phase space* of the gravity/metric effects from the phase space Gauß law constraint which allows to gain plenty of insight into brane configurations from purely cohomological analysis of fluxes on Cauchy surfaces, disregarding the full solution of the coupled (super-)gravity equations of motion, cf. the examples in §1.1.3 and §2.