Outlook. While higher topos theory has general relevance in physics, as described, a key application is to differential generalized cohomology theories, and these in turn enter the formulation of strongly-coupled ("non-perturbative") quantum systems, in the form of *flux quantization laws*.

Here is a brief outlook on a broad picture that seems to be emerging:

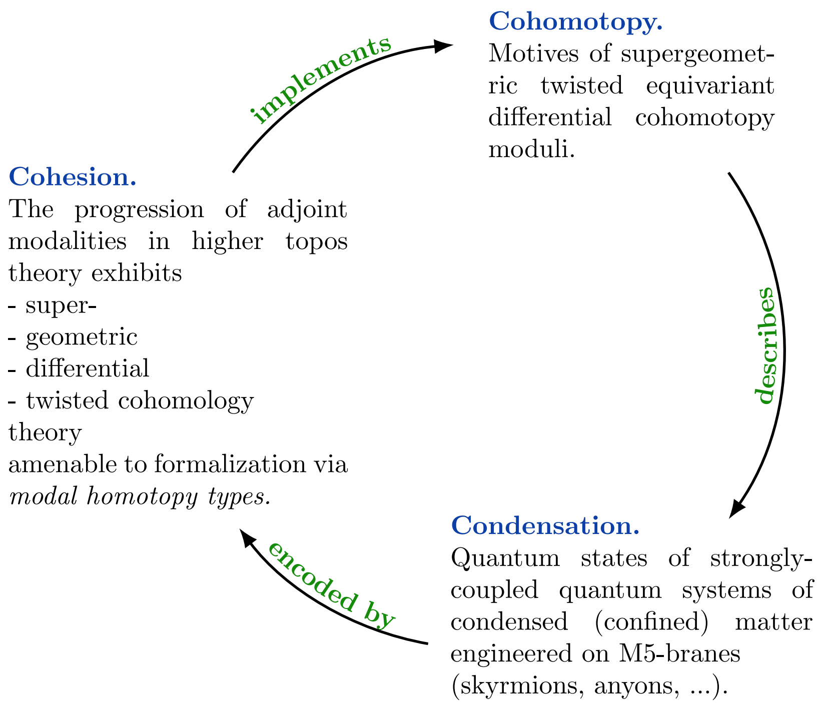
(0.) Strongly-Coupled quantum systems. The glaring open problem of contemporary fundamental physics is a general analytic understanding of *strongly-coupled* ("non-perturbative") quantum systems, such as the standard model of particle physics at room temperature ("confinement") as well as strongly-correlated quantum materials thought to be relevant for realistic quantum computers ("topological order").

(1.) Condensation. No matter what else string theory has been motivated by at any time, it provides a glimpse of a missing general theory of non-perturbative quantum physics, famous under the working title "M-theory". For instance the hypothetical worldvolume theory of "M5-branes" shows indications of reflecting otherwise elusive non-perturbative phenomena such as S-duality, confinement, Skyrmions and anyons in gauge theories.

(2.) Cohomotopy. Remarkably, the local on-shell field content of M5-branes (in 11d supergravity backgrounds) is concisely expressed by basic constructions in Cartan super-geometry, and – as with any higher gauge theory – the global completion of the field content is provided by "flux quantization laws" in twisted differential generalized cohomology theories.

(3.) Cohesion. Exactly these ingredients — which have been largely missing from the traditional toolbox of mathematical physics — are naturally provided by the "gros" higher toposes discussed above, namely by *cohesive* ∞*-toposes*. In fact, the systematic *progression of adjoint modalities* in cohesive higher topos theory produces in a precise sense, starting from "nothing", first the theory of twisted differential cohomology, then (higher) Cartan geometry and finally its super-geometric enhancement.

What this means is that – in stark contrast to traditional geometry – the higher geometry provided by cohesive higher topos theory not only provides previously missing tools for plausibly formulating the outstanding general theory of strongly-coupled quantum systems, but it provides it in an "elementary" way, in the technical sense that the relevant constructions lend themselves to formalization in the "internal logic" of these toposes. Concretely, this means that there can exist natural *programming languages* (certification languages) for processes in (strongly-coupled) quantum systems which serve both as a formal theory for the dynamics of these systems as well as a *machine language* for computational processes exhibited by them.



Cohomotopy.
Motives of supergeometric twisted equivariant differential cohomotopy moduli.
describes
Condensation.
Quantum states of strongly-coupled quantum systems of condensed (confined) matter engineered on M5-branes (skyrmions, anyons, ...).
encoded by
Cohesion.
The progression of adjoint modalities in higher topos theory exhibits
- super-
- geometric
- differential
- twisted cohomology
theory
amenable to formalization via *modal homotopy types.*
implements