Urs Schreiber on joint work with Hisham Sati:

surveying our preprint: [arXiv:2507.00138]

Non-Lagrangian construction of abelian CS/FQH-theory

Chern-Simons

fractional quantum Hall

anyons!

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS @ جامعة نيويورك أبوظبي NYU ABU DHABI

(July 2025) find these slides at: [ncatlab.org/schreiber/show/ISQS29]

Urs Schreiber on joint work with Hisham Sati:

surveying our preprint: [arXiv:2507.00138]

Non-Lagrangian construction of abelian CS/FQH-theory

via Flux Quantization in 2-Cohomotopy

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS @ NYU ABU DHABI
جامعة نيويورك أبوظبي

(July 2025) find these slides at: [ncatlab.org/schreiber/show/ISQS29]

Motivation.

Motivation.

fractional quantum Hall systems (FQH)
are *known* to feature anyons

Motivation.

fractional quantum Hall systems (FQH)

are *known* to feature anyons

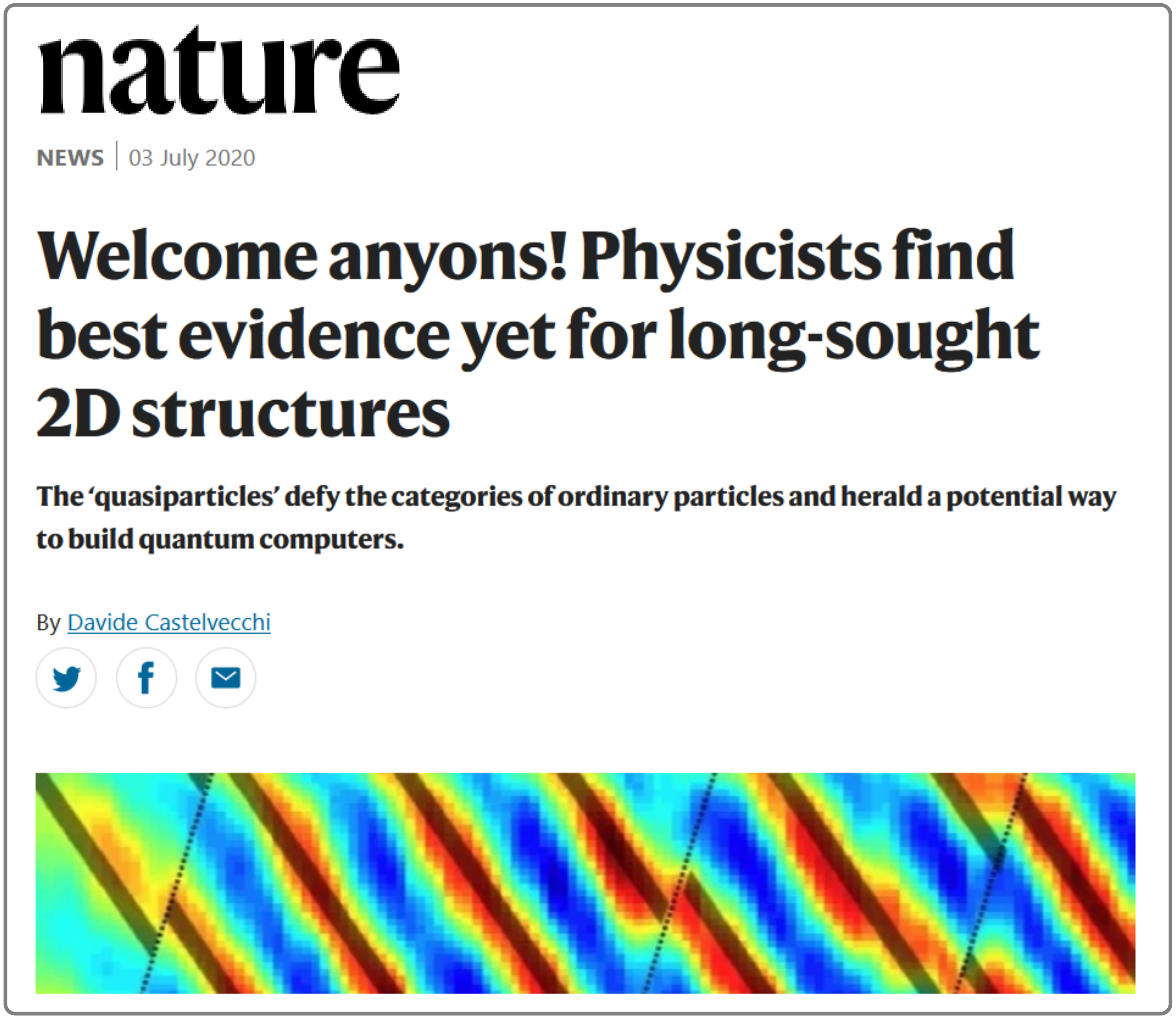
[Nakamura et al. 2020]
[Nakamura et al. 2023]
[Ruelle et al. 2023]
[Glidic et al. 2023]
[Kundu et al. 2023]
[Veillon et al. 2024]
[Ghosh et al. 2025]

Motivation.

fractional quantum Hall systems (FQH)

are *known* to feature anyons

[Nakamura et al. 2020]
[Nakamura et al. 2023]
[Ruelle et al. 2023]
[Glidic et al. 2023]
[Kundu et al. 2023]
[Veillon et al. 2024]
[Ghosh et al. 2025]



nature

NEWS | 03 July 2020

Welcome anyons! Physicists find best evidence yet for long-sought 2D structures

The 'quasiparticles' defy the categories of ordinary particles and herald a potential way to build quantum computers.

By Davide Castelvecchi

Motivation.

fractional quantum Hall systems (FQH)
are *known* to feature anyons

Motivation.

fractional quantum Hall systems

are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,

Motivation.

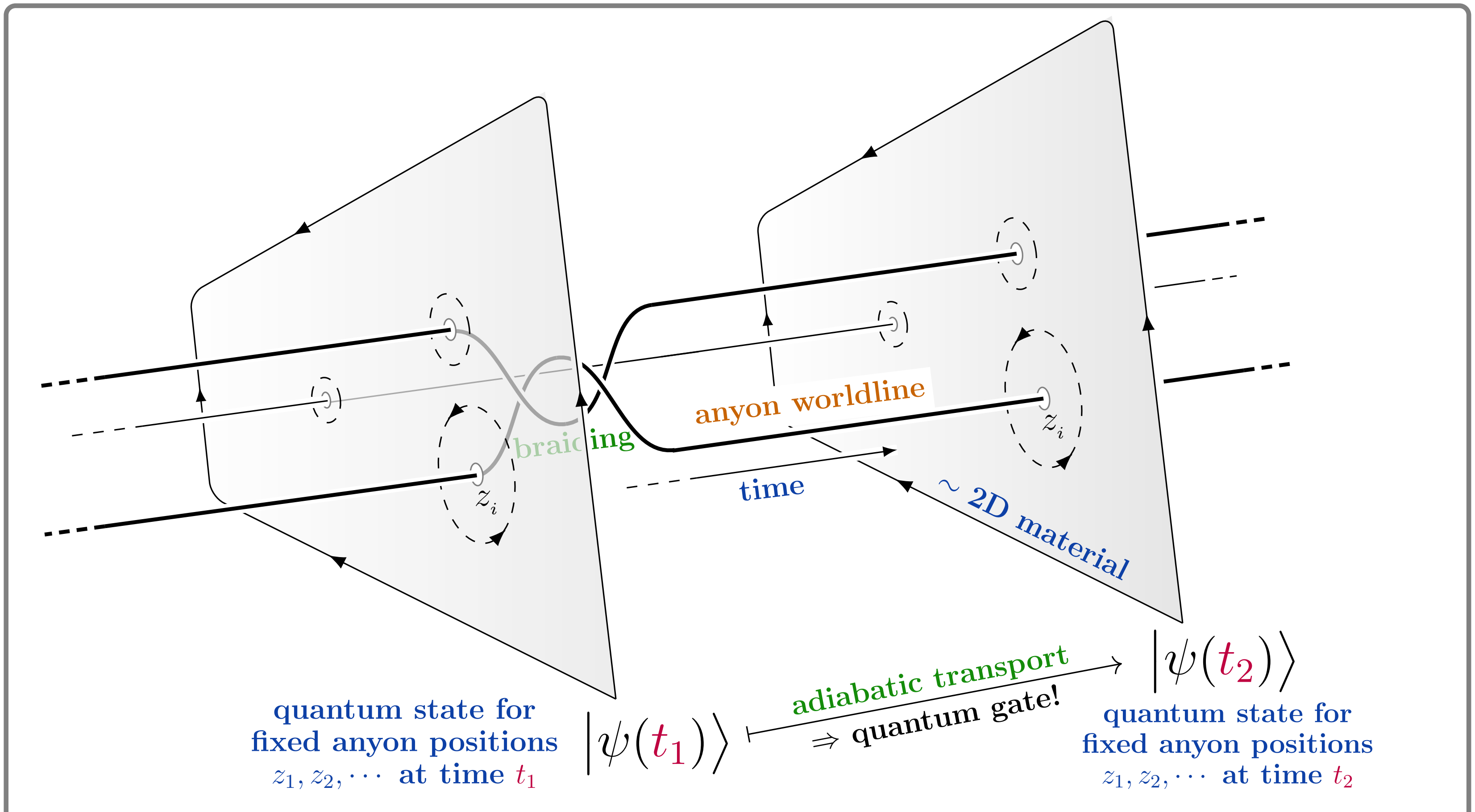
fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

Motivation.

fractional quantum Hall systems
are *known* to feature anyons

⇒ potential hardware for topological quantum,
if external control of anyons be achieved



Motivation.

fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

Motivation.

fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

but despite successful FQH *model building*...

Motivation.

fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

but despite successful FQH *model building*...

$$\left(\begin{array}{c} \text{trial wavefunctions,} \\ \text{composite particle lore,} \\ \text{effective } \underbrace{\text{CS}}_{\text{Chern-Simons field theory}} \text{ field matching} \end{array} \right)$$

Motivation.

fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

but despite successful FQH *model building*...

$$\left(\begin{array}{c} \text{trial wavefunctions,} \\ \text{composite particle lore,} \\ \text{effective } \underbrace{\text{CS}}_{\text{Chern-Simons field theory}} \text{ field matching} \end{array} \right)$$

...*theory* of FQH anyons remained a mystery
cf. [Jain 2007 §5.1], [Jain 2020 §1]

Motivation.

fractional quantum Hall systems
are *known* to feature anyons

$\Rightarrow$ potential hardware for topological quantum,
if external control of anyons be achieved

but despite successful FQH *model building*...

$$\left(\begin{array}{c}\text{trial wavefunctions,}\\ \text{composite particle lore,}\\ \text{effective } \underbrace{\text{CS}}_{\text{Chern-Simons field theory}} \text{ field matching}\end{array}\right)$$

...*theory* of FQH anyons remained a mystery

cf. [Jain 2007 §5.1], [Jain 2020 §1]

Motivation.

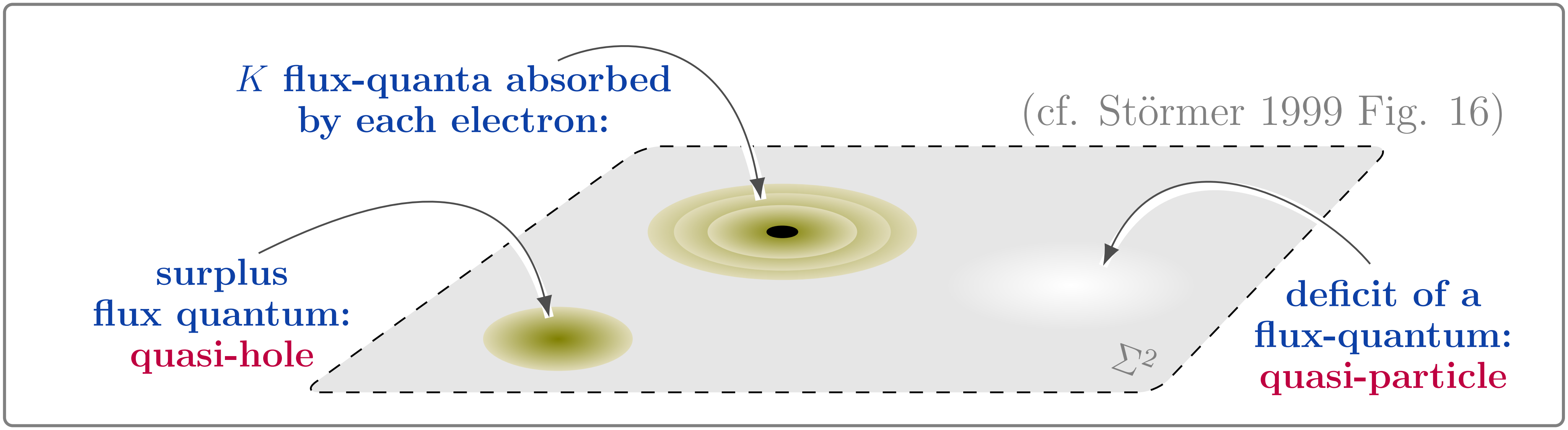
but observe:

the surplus **flux quanta**, aka: quasi-holes

Motivation.

but observe:

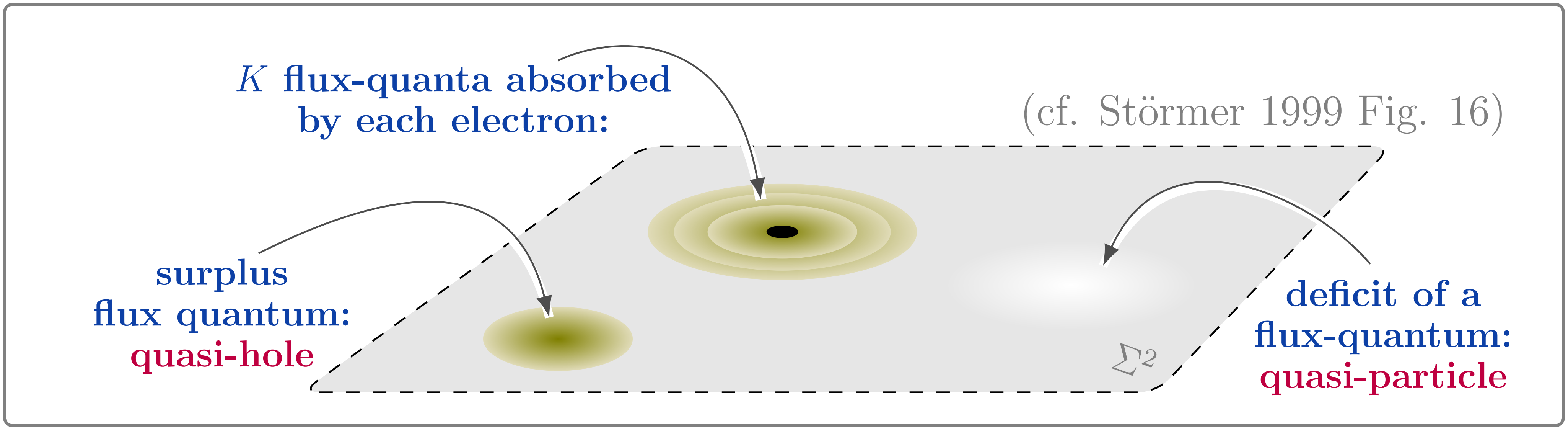
the surplus **flux quanta**, aka: quasi-holes



Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

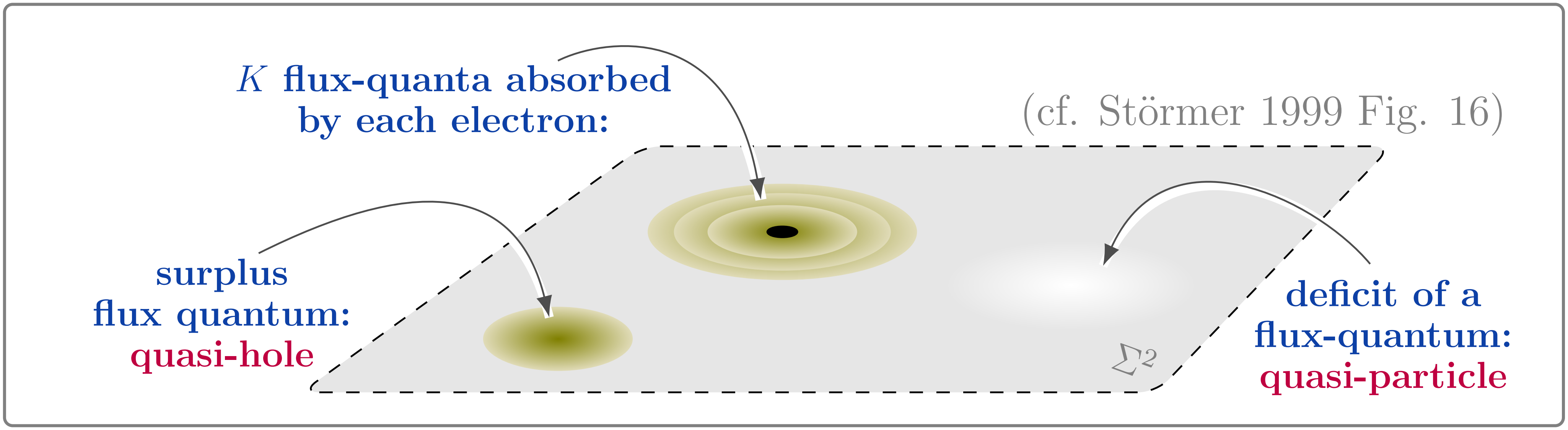


are the FQH anyons!

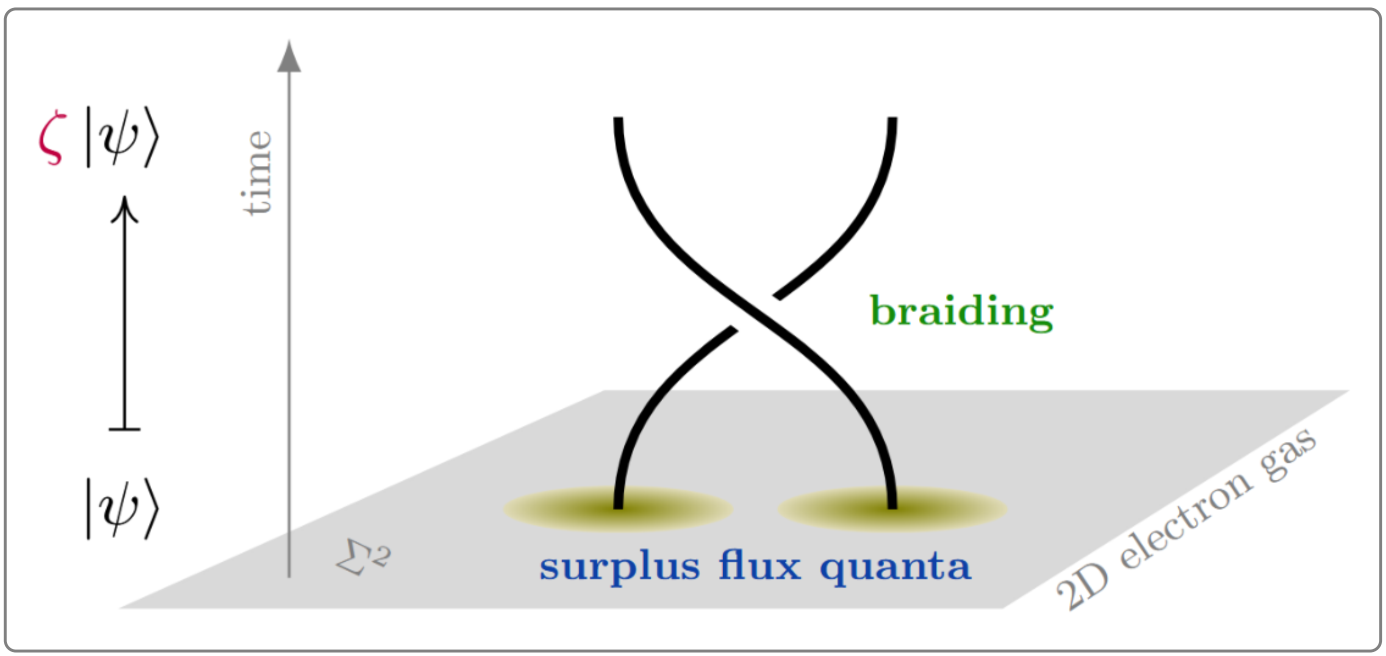
Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes



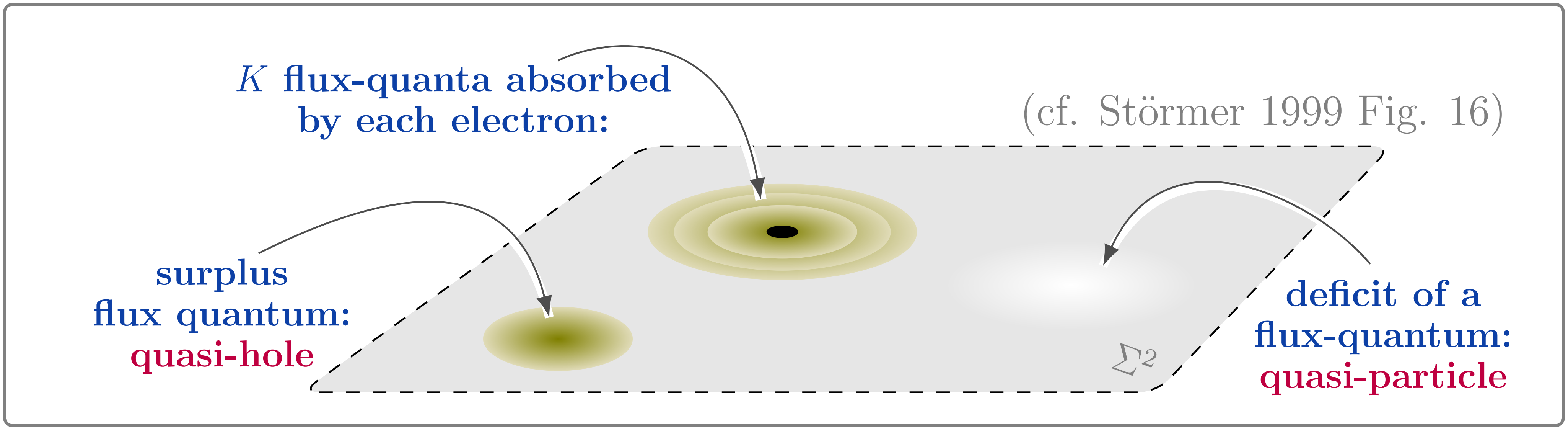
are the FQH anyons!



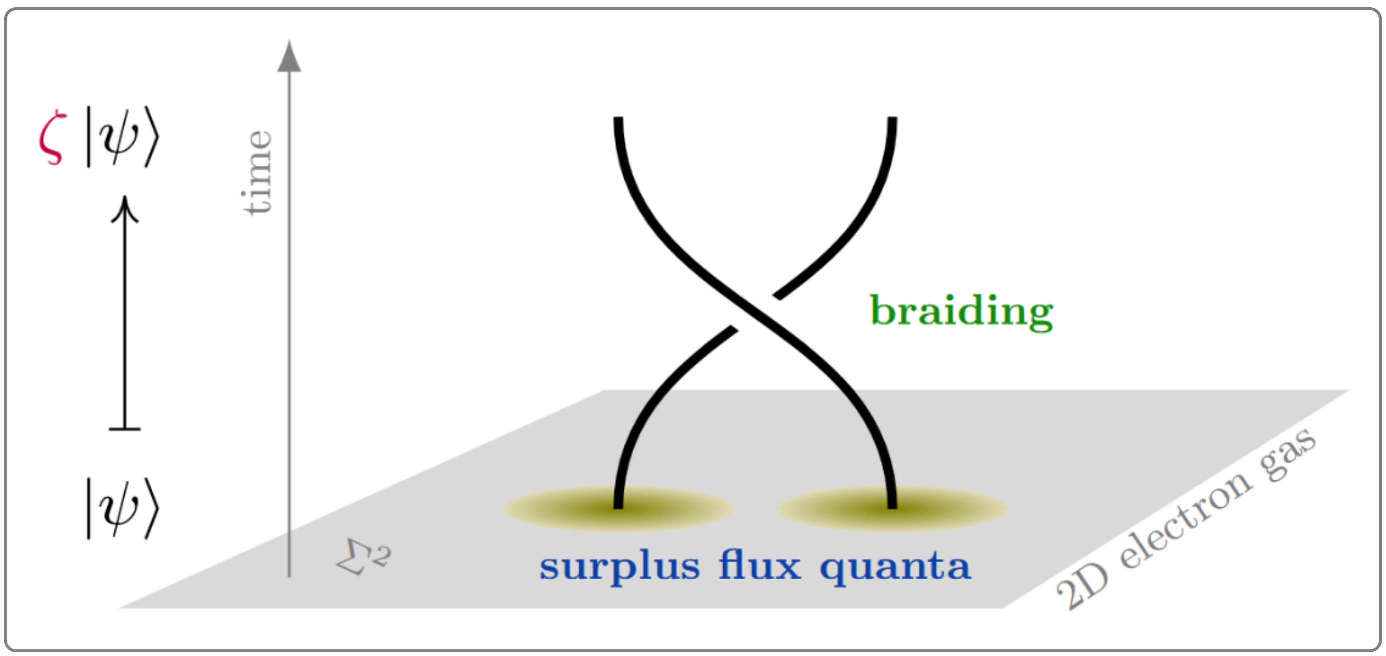
Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes



are the FQH anyons!

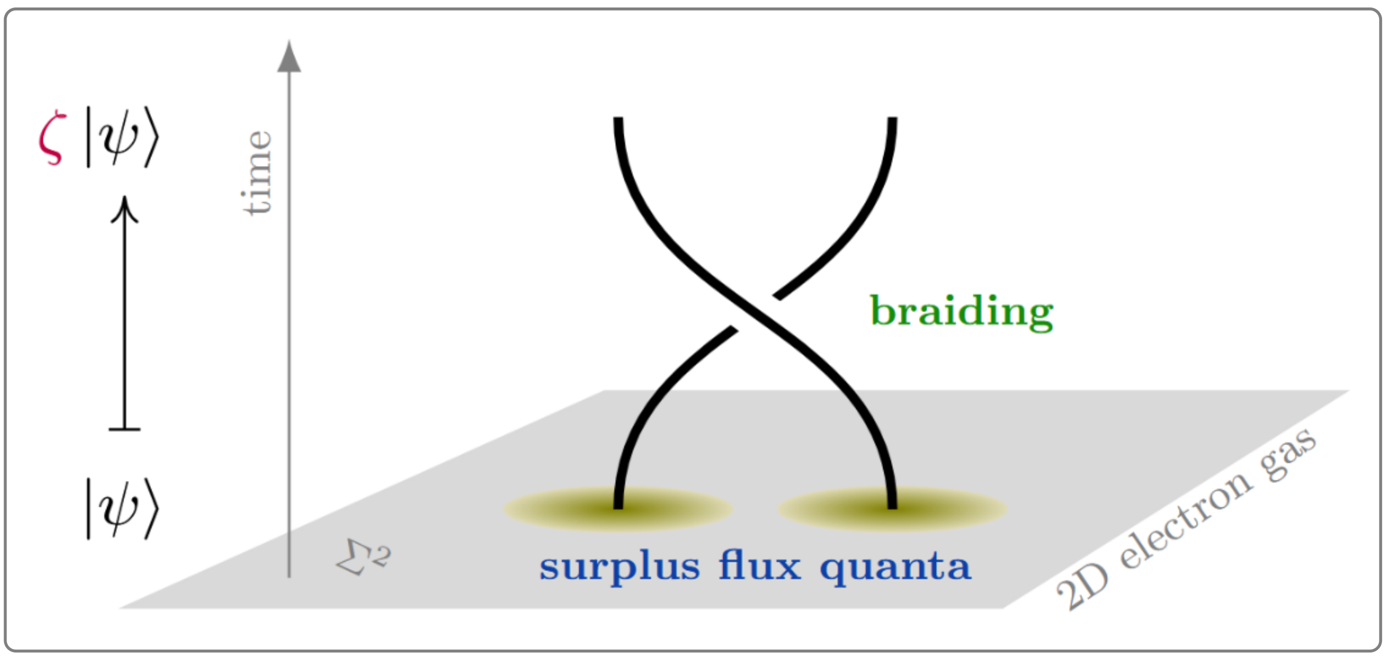


Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!

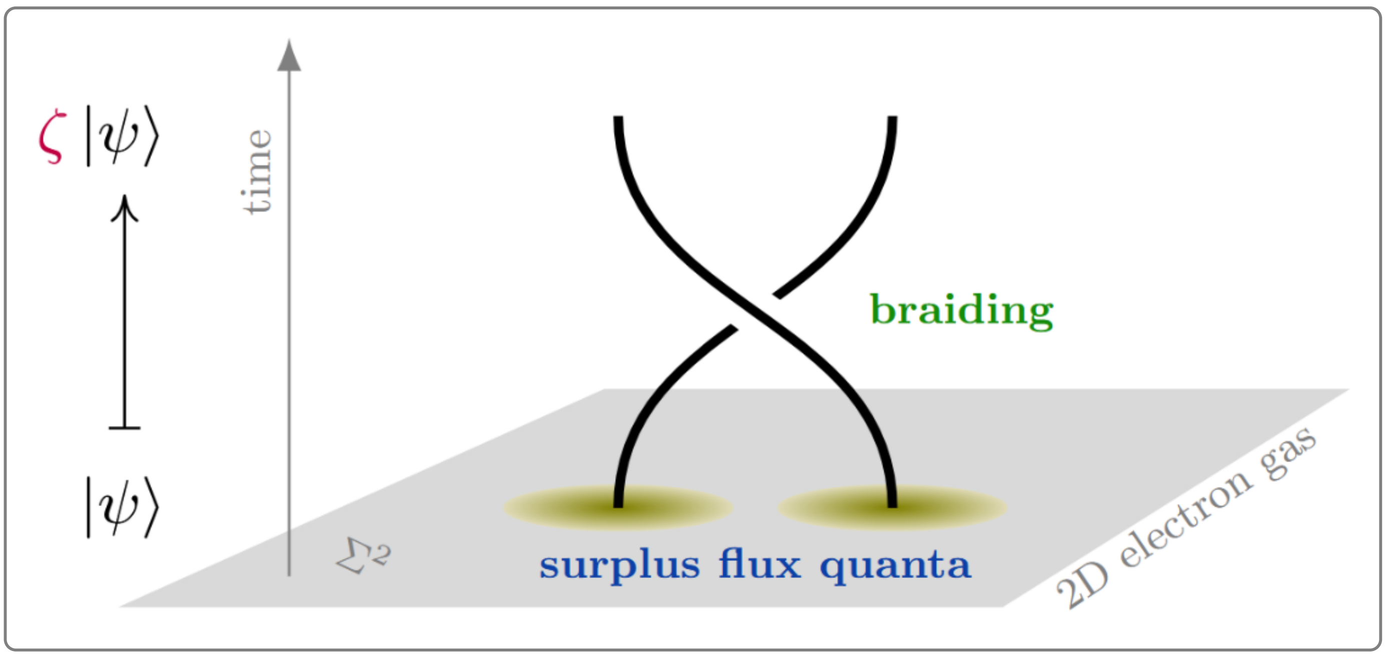


Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!

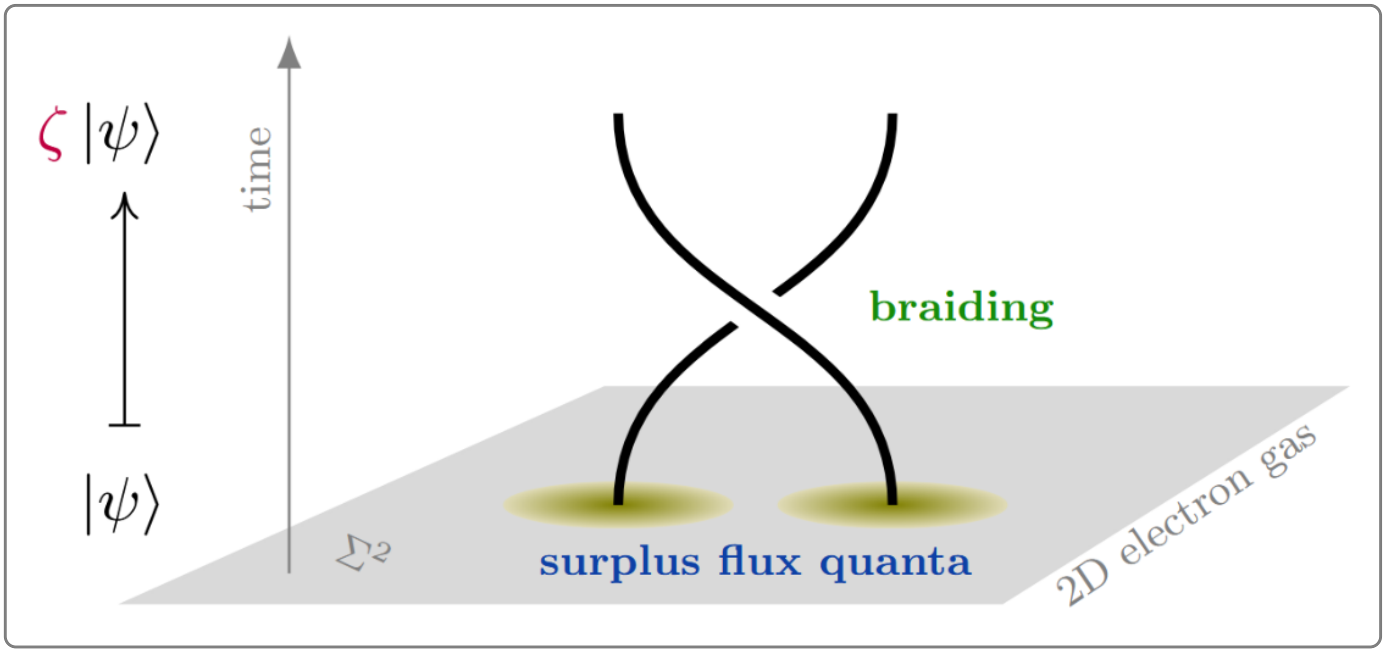


Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!

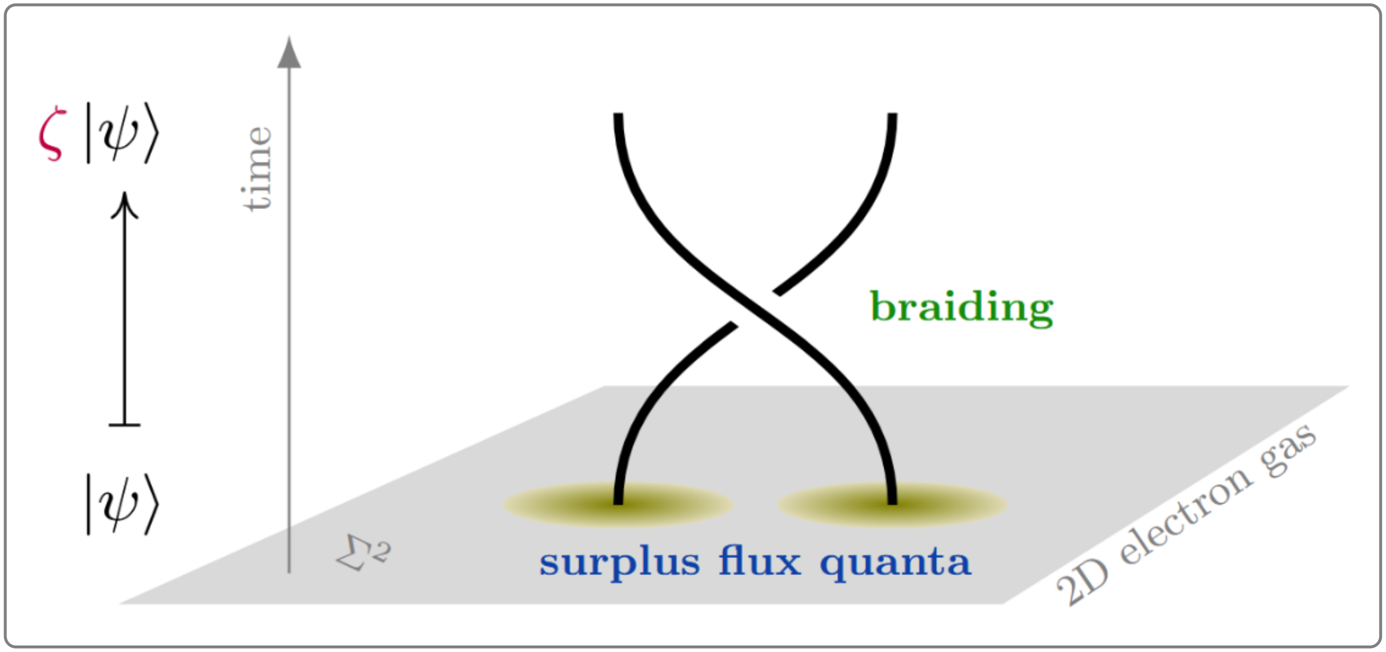


Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!

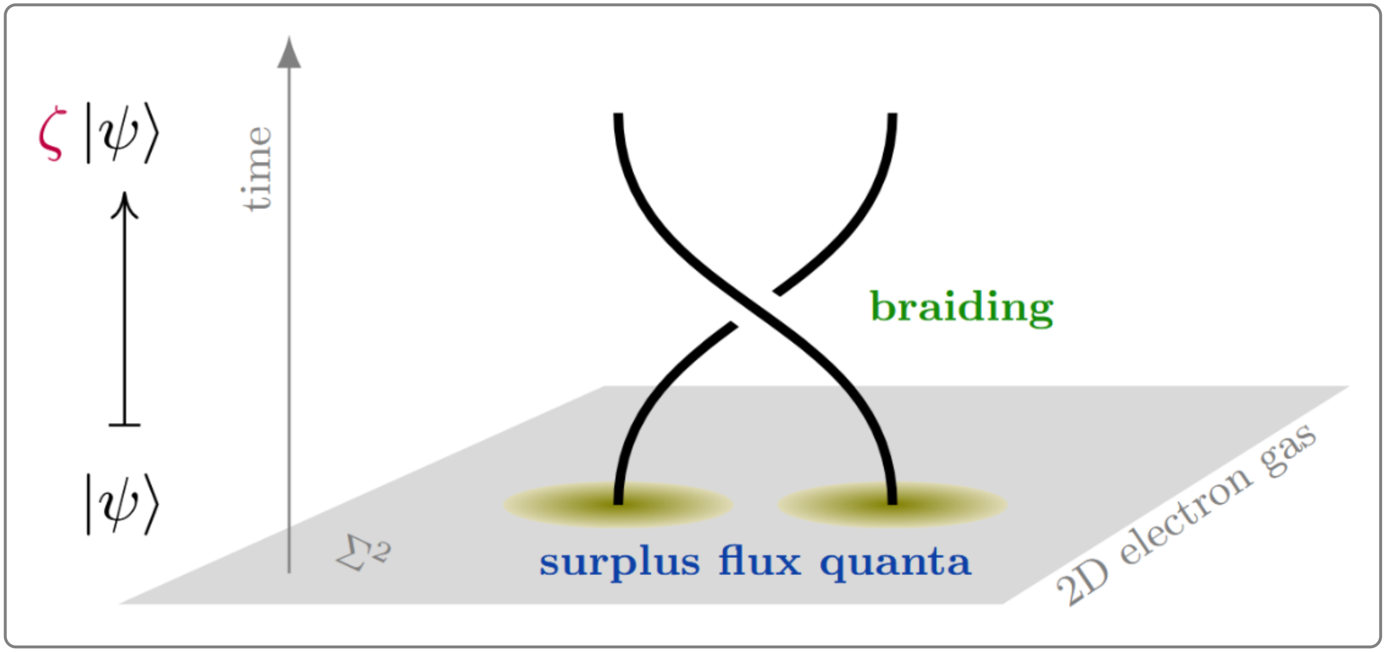


Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!



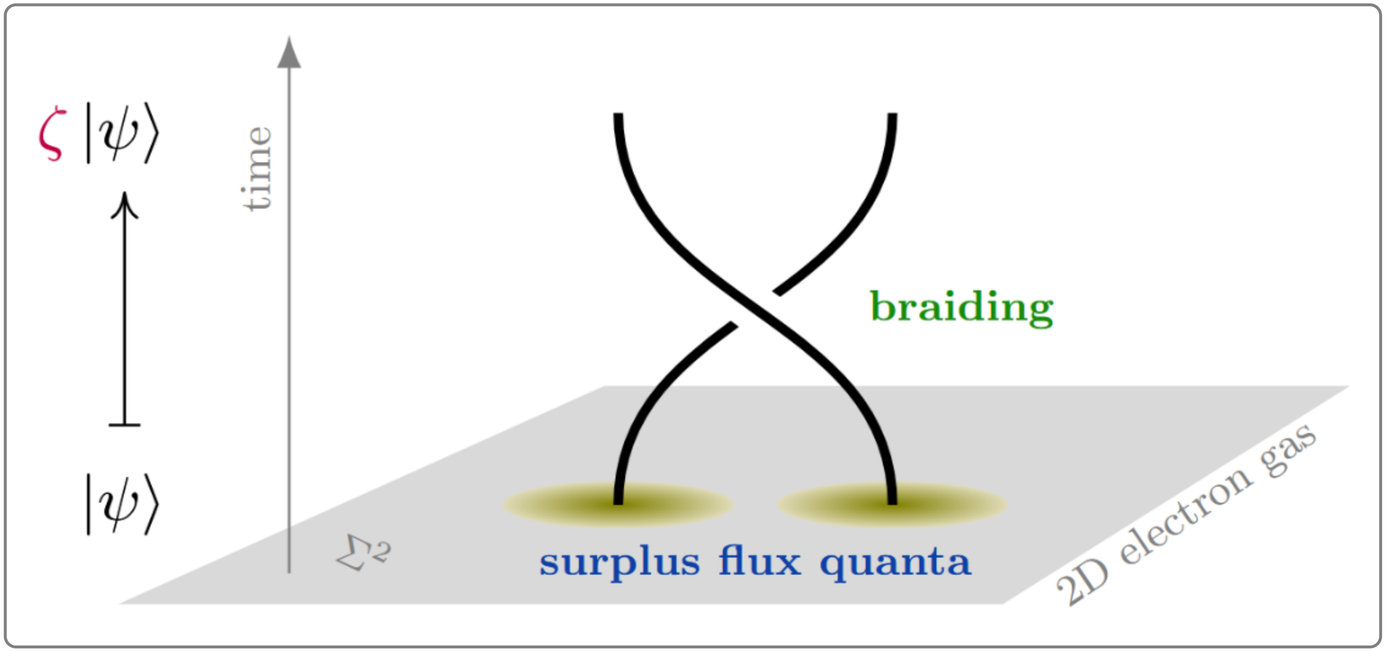
suggests: FQH effect rests on a form of **exotic flux quantization**

Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!



suggests: FQH effect rests on a form of **exotic flux quantization**

but just the magnetic flux quantization appears *inconsistent* in the CS description!

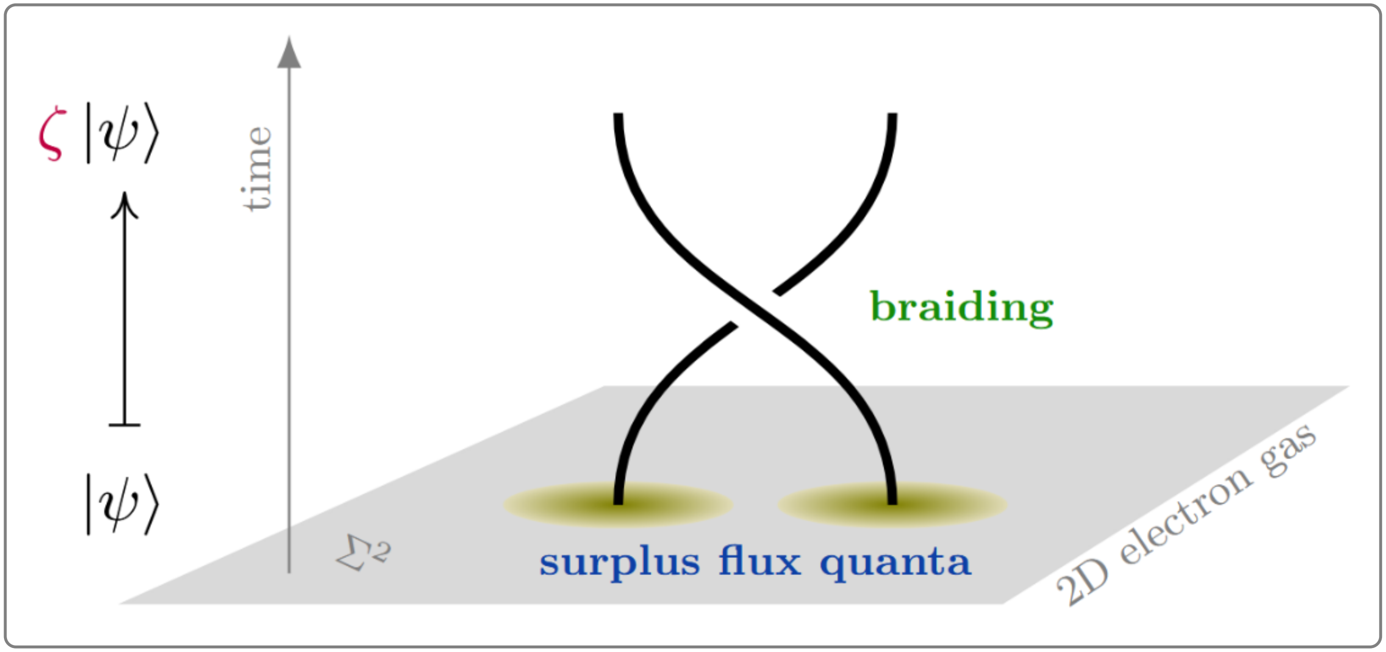
cf. [Witten 2016 p 35], [Tong 2016 p 159]

Motivation.

but observe:

the surplus **flux quanta**, aka: quasi-holes

are the FQH anyons!



suggests: FQH effect rests on a form of **exotic flux quantization**

but just the magnetic flux quantization appears *inconsistent* in the CS description!

cf. [Witten 2016 p 35], [Tong 2016 p 159]

Outline.

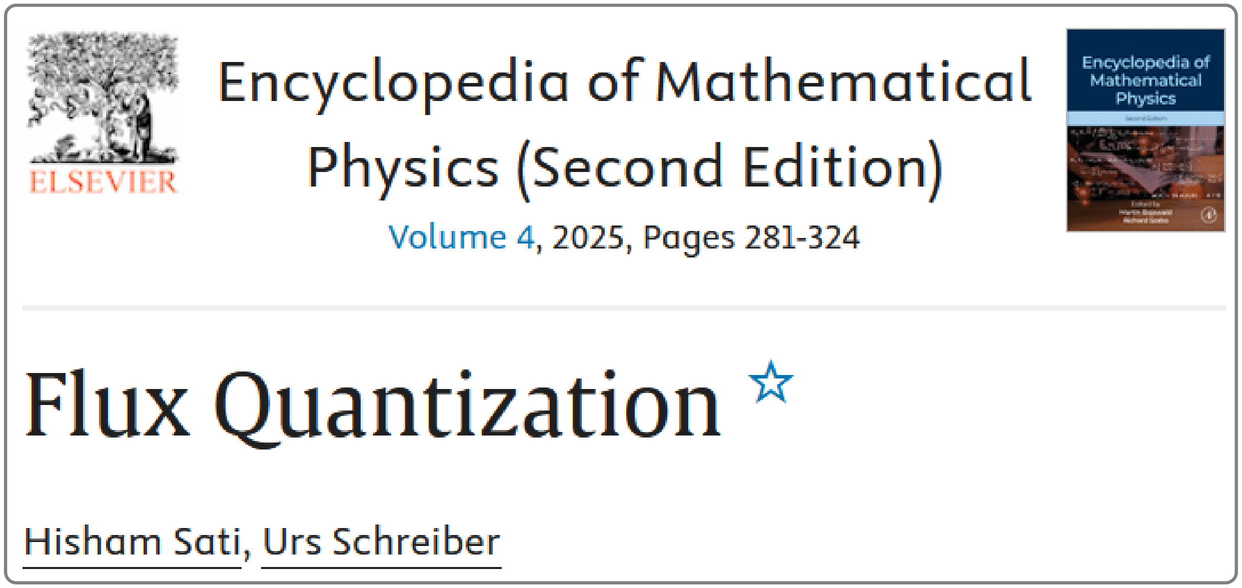
our approach:

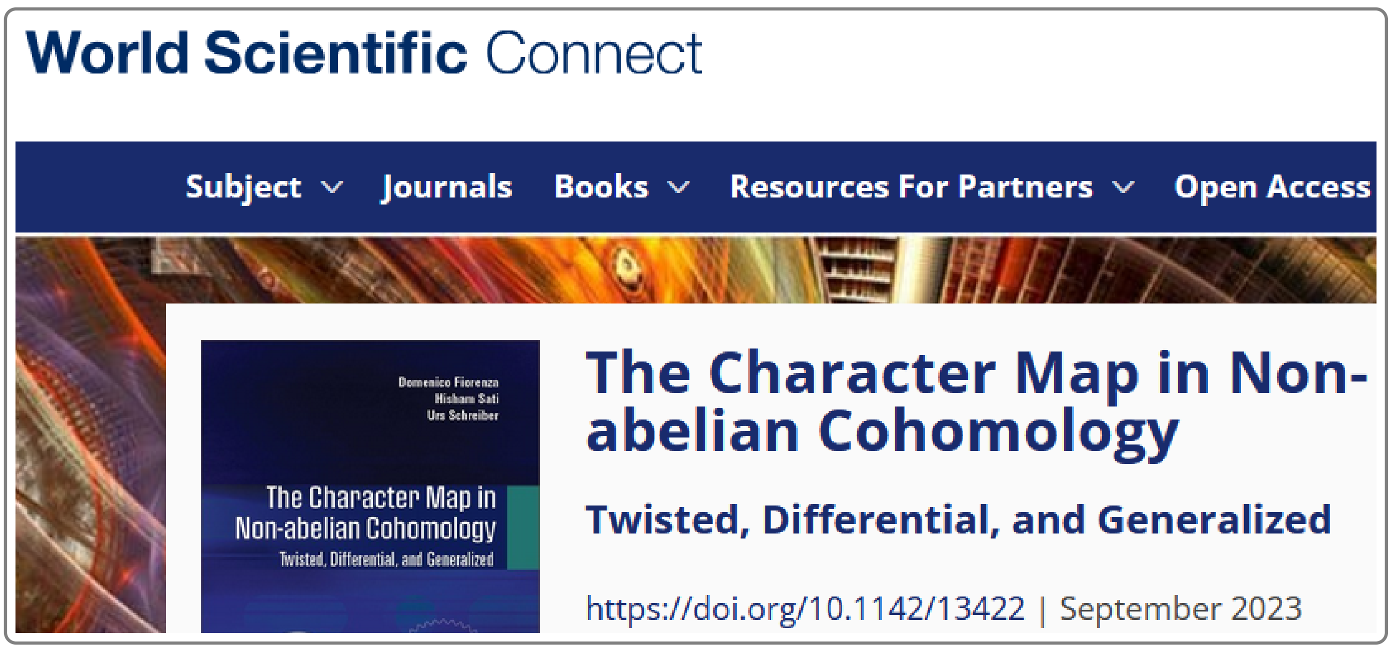
(1.) find mathematics of exotic flux quantization

Outline.

our approach:

(1.) find mathematics of exotic flux quantization





Outline.

our approach:

(1.) find mathematics of exotic flux quantization

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

arXiv > cond-mat > arXiv:2505.22144

Search...

Help

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 28 May 2025 (v1), last revised 2 Jul 2025 (this version, v2)]

Fractional Quantum Hall Anyons via the Algebraic Topology of Exotic Flux Quanta

Hisham Sati, Urs Schreiber

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

approach recovers $\left\{ \begin{array}{c} \text{fractional statistics,} \\ \text{topological order,} \\ \text{edge modes} \end{array} \right\}$ as for FQH

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

approach recovers { fractional statistics, topological order, edge modes } as for FQH

up to subtle details: experimentally discernible

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

approach recovers { fractional statistics, topological order, edge modes } as for FQH

up to subtle details: experimentally discernible

also predicts anyonic *defects* where flux is expelled,
(such as for superconducting islands within 2DEG)

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

approach recovers { fractional statistics, topological order, edge modes } as for FQH

up to subtle details: experimentally discernible

also predicts anyonic *defects* where flux is expelled,
(such as for superconducting islands within 2DEG)
potentially non-abelian and *controllable*

Outline.

our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

our result:

also predicts anyonic *defects* where flux is expelled,
(such as for superconducting islands within 2DEG)
potentially non-abelian and *controllable*

Outline.

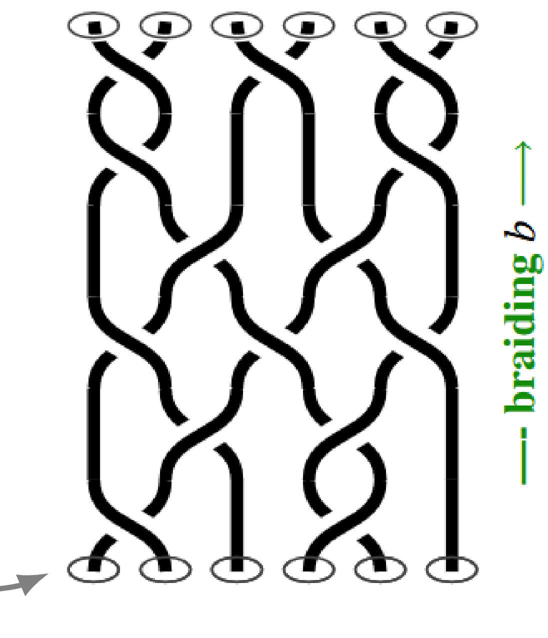
our approach:

(1.) find mathematics of exotic flux quantization

(2.) hypothesize quantization law for FQH flux

(3.) derive predictions using algebraic topology

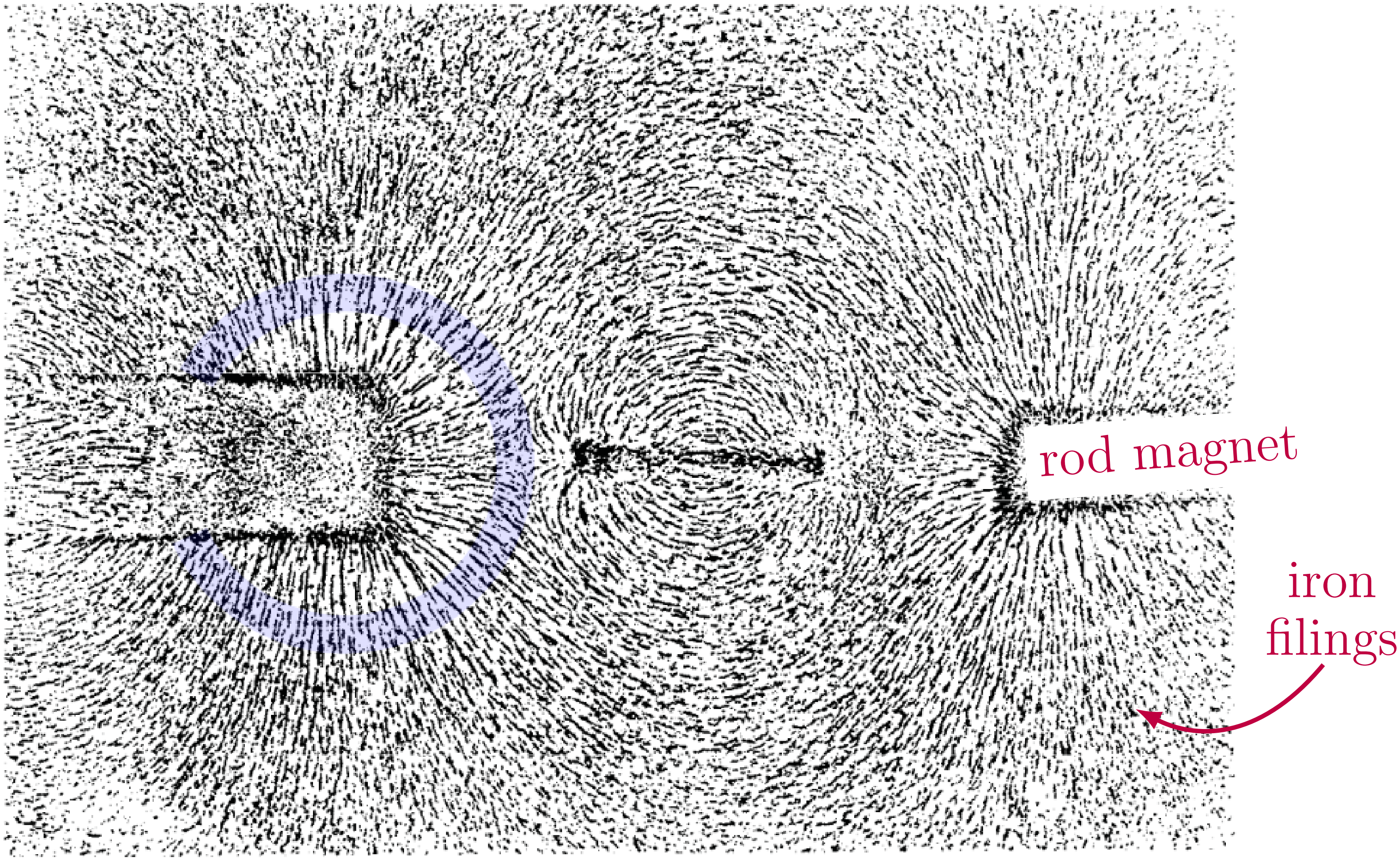
our result:



also predicts anyonic *defects* where flux is expelled,
(such as for superconducting islands within 2DEG)
potentially non-abelian and *controllable*

Recap flux.

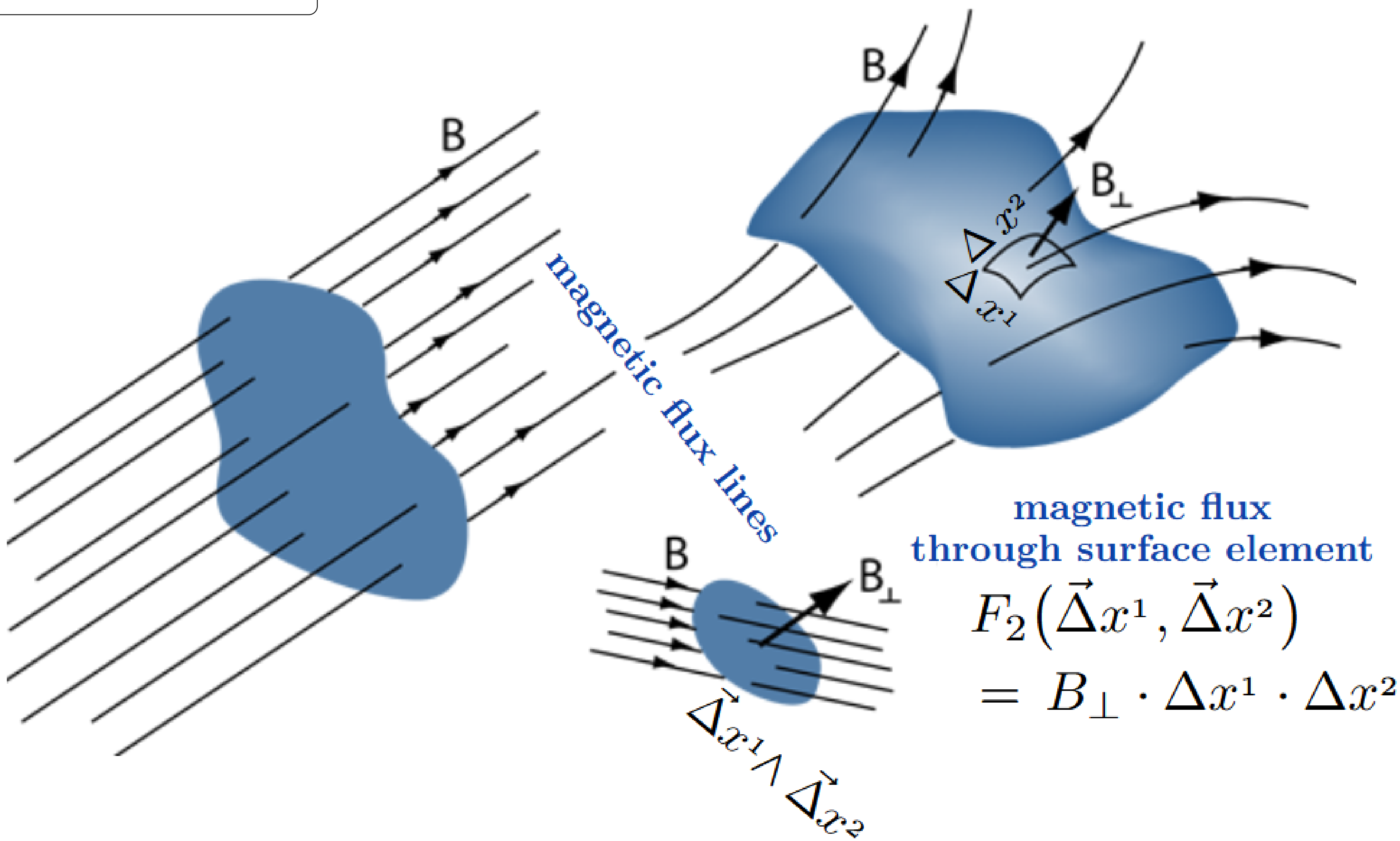
Recap flux.



From Faraday's *Diary of experimental investigation*, vol VI, entry from 11th Dec. 1851, as reproduced in [Martin09]; the colored arc is our addition, for ease of comparison with the next graphics.

Recap flux.

Recap flux.



The density and orientation of magnetic field flux lines are encoded in a differential 2-form F_2 whose integral over a given surface is proportional to the total magnetic flux through that surface. (Graphics adapted from [Hyperphysics].)

Recap flux.

recall ordinary magnetic flux quantization:

Recap flux.

recall ordinary magnetic flux quantization:

1985

Topological quantization and cohomology

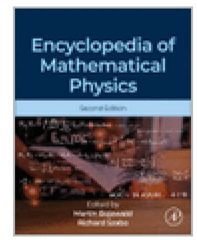
Orlando Alvarez

Comm. Math. Phys. 100(2): 279-309 (1985).

ELSEVIER

Encyclopedia of Mathematical Physics (Second Edition)

Volume 4, 2025, Pages 281-324



Flux Quantization ☆

Hisham Sati, Urs Schreiber

Recap flux.

recall ordinary magnetic flux quantization:

Recap flux.

recall ordinary magnetic flux quantization:

$\exists$ *classifying space* $B\mathrm{U}(1) \simeq \mathbb{C}P^\infty := \bigcup_n \mathbb{C}P^n$ s.t.:

Recap flux.

recall ordinary magnetic flux quantization:

$\exists$ *classifying space* $B\mathrm{U}(1) \simeq \mathbb{C}P^\infty := \bigcup_n \mathbb{C}P^n$ s.t.:

(1.) $$\pi_0 \underbrace{\mathrm{Map}\big(X, \mathbb{C}P^\infty\big)}_{\text{space of maps } X \to \mathbb{C}P^\infty} \simeq \underset{\text{ordinary cohomology}}{H^2(X;\mathbb{Z})}$$

Recap flux.

recall ordinary magnetic flux quantization:

$\exists$ *classifying space* $B\mathrm{U}(1) \simeq \mathbb{C}P^\infty := \bigcup_n \mathbb{C}P^n$ s.t.:

(1.) $\pi_0 \underbrace{\mathrm{Map}\big(X, \mathbb{C}P^\infty\big)}_{\text{space of maps } X \to \mathbb{C}P^\infty} \simeq H^2(X;\mathbb{Z})$ ordinary cohomology

(2.) *global* EM-field is $\left\{ \begin{array}{c} \text{flux density } F_2 = E\,\mathrm{d}t + B \\ \text{charge } \chi : X \to \mathbb{C}P^\infty \\ \text{potential } \widehat{A} : F_2 \Rightarrow \mathrm{ch}(\chi) \end{array} \right\}$

Recap flux.

recall ordinary magnetic flux quantization:

$\exists$ *classifying space* $B\mathrm{U}(1) \simeq \mathbb{C}P^\infty := \bigcup_n \mathbb{C}P^n$ s.t.:

(1.) $\pi_0 \underbrace{\mathrm{Map}\big(X, \mathbb{C}P^\infty\big)}_{\text{space of maps } X \to \mathbb{C}P^\infty} \simeq H^2(X;\mathbb{Z})$ ordinary cohomology

(2.) *global* EM-field is $\left\{ \begin{array}{c} \text{flux density } F_2 = E\,\mathrm{d}t + B \\ \text{charge } \chi : X \to \mathbb{C}P^\infty \\ \text{potential } \widehat{A} : F_2 \Rightarrow \mathrm{ch}(\chi) \end{array} \right\}$

so that, in particular:

$$\begin{array}{ccccccc} \Omega^2_{\mathrm{dR}}(X) & \longrightarrow & H^2_{\mathrm{dR}}(X) & \xleftarrow{\mathrm{ch}} & H^2(X;\mathbb{Z}) & \longleftarrow & \mathrm{Map}\big(X, \mathbb{C}P^\infty\big) \\ F_2 & \longmapsto & [F_2] = [\chi] & \longleftarrow & & \longmapsto & \chi \end{array}$$

Recap flux.

recall ordinary magnetic flux quantization:

$\exists$ *classifying space* $B\mathrm{U}(1) \simeq \mathbb{C}P^\infty := \bigcup_n \mathbb{C}P^n$ s.t.:

(1.) $\pi_0 \underbrace{\mathrm{Map}\big(X, \mathbb{C}P^\infty\big)}_{\text{space of maps } X \to \mathbb{C}P^\infty} \simeq H^2(X; \mathbb{Z})$ ordinary cohomology

(2.) *global* EM-field is $\left\{ \begin{array}{c} \text{flux density } F_2 = E\,\mathrm{d}t + B \\ \text{charge } \chi : X \to \mathbb{C}P^\infty \\ \text{potential } \widehat{A} : F_2 \Rightarrow \mathrm{ch}(\chi) \end{array} \right\}$

so that, in particular:

$$\begin{array}{ccccccc} \Omega^2_{\mathrm{dR}}(X) & \longrightarrow & H^2_{\mathrm{dR}}(X) & \xleftarrow{\mathrm{ch}} & H^2(X;\, \mathbb{Z}) & \longleftarrow & \mathrm{Map}\big(X, \mathbb{C}P^\infty\big) \\ F_2 & \longmapsto & [F_2] = [\chi] & \longleftarrow\!\!\!\shortmid & & & \chi \end{array}$$

total flux = charge character

Recap flux.

EM classifying space knows

Recap flux. EM classifying space knows

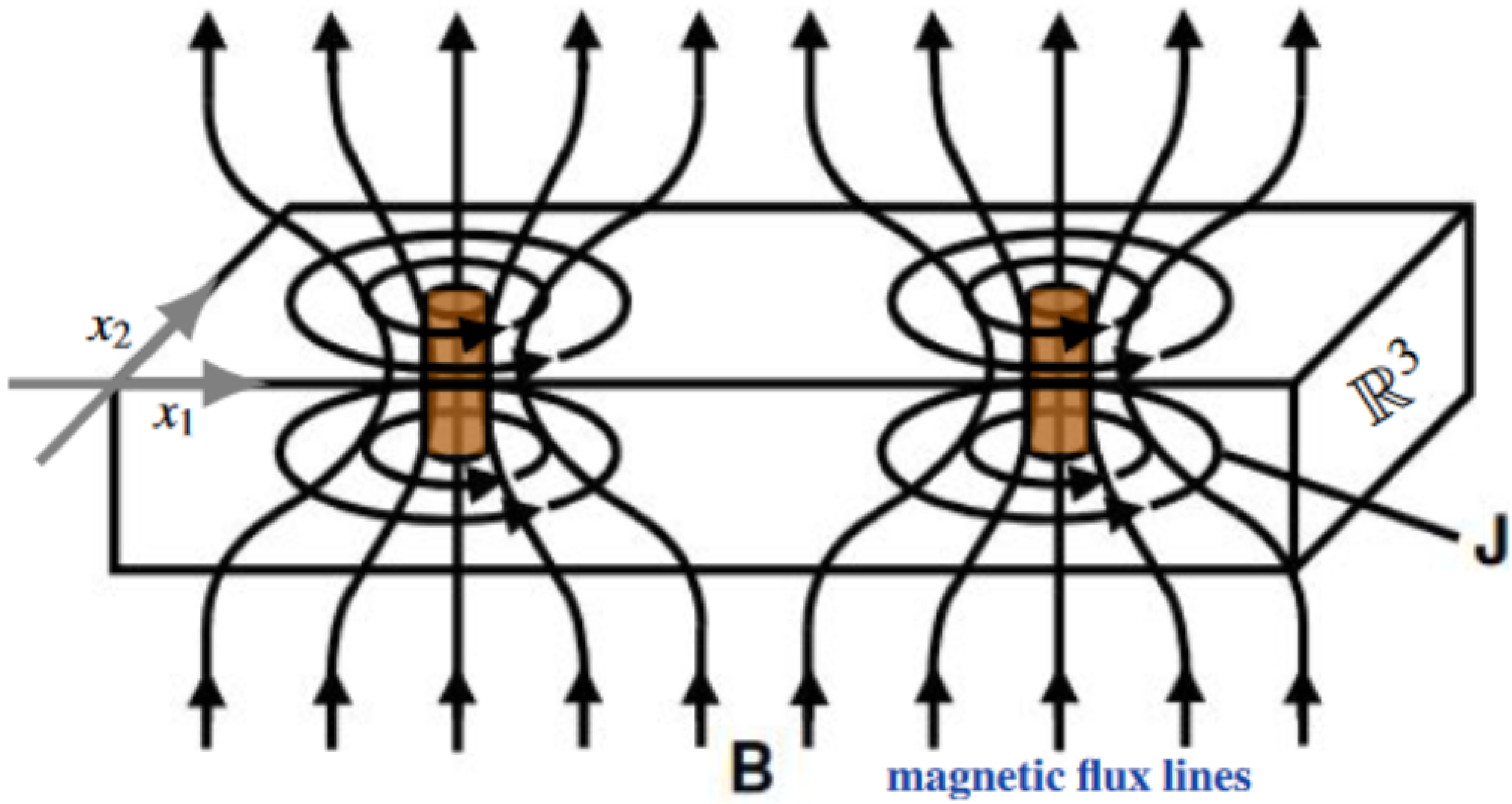
(1.) integrality of flux quanta:

$$\pi_0 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big) \;\simeq\; \mathbb{Z}$$

Recap flux. EM classifying space knows

(1.) integrality of flux quanta:

$$\pi_0 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big) \;\simeq\; \mathbb{Z}$$



Recap flux. EM classifying space knows

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables of topological flux through surface:

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

adjoin the *point-at-infinity*

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1 \,\mathrm{Map}^\ast\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

pointed mapping space
makes flux *vanish-at-infinity*
(the soliton condition)

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^\ast\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

fundamental group (monodromy of flux)

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables of topological flux through surface:

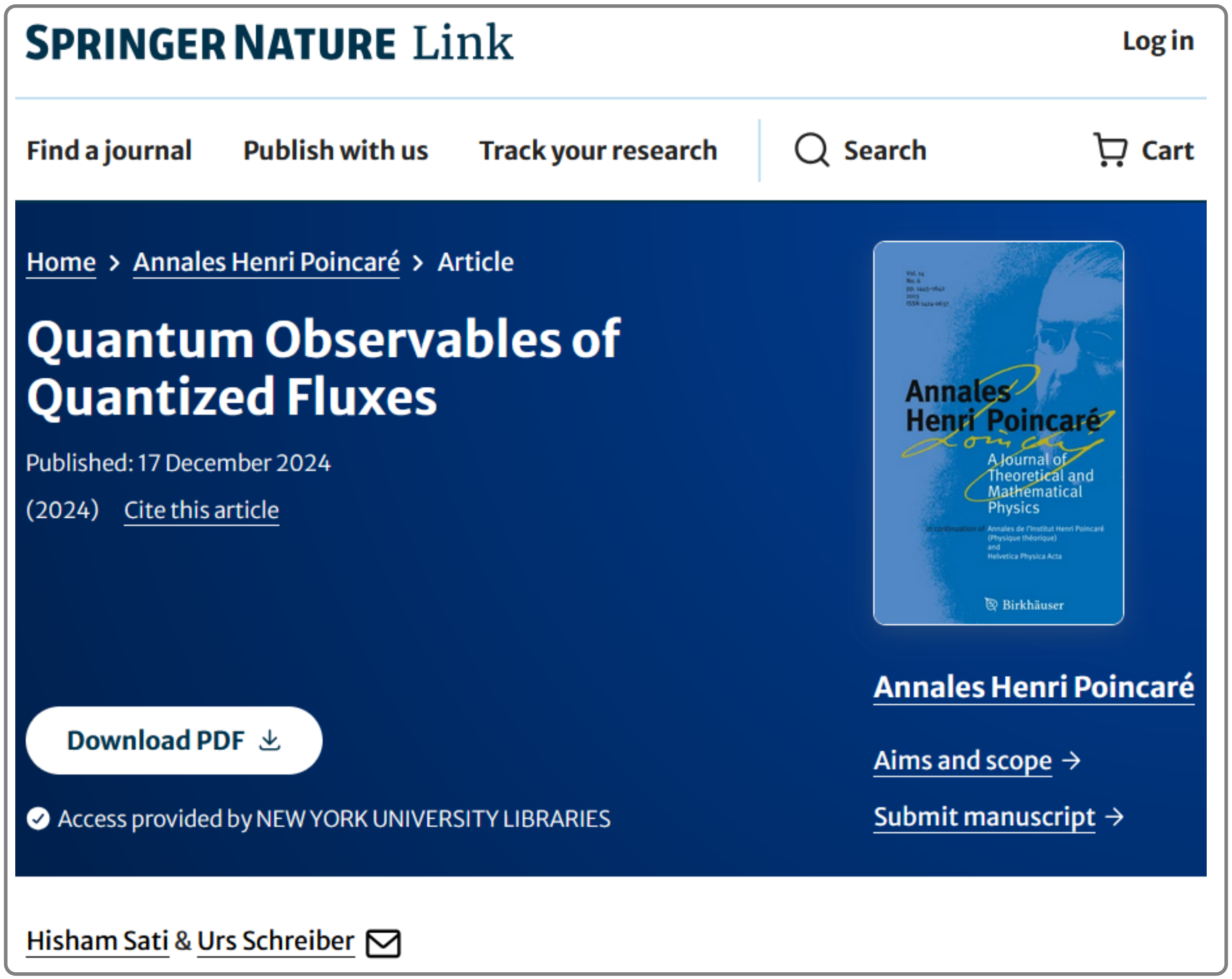
$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

group algebra (flux operators)

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\,\mathbb{C}P^\infty\big)\Big]$$



Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\, \mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^\ast\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

Example: On torus $\Sigma^2 \equiv T^2$,

commuting Wilson line observables:

$$\mathrm{Obs}(T^2)^{\mathrm{EM}} \;=\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;\Big\vert\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} = \mathbb{C}\Big[\pi_1\, \mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, \mathbb{C}P^\infty\big)\Big]$$

Example: On torus $\Sigma^2 \equiv T^2$,

commuting Wilson line observables:

$$\mathrm{Obs}(T^2)^{\mathrm{EM}} = \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;\Big\vert\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} = \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

BUT in FQH systems one expects deformation:

$$\widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} = \zeta^2\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}$$

Recap flux. EM classifying space knows

(2.) the algebra of quantum observables

of topological flux through surface:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{EM}} \;=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\,\mathbb{C}P^\infty\big)\Big]$$

Example: On torus $\Sigma^2 \equiv T^2$,

commuting Wilson line observables:

$$\mathrm{Obs}(T^2)^{\mathrm{EM}} \;=\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;\Big|\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

BUT in FQH systems one expects deformation:

$$\widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \zeta^2\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}\widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}$$

What gives?

Exotic flux.

Exotic flux.

Exotic Flux Quantization:

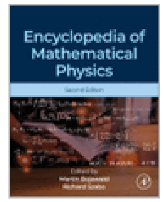
use another classifying space!

Exotic flux. Exotic Flux Quantization:

use another classifying space!

Encyclopedia of Mathematical Physics (Second Edition)

Volume 4, 2025, Pages 281-324

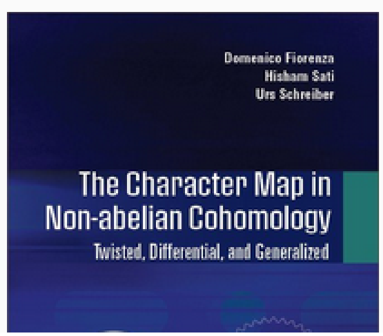


Flux Quantization ☆

Hisham Sati, Urs Schreiber

World Scientific Connect

Subject ⌄ Journals Books ⌄ Resources For Partners ⌄ Open Access



The Character Map in Non-abelian Cohomology

Twisted, Differential, and Generalized

https://doi.org/10.1142/13422 | September 2023

Exotic flux. Exotic Flux Quantization:

use another classifying space!

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \, \mathrm{Map}\big(X,\, S^2\big) \underset{\text{“characcter”}}{\xrightarrow{\;\mathrm{ch}\;}} \left\{ \begin{array}{l|l} F_2 \,\in\, \Omega^2_{\mathrm{dR}}(X) & \mathrm{d}\, F_2 \,=\, 0 \\ H_3 \,\in\, \Omega^3_{\mathrm{dR}}(X) & \mathrm{d}\, H_3 \,=\, F_2 \wedge F_2 \end{array} \right\}_{\big/\sim}$$

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \, \mathrm{Map}\big(X,\, S^2\big) \xrightarrow[\text{“characcter”}]{\mathrm{ch}} \left\{ \begin{array}{l|l} F_2 \,\in\, \Omega^2_{\mathrm{dR}}(X) & \mathrm{d}\, F_2 \,=\, 0 \\ H_3 \,\in\, \Omega^3_{\mathrm{dR}}(X) & \mathrm{d}\, H_3 \,=\, F_2 \wedge F_2 \end{array} \right\}_{/\sim}$$

Reviews in Mathematical Physics | Vol. 34, No. 05, 2250013 (2022) | Research

Twistorial cohomotopy implies Green–Schwarz anomaly cancellation

Domenico Fiorenza, Hisham Sati, and Urs Schreiber

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \operatorname{Map}\big(X,\, S^2\big) \xrightarrow[\text{"characcter"}]{\text{ch}} \left\{ \begin{array}{l} F_2 \,\in\, \Omega^2_{\mathrm{dR}}(X) \\ H_3 \,\in\, \Omega^3_{\mathrm{dR}}(X) \end{array} \,\middle\vert\, \begin{array}{l} \mathrm{d}\, F_2 \,=\, 0 \\ \mathrm{d}\, H_3 \,=\, F_2 \wedge F_2 \end{array} \right\}_{\big/\sim}$$

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \, \mathrm{Map}\big(X,\, S^2\big) \xrightarrow[\text{“characcter”}]{\mathrm{ch}} \left\{ \begin{array}{l} F_2 \,\in\, \Omega^2_{\mathrm{dR}}(X) \\ H_3 \,\in\, \Omega^3_{\mathrm{dR}}(X) \end{array} \middle| \begin{array}{l} \mathrm{d}\, F_2 \,=\, 0 \\ \mathrm{d}\, H_3 \,=\, F_2 \wedge F_2 \end{array} \right\}_{/\sim}$$

here the flux species

$\updownarrow$

free homotopy groups

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \, \mathrm{Map}\big(X,\, S^2\big) \xrightarrow[\text{"character"}]{\mathrm{ch}} \left\{ \begin{array}{l} F_2 \,\in\, \Omega^2_{\mathrm{dR}}(X) \\ H_3 \,\in\, \Omega^3_{\mathrm{dR}}(X) \end{array} \middle\vert \begin{array}{l} \mathrm{d}\, F_2 \,=\, 0 \\ \mathrm{d}\, H_3 \,=\, F_2 \wedge F_2 \end{array} \right\}_{\big/\sim}$$

here the flux species

$\updownarrow$

free homotopy groups

unit class — as usual

$$\mathbb{Z} \simeq \pi_2\big(\mathbb{C}P^1\big) = \pi_2\big(\mathbb{C}P^\infty\big)$$

$$\mathbb{Z} \simeq \pi_3\big(\mathbb{C}P^1\big) \xrightarrow{0} \pi_3\big(\mathbb{C}P^\infty\big)$$

Hopf generator — is new!

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \,\mathrm{Map}\big(X,\, S^2\big) \xrightarrow{\;\mathrm{ch}\;} \left\{ \begin{array}{l} F_2 \in \Omega^2_{\mathrm{dR}}(X) \\ H_3 \in \Omega^3_{\mathrm{dR}}(X) \end{array} \middle| \begin{array}{l} \mathrm{d}\, F_2 = 0 \\ \mathrm{d}\, H_3 = F_2 \wedge F_2 \end{array} \right\}\Big/\sim$$

"characcter"

unit class — as usual

here the flux species $\updownarrow$ free homotopy groups

$$\mathbb{Z} \simeq \pi_2\big(\mathbb{C}P^1\big) = \pi_2\big(\mathbb{C}P^\infty\big)$$

$$\mathbb{Z} \simeq \pi_3\big(\mathbb{C}P^1\big) \xrightarrow{\;0\;} \pi_3\big(\mathbb{C}P^\infty\big)$$

Hopf generator — is new!

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \,\mathrm{Map}\big(X,\, S^2\big) \xrightarrow{\mathrm{ch}} \left\{ \begin{array}{l} F_2 \in \Omega^2_{\mathrm{dR}}(X) \\ H_3 \in \Omega^3_{\mathrm{dR}}(X) \end{array} \middle| \begin{array}{l} \mathrm{d}\,F_2 = 0 \\ \mathrm{d}\,H_3 = F_2 \wedge F_2 \end{array} \right\}_{\big/\sim}$$

"characcter"

here the flux species ↕ free homotopy groups

unit class / as usual

$$\mathbb{Z} \simeq \pi_2\big(\mathbb{C}P^1\big) = \pi_2\big(\mathbb{C}P^\infty\big)$$

$$\mathbb{Z} \simeq \pi_3\big(\mathbb{C}P^1\big) \xrightarrow{0} \pi_3\big(\mathbb{C}P^\infty\big)$$

Hopf generator / is new!

not quite ordinary CS theory

(where H_3 is not flux but Lagrangian)

but something similar...

Exotic flux. Exotic Flux Quantization:

use another classifying space!

such as the 2-sphere $S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

Fact. This quantizes EM-flux *with* Chern-Simons flux:

$$\pi_0 \,\mathrm{Map}\big(X,\, S^2\big) \xrightarrow{\mathrm{ch}} \left\{ \begin{array}{l} F_2 \in \Omega^2_{\mathrm{dR}}(X) \\ H_3 \in \Omega^3_{\mathrm{dR}}(X) \end{array} \middle| \begin{array}{l} \mathrm{d}\, F_2 = 0 \\ \mathrm{d}\, H_3 = F_2 \wedge F_2 \end{array} \right\} \Big/ \sim$$

"characcter"

unit class — as usual

here the flux species

$\updownarrow$

free homotopy groups

$$\mathbb{Z} \simeq \pi_2\big(\mathbb{C}P^1\big) = \pi_2\big(\mathbb{C}P^\infty\big)$$

$$\mathbb{Z} \simeq \pi_3\big(\mathbb{C}P^1\big) \xrightarrow{0} \pi_3\big(\mathbb{C}P^\infty\big)$$

Hopf generator — is new!

not quite ordinary CS theory

(where H_3 is not flux but Lagrangian)

but something similar...

Hypothesis h:

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} := \mathbb{C}\Big[\pi_1 \, \mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}}, \, S^2\big)\Big]$$

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\; S^2\big)\Big]$$

$\mathbb{C}P^1$ instead of the usual $\mathbb{C}P^\infty$

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} := \mathbb{C}\Big[\pi_1 \,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$

little cousin of **Hypothesis H** in super-gravity

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\, \mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$

little cousin of
Hypothesis H
in super-gravity

Volume 59, Issue 6
June 2018

RESEARCH ARTICLE | JUNE 19 2018

Framed M-branes, corners, and topological invariants

Hisham Sati

Check for updates

+ Author & Article Information

J. Math. Phys. 59, 062304 (2018)
https://doi.org/10.1063/1.5007185 **Article history**

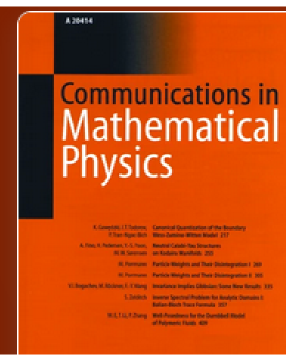
CHORUS

;

Home › Communications in Mathematical Physics › Article

Twisted Cohomotopy Implies M-Theory Anomaly Cancellation on 8-Manifolds

Published: 06 April 2020
Volume 377, pages 1961–2025, (2020) Cite this article



;

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1 \,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$


SPRINGER NATURE Link

Find a journal Publish with us Track your research Search

Home > Letters in Mathematical Physics > Article

Anyons on M5-probes of Seifert 3-orbifoldsvia flux quantization

Published: 24 March 2025

Volume 115, article number 36, (2025) Cite this article

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\; S^2\big)\Big]$$

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} := \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$

or rather, made generally covariant:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} := \mathbb{C}\Big[\pi_1\Big(\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\, \overbrace{/\!\!/ \mathrm{Diff}(\Sigma^2)}^{\text{homotopy quotient by diffeomorphisms}}\Big)\Big]$$

$$= \mathbb{C}\Big[\pi_1\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}}, S^2\big) \underbrace{\rtimes\, \pi_0\mathrm{Diff}(\Sigma^2)}_{\text{semidirect product by mapping classes}}\Big)\Big]$$

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$

or rather, made generally covariant:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\Big(\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\,\overbrace{/\!\!/\mathrm{Diff}(\Sigma^2)}^{\substack{\text{homotopy quotient}\\ \text{by diffeomorphisms}}}\Big)\Big]$$

$$=\; \mathbb{C}\Big[\pi_1\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},S^2\big)\,\underbrace{\rtimes\,\pi_0\mathrm{Diff}(\Sigma^2)}_{\substack{\text{semidirect product}\\ \text{by mapping classes}}}\Big)\Big]$$

$\Rightarrow$ FQH flux state spaces are unitary reps:

$$\mathcal{H}^{\mathrm{h}}_{\Sigma^2} \;\in\; \mathrm{URep}\Big(\underbrace{\pi_1\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},S^2\big)\rtimes\pi_0\mathrm{Diff}(\Sigma^2)}_{\text{covariant flux monodromy group}}\Big)$$

Hypothesis h: FQH flux observables are

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\,\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\Big]$$

or rather, made generally covariant:

$$\mathrm{Obs}(\Sigma^2)^{\mathrm{h}} \;:=\; \mathbb{C}\Big[\pi_1\Big(\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\,\overbrace{/\!\!/\mathrm{Diff}(\Sigma^2)}^{\substack{\text{homotopy quotient}\\ \text{by diffeomorphisms}}}\Big)\Big]$$

$$=\; \mathbb{C}\Big[\pi_1\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\,\underbrace{\rtimes\,\pi_0\mathrm{Diff}(\Sigma^2)}_{\substack{\text{semidirect product}\\ \text{by mapping classes}}}\Big)\Big]$$

$\Rightarrow$ FQH flux state spaces are unitary reps:

$$\mathcal{H}^{\mathrm{h}}_{\Sigma^2} \;\in\; \mathrm{URep}\Big(\underbrace{\pi_1\mathrm{Map}^*\big(\Sigma^2_{\cup\{\infty\}},\, S^2\big)\rtimes\pi_0\mathrm{Diff}(\Sigma^2)}_{\text{covariant flux monodromy group}}\Big)$$

Hypothesis h.

Hypothesis h.

novel effective theory of FQH

- non-Lagrangian
- non-perturbative
- globally well-defined

Hypothesis h.

novel effective theory of FQH $\left\{ \begin{array}{l} \text{non-Lagrangian} \\ \text{non-perturbative} \\ \text{globally well-defined} \end{array} \right.$

predictions follow by rigorous analysis:

Hypothesis h.

novel effective theory of FQH $\begin{cases} \text{non-Lagrangian} \\ \text{non-perturbative} \\ \text{globally well-defined} \end{cases}$

predictions follow by rigorous analysis:

Algebraic Topology to compute the flux monodromy

Hypothesis h.

novel effective theory of FQH $\left\{ \begin{array}{l} \text{non-Lagrangian} \\ \text{non-perturbative} \\ \text{globally well-defined} \end{array} \right.$

predictions follow by rigorous analysis:

Algebraic Topology to compute the flux monodromy

Representation Theory to classify its irreps

Hypothesis h.

novel effective theory of FQH $\begin{cases} \text{non-Lagrangian} \\ \text{non-perturbative} \\ \text{globally well-defined} \end{cases}$

predictions follow by rigorous analysis:

Algebraic Topology to compute the flux monodromy

Representation Theory to classify its irreps

so let's check the predictions of Hypothesis h...

Hypothesis h.

novel effective theory of FQH $\begin{cases} \text{non-Lagrangian} \\ \text{non-perturbative} \\ \text{globally well-defined} \end{cases}$

predictions follow by rigorous analysis:

Algebraic Topology to compute the flux monodromy

Representation Theory to classify its irreps

so let's check the predictions of Hypothesis h...

or, for lack of time: jump to conclusions

Generally:

the **Pontrjagin theorem**
entails that spheres classify

$$\left.\begin{array}{r}\text{submanifolds } Q \text{ with} \\ \text{normal framing } NQ\end{array}\right\}$$

Generally:

the **Pontrjagin theorem**
entails that spheres classify

submanifolds Q with normal framing NQ

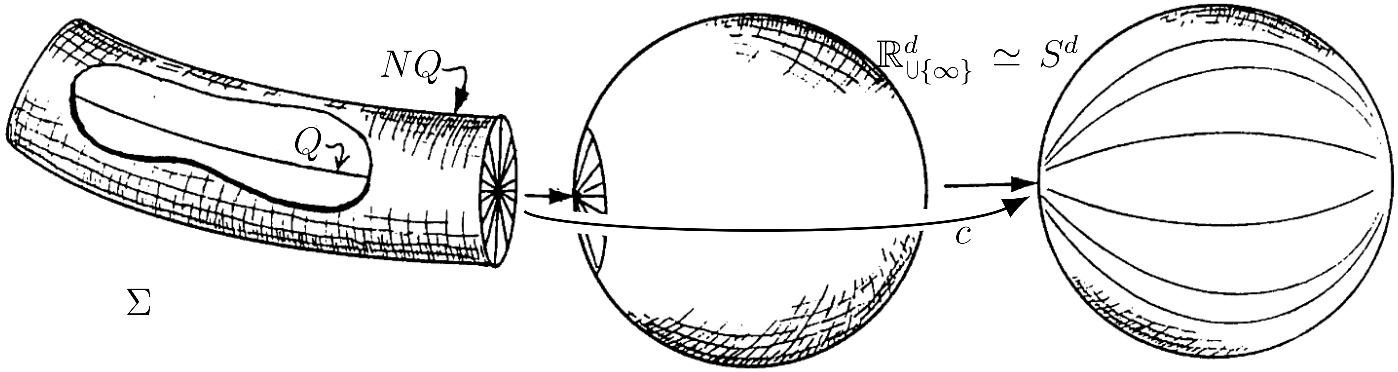


figure adapted from [Bredon 1993 Fig. II-13]

Generally:

the **Pontrjagin theorem**
entails that spheres classify

submanifolds Q with normal framing NQ $\Big\}$

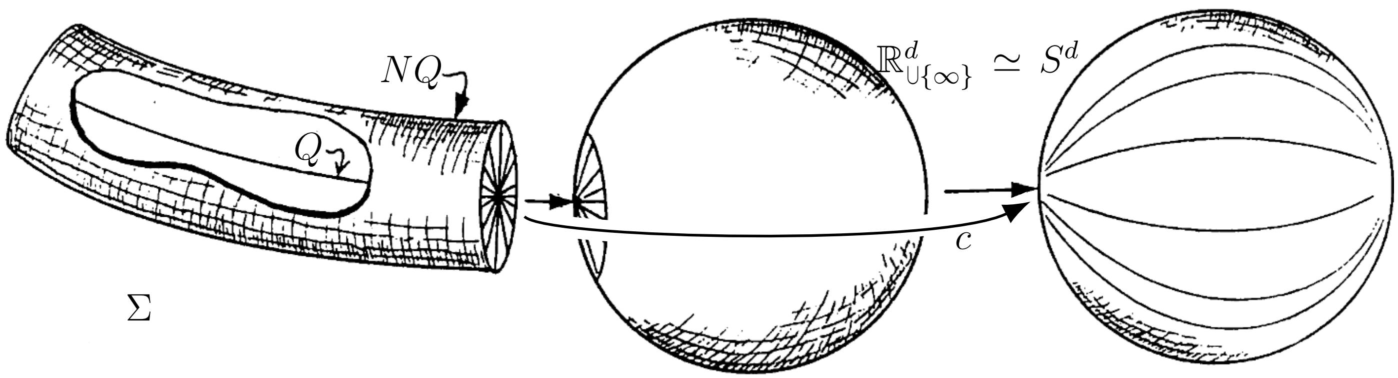


figure adapted from [Bredon 1993 Fig. II-13]

Series on Knots and Everything

| Topological Library, pp. 1-130 (2007)

Smooth manifolds and their applications in homotopy theory

Л. С. Понтрягин, Гладкие многообразия и и применения в теории гомотопий, Москва, 1976. Translated by V.O.Manturov.

L. S. Pontrjagin (original: 1955)

Generally:

the **Pontrjagin theorem**
entails that spheres classify

$$\left.\begin{array}{r}\text{submanifolds } Q \text{ with} \\ \text{normal framing } NQ\end{array}\right\} \leftrightarrow \left\{\begin{array}{l}\text{soliton cores within} \\ \text{flux concentrations}\end{array}\right.$$

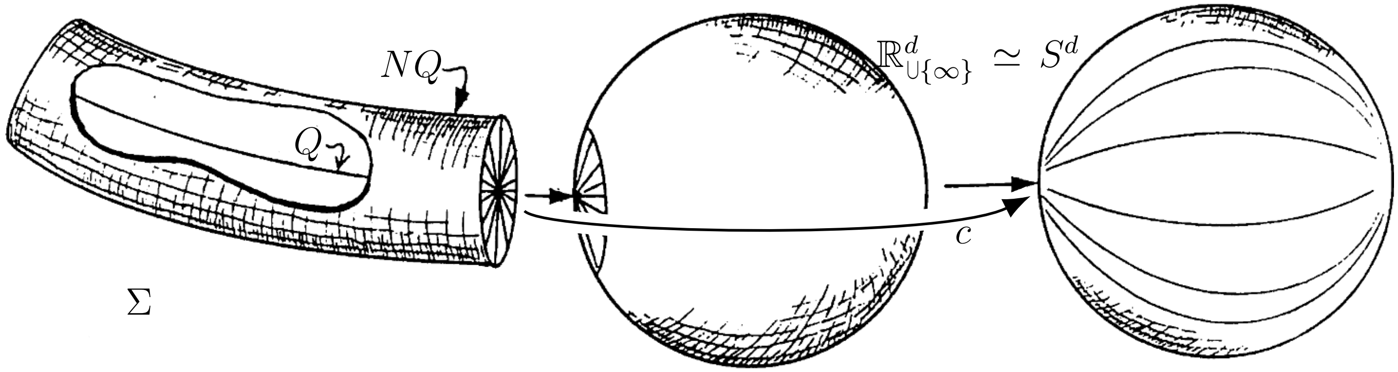


figure adapted from [Bredon 1993 Fig. II-13]

Series on Knots and Everything
| **Topological Library, pp. 1-130 (2007)**

Smooth manifolds and their applications in homotopy theory

Л. С. Понтрягин, Гладкие многообразия и и применения в теории гомотопий, Москва, 1976. Translated by V.O.Manturov.

L. S. Pontrjagin (original: 1955)

Journal of Geometry and Physics
Volume 156, October 2020, 103775

Equivariant Cohomotopy implies orientifold tadpole cancellation

Hisham Sati, Urs Schreiber [1]

Reviews in Mathematical Physics | Vol. 35, No. 10, 2350028 (2023)

M/F-theory as Mf-theory

Hisham Sati and Urs Schreiber

Generally:

the **Pontrjagin theorem**
entails that spheres classify

$$\left.\begin{array}{r}\text{submanifolds } Q \text{ with}\\ \text{normal framing } NQ\end{array}\right\} \leftrightarrow \left\{\begin{array}{l}\text{soliton cores within}\\ \text{flux concentrations}\end{array}\right.$$

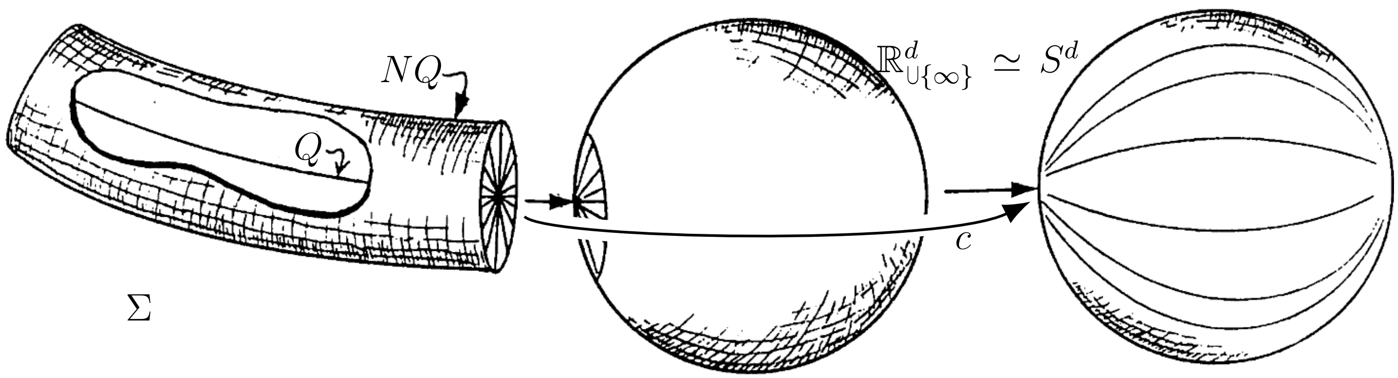
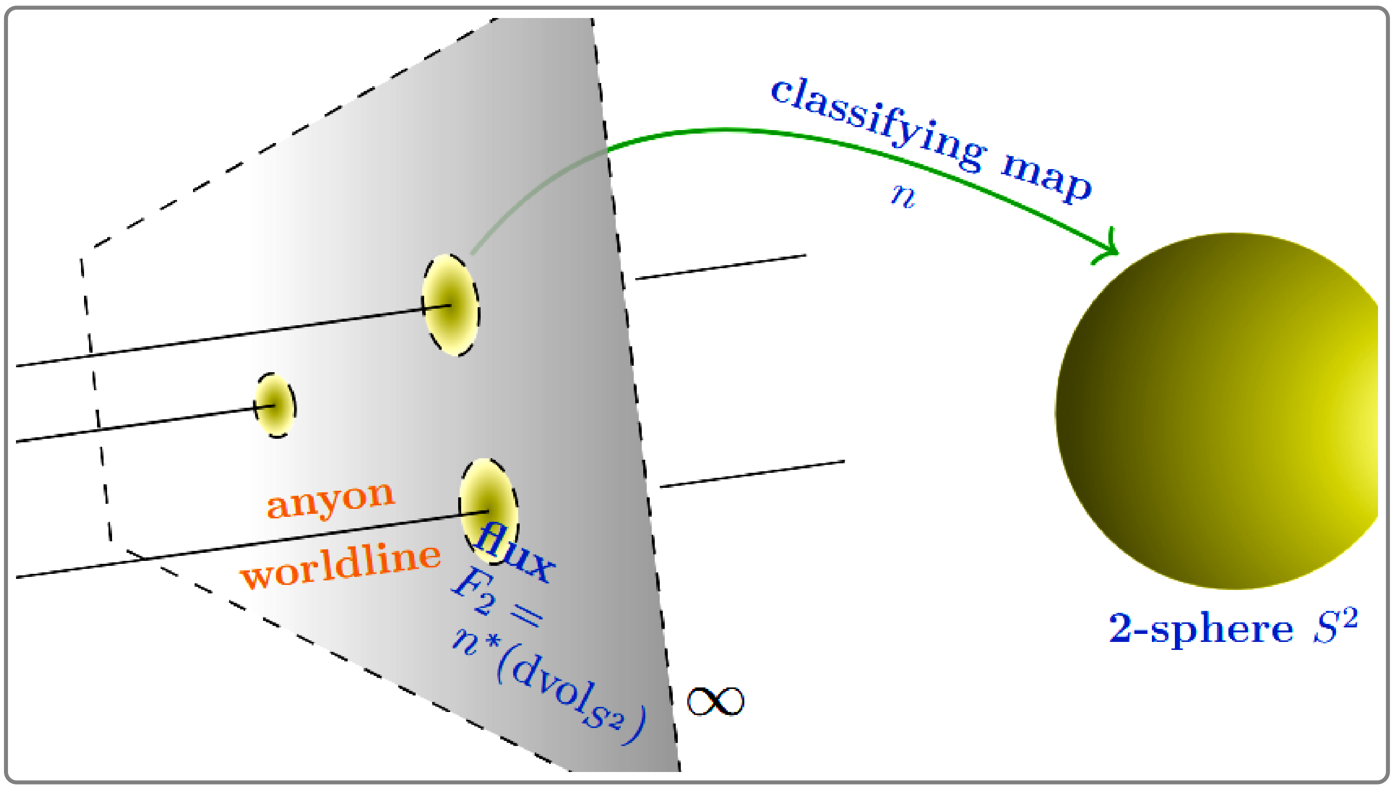


figure adapted from [Bredon 1993 Fig. II-13]



First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

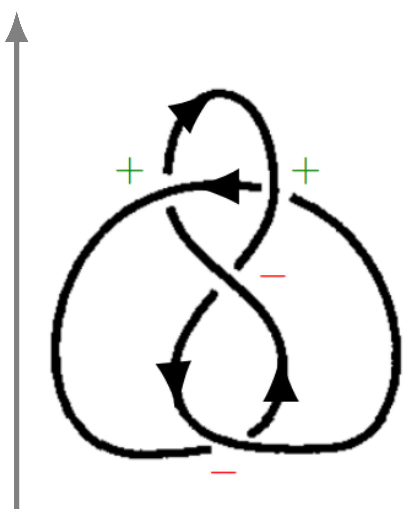
Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega \,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

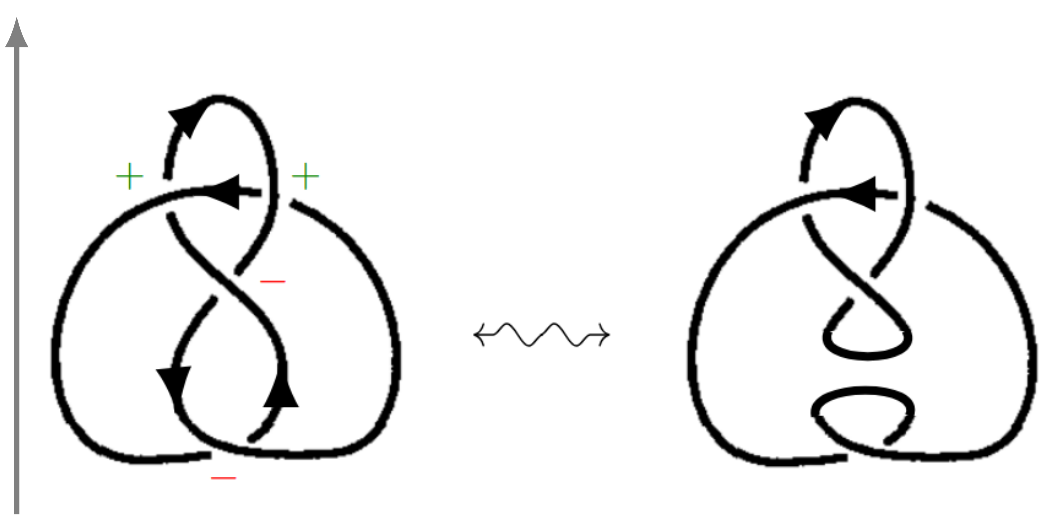


vacuum-to-vacuum process
of exotic flux quanta

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

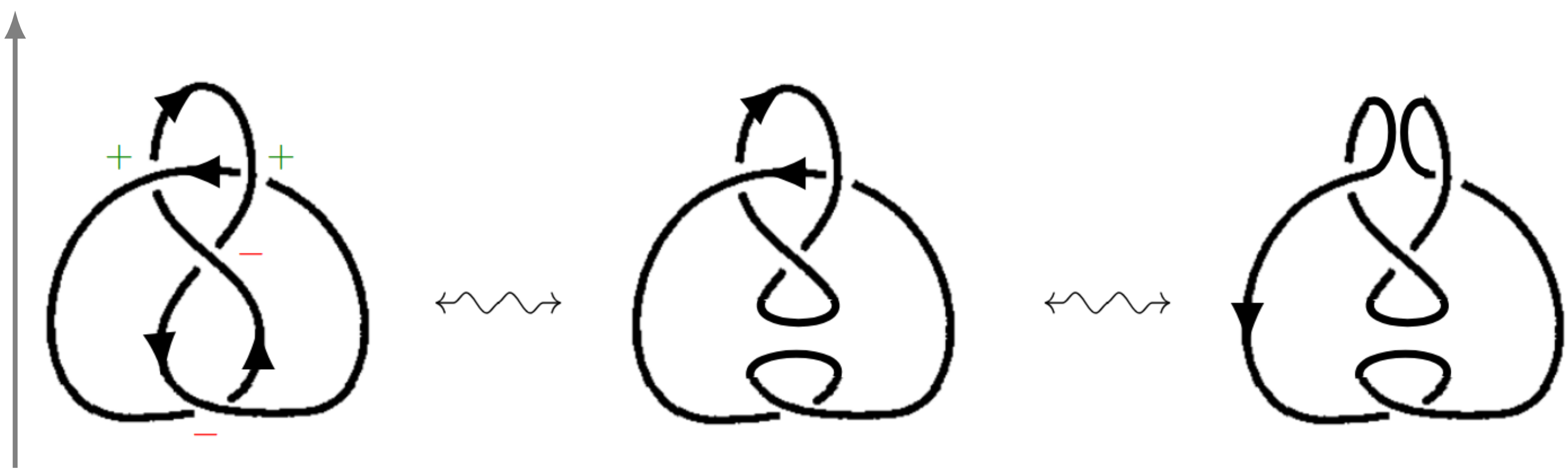


observationally
equivalent process

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$



observationally
equivalent processes

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

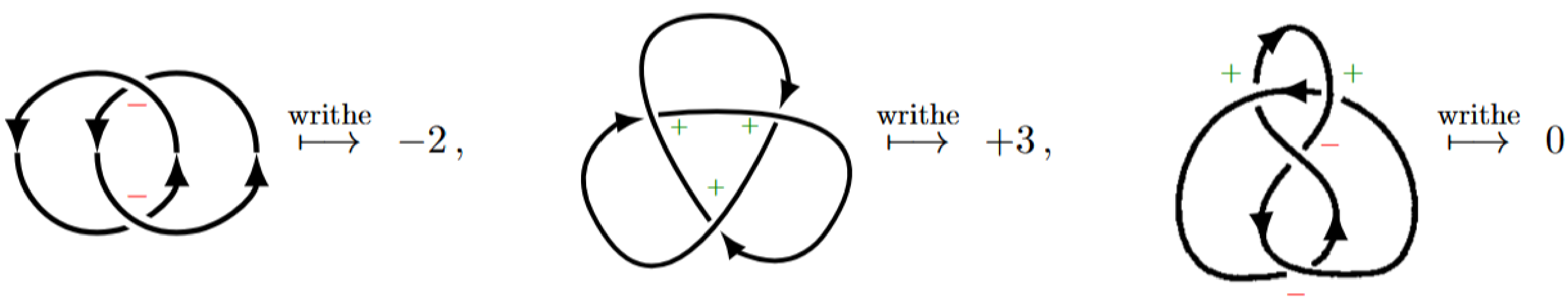
$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$



Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \longrightarrow\!\!\!\!\!\rightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega \,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega \,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

in which Wilson loop observable L evaluates to:

$$\langle \zeta | L | \zeta \rangle = \zeta^{\mathrm{writhe}(L)} = \zeta^{\left(\sum_i \mathrm{frm}(L_i) \,+\, \sum_{i \neq j} \mathrm{lnk}(L_i, L_j)\right)}$$

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

in which Wilson loop observable L evaluates to:

$$\langle \zeta | L | \zeta \rangle = \zeta^{\mathrm{writhe}(L)} = \zeta^{\left(\sum_i \mathrm{frm}(L_i) \,+\, \sum_{i \neq j} \mathrm{lnk}(L_i, L_j)\right)}$$

= result of abelian CS, including framing regularization!

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

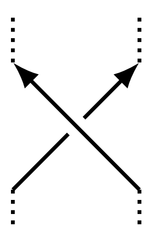
$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

in which Wilson loop observable L evaluates to:

$$\langle \zeta | L | \zeta \rangle = \zeta^{\mathrm{writhe}(L)} = \zeta^{\left(\sum_i \mathrm{frm}(L_i) \,+\, \sum_{i \neq j} \mathrm{lnk}(L_i, L_j)\right)}$$

= result of abelian CS, including framing regularization!

$\langle \zeta |$  $| \zeta \rangle = \zeta \cdots,$

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

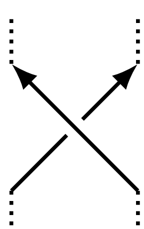
$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

in which Wilson loop observable L evaluates to:

$$\langle \zeta | L | \zeta \rangle = \zeta^{\mathrm{writhe}(L)} = \zeta^{\left(\sum_i \mathrm{frm}(L_i) \,+\, \sum_{i \neq j} \mathrm{lnk}(L_i, L_j)\right)}$$

= result of abelian CS, including framing regularization!

$\langle \zeta |$  $| \zeta \rangle = \zeta \cdots,$ flux quanta braiding phase!

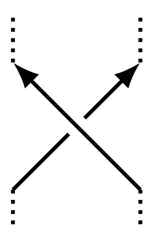
First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \longrightarrow\!\!\!\!\rightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

$\Rightarrow$ fractional statistics comes from Hopf generator $\in \pi_3(\mathbb{C}P^1)$ which disappears along $\mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

$\langle \zeta |$  $| \zeta \rangle \;=\; \zeta \cdots,$ flux quanta braiding phase!

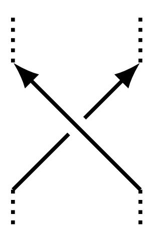
First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

Thm. [Sati-S.'24 based on Okuyama'05 based on Segal'73]:

$$\Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) = \left\{ \begin{array}{l} \text{points: framed links} \\ \text{curves: link cobordism} \end{array} \right\}$$

$$\begin{array}{ccc} \Omega\,\mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) & \twoheadrightarrow & \pi_1 \mathrm{Map}^*\big(\mathbb{R}^2_{\cup\{\infty\}}, S^2\big) \simeq \mathbb{Z} \\ L & \longmapsto & \mathrm{writhe}(L) \end{array}$$

$\Rightarrow$ fractional statistics comes from Hopf generator $\in \pi_3(\mathbb{C}P^1)$ which disappears along $\mathbb{C}P^1 \hookrightarrow \mathbb{C}P^\infty$

$\langle\zeta|$  $|\zeta\rangle = \zeta \cdots,$ flux quanta braiding phase!

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2,\, S^2\big) \;\simeq\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]},\, \underset{\text{central}}{\widehat{\zeta}} \;\Big|\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{\zeta}^{\,2}\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2,\, S^2\big) \;\simeq\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]},\, \underset{\text{central}}{\widehat{\zeta}} \;\Big|\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{\zeta}^{\,2}\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the $\overbrace{\text{covariantization}}^{\text{here: modularity}}$ by diffeos preserving fermionic (aa) spin structure

have $\Big\{$

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2, S^2\big) \simeq \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}, \underset{\text{central}}{\widehat{\zeta}} \,\Big|\, \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} = \widehat{\zeta}^2\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the $\overbrace{\text{covariantization}}^{\text{here: modularity}}$ by diffeos preserving fermionic (aa) spin structure

have $\Big\{$ braiding phase a primitive root of unity $\boxed{\zeta = e^{\pi \mathrm{i} \frac{p}{q}}}$

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2,\, S^2\big) \;\simeq\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]},\, \underset{\text{central}}{\widehat{\zeta}} \;\Big|\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{\zeta}^{2}\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the covariantization (here: modularity) by diffeos preserving fermionic (aa) spin structure

have { braiding phase a primitive root of unity $\zeta = e^{\pi \mathrm{i} \frac{p}{q}}$
and dimension q — ground state degeneracy!

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2, S^2\big) \simeq \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]}, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]}, \underset{\text{central}}{\widehat{\zeta}} \,\Big|\, \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} = \widehat{\zeta}^{2} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the $\overbrace{\text{covariantization}}^{\text{here: modularity}}$ by diffeos preserving fermionic (aa) spin structure

have $\left\{ \begin{array}{l} \text{braiding phase a primitive root of unity} \quad \boxed{\zeta = e^{\pi \mathrm{i} \frac{p}{q}}} \\ \text{and dimension } q \text{ — ground state degeneracy!} \end{array} \right.$

Remarks. The proof shows...

(1.) ...braiding phase ζ is again Hopf generator of $\pi_3(S^2)$.

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2,\, S^2\big) \;\simeq\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]},\, \underbrace{\widehat{\zeta}}_{\text{central}} \,\Big|\, \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} = \widehat{\zeta}^2\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the $\overbrace{\text{covariantization}}^{\text{here: modularity}}$ by diffeos preserving fermionic (aa) spin structure

have $\begin{cases} \text{braiding phase a primitive root of unity } \boxed{\zeta = e^{\pi \mathrm{i} \frac{p}{q}}} \\ \text{and dimension } q \text{ — ground state degeneracy!} \end{cases}$

Remarks. The proof shows...

(1.) ...braiding phase ζ is again Hopf generator of $\pi_3(S^2)$.

(2.) ...FQH must be described by "Spin-CS" (rarely admitted).

Second case: $\Sigma^2 \equiv T^2$ the torus — topological order

Thm. (1.) Bare flux monodromy gives torus Wilson loop observables $\widehat{W}$ as in CS and as expected for FQH anyons:

$$\pi_1 \mathrm{Map}\big(T^2,\, S^2\big) \;\simeq\; \Big\langle \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]},\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]},\, \underset{\text{central}}{\widehat{\zeta}} \;\Big|\; \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \;=\; \widehat{\zeta}^{2}\, \widehat{W}_{\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]} \widehat{W}_{\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]} \Big\rangle$$

(2.) Fin-dim irreps that extend to the covariantization (here: modularity) by diffeos preserving fermionic (aa) spin structure

have braiding phase a primitive root of unity $\zeta = e^{\pi \mathrm{i} \frac{p}{q}}$ and dimension q — ground state degeneracy!

Remarks. The proof shows...

(1.) ...braiding phase ζ is again Hopf generator of $\pi_3(S^2)$.

(2.) ...FQH must be described by "Spin-CS" (rarely admitted).

(3.) ...modularity was not properly accounted in literature.

Third case: $\Sigma^2 \equiv A^2$ the open annulus — edge modes

Thm. (1.) The covariantized flux monodromy is:

$$\pi_1\Big(\mathrm{Map}^*\big(A^2_{\cup\{\infty\}}, S^2\big) /\!\!/ \mathrm{Diff}(A^2)\Big) \simeq \left\langle \widehat{\xi}, \widehat{\sigma} \;\middle|\; \begin{array}{r} \widehat{\sigma}^2 = 1 \\ (\widehat{\sigma}\widehat{\xi})^2 = 1 \end{array} \right\rangle$$

Third case: $\Sigma^2 \equiv A^2$ the open annulus — edge modes

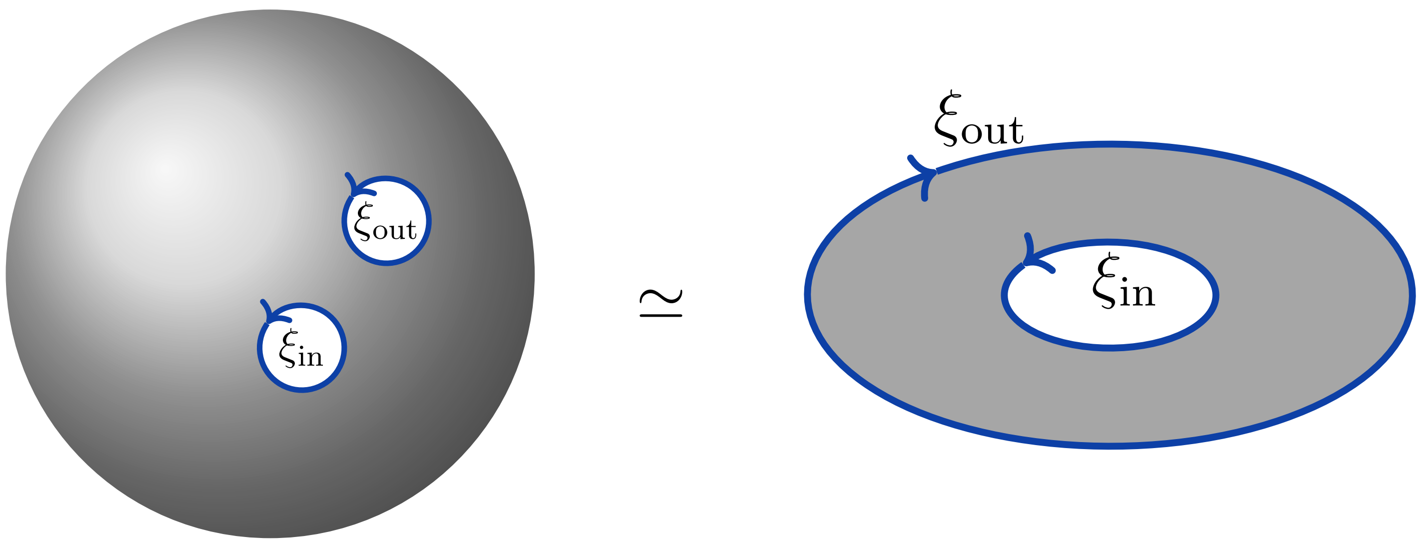
Thm. (1.) The covariantized flux monodromy is:

$$\pi_1\Big(\mathrm{Map}^*\big(A^2_{\cup\{\infty\}}, S^2\big) \sslash \mathrm{Diff}(A^2)\Big) \simeq \left\langle \widehat{\xi},\, \widehat{\sigma} \;\middle|\; \begin{array}{r} \widehat{\sigma}^2 = 1 \\ (\widehat{\sigma}\widehat{\xi})^2 = 1 \end{array} \right\rangle$$

(2.) Fd irreps compatible with $A^2_{\cup\{\infty\}} \simeq S^2 \vee S^1 \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\xi = \xi_{\mathrm{in}}/\xi_{\mathrm{out}}$ with $\zeta = \xi_{\mathrm{in}} \cdot \xi_{\mathrm{out}}$

which is relation of FQH edge mode phases due to tunneling



Third case: $\Sigma^2 \equiv A^2$ the open annulus — edge modes

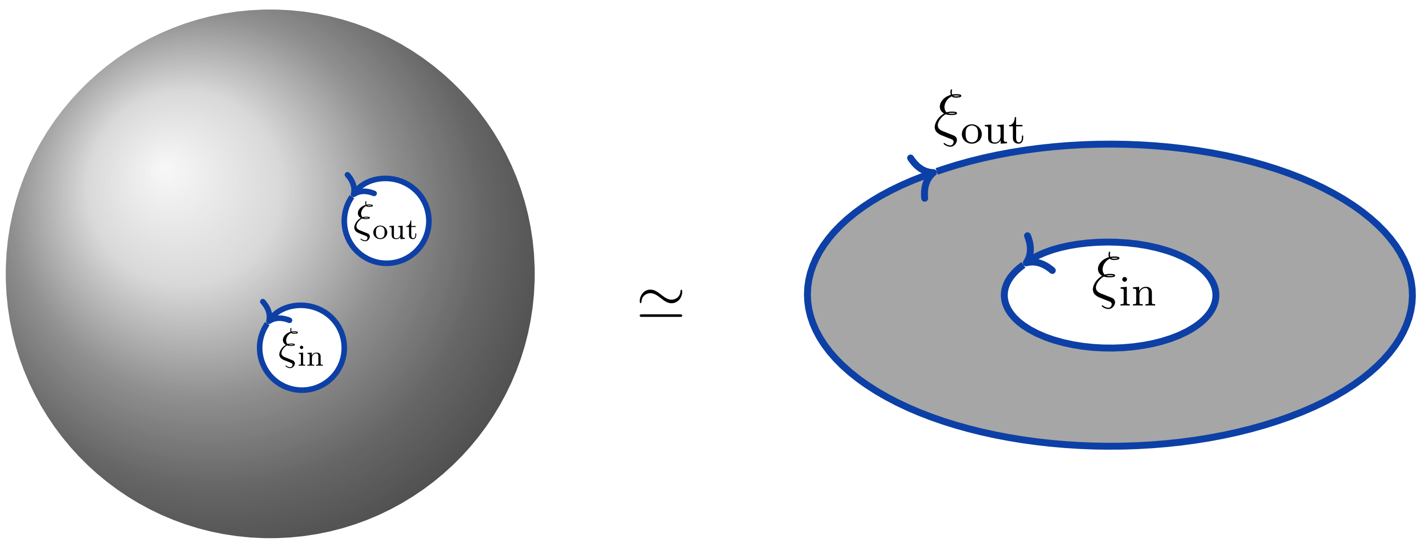
Thm. (1.) The covariantized flux monodromy is:

$$\pi_1\Big(\mathrm{Map}^*\big(A^2_{\cup\{\infty\}}, S^2\big) /\!\!/ \mathrm{Diff}(A^2)\Big) \simeq \left\langle \widehat{\xi},\, \widehat{\sigma} \;\middle|\; \begin{array}{r} \widehat{\sigma}^2 = 1 \\ (\widehat{\sigma}\widehat{\xi})^2 = 1 \end{array} \right\rangle$$

(2.) Fd irreps compatible with $A^2_{\cup\{\infty\}} \simeq S^2 \vee S^1 \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\xi = \xi_{\mathrm{in}}/\xi_{\mathrm{out}}$ with $\zeta = \xi_{\mathrm{in}} \cdot \xi_{\mathrm{out}}$

which is relation of FQH edge mode phases due to tunneling



(theory admits $\xi_{\mathrm{in}} \neq \xi_{\mathrm{out}}$ in which case $\dim(\mathrm{irrep}) = 2$)

Third case: $\Sigma^2 \equiv A^2$ the open annulus — edge modes

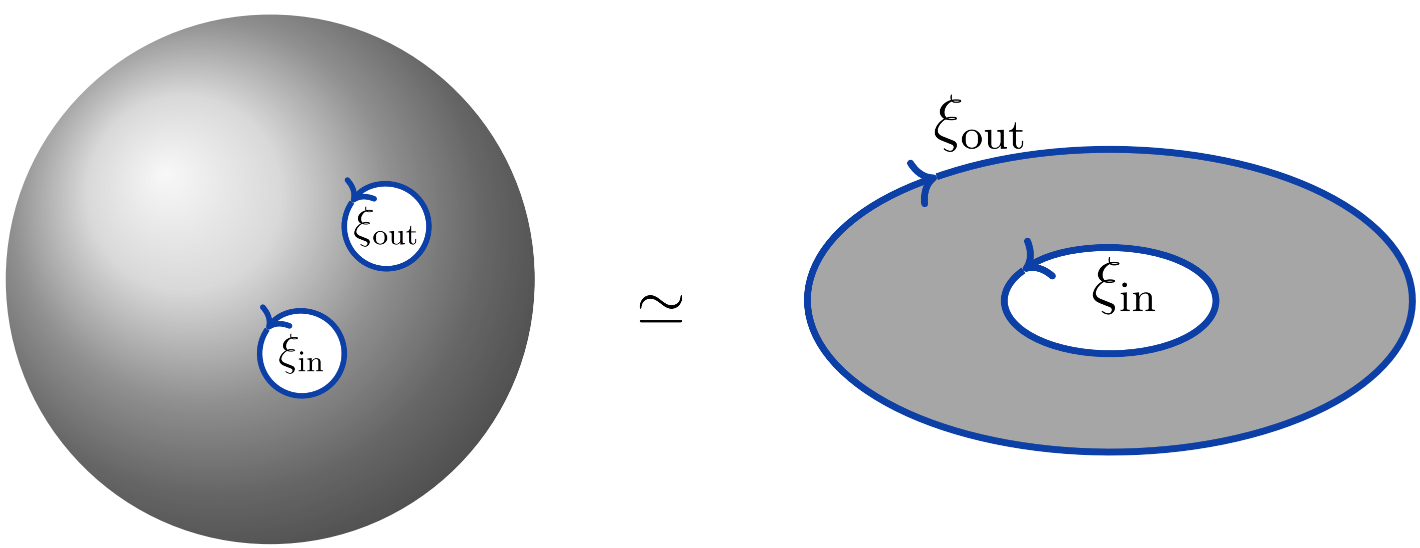
Thm. (1.) The covariantized flux monodromy is:

$$\pi_1\Big(\mathrm{Map}^*\big(A^2_{\cup\{\infty\}}, S^2\big) \,/\!\!/\, \mathrm{Diff}(A^2)\Big) \simeq \left\langle \widehat{\xi}, \widehat{\sigma} \;\middle|\; \begin{array}{r} \widehat{\sigma}^2 = 1 \\ (\widehat{\sigma}\widehat{\xi})^2 = 1 \end{array} \right\rangle$$

(2.) Fd irreps compatible with $A^2_{\cup\{\infty\}} \simeq S^2 \vee S^1 \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\quad \xi = \xi_{\mathrm{in}}/\xi_{\mathrm{out}} \quad$ with $\quad \zeta = \xi_{\mathrm{in}} \cdot \xi_{\mathrm{out}}$

which is relation of FQH edge mode phases due to tunneling



(theory admits $\xi_{\mathrm{in}} \neq \xi_{\mathrm{out}}$ in which case $\dim(\mathrm{irrep}) = 2$)

Taking stock.

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

up to some subtleties
(ground state degeneracy for non-unit filling fraction
may differ from prediction of K-matrix CS formalism)

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

up to some subtleties
(ground state degeneracy for non-unit filling fraction
may differ from prediction of K-matrix CS formalism)

thus supporting Hypothesis h & making it testable

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

up to some subtleties
(ground state degeneracy for non-unit filling fraction
may differ from prediction of K-matrix CS formalism)

thus supporting Hypothesis h & making it testable

Going further.

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

up to some subtleties
(ground state degeneracy for non-unit filling fraction
may differ from prediction of K-matrix CS formalism)

thus supporting Hypothesis h & making it testable

Going further.

Hypothesis h goes further to make

predictions about surfaces with punctures

modelling flux-expelling defects in FQH material →

Taking stock.

these results show that

hallmark properties of FQH systems are reproduced:

(fractional statistics, topological order, edge modes)

up to some subtleties
(ground state degeneracy for non-unit filling fraction
may differ from prediction of K-matrix CS formalism)

thus supporting Hypothesis h & making it testable

Going further.

Hypothesis h goes further to make

predictions about surfaces with punctures

modelling flux-expelling defects in FQH material $\rightarrow$

Fourth case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{2}}$ the 2-pnctrd plane — para-defects

Fourth case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{2}}$ the 2-pnctrd plane — para-defects

Thm. the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\setminus \mathbf{2}})_{\cup\{\infty\}}, S^2\big) \mathbin{/\!\!/} \mathrm{Diff}(\mathbb{R}^2_{\setminus \mathbf{2}})\Big) \subset \mathrm{FSym}_3$$

group of framed permutations with total framing $\in 3\mathbb{Z}$

Fourth case: $\Sigma^2 \equiv \mathbb{R}^2_{\backslash \mathbf{2}}$ the 2-pnctrd plane — para-defects

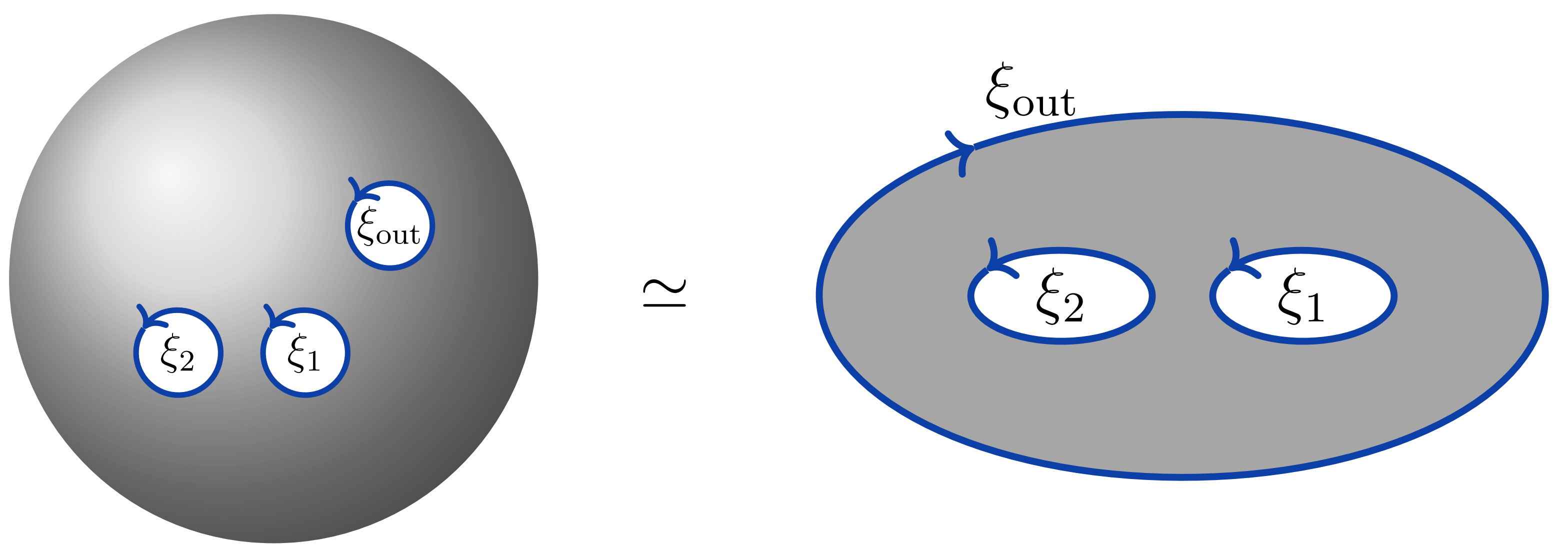
Thm. the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\backslash \mathbf{2}})_{\cup\{\infty\}}, S^2\big) \,/\!\!/\, \mathrm{Diff}(\mathbb{R}^2_{\backslash \mathbf{2}})\Big) \subset \mathrm{FSym}_3$$

group of framed permutations with total framing $\in 3\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\backslash \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee^n} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = (\xi_1 \cdot \xi_2) \,/\, \xi_{\mathrm{out}}$



Fourth case: $\Sigma^2 \equiv \mathbb{R}^2_{\backslash \mathbf{2}}$ the 2-pnctrd plane — para-defects

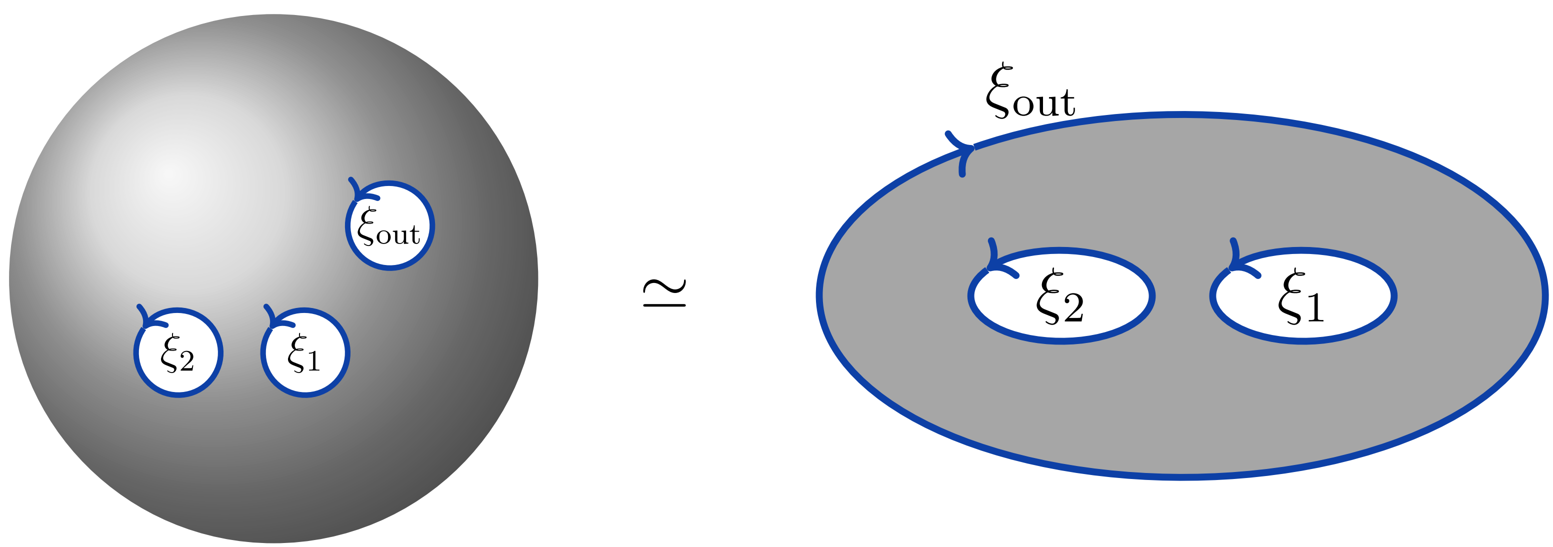
Thm. the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\backslash \mathbf{2}})_{\cup\{\infty\}}, S^2\big) \sslash \mathrm{Diff}(\mathbb{R}^2_{\backslash \mathbf{2}})\Big) \subset \mathrm{FSym}_3$$

group of framed permutations with total framing $\in 3\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\backslash \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee^n} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = (\xi_1 \cdot \xi_2) \,/\, \xi_{\mathrm{out}}$



If $\xi_1 = \xi_2$, $\exists$ 2D para-anyon rep permuting the defects.

Fourth case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{2}}$ the 2-pnctrd plane — para-defects

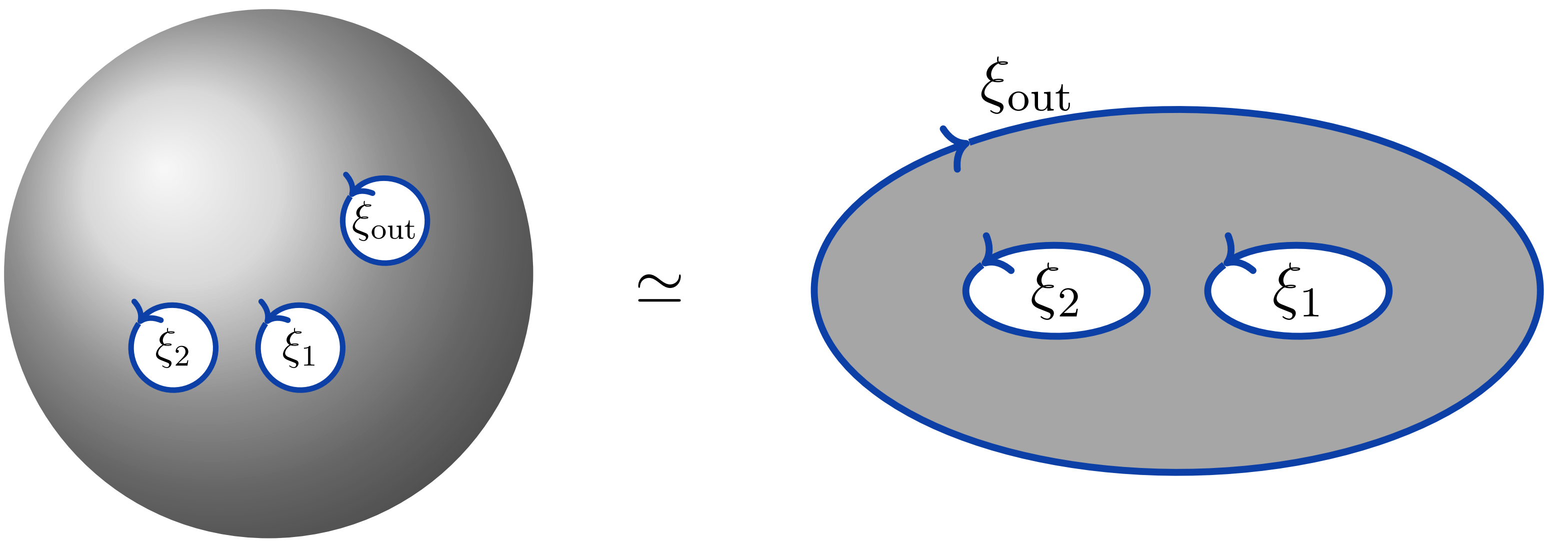
Thm. the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\setminus \mathbf{2}})_{\cup\{\infty\}}, S^2\big) /\!\!/ \mathrm{Diff}(\mathbb{R}^2_{\setminus \mathbf{2}})\Big) \subset \mathrm{FSym}_3$$

group of framed permutations with total framing $\in 3\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee^n} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = (\xi_1 \cdot \xi_2) \,/\, \xi_{\mathrm{out}}$



If $\xi_1 = \xi_2$, $\exists$ 2D para-anyon rep permuting the defects.

Further case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{n}}$ the n-pnctrd plane – defect anyons

Further case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{n}}$ the n-pnctrd plane – defect anyons

Thm. For $n \geq 3$ the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}}, S^2\big) \mathbin{/\!\!/} \mathrm{Diff}(\mathbb{R}^2_{\setminus \mathbf{3}})\Big) \subset \mathrm{FBr}_{n+1}(S^2)/\mathrm{rot}$$

group of framed braids with total framing $\in (n+1)\mathbb{Z}$

Further case: $\Sigma^2 \equiv \mathbb{R}^2_{\backslash \mathbf{n}}$ the n-pnctrd plane – defect anyons

Thm. For $n \geq 3$ the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\backslash \mathbf{n}})_{\cup\{\infty\}}, S^2\big) \sslash \mathrm{Diff}(\mathbb{R}^2_{\backslash \mathbf{3}})\Big) \subset \mathrm{FBr}_{n+1}(S^2)/\mathrm{rot}$$

group of framed braids with total framing $\in (n+1)\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\backslash \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee^2} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = \left(\prod_i \xi_i\right) / \, \xi_{\mathrm{out}}$

Further case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{n}}$ the n-pnctrd plane – defect anyons

Thm. For $n \geq 3$ the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}}, S^2\big) \mathbin{/\!\!/} \mathrm{Diff}(\mathbb{R}^2_{\setminus \mathbf{3}})\Big) \subset \mathrm{FBr}_{n+1}(S^2)/\mathrm{rot}$$

group of framed braids with total framing $\in (n+1)\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee^2} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = \left(\prod_i \xi_i\right) / \, \xi_{\mathrm{out}}$

irreps involve braid representations
braiding worldlines of the defects — defect anyons

Further case: $\Sigma^2 \equiv \mathbb{R}^2_{\setminus \mathbf{n}}$ the n-pnctrd plane – defect anyons

Thm. For $n \geq 3$ the covariantized flux monodromy is

$$\pi_1\Big(\mathrm{Map}^*\big((\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}}, S^2\big) \sslash \mathrm{Diff}(\mathbb{R}^2_{\setminus \mathbf{3}})\Big) \subset \mathrm{FBr}_{n+1}(S^2)/\mathrm{rot}$$

group of framed braids with total framing $\in (n+1)\mathbb{Z}$

fd-irreps compatible with $(\mathbb{R}^2_{\setminus \mathbf{n}})_{\cup\{\infty\}} \simeq S^2 \vee (S^1)^{\vee 2} \twoheadrightarrow \mathbb{R}^2_{\cup\{\infty\}}$

have $\zeta = \left(\prod_i \xi_i\right) / \xi_{\mathrm{out}}$

irreps involve braid representations

braiding worldlines of the defects — defect anyons

Conclusion.

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable

Conclusion.

- FQH is candidate for TQC (topological quantum hardware) *if* anyons controllable

- FQH anyons are exotic flux quanta, but

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well
- turn situation right-side-up: exotic flux quantization

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well
- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well
- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h
- this re-derives all hallmark FQH anyon properties

Conclusion.

- FQH is candidate for TQC (topological quantum hardware) *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well

- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h
- this re-derives all hallmark FQH anyon properties
- but also predicts anyonic *defects* where flux is expelled

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well

- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h
- this re-derives all hallmark FQH anyon properties
- but also predicts anyonic *defects* where flux is expelled

$\Rightarrow$ TQC via superconducting doping of FQH systems??

Conclusion.

- FQH is candidate for $\overbrace{\text{TQC}}^{\text{topological quantum hardware}}$ *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well

- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h
- this re-derives all hallmark FQH anyon properties
- but also predicts anyonic *defects* where flux is expelled

$\Rightarrow$ TQC via superconducting doping of FQH systems??

in any case:

exotic flux quantization provides new understanding

Conclusion.

- FQH is candidate for TQC (topological quantum hardware) *if* anyons controllable
- FQH anyons are exotic flux quanta, but
- effective CS theory does not reflect that well

- turn situation right-side-up: exotic flux quantization
- candidate such law does exist: Hypothesis h
- this re-derives all hallmark FQH anyon properties
- but also predicts anyonic *defects* where flux is expelled

$\Rightarrow$ TQC via superconducting doping of FQH systems??

in any case:

exotic flux quantization provides new understanding

exotic flux quantization provides new understanding

Outlook.

exotic flux quantization provides new understanding

Outlook.

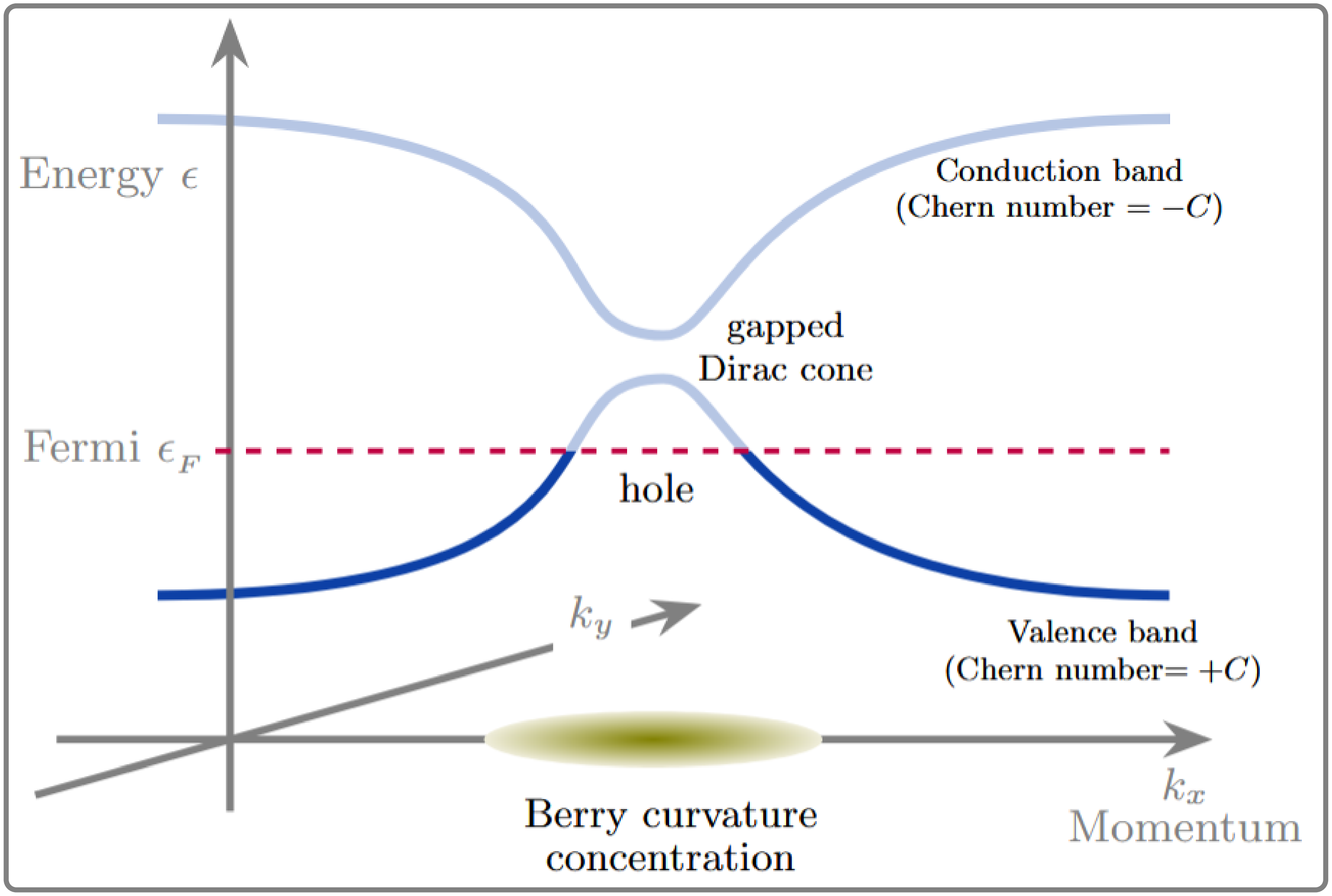
naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH)

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling,

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling,



Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling,

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling,

because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$\mathcal{V} =$

valence bundle

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Eig}_{-1}\Big(\qquad\qquad\qquad\qquad \Big)$$

valence bundle

lower eigen-bundle

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq BU(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Eig}_{-1}\Big(\qquad H/|H| \qquad \Big)$$

valence bundle

lower eigen-bundle

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Eig}_{-1}\Big(\widehat{T}^2 \xrightarrow{\;\; H/|H| \;\;} \mathcal{B}(\mathbb{C}^2) \Big)$$

valence bundle

lower eigen-bundle

Brillouin torus

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq BU(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Eig}_{-1}\left(\widehat{T}^2 \overset{\text{factors}}{\dashrightarrow} \mathbb{C}P^1 \longrightarrow \mathcal{B}(\mathbb{C}^2) \right)$$

valence bundle

lower eigen-bundle

Brillouin torus

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Eig}_{-1}\Big(\widehat{T}^2 \overset{\text{factors}}{\dashrightarrow} \mathbb{C}P^1 \xrightarrow[\text{Pauli matrices}]{\sigma} \mathcal{B}(\mathbb{C}^2) \Big)$$

valence bundle

lower eigen-bundle

Brillouin torus

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}\mathbb{P}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Taut}\left(\widehat{T}^2 \dashrightarrow \mathbb{C}P^1 \qquad\qquad \right)$$

valence bundle

tauto- logical line bundle

Brillouin torus

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq BU(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Taut}\Big(\widehat{T}^2 \dashrightarrow \mathbb{C}P^1 \Big)$$

valence bundle

tauto-logical line bundle

Brillouin torus

Bloch Hamiltonian

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Taut}\left(\widehat{T}^2 \dashrightarrow \mathbb{C}P^1 \longrightarrow \mathbb{C}P^\infty\right)$$

valence bundle

tauto-logical line bundle

Brillouin torus

Bloch Hamiltonian

stable band topology

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{CP}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Taut}\left(\widehat{T}^2 \dashrightarrow \mathbb{C}P^1 \longrightarrow \mathbb{C}P^\infty \right)$$

valence bundle

tauto-logical line bundle

Brillouin torus

Bloch Hamiltonian

stable band topology

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH) namely in Chern insulators at fractional filling, because here the cohomotopical factorization through

$$S^2 \simeq \mathbb{C}P^1 \hookrightarrow \mathbb{C}\mathbb{P}^\infty \simeq B\mathrm{U}(1)$$

reflects the electron's *fragile* band topology

$$\mathcal{V} = \mathrm{Taut}\left(\widehat{T}^2 \dashrightarrow \mathbb{C}P^1 \longrightarrow \mathbb{C}P^\infty \right)$$

valence bundle

tauto-logical line bundle

Brillouin torus

Bloch Hamiltonian

stable band topology

$\Rightarrow$ FQAH anyons classified by adiabatic monodromy in fragile band topology

Outlook.

naturally explains expected but elusive anyons in fractional quantum *anomalous* Hall systems (FQAH)

arXiv > cond-mat > arXiv:2507.00138

Search...

Help |

Condensed Matter > Strongly Correlated Electrons

[Submitted on 30 Jun 2025]

Identifying Anyonic Topological Order in Fractional Quantum Anomalous Hall Systems

Hisham Sati, Urs Schreiber

$\Rightarrow$ FQAH anyons classified by adiabatic monodromy in fragile band topology

Urs Schreiber on joint work with Hisham Sati:

surveying our preprint: [arXiv:2507.00138]

Non-Lagrangian construction of abelian CS/FQH-theory via Flux Quantization in 2-Cohomotopy

Thanks!

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS @ جامعة نيويورك أبوظبي NYU ABU DHABI

(July 2025) find these slides at: [ncatlab.org/schreiber/show/ISQS29]