Knots for Quantum Computation from Defect branes

Urs Schreiber on joint work with Hisham Sati

جامعـة نيويورك أبـوظـي NYU ABU DHABI NYU AD Science Division, Program of Mathematics

& Center for Quantum and Topological Systems

New York University, Abu Dhabi



talk at:

Topological Methods in Mathematical Physics @ Erice, 2 Sep 2022

slides and pointers at: https://ncatlab.org/schreiber/show/Knots+for+Quantum+Computation

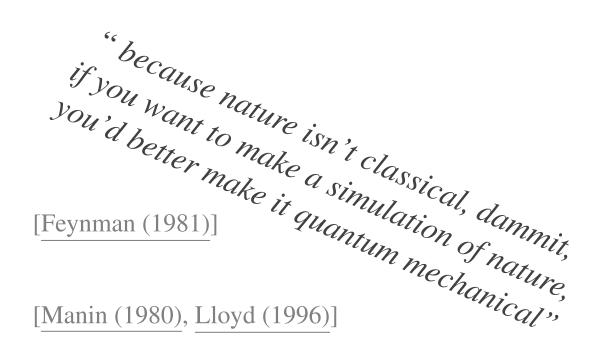
Quantum Supremacy

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analysis of	quantum matter
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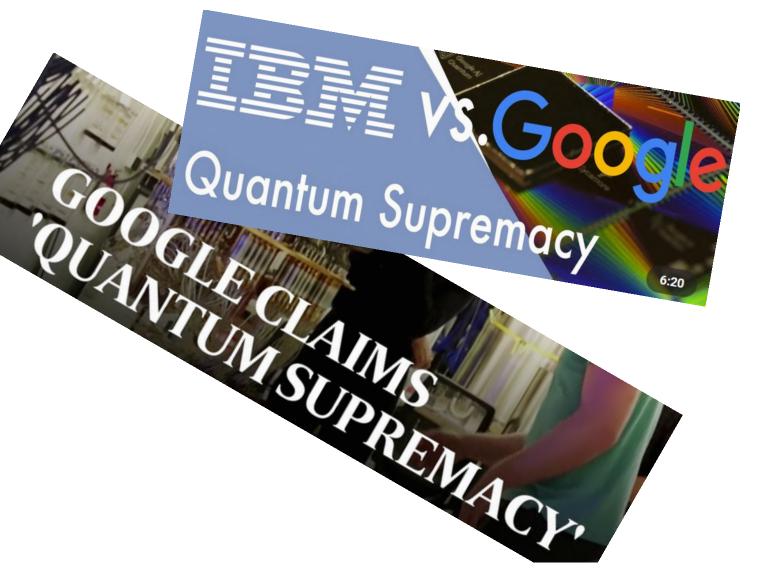
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Quantum Instability:

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so far these demonstrations are contrived and have all been contested

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Quantum Physics

[Submitted on 2 Jan 2018 (v1), last revised 31 Jul 2018 (this version, v3)]

Quantum Computing in the NISQ era and beyond

John Preskill

Noisy Intermediate-Scale Quantum (NISQ) technology will be available in the near future. Quantum computers with 50-100 qubits may be able to perform tasks which surpass the capabilities of today's classical digital computers, but noise in quantum gates will limit the size of quantum circuits that can be executed reliably.

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TOPOLOGICAL QUANTUM COMPUTATION

MICHAEL H. FREEDMAN, ALEXEI KITAEV, MICHAEL J. LARSEN, AND ZHENGHAN WANG

ABSTRACT. The theory of quantum computation can be constructed from the abstract study of anyonic systems. In mathematical terms, these are unitary topological modular functors. They underlie the Jones polynomial and arise in Witten-Chern-Simons theory. The braiding and fusion of anyonic excitations in quantum Hall electron liquids and 2D-magnets are modeled by modular functors, opening a new possibility for the realization of quantum computers. The chief advantage of anyonic computation would be physical error correction

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High Energy Physics - Theory

[Submitted on 22 Mar 2022]

Anyonic Defect Branes and Conformal Blocks in Twisted Equivariant Differential (TED) K-theory

Hisham Sati, Urs Schreiber

We demonstrate that twisted equivariant differential K-theory of transverse complex curves accommodates exotic charges of the form expected of codimension=2 defect branes, such as of D7-branes in IIB/F-theory on A-type orbifold singularities, but also of their dual 3-brane defects of class-S theories on M5-branes.

We claim that we have made some progress on the problem.



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Here we provide a detailed argument for the classification of symmetry protected/enhanced su(2)anyonic topological order, specifically in interacting 2d semi-metals, by the twisted equivariant differential (TED) K-theory of configuration spaces of points in the complement of nodal points inside the crystal's Brillouin torus orbi-orientifold.

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A PLanQC 2022

Thu 15 Sep 2022 11:00 - 11:25 at M2 - Hardware-aware quantum programming

★ Topological Quantum Programming in TED-K

While the realization of scalable quantum computation will arguably require topological stabilization and, with it, topological-hardwareaware quantum programming and topological-quantum circuit verification, the proper combination of these strategies into dedicated *it topological quantum programming* languages has not yet received attention.

Here we describe a fundamental and natural scheme that we are developing, for typed functional (hence verifiable) topological quantum programming which is *topological-hardware aware* – in that it natively reflects the universal fine technical detail of topological q-bits, namely of symmetry-protected (or enhanced) topologically ordered Laughlin-type anyon ground states in topological phases of quantum materials.

What follows is general motivation & gentle exposition.



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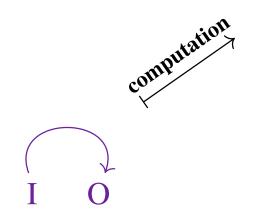
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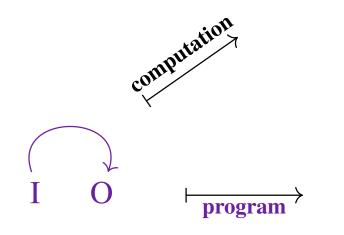
To compute is

cf. [van Leeuwen & Wiedermann (2017)]



To compute is to execute

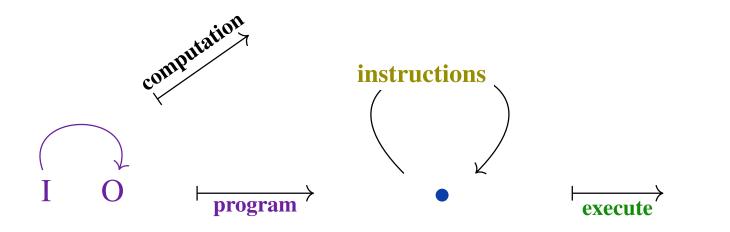
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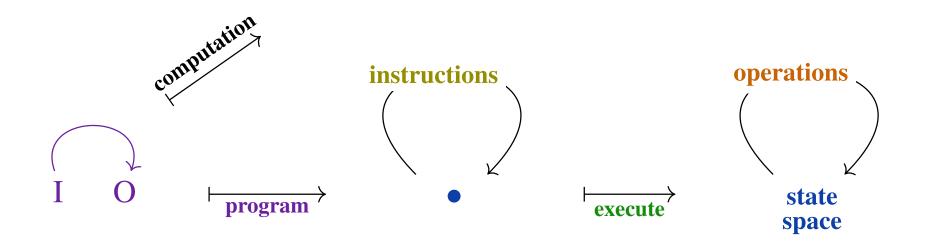
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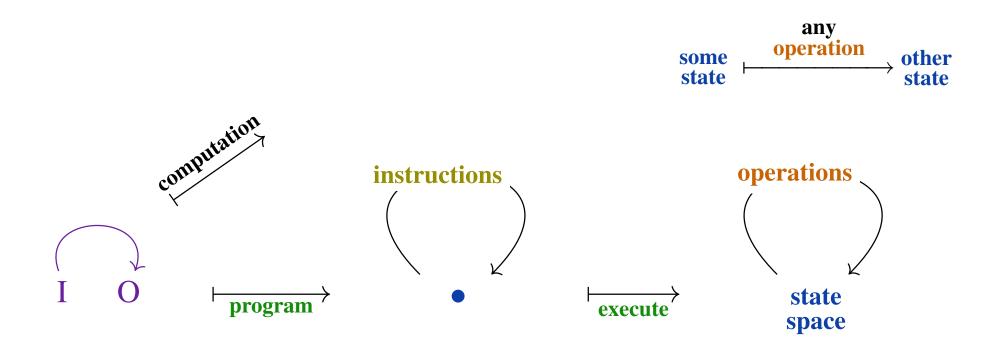


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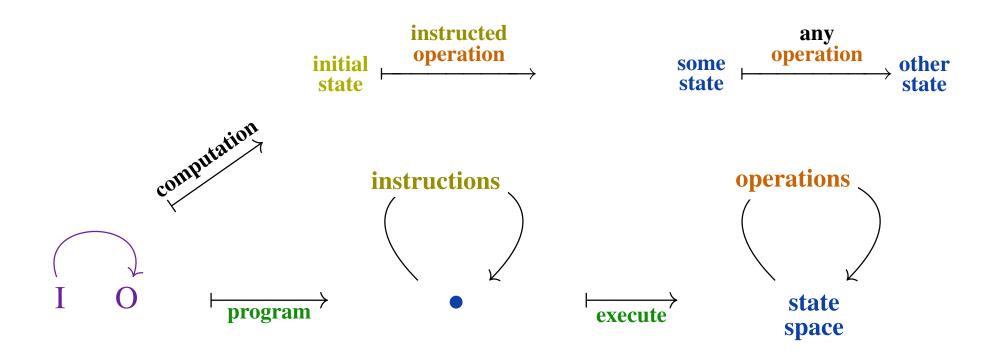


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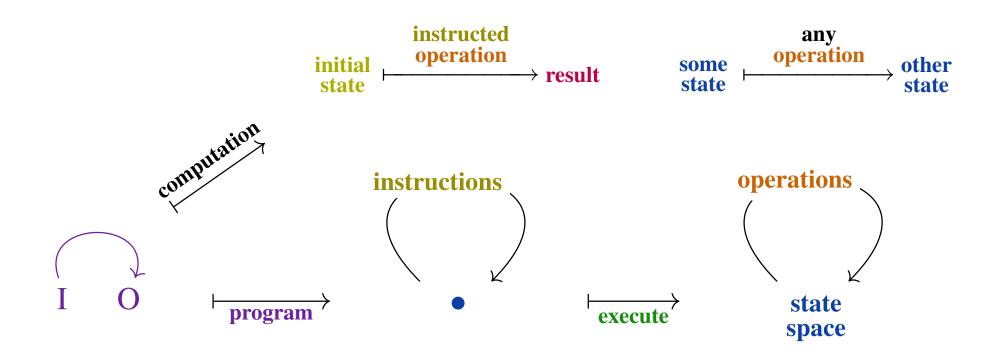
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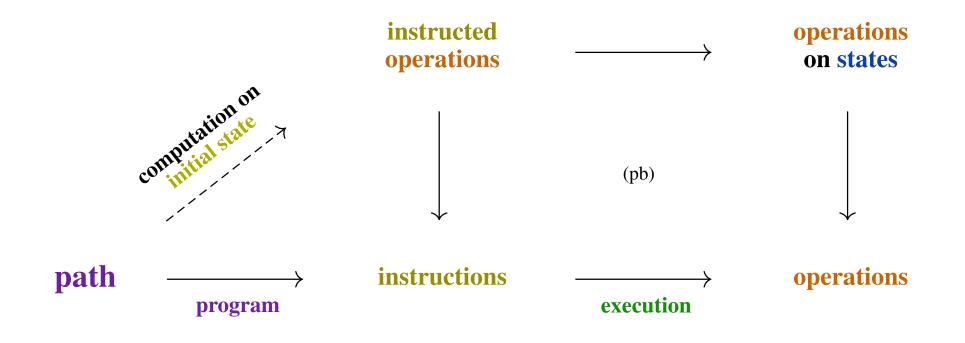
turning a given **initial state** into the computed **result**.

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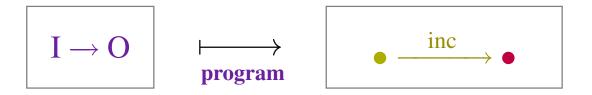
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Aside: Formalization by transport in Homotopy Type Theory:

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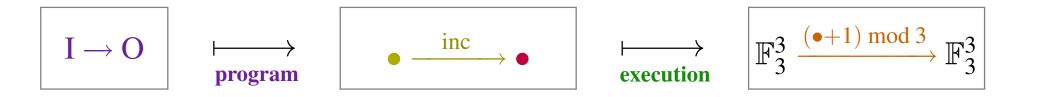
[Sati & Schreiber, PlanQC 2022 33 (2022)]



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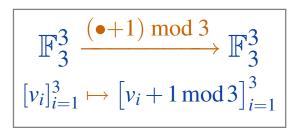
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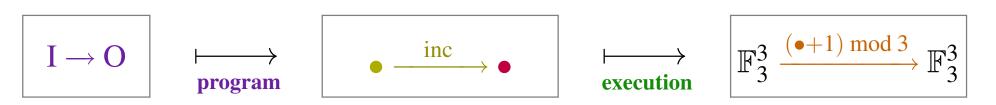
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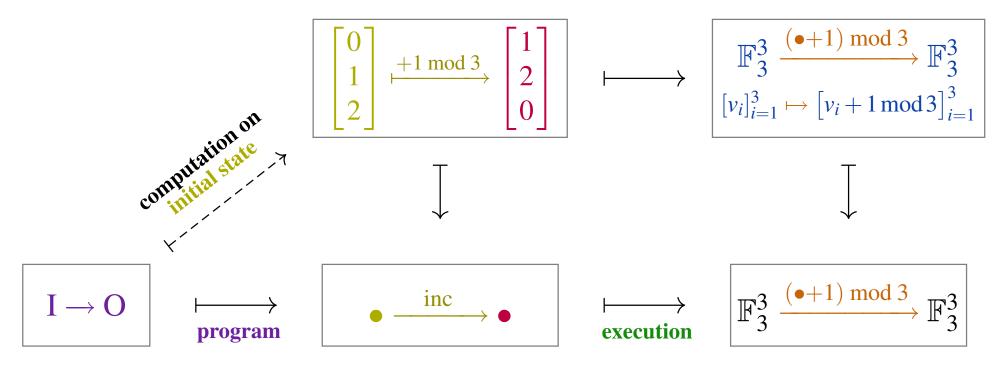




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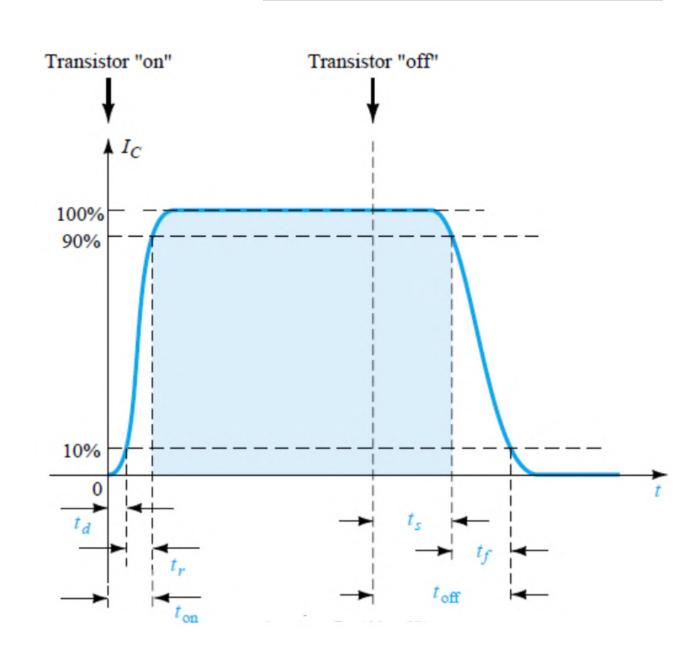


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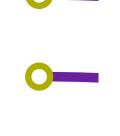
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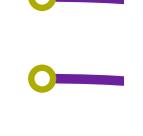


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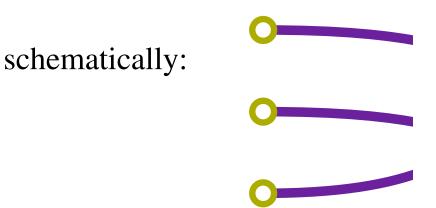




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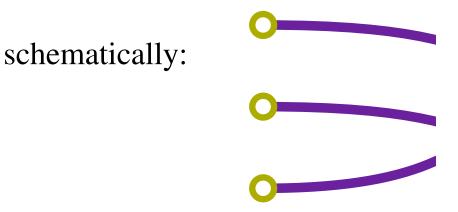
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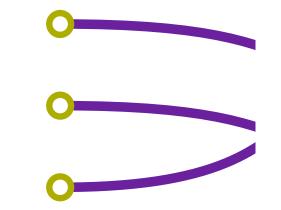


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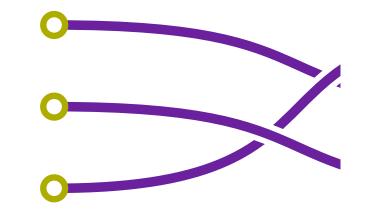
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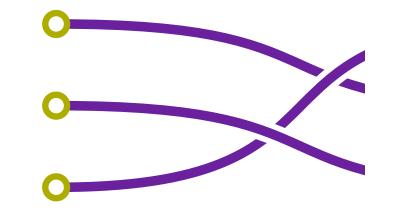


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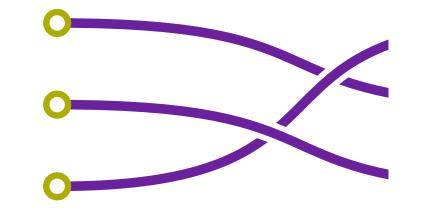


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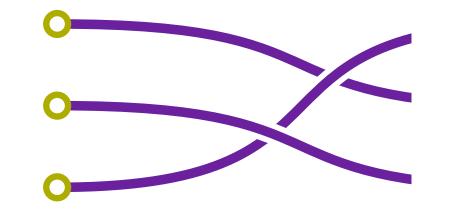


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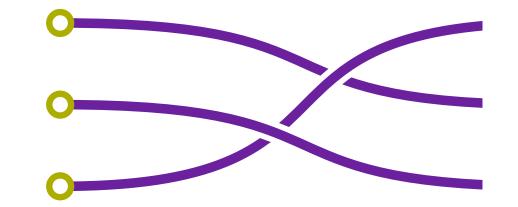


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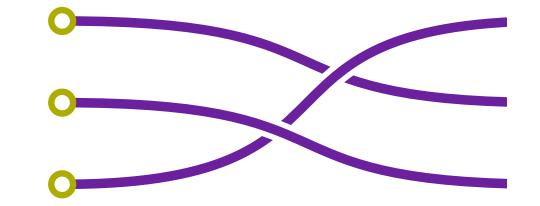


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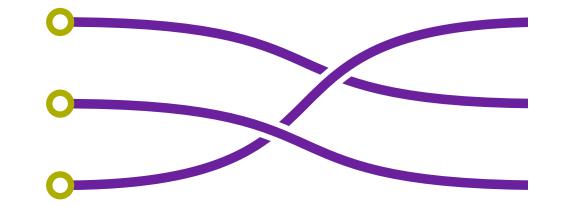


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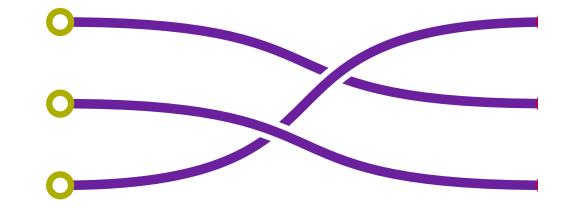


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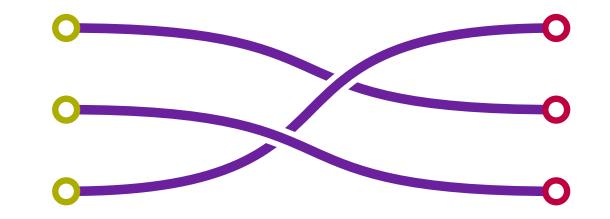


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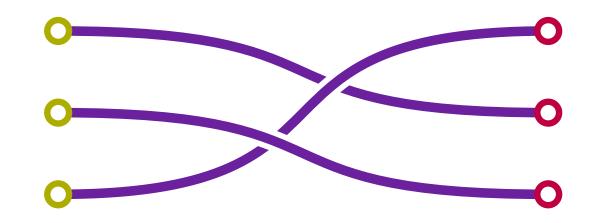
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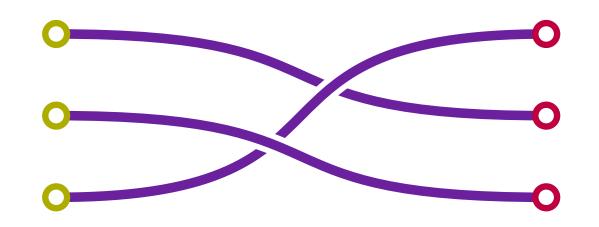








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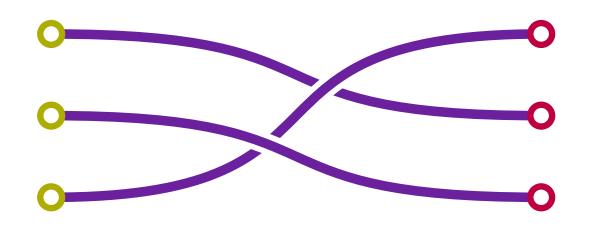
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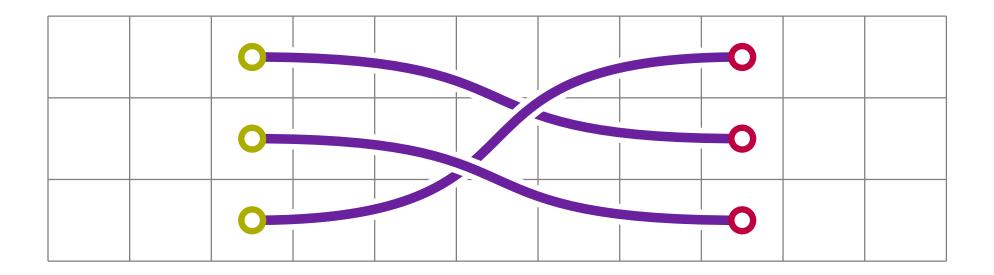
Strategy of **Classical Digital Computation:** Coarse-grain state space into a bit lattice.





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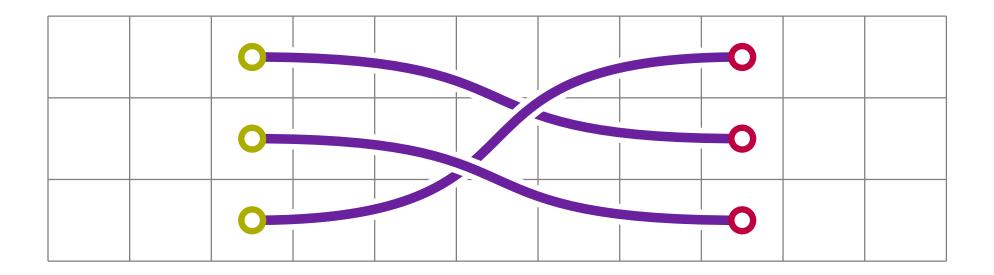
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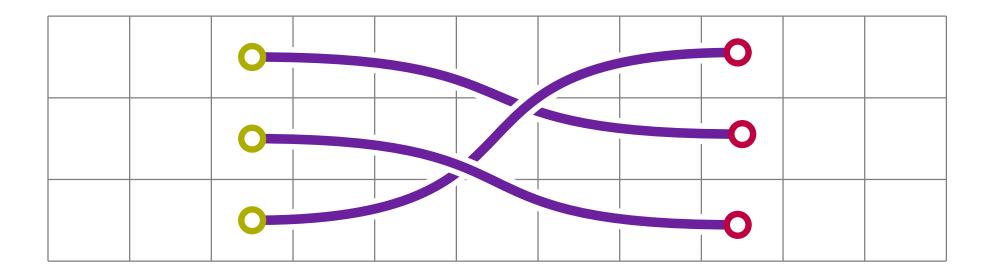
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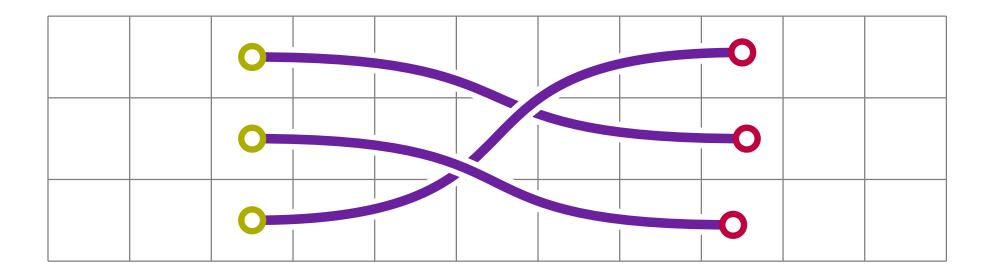
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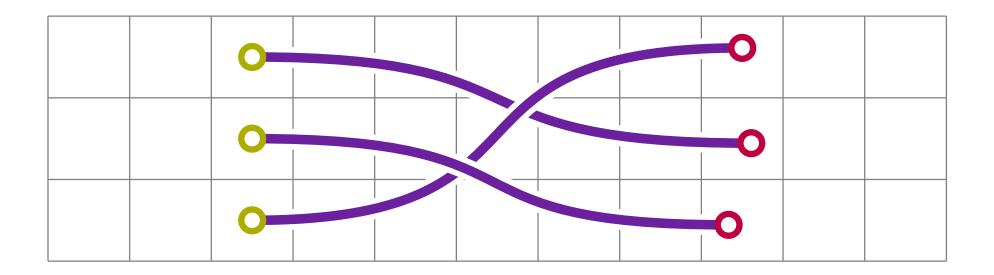
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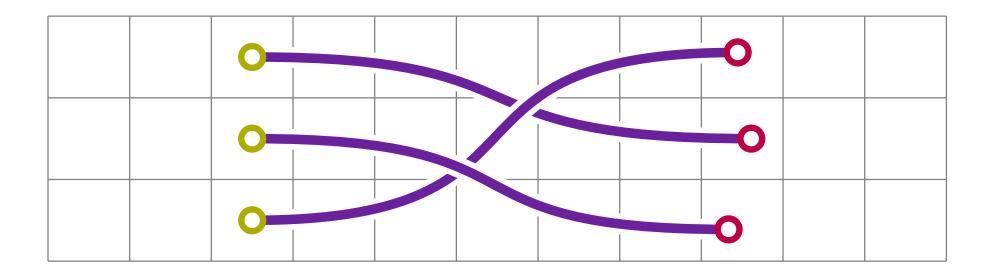
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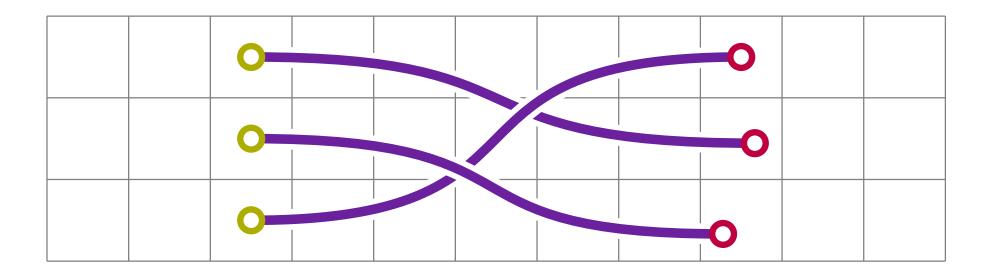
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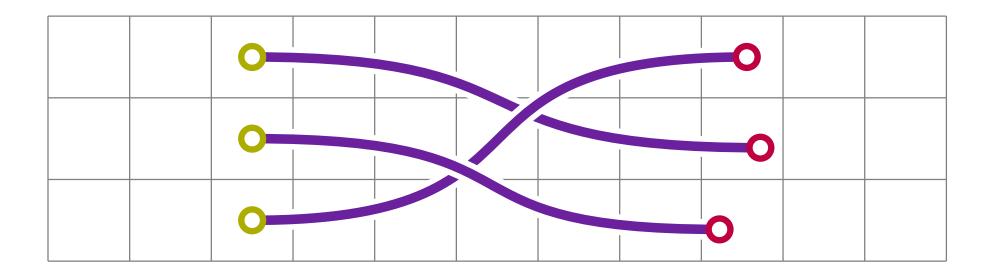
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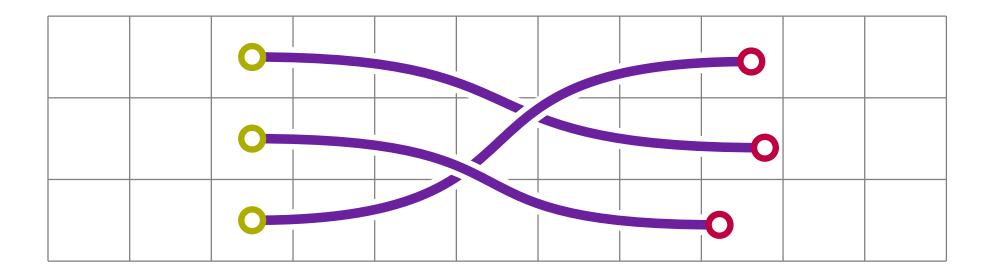
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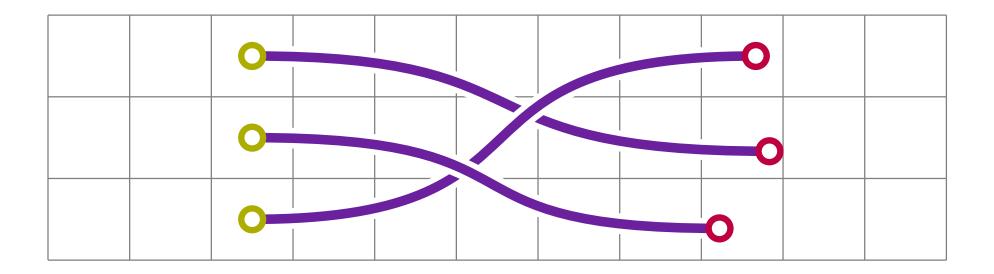
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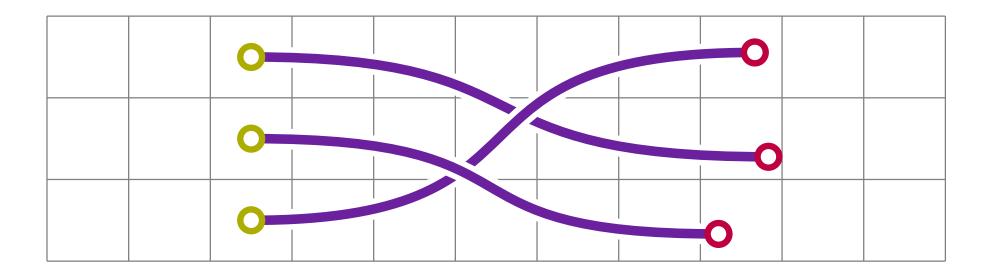
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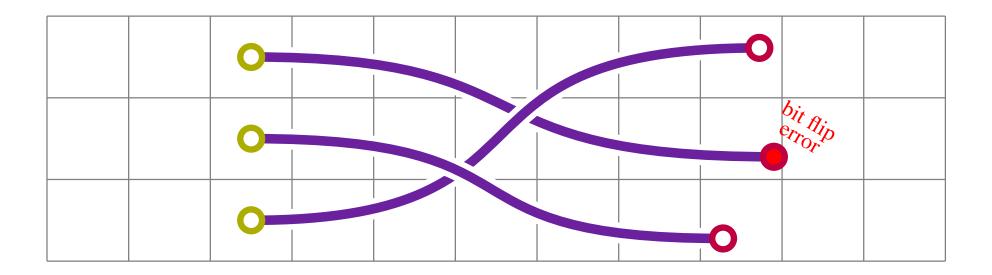
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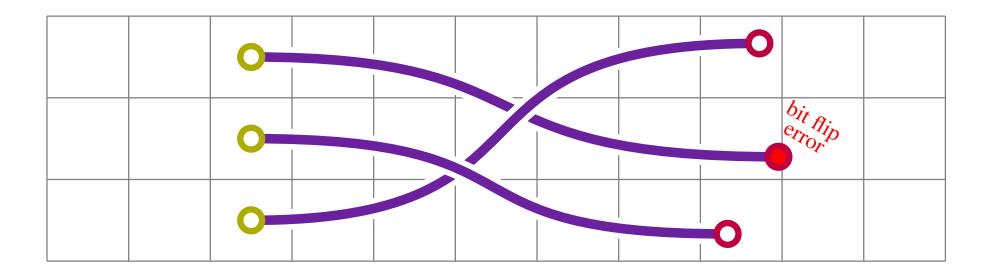
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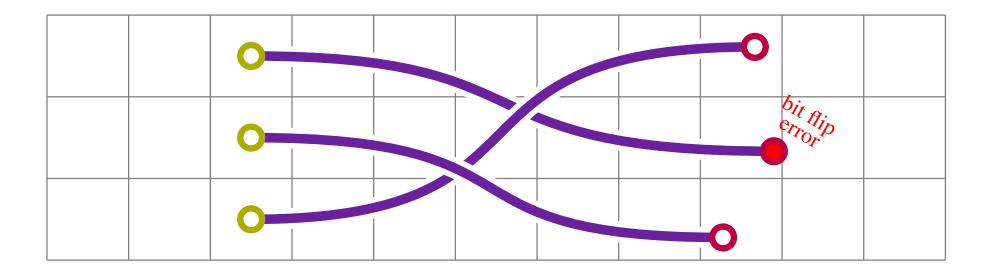
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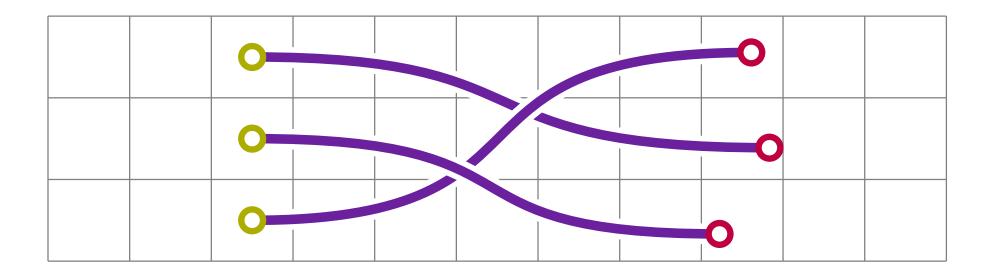
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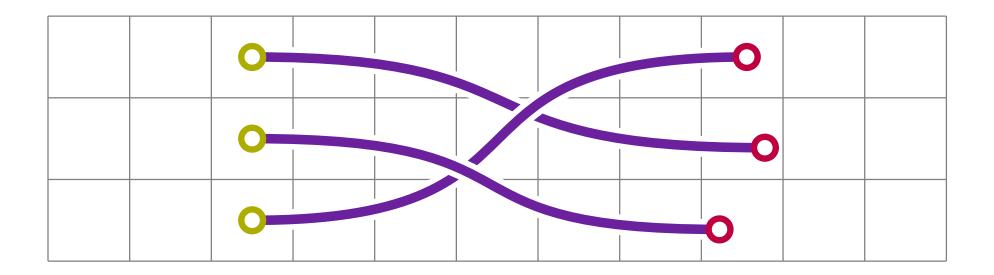
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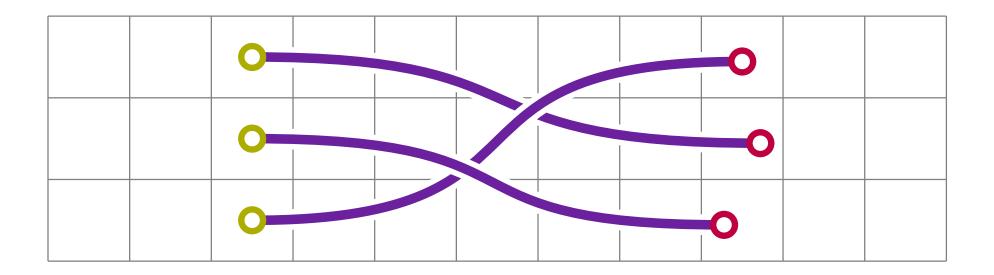
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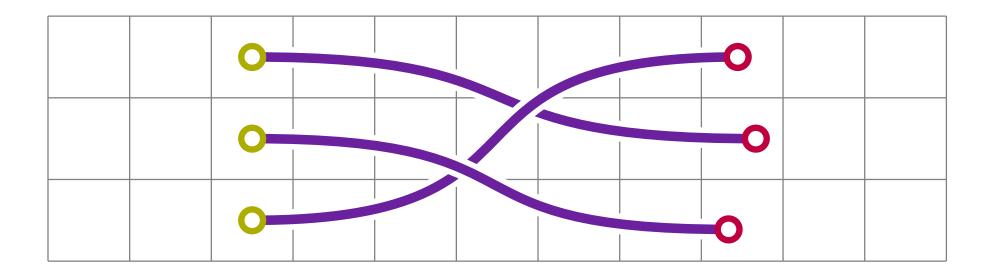
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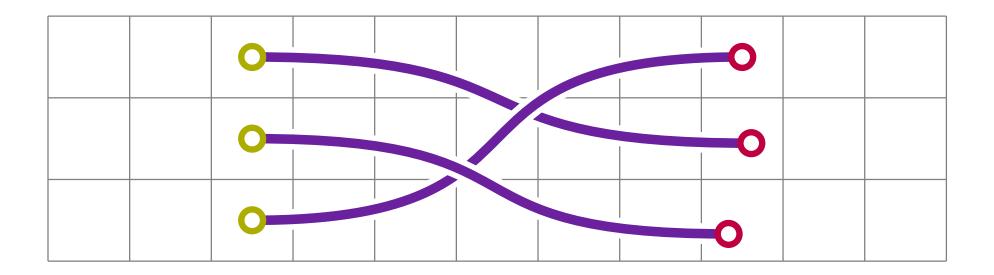
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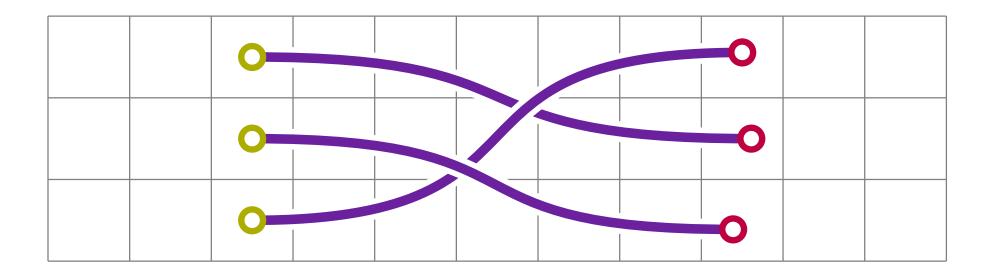
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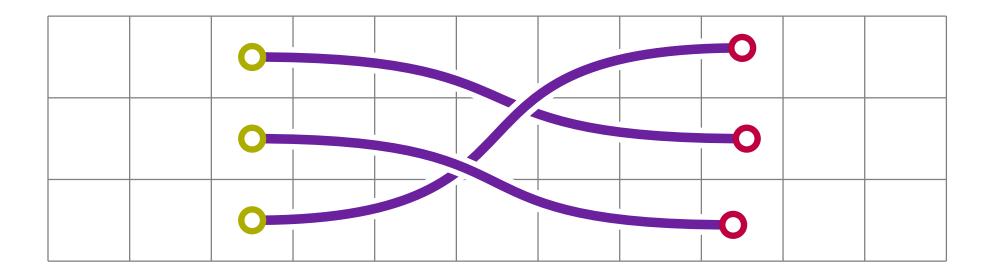
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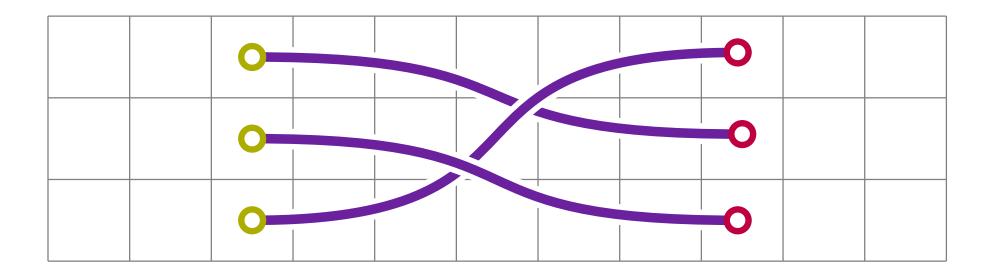
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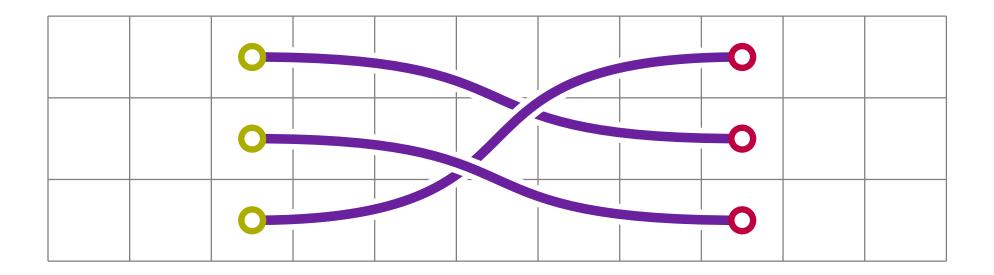
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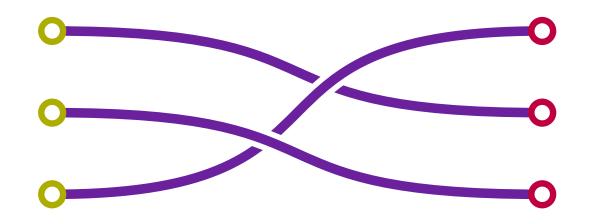
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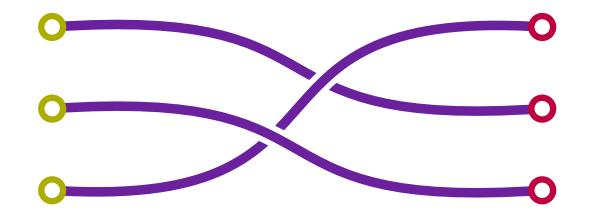
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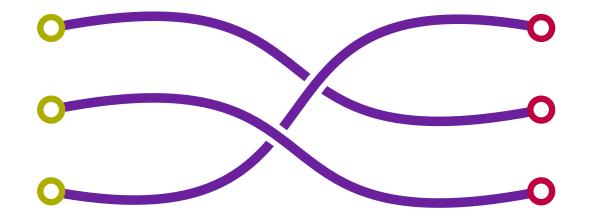
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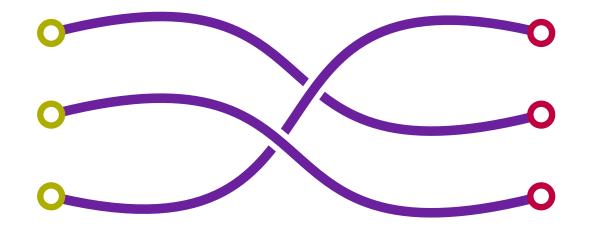
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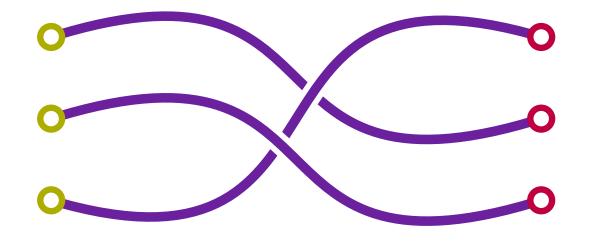
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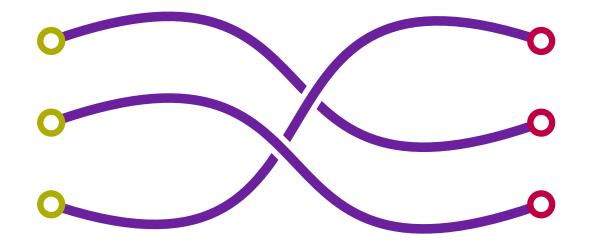
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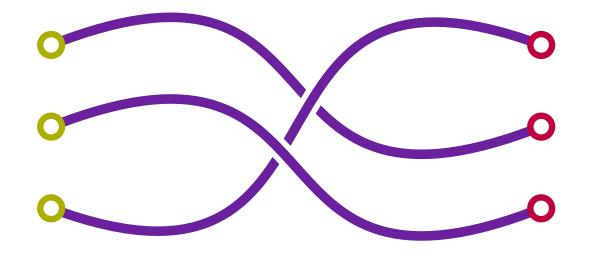
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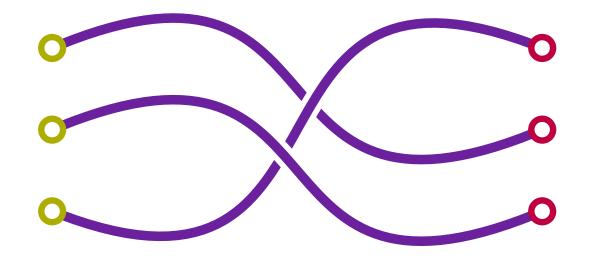
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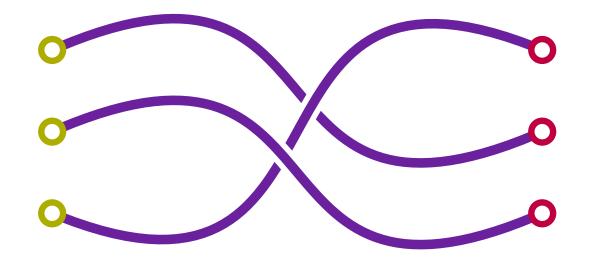
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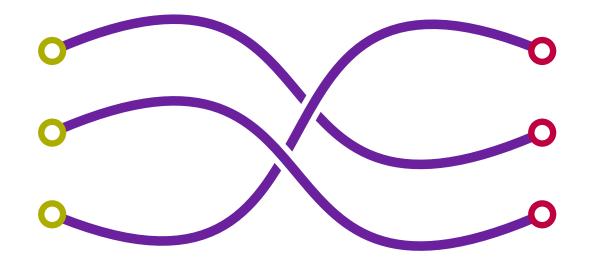
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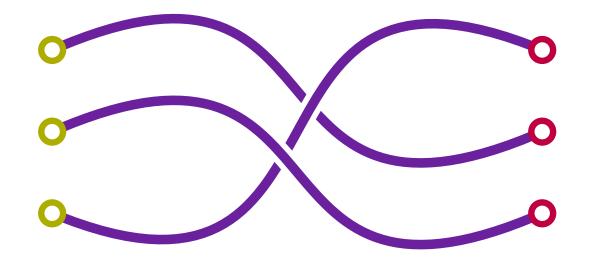
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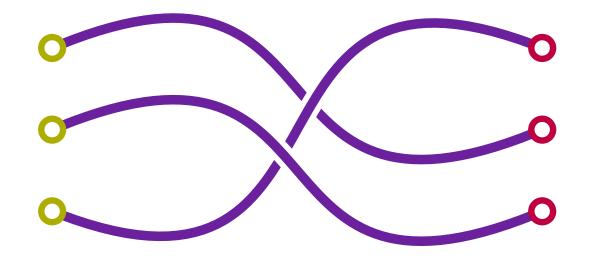
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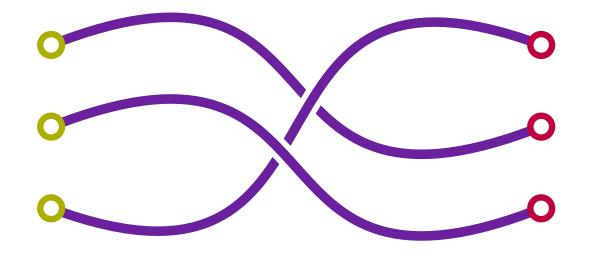
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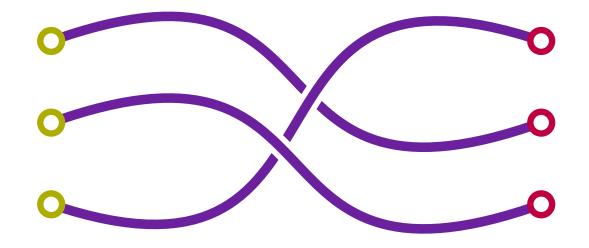
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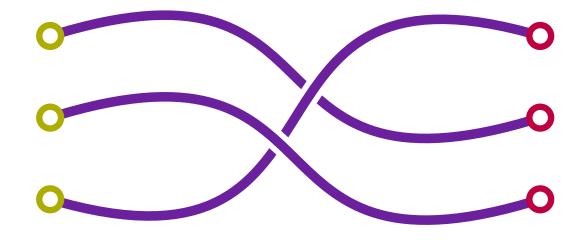
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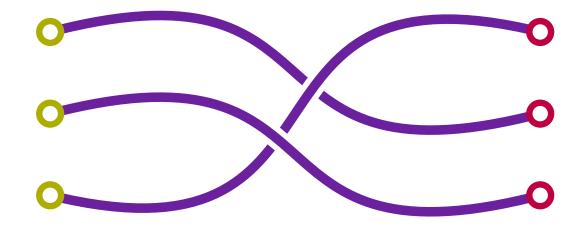
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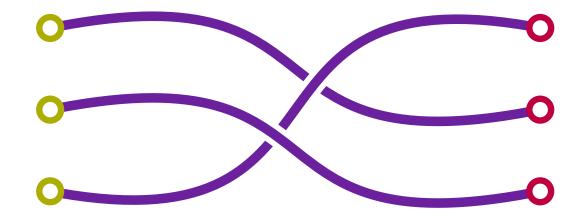
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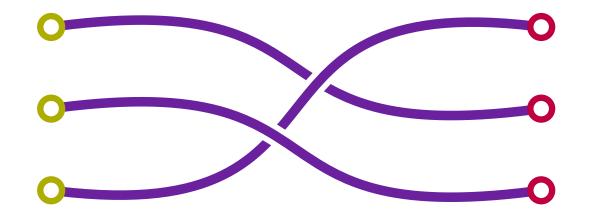




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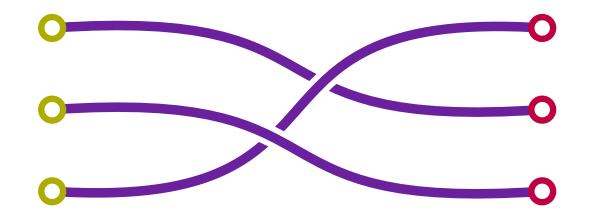
 \Rightarrow knots as quantum gates





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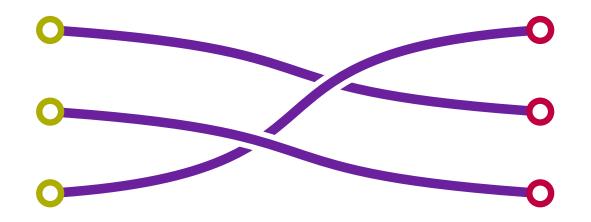
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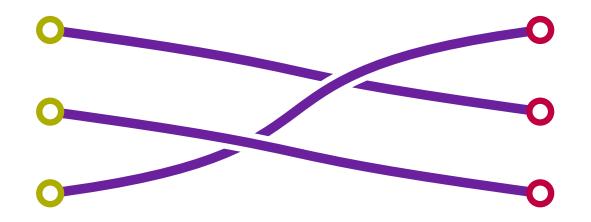
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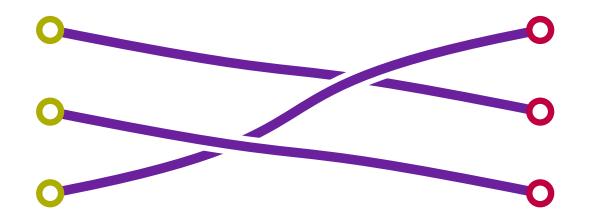
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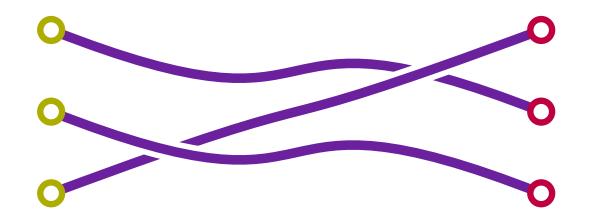
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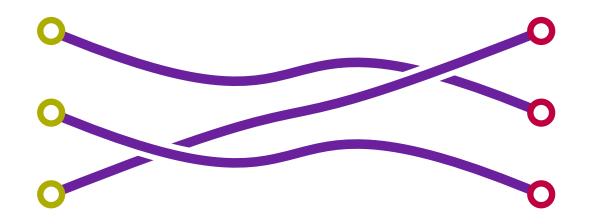
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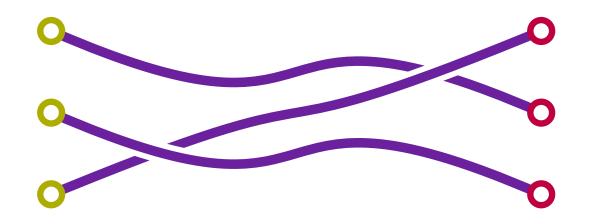
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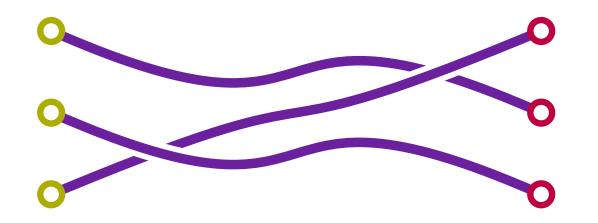
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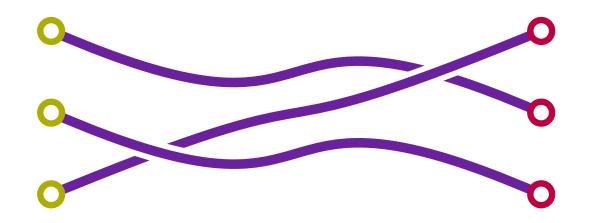
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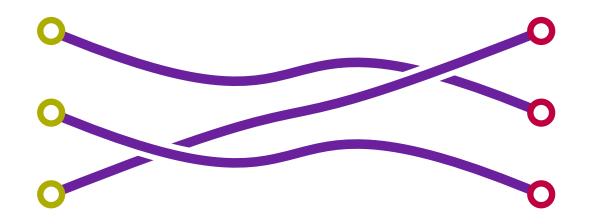
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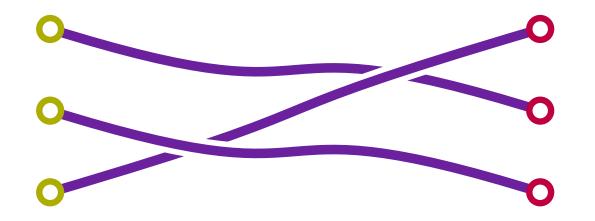
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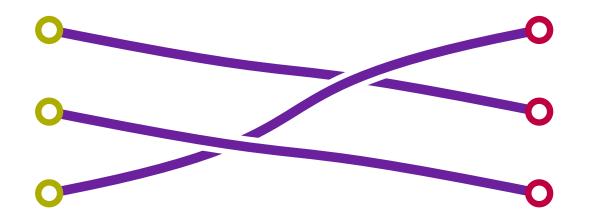
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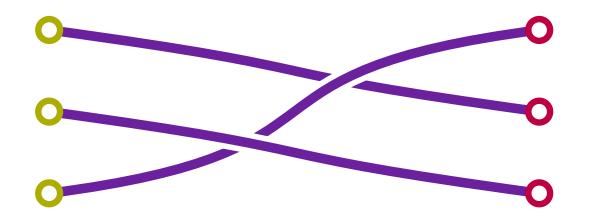
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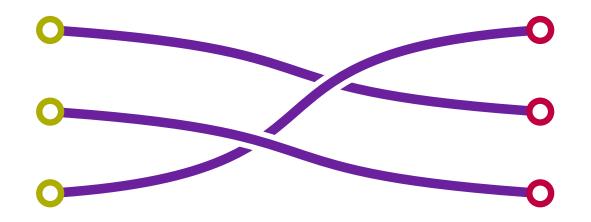
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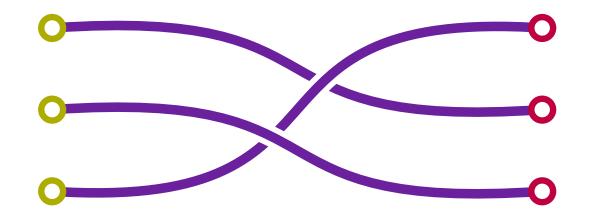
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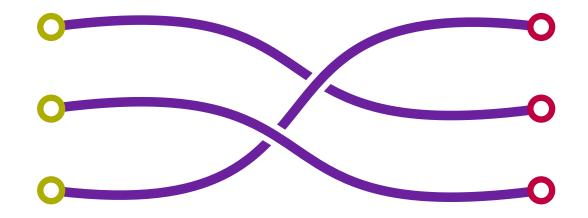
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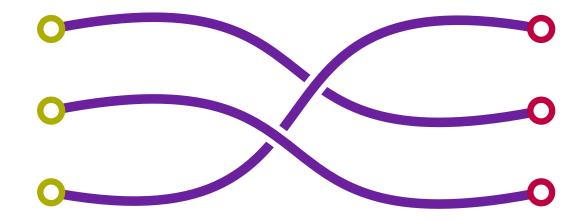
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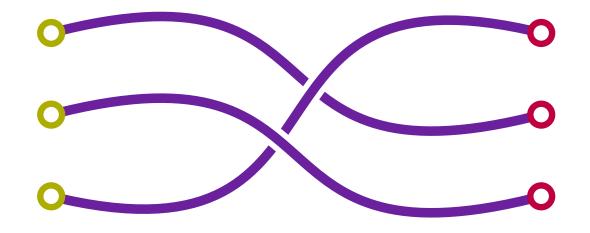
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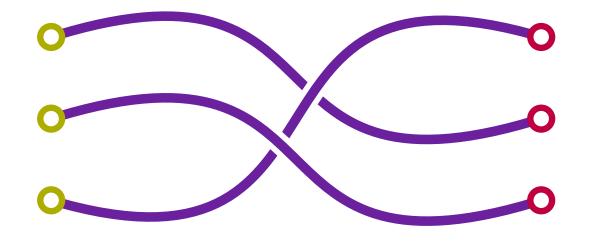
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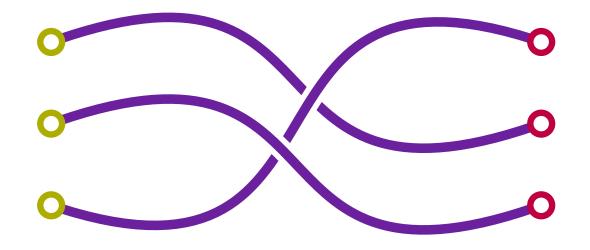
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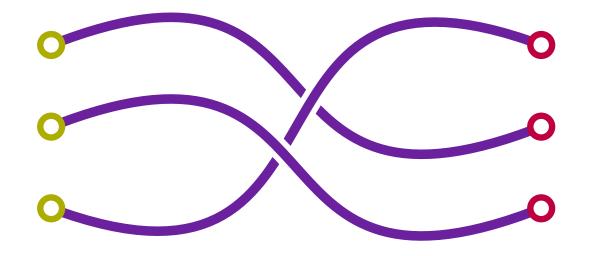
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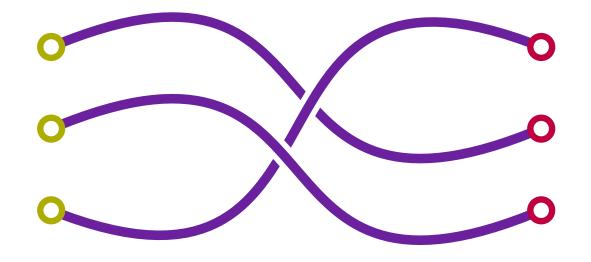
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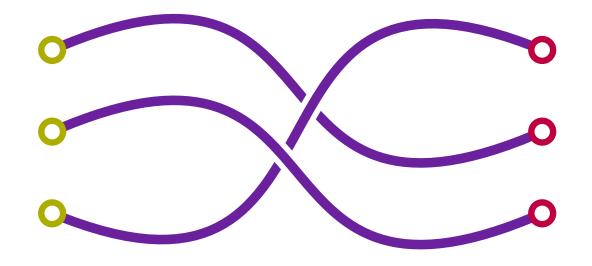
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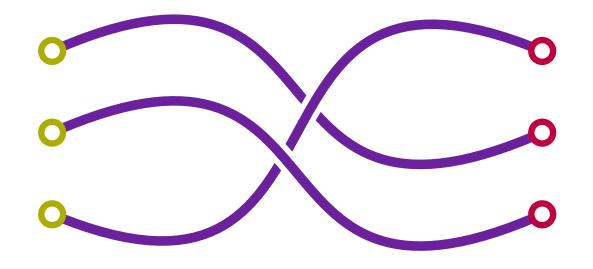
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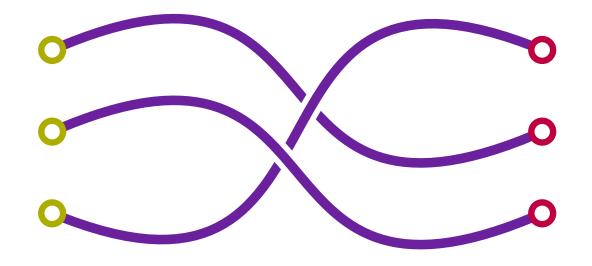
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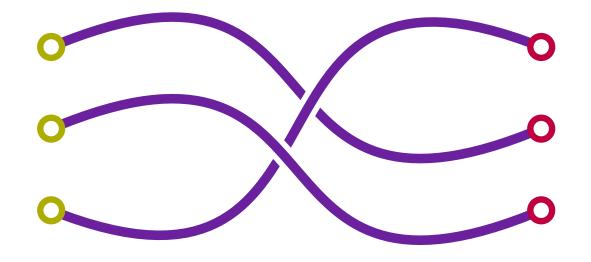
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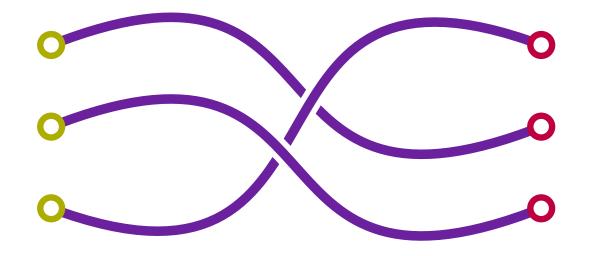
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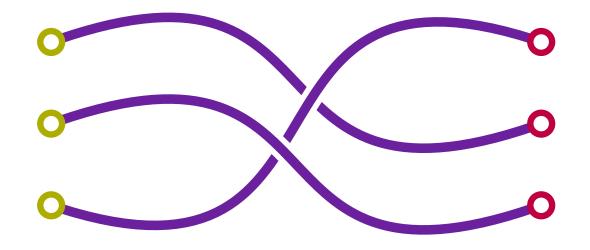
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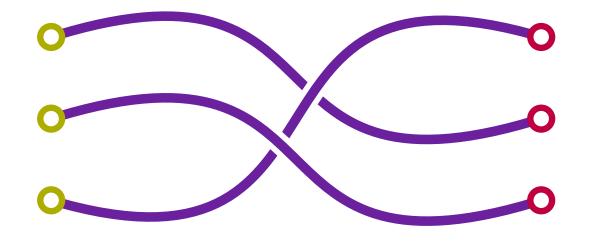
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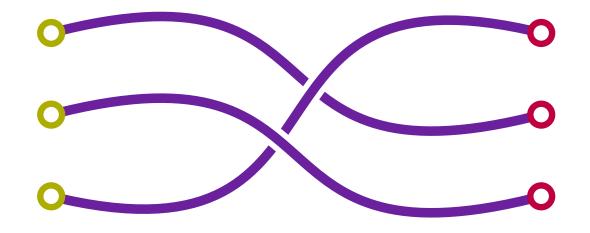
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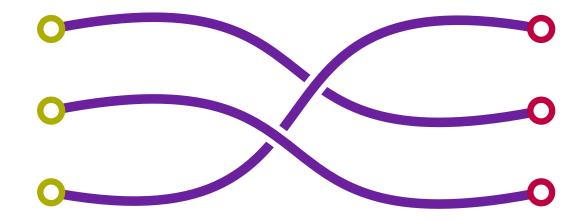
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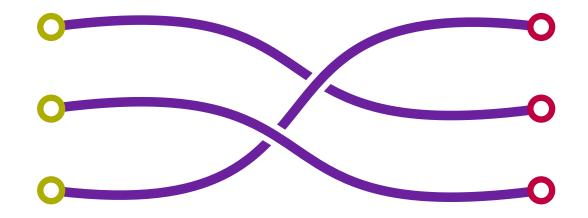
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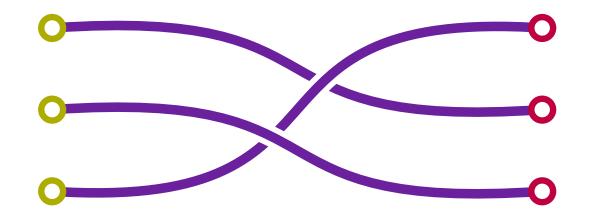
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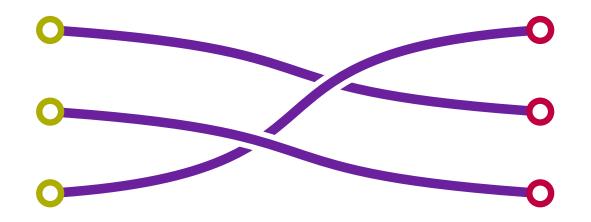
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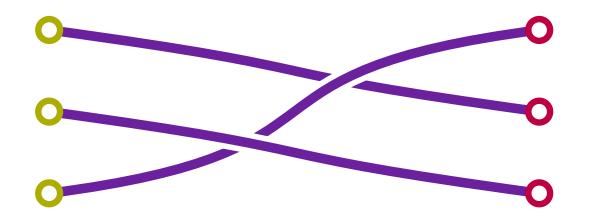
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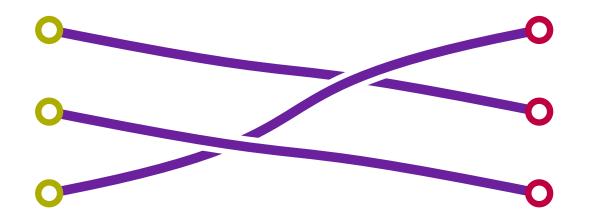
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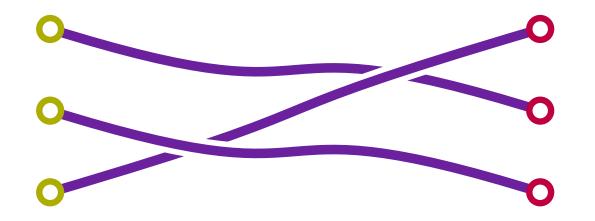
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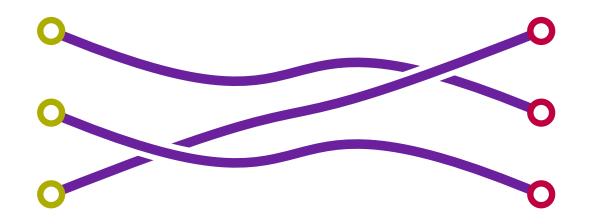
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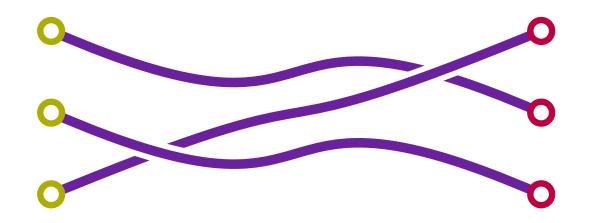
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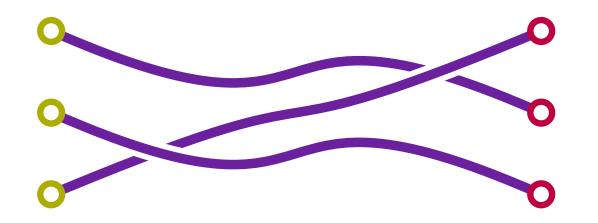
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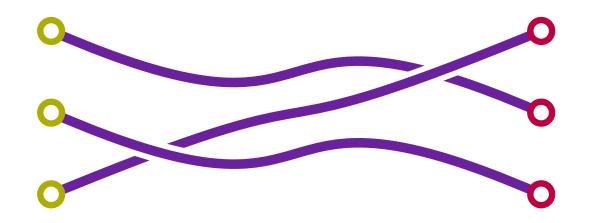
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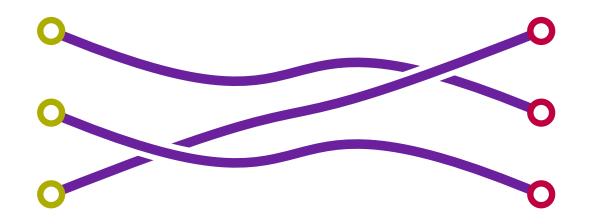
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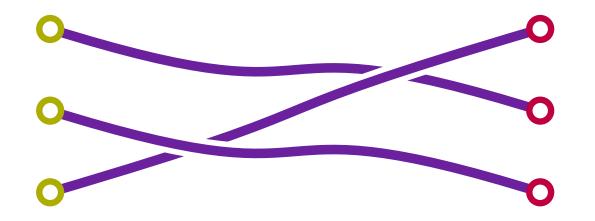
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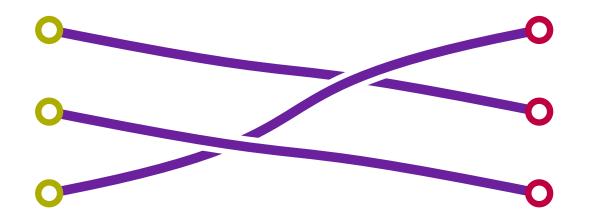
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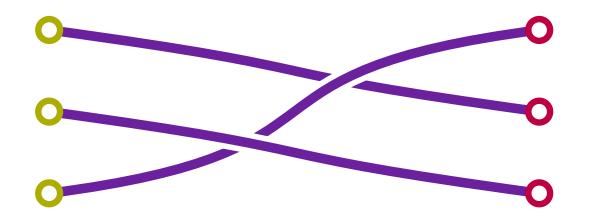
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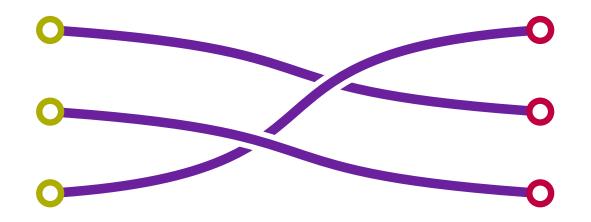
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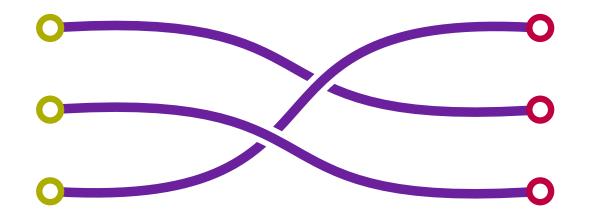
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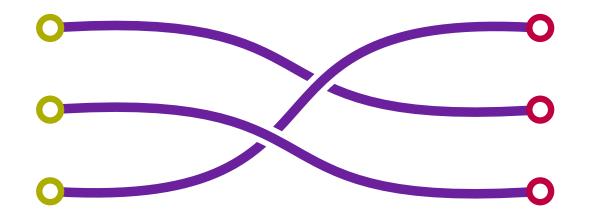
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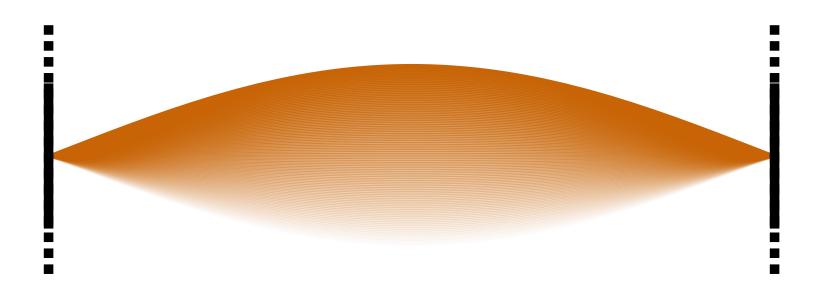


On atomic scales, particles are waves; whose energy is quantized.



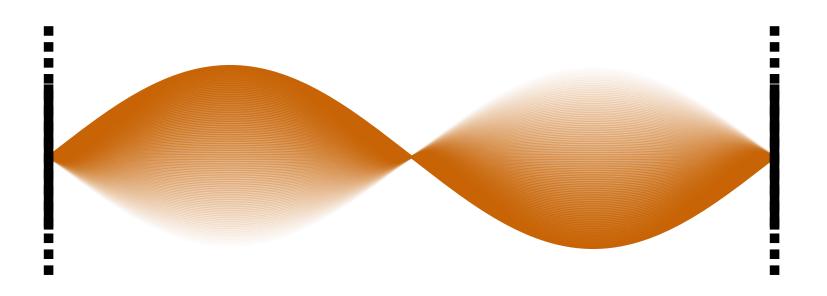
ground state: E = 0

On atomic scales, particles are waves; whose energy is quantized.



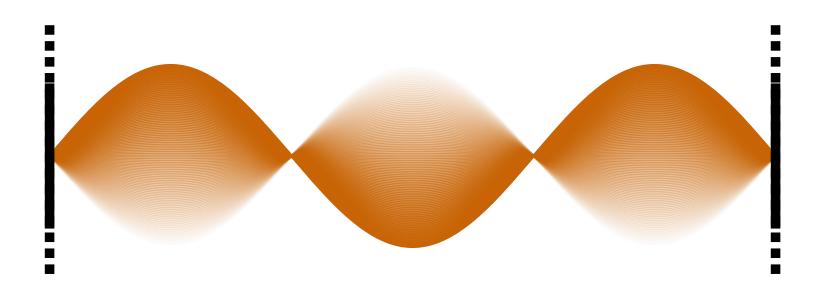
first excited state: $E = \hbar \omega$

On atomic scales, particles are waves; whose energy is quantized.



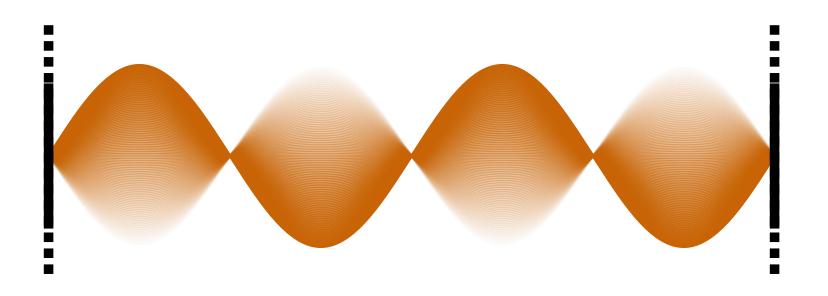
second excited state: $E = 2\hbar\omega$

On atomic scales, particles are waves; whose energy is quantized.



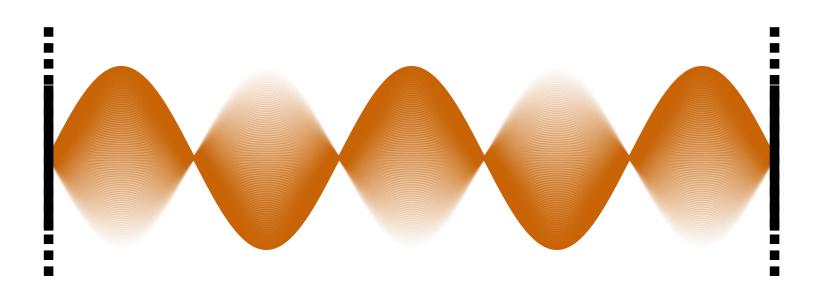
third excited state: $E = 3\hbar\omega$

On atomic scales, particles are waves; whose energy is quantized.



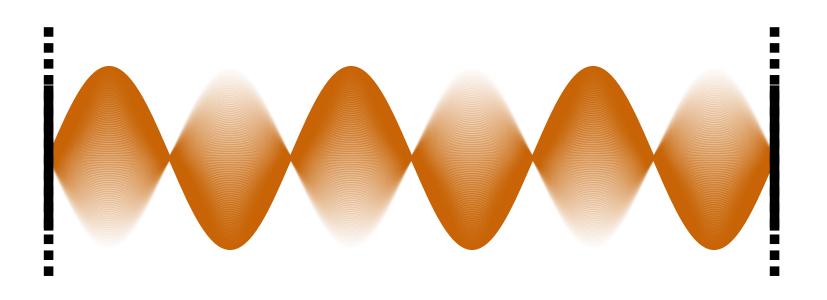
fourth excited state: $E = 4\hbar\omega$

On atomic scales, particles are waves; whose energy is quantized.



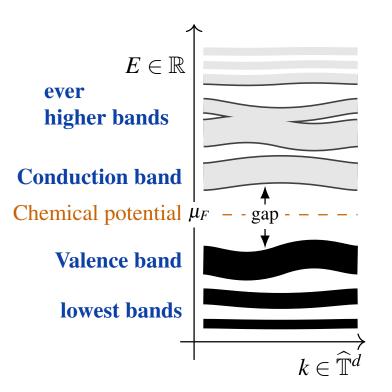
fifth excited state: $E = 5\hbar\omega$

On atomic scales, particles are waves; whose energy is quantized.

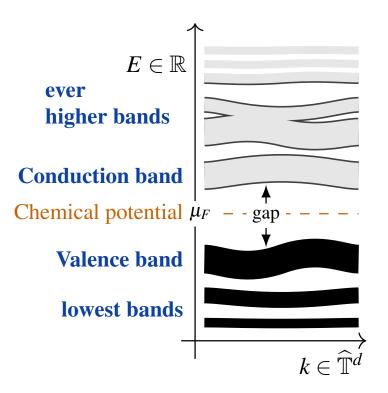


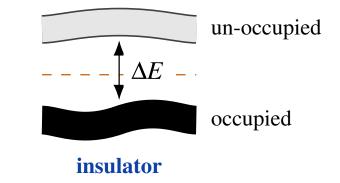
sixth excited state: $E = 6\hbar\omega$

As very many particles come together in a crystal their excitation energies accumulate in "bands" but energy gaps *may* remain.

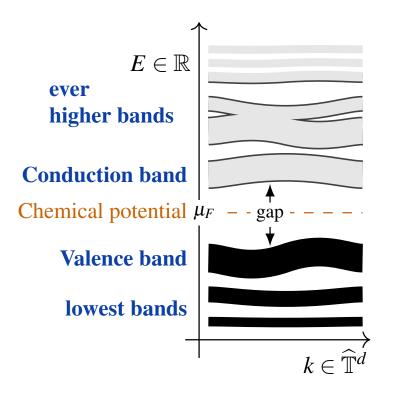


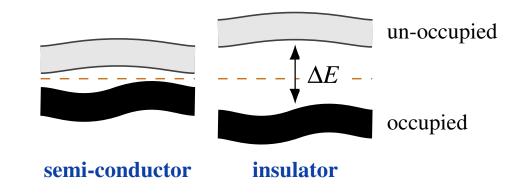
If the ground state remains separated by an energy gap ΔE then it is *completely* undisturbed by disturbances $< \Delta E$.



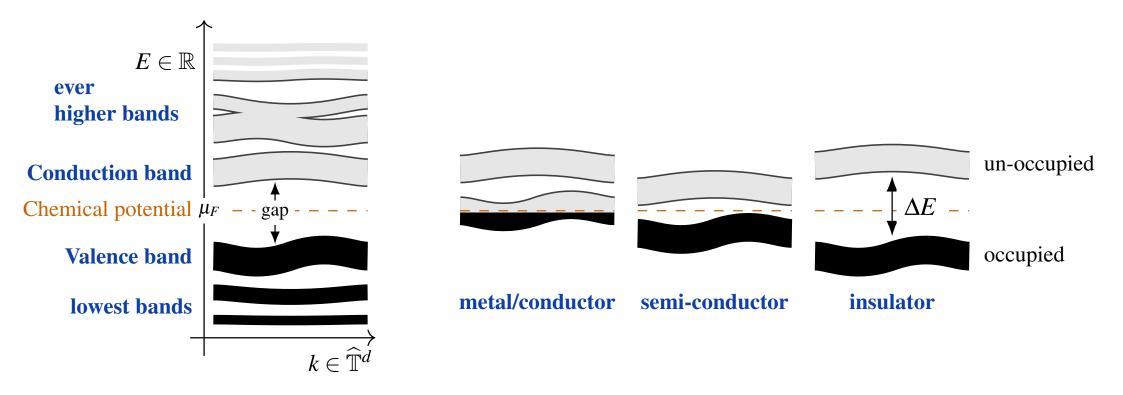


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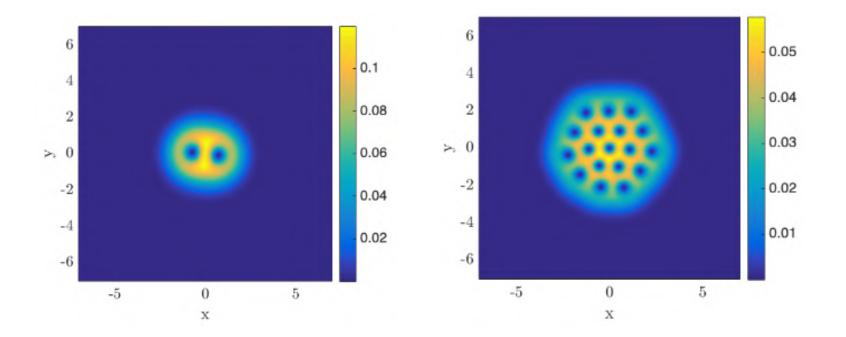




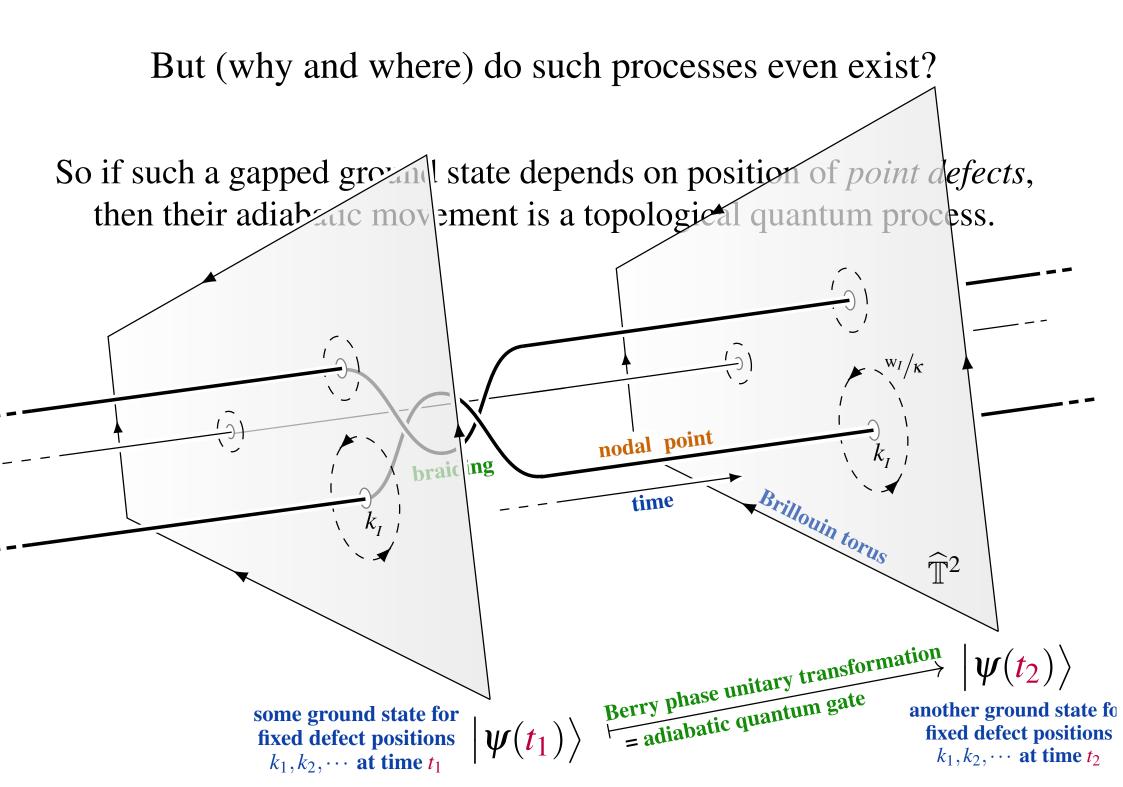
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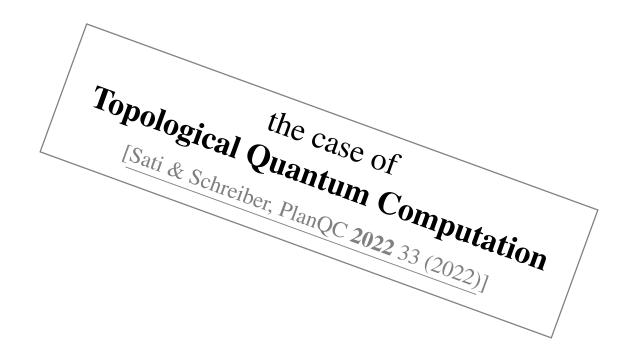
So if such a gapped ground state depends on position of *point defects*, then their adiabatic movement is a topological quantum process.

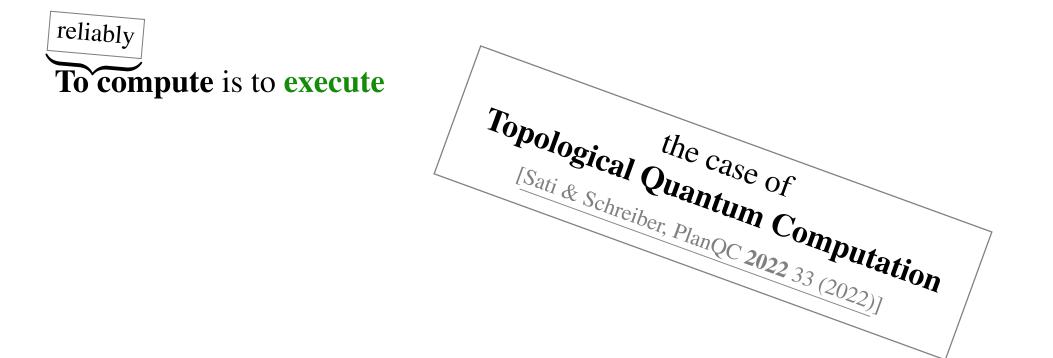


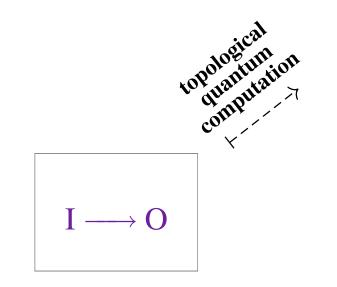
(numerical simulation from arXiv:1901.10739)







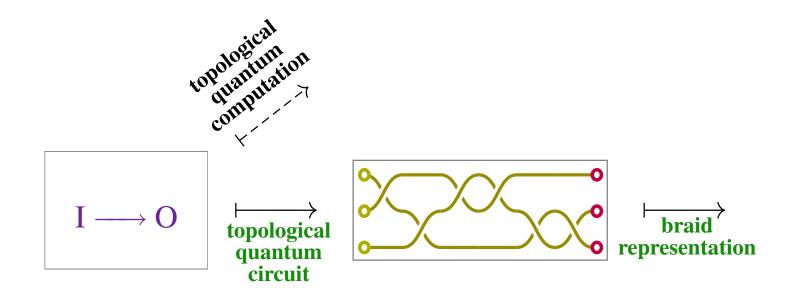






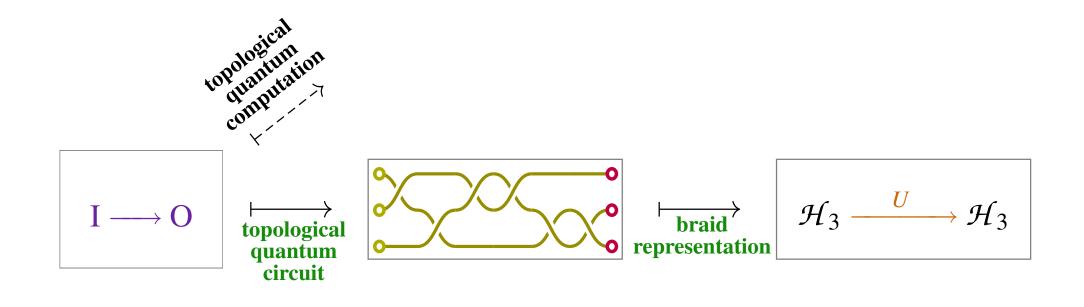
To compute is to **execute** sequences of **instructions**

Topological Quantum Computation [Sati & Schreiber, PlanQC 2022 33 (2022)]

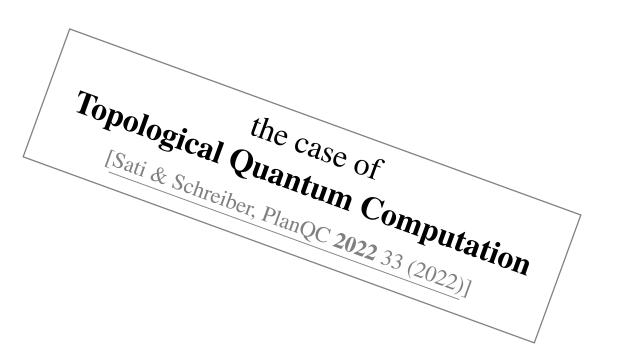


To compute is to **execute** sequences of **instructions** as composable **operations**

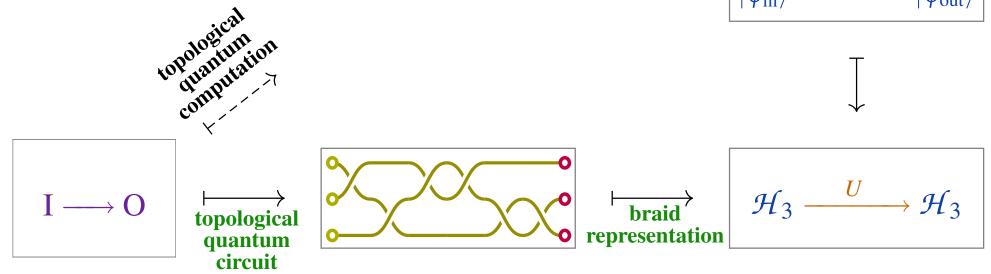
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		(2022)]



To compute is to **execute** sequences of **instructions** as composable **operations** on a chosen **state space**,

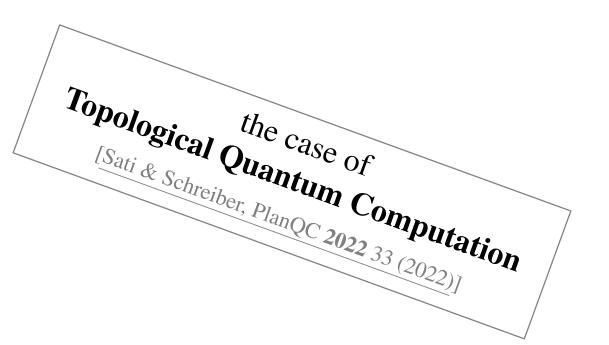


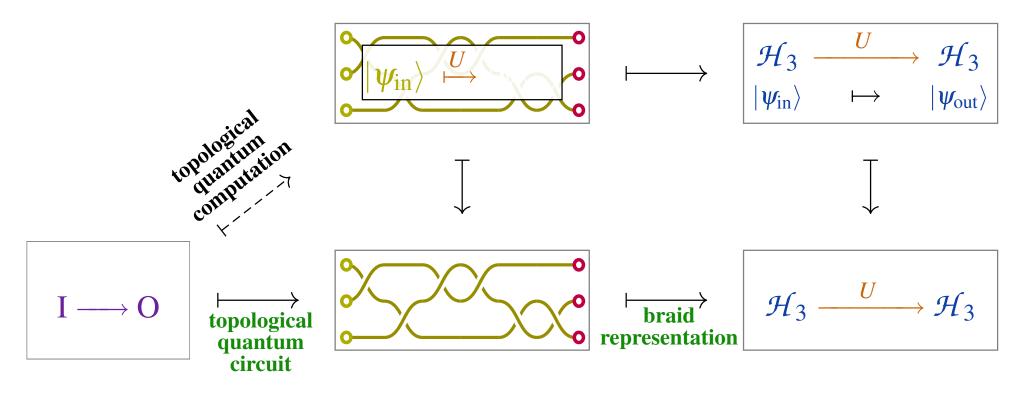
 \mathcal{H}_3 \mathcal{H}_3 $|\psi_{
m in}
angle$ $|\psi_{\rm out}\rangle$ \mapsto



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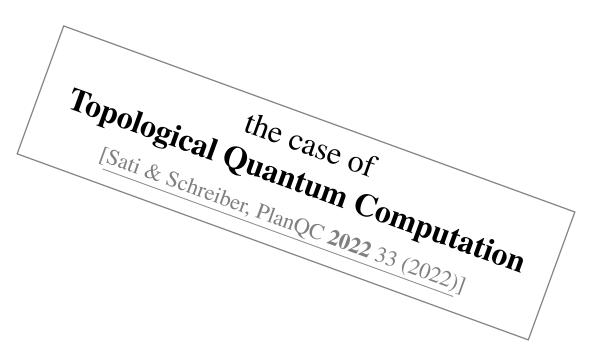
turning a given **initial state**

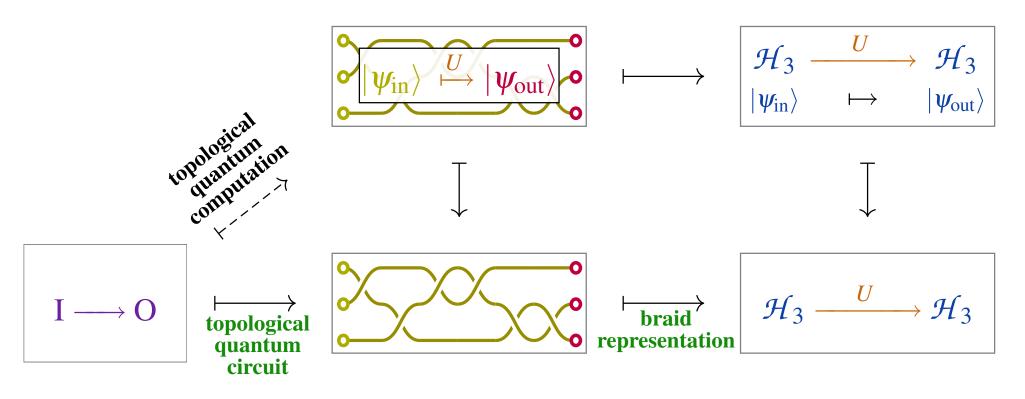




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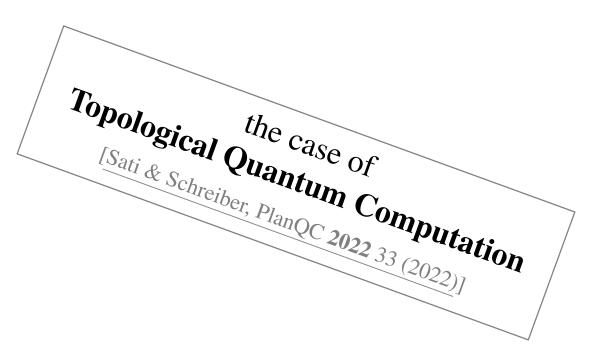
turning a given **initial state** into the computed **result**.

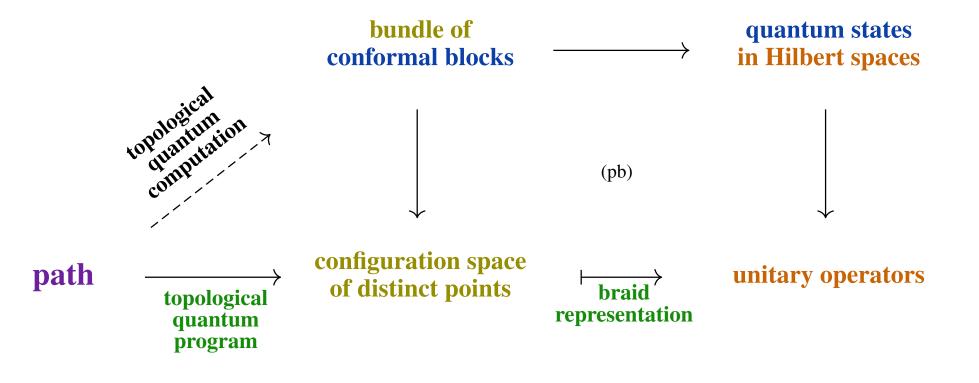




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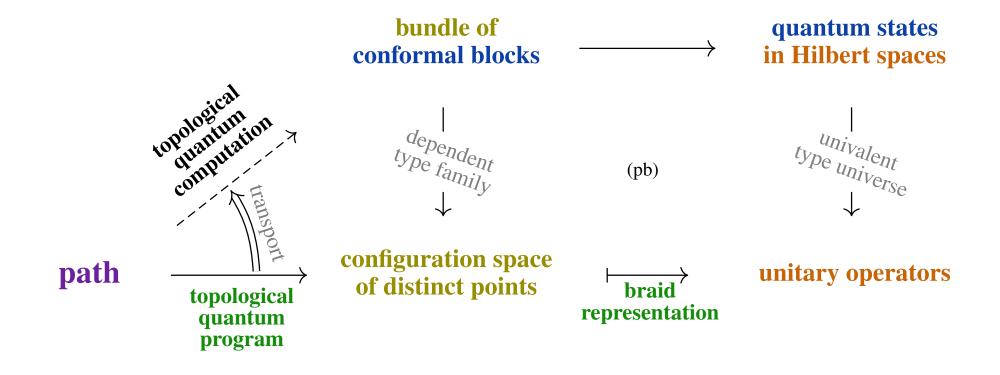




To compute is to execute sequences of instructions as composable operations on a chosen state space,

turning a given initial state into the computed **result**.

Claim: This has natural construction in Homotopy Type Theory:



Topological Quantum Computation

[Sati & Schreiber, PlanQC 2022 33 (2022)]

Quantum materials with these properties are called topological phases of matter

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International Journal of Modern Physics B

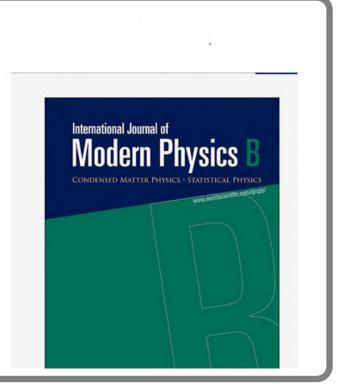
| Vol. 05, No. 10, pp. 1641-1648 (1991)

IV. CHERN-SIMONS FIELD ...

TOPOLOGICAL ORDERS AND CHERN-SIMONS THEORY IN STRONGLY CORRELATED QUANTUM LIQUID

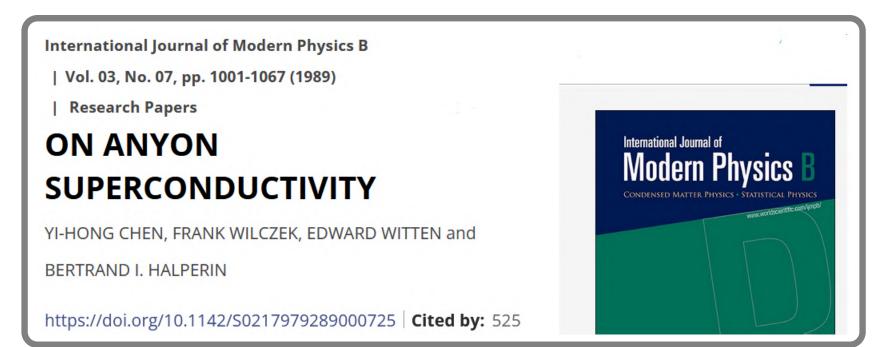
XIAO-GANG WEN

https://doi.org/10.1142/S0217979291001541 | Cited by: 98



Their point defects are known as anyons.

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Such materials are expected to exists but have remained somewhat elusive:

experimentally as well as theoretically:

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search for "<u>Majorana zero modes</u>" had no reproducible success, and would lack topological braiding;

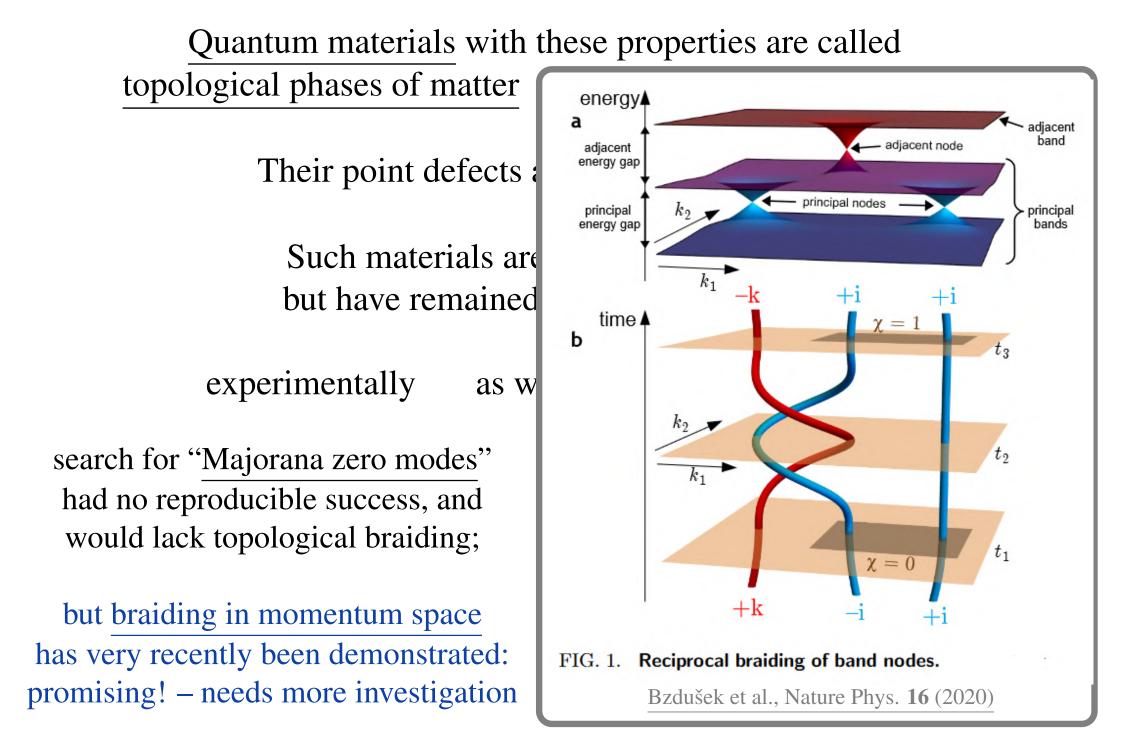
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 $ar \times iv > cond-mat > ar \times iv:0901.2686$

Condensed Matter > Mesoscale and Nanoscale Physics

[Submitted on 18 Jan 2009 (v1), last revised 20 Jan 2009 (this version, v2)]

Periodic table for topological insulators superconductors

Alexei Kitaev

Gapped phases of noninteracting fermions, with and without charge conservation and time-reversal symmetry, are classified using Bott periodicity. The symmetry and spatial dimension determines a general universality class, which corresponds to one of the 2 types of complex and 8 types of real Clifford algebras. The phases within a given class are further characterized by a topological invariant, an element of some Abelian group that can be 0, Z, or Z_2. The interface between two infinite phases with different topological numbers must carry some gapless mode. Topological properties of finite systems are described in terms of K-homology. This classification is robust with respect to disorder, provided electron states near the Fermi energy are absent or localized. In some cases (e.g., integer quantum Hall systems) the K-theoretic classification is stable to interactions, but a counterexample is also given.

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our new theory in fact *predicts* anyon braiding in momentum space of topological semi-metals.

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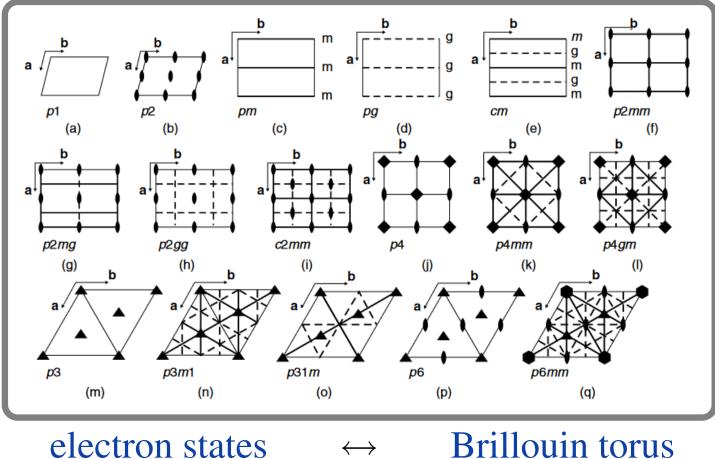
our new theory in fact *predicts* anyon braiding in momentum space of topological semi-metals. PhysicsTheoryunderlyingcontrollingTopological Quantum Computation

Concretely, we arrive at the following resolution. PhysicsTheoryunderlyingcontrollingTopological Quantum Computation

electron states \leftrightarrow Brillouin torus

[Brillouin (1930)]





electron states \leftrightarrow

[Brillouin (1930)]

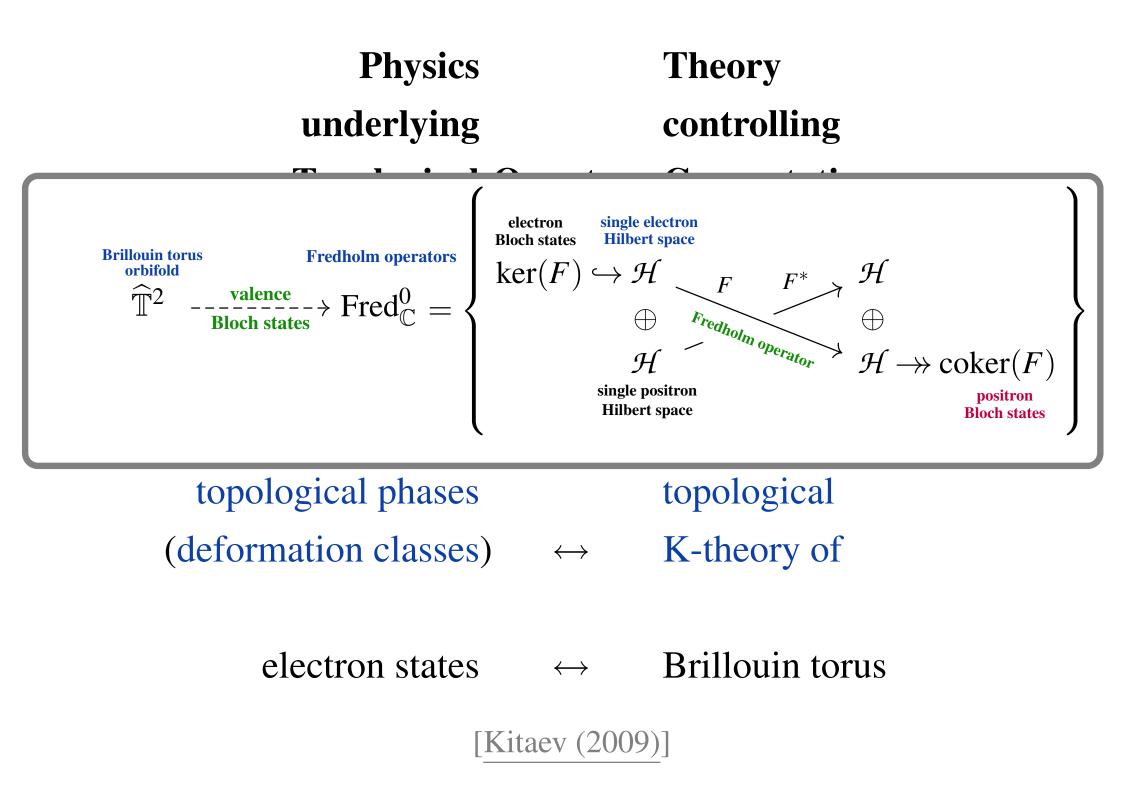
electron states \leftrightarrow Brillouin torus

[Brillouin (1930)]

topological phasestopological(deformation classes) \leftrightarrow K-theory of

electron states \leftrightarrow Brillouin torus

[Kitaev (2009)]



topological phasestopological(deformation classes) \leftrightarrow K-theory of

electron states \leftrightarrow Brillouin torus

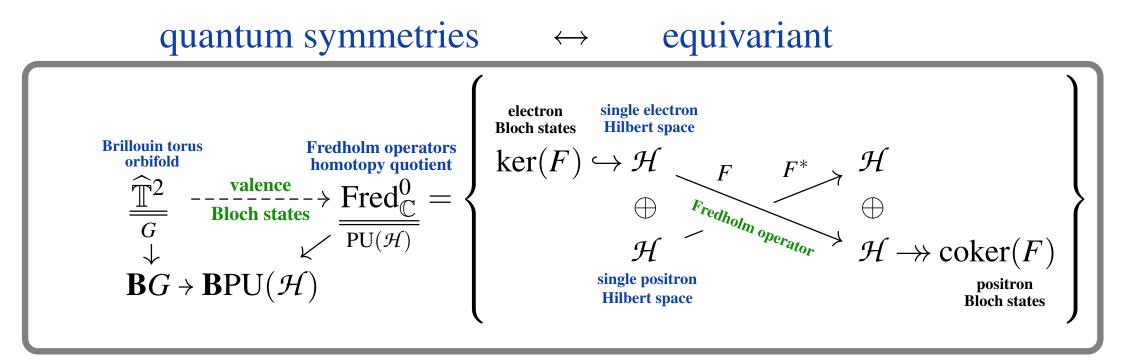
[Kitaev (2009)]

quantum symmetries \leftrightarrow equivariant

topological phases \leftrightarrow topologicaldeformation classes \leftrightarrow K-theory of

electron states \leftrightarrow Brillouin torus

[Freed & Moore (2013)]



[Freed & Moore (2013)]

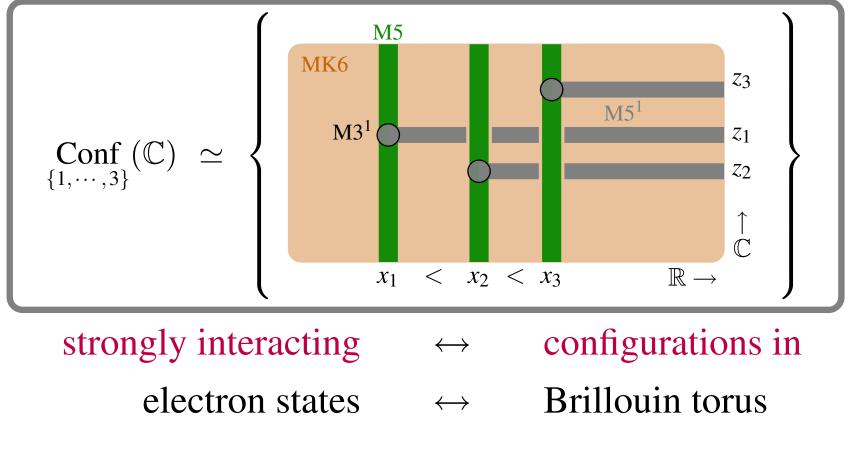
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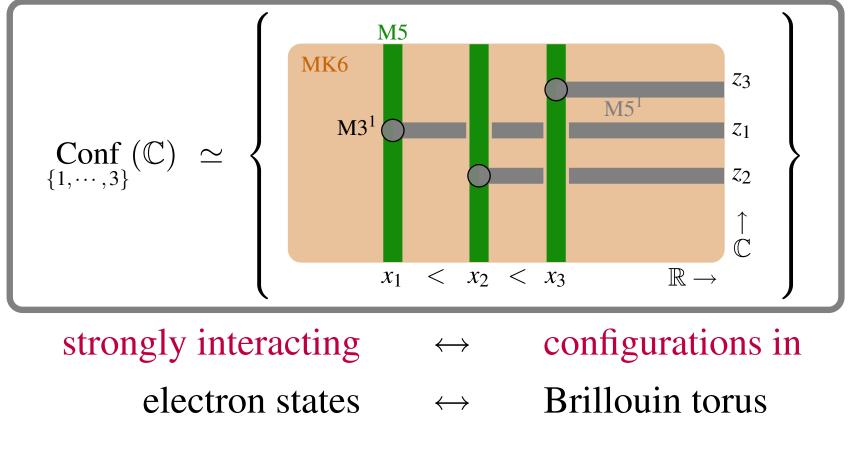
topological phases \leftrightarrow topologicaldeformation classes \leftrightarrow K-theory of

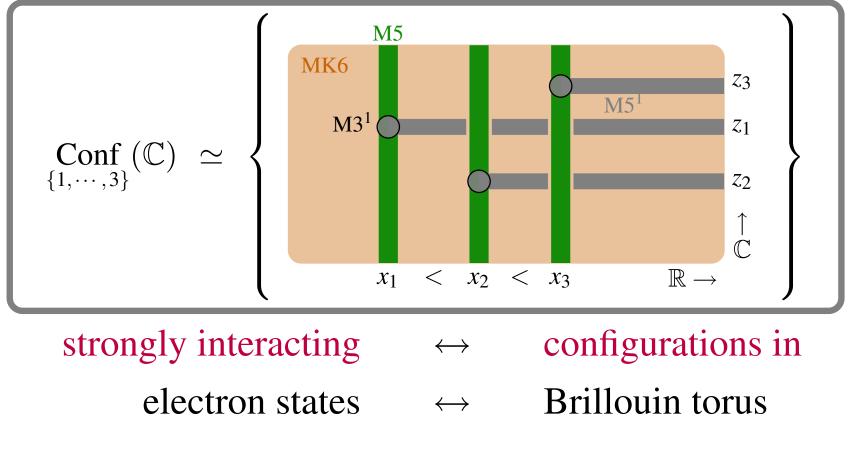
electron states \leftrightarrow Brillouin torus

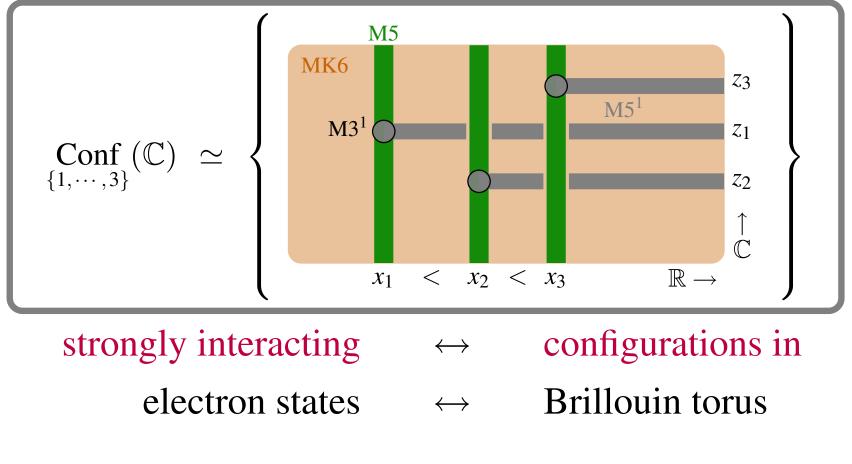
[Freed & Moore (2013)]

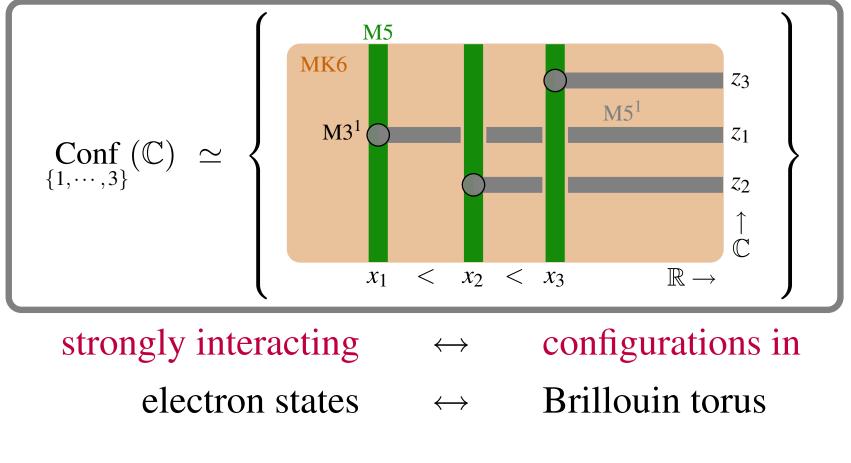
- quantum symmetries \leftrightarrow equivariant
 - topological phases \leftrightarrow topological
 - deformation classes \leftrightarrow K-theory of
 - strongly interacting \leftrightarrow configurations in
 - electron states \leftrightarrow Brillouin torus

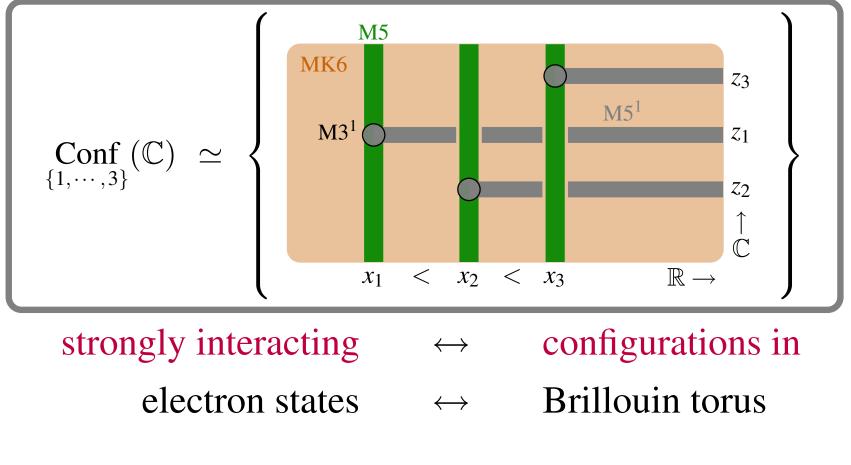


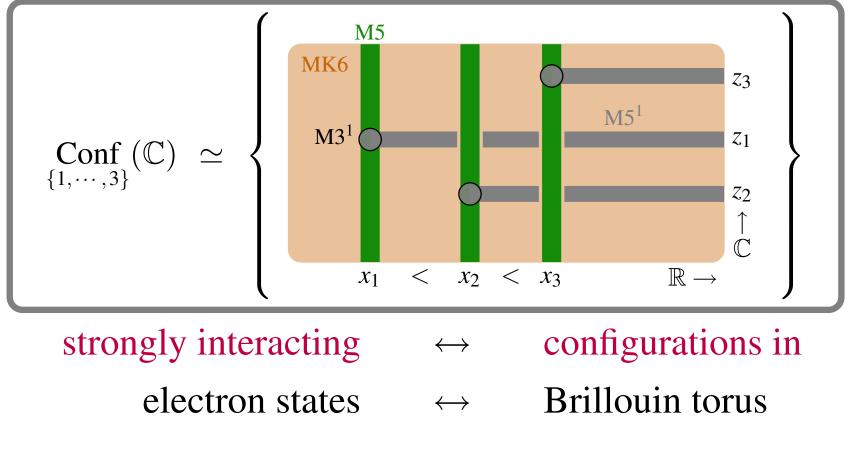


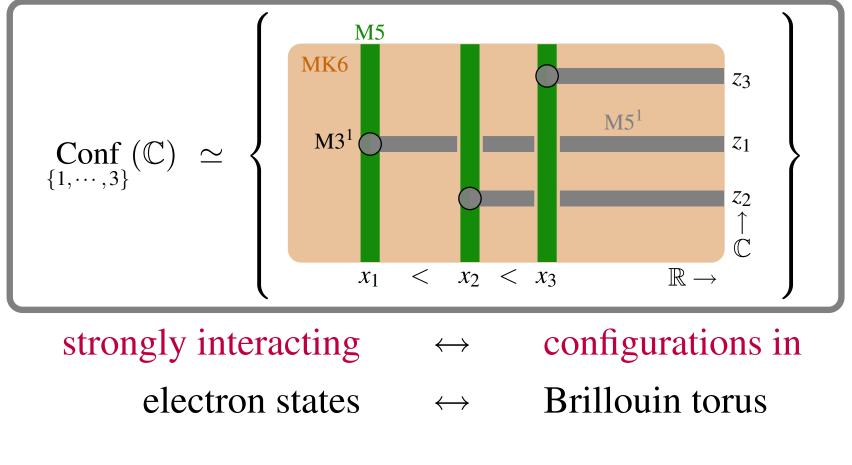


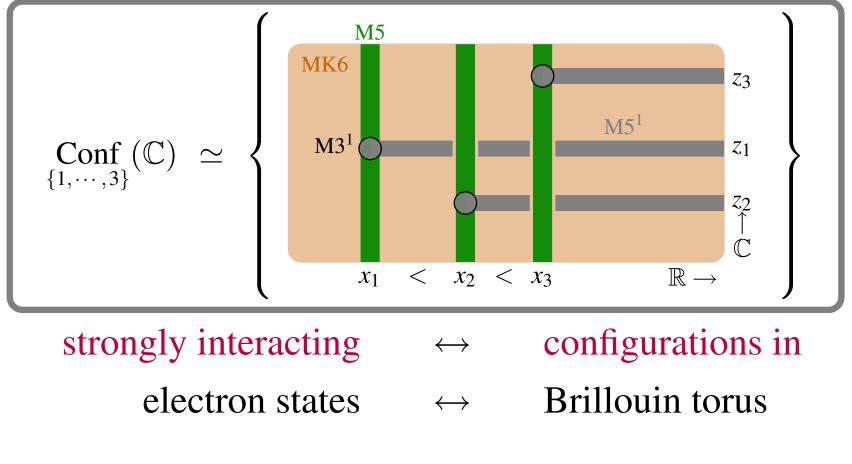


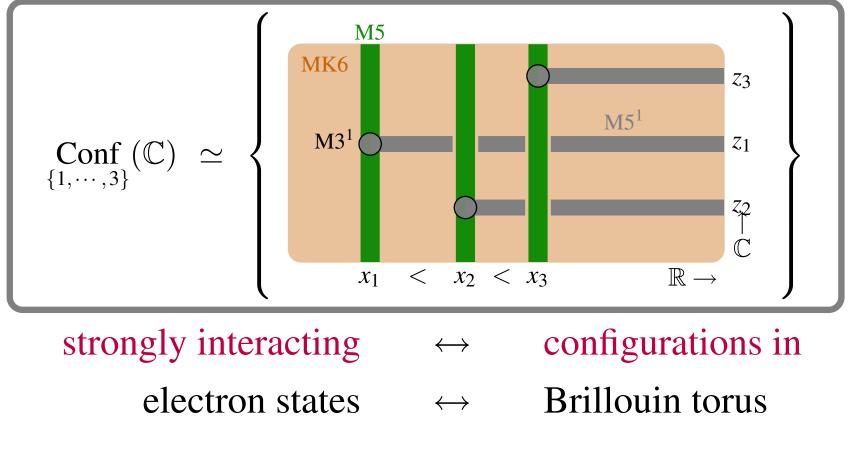


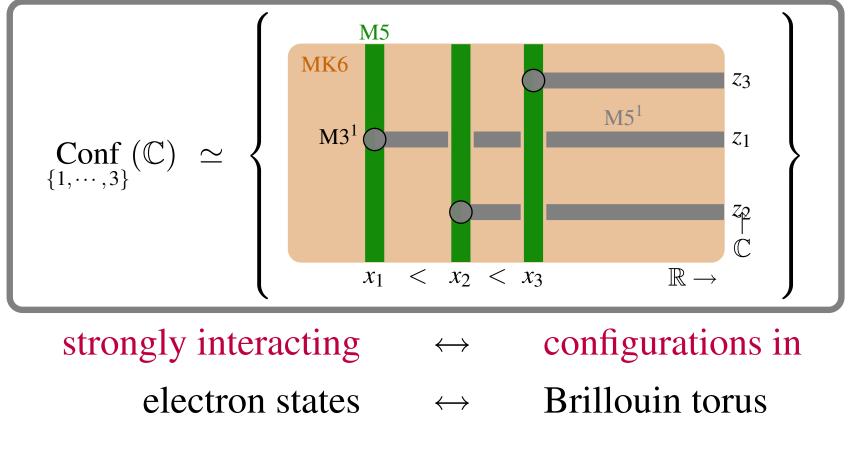


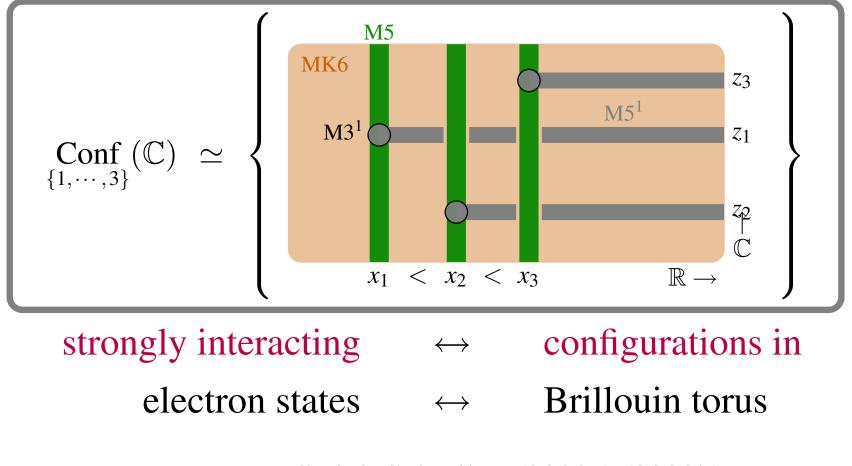












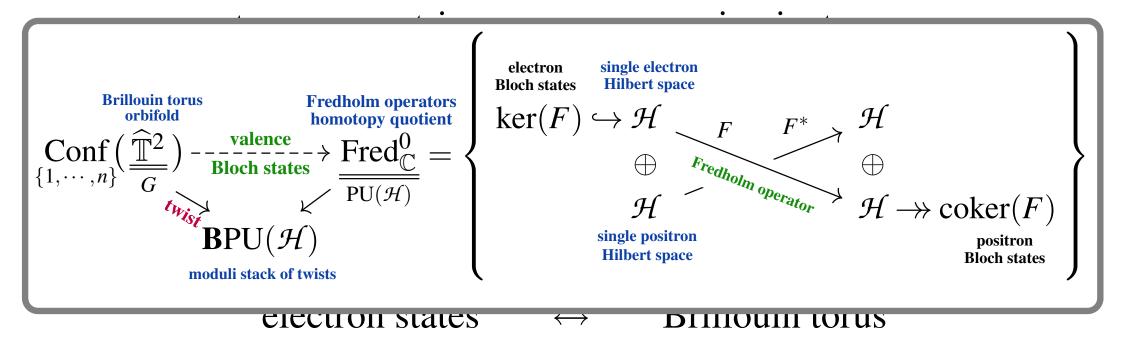
- quantum symmetries \leftrightarrow equivariant
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 - electron states \leftrightarrow Brillouin torus

- anyon species \leftrightarrow twisted
- quantum symmetries \leftrightarrow e
- anyon wavefunctions
 - topological phases
 - deformation classes
 - strongly interacting
 - electron states

- · · · ·
- → equivariant
- \leftrightarrow differential
- \leftrightarrow topological
- $\leftrightarrow \qquad \text{K-theory of} \qquad$
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[Sati & Schreiber (2022b)]

anyon species \leftrightarrow twisted



[Sati & Schreiber (2022b)]

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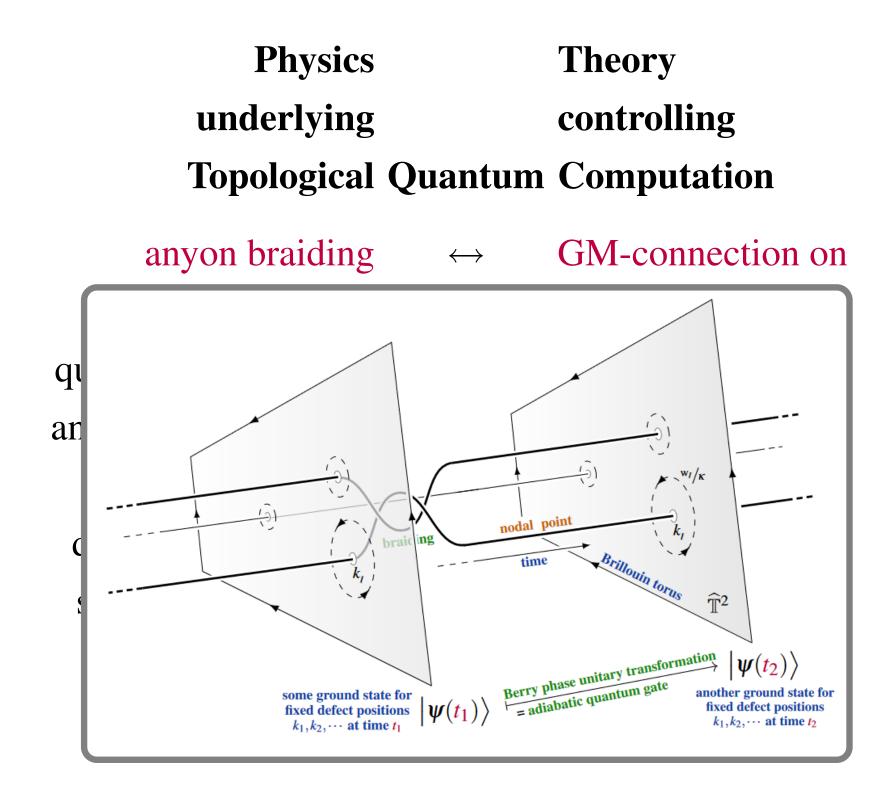
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[Sati & Schreiber (2022b)]

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[Sati & Schreiber, PlanQC 2022 33 (2022)]



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- $\leftrightarrow \qquad \text{K-theory of} \qquad$
- \leftrightarrow configurations in
- $\leftrightarrow \qquad \text{Brillouin torus}$

[Sati & Schreiber, PlanQC 2022 33 (2022)]

- anyon braiding \leftrightarrow GM-connection on
- anyon species +
- quantum symmetries
- anyon wavefunctions
 - topological phases
 - deformation classes
 - strongly interacting
 - electron states \leftrightarrow

- \leftrightarrow twisted
- \leftrightarrow equivariant
- \leftrightarrow differential
- \leftrightarrow topological
- $\leftrightarrow \qquad \text{K-theory of} \qquad$
- \leftrightarrow configurations in
 - → Brillouin torus

[Kitaev (2003)] [Freedman, Kitaev, Larsen & Wang (2003)]

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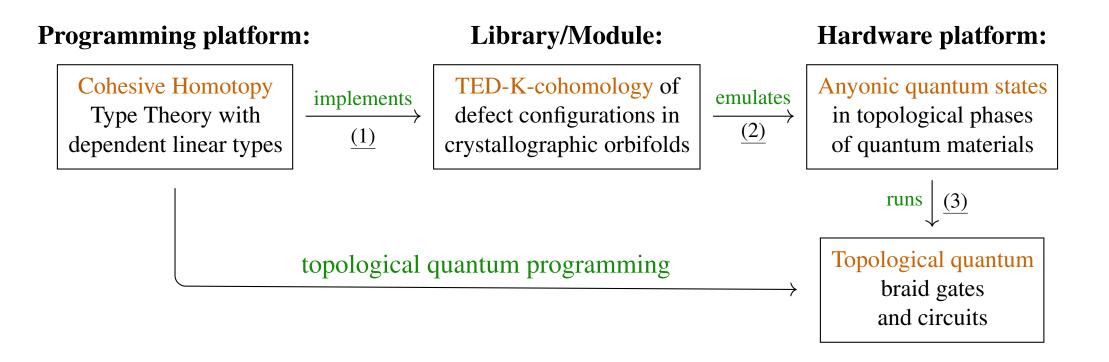
 \leftrightarrow equivariant

twisted

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[Sati & Schreiber (2022a) (2022b) (2022c)]

Claim: This insight yields a striking prospect for naturally implementing quantum simulators that actually reflect fine detail of braid grates for anyonic topological quantum computation.



[Sati & Schreiber, PlanQC 2022 33 (2022)]

We are developing this program at our newly launched research center...



جامعـة نيويورك أبوظـي NYU ABU DHABI

We are developing this program at our newly launched research center...

Center for Quantum and Topological Systems

CQTS just launched

جامعية نيويورك ابوظي

NYU Abu Dhabi Research Institute Announces Four New Research Abb Uhata Nature Network of whole with up house for proposed in first one second and whole is a feasure who we will be a second and whole is a feasure of the second and whole is a feasure who and our proposed of the second and the US and whole is a feasure and cultural activity. It is a taddress questions of ice and local relevance and is faculty members from across innes to carry out creative ship and high-level research on a range complex issues with depth, scale, and ongevity that otherwise would not be possible. --h phase: --h phase: --h phase: Centers The NYU Abu Dhabi Research Institute is pleased to announce the funding decisions from eaulity of the proposals this vear was super-

Faculty

November 30, 2021

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Campus Life

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Center for Quantum and Topological

AII NYU

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Principal Investigator





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Saurabh Ray

Associate Professor of Computer Science



Farah Shamout

Assistant Professor and Emerging Scholar of Electrical and Computer Engineering

Hisham Sati Professor of Mathematics



Tuka Alhanai

Assistant Professor and Emerging Scholar of Electrical and Computer Engineering



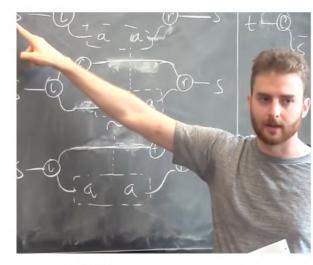
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Pilkyung Moon Assistant Professor of Physics (NYUSH)

Researchers





Mitchell Riley



David Myers



Tatiana Ezubova

Urs Schreiber



Marwa Mannai



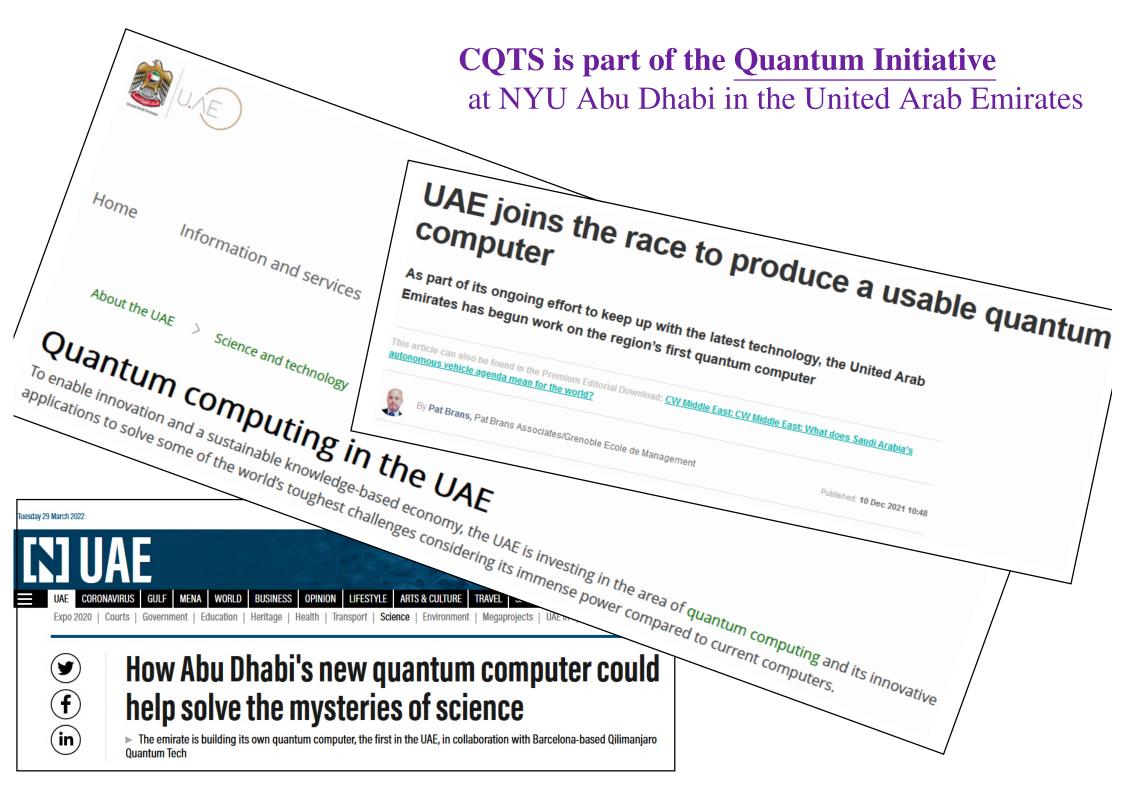
Sachin Valera



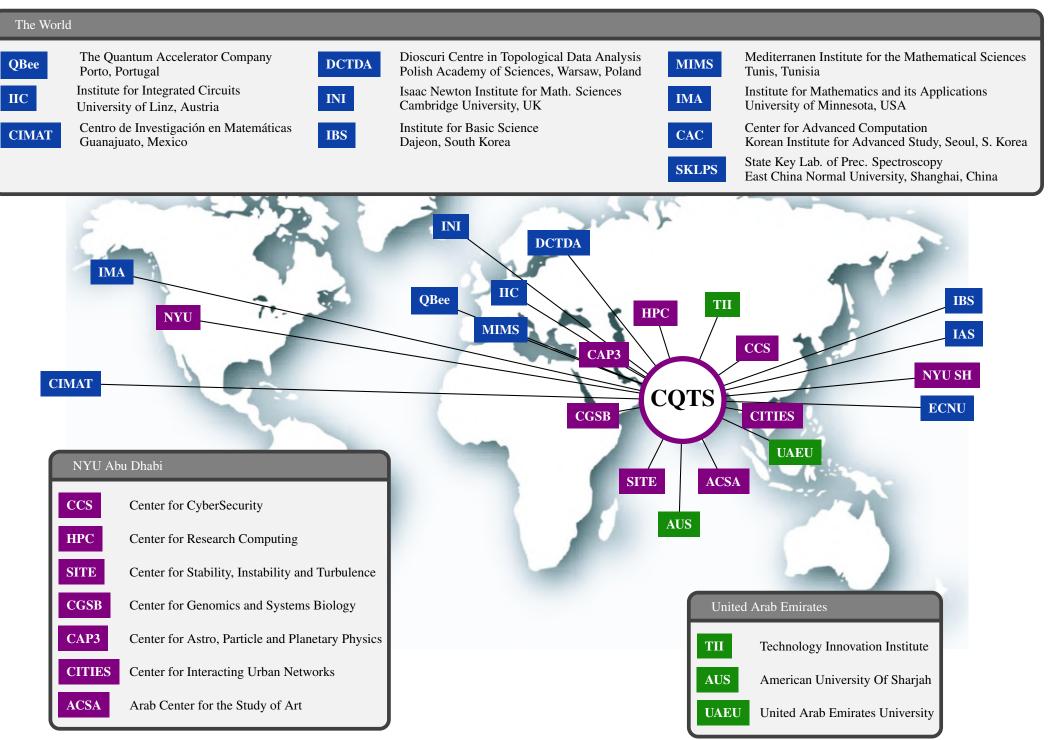
Amaria Javed



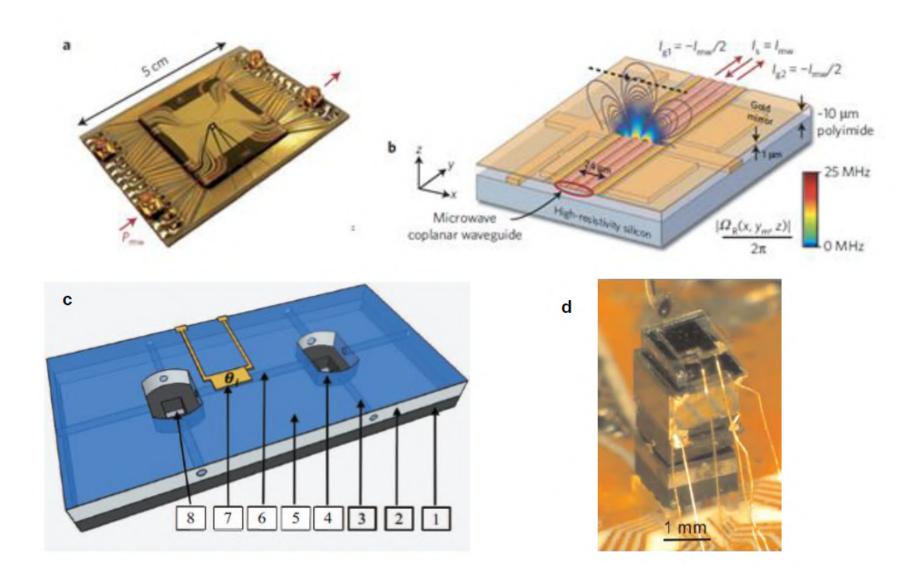
Adrian Clough



Collaborations and Partnerships: NYU-global, local, and international

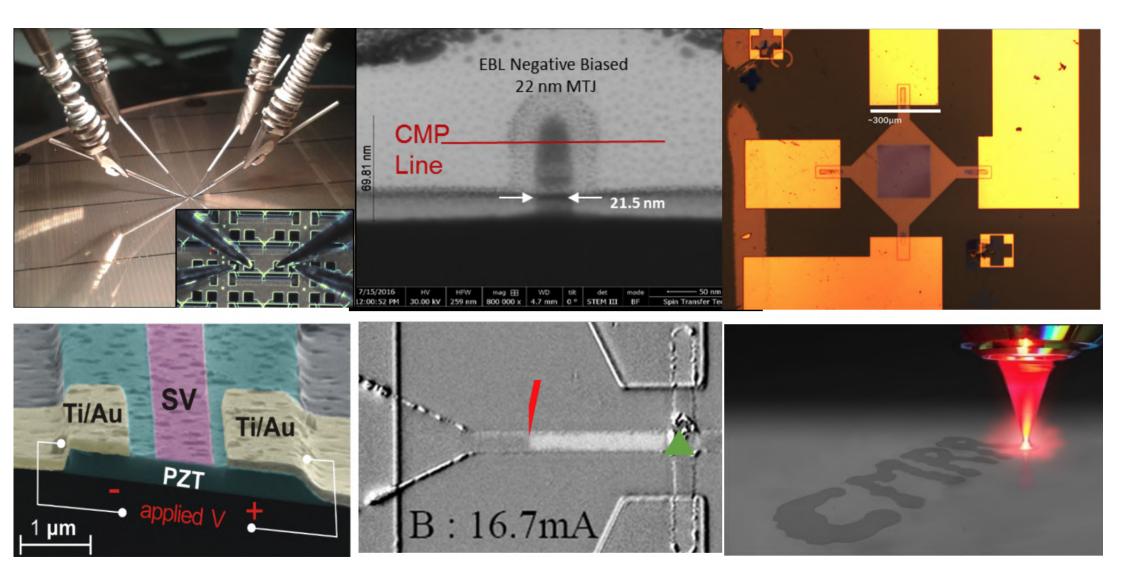


Partnership with NYU Shanghai, Quantum Technology Lab



Quantum computing, Quantum information, Bose-Einstein condensates, ...

Partnership with Center for Quantum Phenomena (CQP), NYU



Condensed matter physics, quantum materials, and quantum information technology, ...

Partnership with the Technology Innovation Institute (TII) in Abu Dhabi





Through their **Quantum Research Center (QRC)**, TII is building the UAE's first NISQ quantum computer. **Quantum Supremacy**:

Quantum Computation is expected to
be enormously more powerful
than Class
(for special but crucial applications).But Quantum Computation is prone to
instability and hence
bring in Topology
or Topology
dition is prone to
o errors
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Topological Quantum Computation:

is an ambitious but plausible strategy for retaining supremacy while defeating instability.Both its math & physics need further development.

Das Sarma, MIT Tech Rev (2022):

"The quantum-bit systems we have today are a tremendous scientific achievement, but they take us no closer to having a quantum computer that can solve a problem that anybody cares about.

What is missing is the breakthrough bypassing quantum error correction by using farmore-stable quantum-bits, in an approach called topological quantum computing." **Quantum Supremacy**:

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Topological K-theory

fully Twisted & Equivariant & Differential (TED)

classifies

free topological phases in condensed matter theory stable D-branes in string theory

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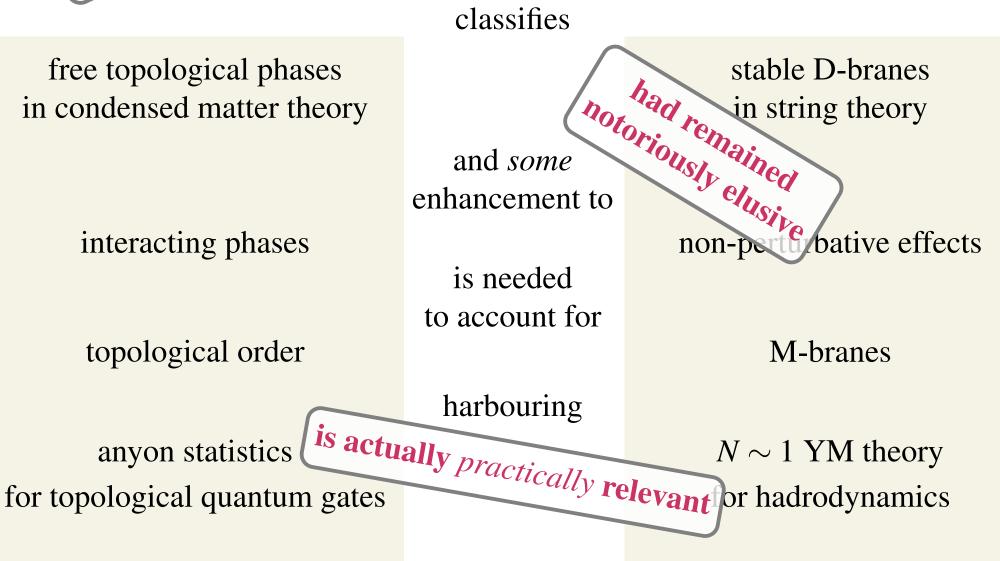
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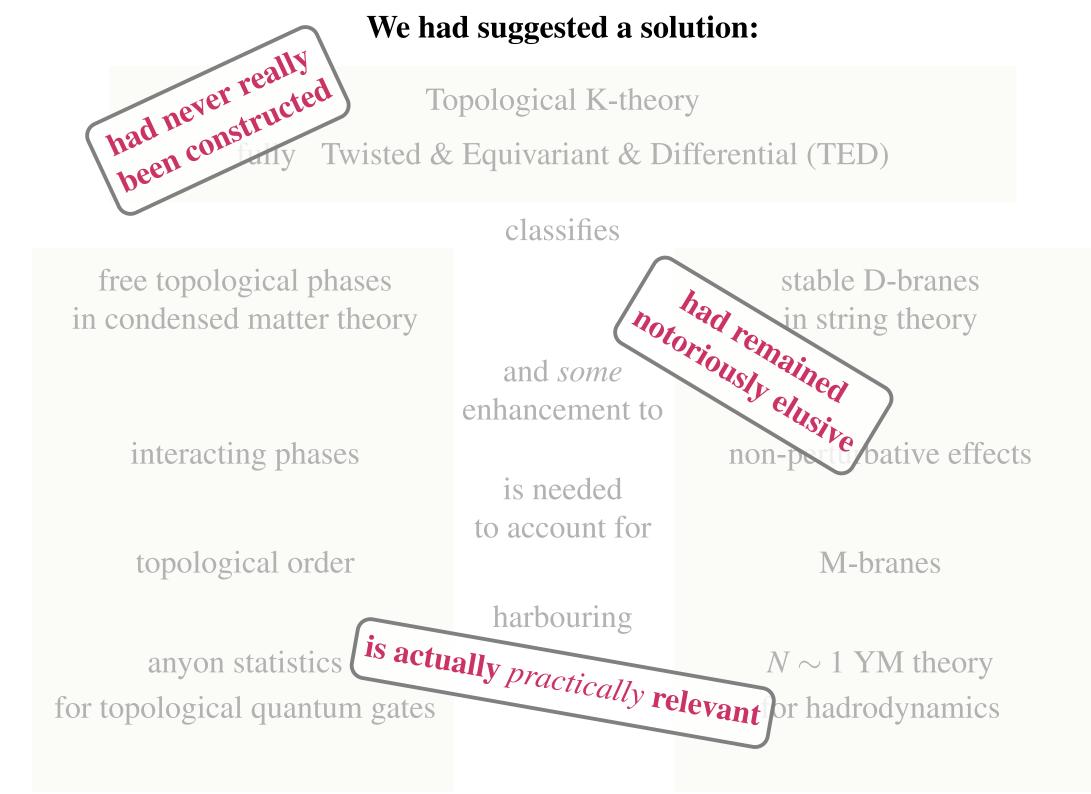
classifies had remained notoriously emisive elusive bative effects stable D-branes and *some* enhancement to is needed to account for **M**-branes harbouring $N \sim 1$ YM theory for hadrodynamics

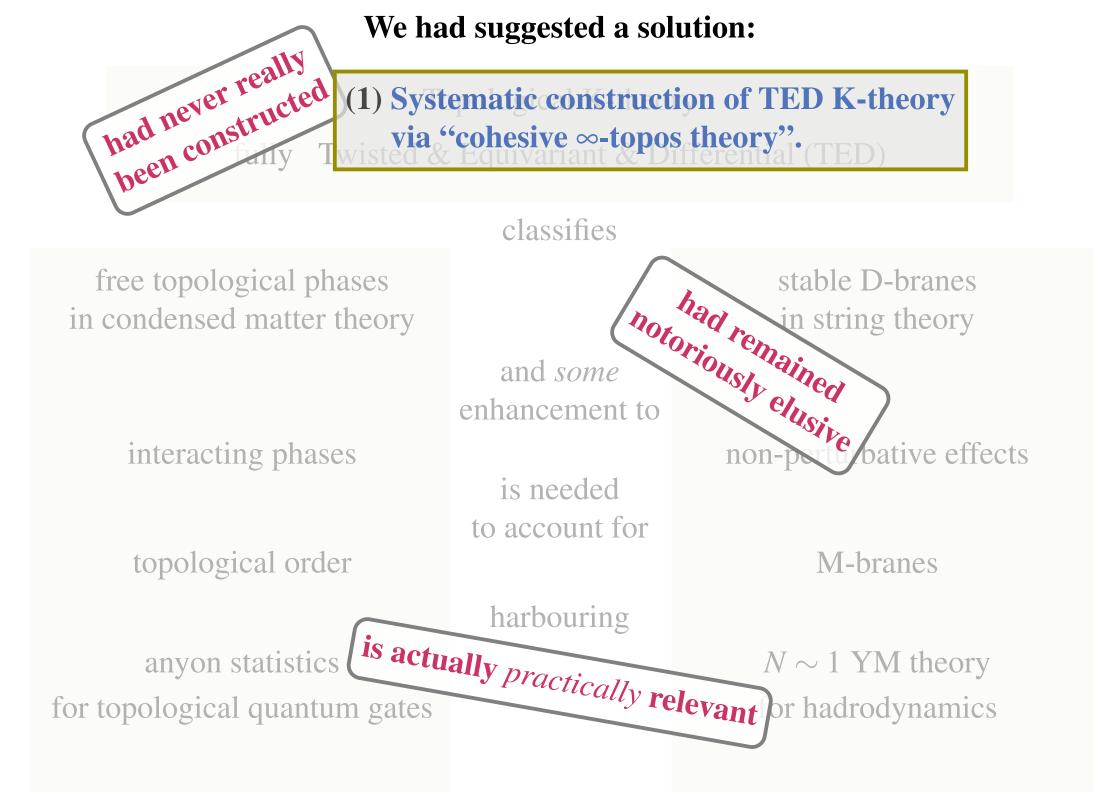


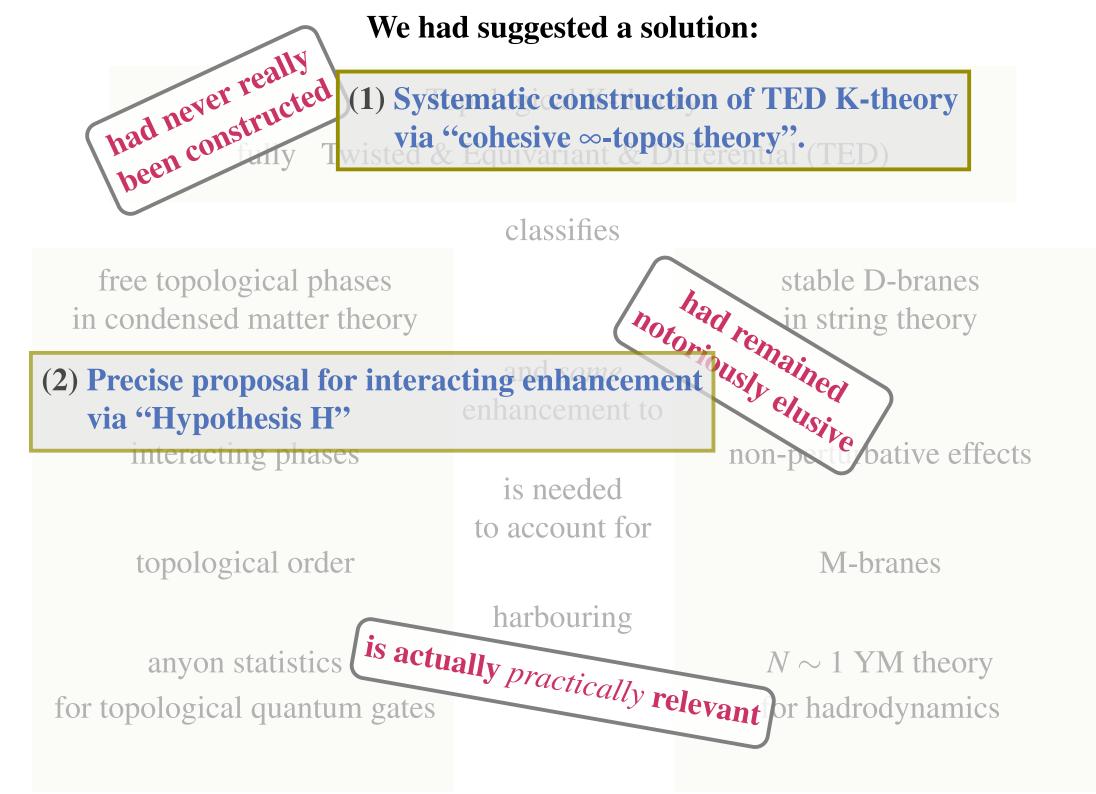
Topological K-theory

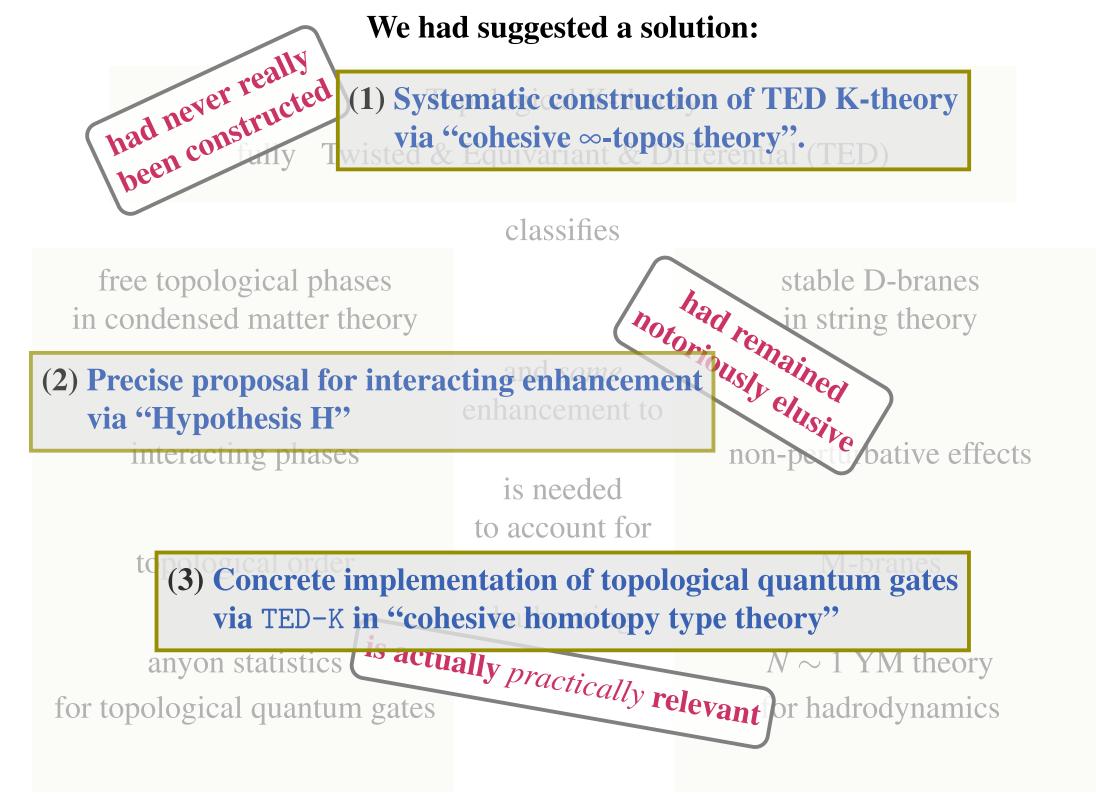
Twisted & Equivariant & Differential (TED)











(1) Systematic construction of TED K-theory via "cohesive ∞-topos theory".

(2) Precise proposal for interacting enhancement via "Hypothesis H"

> (3) Concrete implementation of topological quantum gates via TED-K in "cohesive homotopy type theory"

(for finite equivariance as befits the "very good" orbifolds appearing in CMT and ST)

[arX:2008.01101][arX:2009.11909][arX:2011.06533][arX:2203.11838][SS22-TEC]

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key technicality:

constructing twisted equivariant Chern character as map of equivariant moduli stacks (\Rightarrow flat TED K-theory is homotopy fiber of TE Chern character in equivariant stacks)

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But the Galois-theoretic effect hidden in this technicality is responsible for the appearance of conformal blocks and braid group statistics in TED-K (more <u>below</u>)

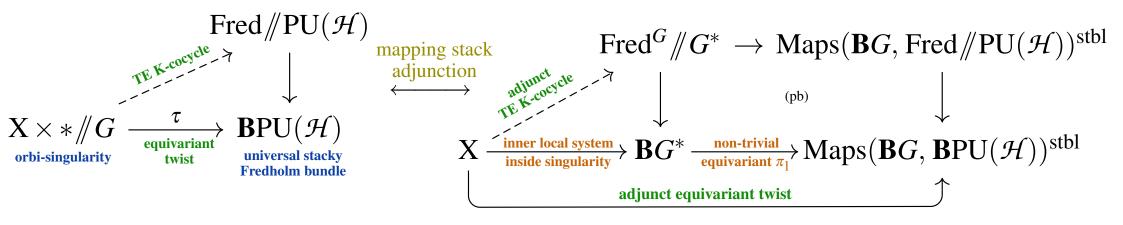
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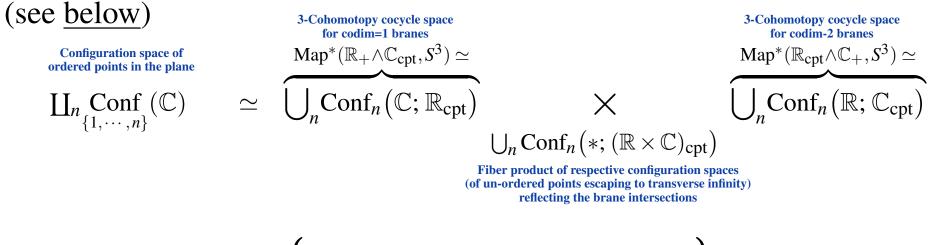
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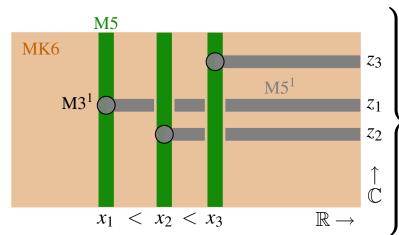


Evaluate TED K-cohomology not on Brillouin torus/spacetime-orbifold itself, but on its configuration space of points, and generally: on its Cohomotopy moduli [CMP **377** (2020)] [JMP **62** (2021)] [ATMP **26** 4 (2022)] [RMP **34** 5 (2022)] [arX:2103.01877] (see <u>below</u>)

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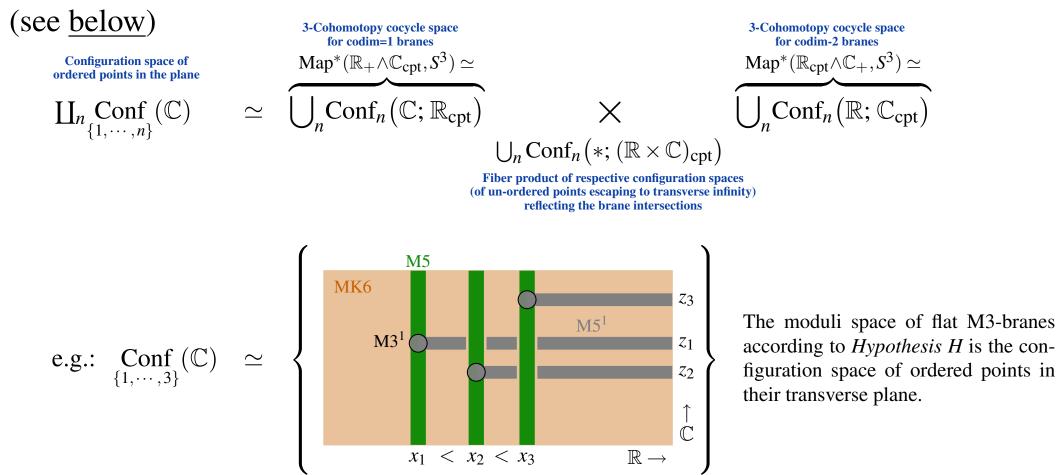


e.g.:
$$\operatorname{Conf}_{\{1,\cdots,3\}}(\mathbb{C}) \simeq$$



The moduli space of flat M3-branes according to *Hypothesis H* is the configuration space of ordered points in their transverse plane.

Evaluate TED K-cohomology not on Brillouin torus/spacetime-orbifold itself, but on its configuration space of points, and generally: on its Cohomotopy moduli [CMP **377** (2020)] [JMP **62** (2021)] [ATMP **26** 4 (2022)] [RMP **34** 5 (2022)] [arX:2103.01877]



Claim: The TED K-cohomology of *n*-point configurations in Brillouin torus classifies valence bundle of *n*-electron interacting states [arX:2206.13563]

via TED-K in cohesive homotopy type theory:

[PlanQC **2022** 33] [arX:2206.13563]

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[PlanQC **2022** 33] [arX:2206.13563]

Claim: The TED K-theoretic Chern characters of configuration spaces of points contain the $\mathfrak{su}(2)$ -affine conformal blocks at admissible fractional levels & genus=0 (see <u>below</u>)

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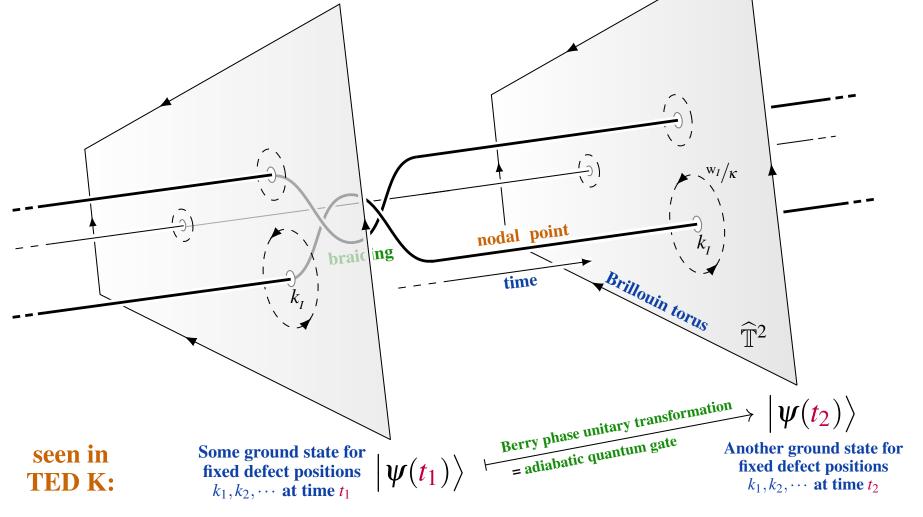
[PlanQC **2022** 33] [arX:2206.13563]

Claim: The TED K-theoretic Chern characters of configuration spaces of points contain the $\mathfrak{su}(2)$ -affine conformal blocks at admissible fractional levels & genus=0 and the Gauss-Manin connection is (see <u>below</u>) the KZ equation for braid monodromy:

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[PlanQC **2022** 33] [arX:2206.13563]

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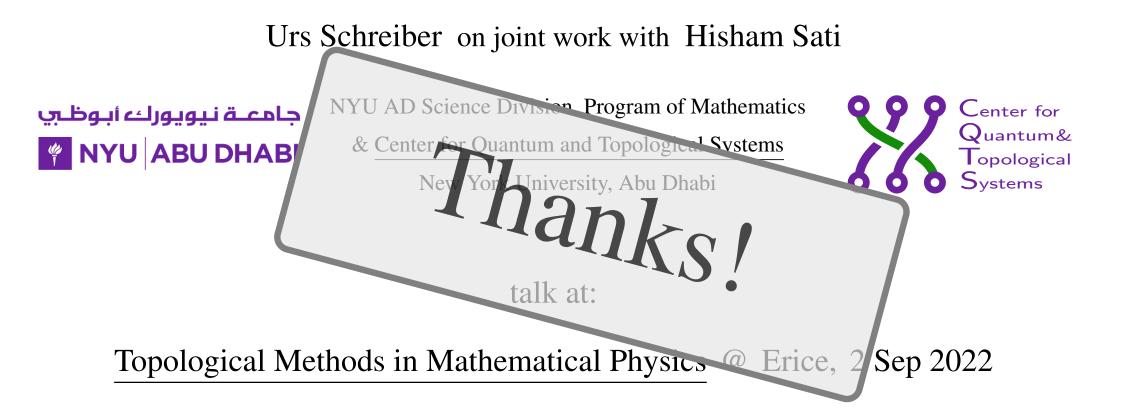
via TED-K in cohesive homotopy type theory:

[PlanQC **2022** 33] [arX:2206.13563]

Remarkably, for such constructions in cohesive ∞-topos theory there is developed a programming language: "cohesive HoTT"

[EPTCS **158** (2014)] [arX:1402.7041]

Knots for Quantum Computation from Defect branes



slides and pointers at: https://ncatlab.org/schreiber/show/Knots+for+Quantum+Computation