Renormalisation and Randomness

Mini-Workshop: Renormalisation and Randomness

2539b

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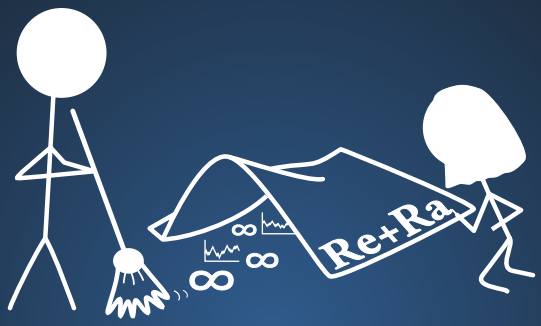
Sylvie Paycha, Potsdam
Kasia Rejzner, York

Contents

Monday, September 22th

1 hour talks + 15 min questions



Inspired by https://www.youtube.com/@zapphysics

Opening 9:00–09:15

Ilya Chevyrev **(Sissa, Trieste)** **9:15–10:30**

Rough analysis and renormalisation

Rough analysis is a collection of methods that allows one to make sense of singular stochastic differential equations. Its key idea is to factor the solution map into a probabilistic step, in which one constructs certain stochastic objects, followed by an analytic and entirely deterministic analysis. In this talk, I will give a gentle introduction to rough analysis, focusing on rough path theory and regularity structures. I will especially show how renormalisation enters into the construction of the stochastic objects, mentioning some of the algebraic structures that appear.

Break **10:30–11:00**

Manfred Salmhofer **(Heidelberg, Germany)** **11:00–12:15**

Wilsonian renormalisation: ideas, techniques, and applications

The renormalisation group, as invented by Kadanoff, Wilson, and Wegner, is ubiquitous in physics and has since the early 1980s been developed and used as a mathematical tool for the construction of models of quantum field theory and classical and quantum statistical mechanics. I give a brief overview of the basic concepts, their implementation via flow equations for effective actions, and the solution of these flow equations by formal and by convergent expansions, followed by a discussion of some applications and open questions.

Lunch break 12:30-14:00

Francesco Bonechi **(Firenze, Italy)** **14:00–15:15**

The extension to higher codimension of local field theories

The idea of extending local field theory to nonzero codimension strata is ubiquitous. A concrete realisation was given in the Cattaneo-Mnev-Reshetikhin (CMR) approach where to each codimension k stratum of the spacetime is associated a (BF^{k}V) theory. In particular for $k = 0$ (the bulk) one has a Batalin Vilkovisky (BV) theory and for $k = 1$ (the boundary) a Batalin-Fradkin-Vilkovisky (BFV) theory. In general there are obstructions to the extension of a given BV theory on the bulk. In this talk we try to discuss the underlying geometry of this extension, focusing to the bulk to boundary ($k = 1$) and boundary to corner ($k = 2$) cases. The AKSZ solutions of the classical master equation are the main examples.

Cake break 15:15-16:00

***Joscha Diehl* (Greifswald, Germany) and *Kurusch Ebrahimi-Fard* (Trondheim, Norway) 16:00–17:15**

Combinatorial and algebraic structures underlying renormalisation

Our aim is to motivate/formulate the algebraic viewpoint on renormalisation in perturbative quantum field theory by framing it as a reparametrisation problem, using the Faà di Bruno Hopf algebra framework (following essentially B. Delamotte). From there, we derive the combinatorial formulation via the Connes-Kreimer Hopf algebras of Feynman graphs (and rooted trees). This naturally leads to the renormalisation of branched rough paths through translation, in the sense of Bruned et al. Time permitting, we want to also touch on regularity structures and briefly comment on recent developments involving multi-indices (as in the work of Bruned et al. and Tapia et al.).

***Kasia Rejzner* (York, England) 17:15–18:30**

Introduction to (perturbative) algebraic QFT

In this talk I will review the basics of algebraic QFT (AQFT) and its perturbative version, pAQFT. The underlying principle is that one associates algebras of observables to regions of a given spacetime, requiring physically motivated axioms to be satisfied. This allows one to combine concepts from quantum mechanics with causality, forming a relativistic quantum theory.

Dinner 18:30

Tuesday, September 23th

50 min talks + 10 min questions

QFT and Re+Ra

Claudio Dappiaggi **(Pavia, Italy)** **9:00–10:00**

Stochastic Partial Differential Equations and Renormalization à la Epstein-Glaser

We present a novel framework for the study of a large class of nonlinear stochastic partial differential equations, which is inspired by the algebraic approach to quantum field theory. The main merit is that, by realising random fields within a suitable algebra of functional-valued distributions, we are able to use specific techniques proper of microlocal analysis. These allow us to deal with renormalization using an Epstein-Glaser perspective, hence without resorting to any specific regularisation scheme. As a concrete example we shall use this method to discuss the stochastic Φ^3_d model and we shall comment on its applicability to the stochastic nonlinear Schrödinger equation as well as to the stochastic Thirring model.

Based on joint works with A. Bonicelli, B. Costeri, N. Drago, P. Rinaldi and L. Zambotti.

Dario Benedetti **(CPHT, France)** **10:00–11:00**

A strong–weak duality for the 1d long-range Ising model

I will present recent work on the 1D Ising model with long-range interactions decaying as $1/r^{1+s}$. It is known since a long time that this model admits a phase transition for $0 \leq s \leq 1$, described by meanfield theory for $s \leq 1/2$, and by a nontrivial family of 1D conformal field theories for $1/2 \leq s \leq 1$. Above $s = 1$, there is no phase transition at finite temperature, as in the shortrange 1d Ising model. Therefore, the point $s = 1$ corresponds to a

crossover between long-range and short-range behavior, and over the years it has been found to present several interesting features and links to other models. The model can be studied perturbatively near $s = 1/2$, as a generalized free field perturbed by a quartic interaction, but such description becomes strongly coupled near the short-range crossover at $s = 1$. We propose a dual description that instead becomes weakly coupled near $s = 1$. We have performed a number of consistency checks of our proposal, in particular calculating the perturbative conformal field theory data around $s = 1$ analytically using both (1) our proposed field theory and (2) the analytic conformal bootstrap. Our results show complete agreement between the two methods.
https://www.arxiv.org/abs/2509.05250

Break **11:00–11:30**

Sumati Surya **RRI, India** **11:30–12:30**

Lunch break **12:30–14:00**

Algebra and Re+Ra

Scientific discussion 14:00–15:30

Cake break 15:30–16:00

***Joscha Diehl* (Greifswald, Germany)** 16:00–17:00

A multiplicative surface signature through its Magnus expansion
We present a two-parameter analog to Chen's iterated-integrals signature, which can be applied to image data. It is based on 'surface development' from higher category and Kapranov's construction of the analog of the free Lie algebra. Its crucial property is a two-parameter Chen's identity. This enables efficient, and parallelizable, computation of the signature. Our approach is based on the Magnus expansion, which allows us to compute, up to a certain order, explicit expressions for the integrals appearing. On the analytic side, we provide a sewing lemma for surface development, which allows to go beyond the smooth case. This talk is based on joint work with I. Chevyrev, K. Ebrahimi-Fard, N. Tapia.

Talk 5 17:00–18:00

Dinner 18:30

Presentation of the exhibition 19:30
Women of mathematics from around the world

Wednesday, September 24th

50 min talks + 10 min questions

Geometry and Re+Ra

***Francesco Bonechi* (INFN, Italy) 9:00–10:00**

BV pushforward and equivariant Yang Mills theory
The BV pushforward of solutions of Classical Master Equation is a very simple construction that formally adapts the idea of Wilson renormalization group to the Batalin Vilkovisky setting. Topological Yang Mills theory in two dimensions is an example where the computations can be easily performed. We discuss the equivariant extension and describe the computation leading to the equivariant extension of abelian Yang Mills in four dimensions. Joint work with A.Cattaneo and M. Zabzine.

***Ilya Chevyrev* (SISSA, Italy) 10:00–11:00**

Observables and gauge covariant renormalisation of stochastic 3D Yang-Mills
In this talk, I will describe a family of observables for 3D quantum Yang-Mills theory based on regularising connections with the YM heat flow. I will describe how these observables can be used to show that there is a unique renormalisation of the stochastic quantisation equation of YM in 3D that preserves gauge symmetries. This complements a recent result on the existence of such renormalisations. Our analysis is based on short time expansions of SPDEs and of regularised Wilson loops, and requires a careful balance between the running time of the dynamic and the regularisation parameter coming from the YM heat flow. Based on joint work with Hao Shen.

Break 11:00–11:30

***Alessandra Frabetti* (Lyon, France)** **11:30–12:30**

Covariant transport and Poincaré symmetry with groupoids and direct connections

Transport maps and Poincaré symmetry, which includes translation invariance, typically require a flat base manifold. A covariant presentation on (pseudo)-Riemannian manifold was given by Dahlqvist-Diehl-Driver for transport maps, adding a transport precision, and by Costa-Forger-Pegas for Poincaré symmetry, using a jet presentation of the orthogonal frame groupoid. We explain how groupoids relate these results, and how transport maps lead to a generalization of gauge fields as direct connections on jet groupoids. The talk is based on works in progress with S. Azzali, Y. Boutaib, S. Paycha and S. Amiel, A. Miti.

Lunch break **12:30–14:00**

Free Afternoon

Excursion to enjoy the famous
Schwarzwalder Kirschtorte

Dinner **18:30**

Thursday, September 25th

50 min talks + 10 min questions

Category Theory and Re+Ra

Konrad Waldorf (Greifswald, Germany) 9:00–10:00

Introduction to Higher-Categorical Differential Geometry
In this expository talk, I provide an overview of higher-categorical differential geometry, with a focus on higher-categorical bundles, connections, and parallel transport. I discuss the motivation behind the development of the field, highlight the established aspects of the theory, and outline some recent developments and directions.

Urs Schreiber (NYU Abu Dhabi, UAE) 10:00–11:00

Renormalization and Complete QFTs
In the practice of physics model building, renormalization, resummation and anomaly cancellation are steps to incrementally repair initially ill-defined Lagrangian quantum field theories. Impressive as this process is, one would rather have concisely defined complete theories to begin with and understand the effective renormalization choices as implied from first principles. As an instructive example, I will recall renormalization choices for Wilson loop observables in abelian 3D Chern-Simons theory, one due to Polyakov, the other due to Witten. Then I explain how Witten's choice is a derivable consequence of a novel non-Lagrangian completion of 5D Chern-Simons QFT by means of proper flux-quantization in 2-Cohomotopy. This is based on joint work with H. Sati.

Break 11:00–11:30

Gong Talks 1

Peter Paulovics (Oxford) 11:30
Martin Peev (Imperial College, London) 11:50
Fabrizio Zanello (Potsdam) 12:10

Lunch break 12:30–14:00

Category Theory and Re+Ra

Branislav Jurco (Prague, Czech Republic) 14:00–15:00

Higher Gauge Theory
Higher gauge theory and connections will be discussed from the simplicial perspective.

Cake break 15:00–15:30

Scientific discussion 15:30–17:00

Gong Talks 2

Matteo Ravot Licheri (TU Berlin) 17:00
Felix Medwed (Potsdam) 17:20
Hannes Keppler (Heidelberg) 17:40
ONLINE: *Léonard Ferdinand* (Berkeley) 18:00

Dinner 18:30

Podium discussion 19:30
Conflicts and harassment in academia

Friday, September 26th

50 min talks + 10 min questions

Analysis and Re+Ra

Talk 12 — **8:30–09:30**

Harprit Singh **(Vienna, Austria)** — **09:30–10:30**

Renormalisation of Singular SPDEs with Correlated Coefficients
I shall first motivate and discuss the solution theory for parabolic singular SPDEs with (deterministic) variable coefficients driven by white noise. Then, I shall focus on what happens if the coefficient field is itself random and correlated to the driving noise.

Break — **10:30–10:50**

Talk 14 — **10:50–11:50**

Brainstorming ideas for the future — **11:50–12:30**

Lunch break — **12:30–14:00**

End of the first episode

We wish you a safe trip back!

GONG Talks Th., Sep. 25th

Peter Paulovics **(Oxford)** **11:30–11:50**

Composite observables and nonperturbative operator product expansion via stochastic analysis

We begin to develop a framework for the rigorous description of general composite observables in Euclidean quantum field theories. The approach is fully nonperturbative and combines renormalisation group ideas with techniques of stochastic analysis. As a proof of concept, we construct various local observables of the sine–Gordon model on the plane up to the third threshold $\beta^2 < 6\pi$ and prove existence of the operator product expansion for them. Based on ongoing work with M. Gubinelli.

Martin Peev **(Imperial College, London)** **11:50–12:10**

Introduction to q-Mezdons

q-Gaussian processes, first introduced by Bożejko and Speicher, form an interesting family of noncommutative algebras interpolating between Fermions and Bosons, including Free Probability. In this talk, we will give a brief overview of their definition and their ultracontractivity property. Using this, we will show how to equip the q-Gaussian processes with the novel q-Mezdonic topology and how to derive operator insertion estimates for multilinear functionals of q-Gaussian noises. These are crucial in solving Mezdonic singular SPDEs.

***Fabrizio Zanello* (Potsdam)** **12:10–12:30**

Geometric and analytic properties of the renormalisation group in pAQFT

The rigorous mathematical formulation of the renormalisation problem and of a general procedure to solve it at a very general level are among the greatest successes of perturbative algebraic quantum field theory (pAQFT). Nevertheless, some of the fundamental properties of the arising renormalisation group are not well-understood yet. In this talk I will recall the basic relevant features of the renormalisation picture in pAQFT and outline the missing pieces of the puzzle.

***Matteo Ravot Licheri* (TU Berlin)** **17:00–17:20**

Towards a Noncommutative Marcinkiewicz Theorem

The Marcinkiewicz theorem is a classical result that provides a characterisation of (possibly degenerate) Gaussian measures: these are the only probability measures whose characteristic functions are exponentials of polynomials. A fortiori, no real-valued random variable can have as cumulant generating function a polynomial of degree > 2.
The goal of this talk is to reframe the notions of moments (and cumulants) of real-valued random variables in terms of grouplike-valued random variables of the Tensor Series Algebra, which is a complete Hopf algebra, and show that the classical Marcinkiewicz theorem finds its natural reformulation on the Symmetric Series Algebra. The next immediate question is removing the commutativity hypothesis to see what happens directly on the Tensor Series Algebra. Along the way connections to PDE and SDE are explored.

Felix Medwed **(Potsdam)** **17:20–17:40**

A geometric perspective on the signature group

It was shown by B. Hambly and T. Lyons (2010) that the signature of a continuous path of bounded variation is in bijection with the path up to thin homotopy, also known as tree-like equivalence. This correspondence was generalised by Boerdihardjo et al. (2016) to continuous paths of finite p-variation (with $p > 1$) called weakly geometric rough paths, with values in the free real nilpotent Lie group whose step is the integer part of p. Their signatures, equipped with tensor multiplication form what is called the p-signature group. It turns out that the signatures of the equivalence classes under tree-like equivalence of weakly geometric p-rough paths form a group under the tensor product, called the p-signature group. In this talk I will highlight some interesting features of the p-signature group or a subgroup of it therein.

Hannes Keppler **(Heidelberg)** **17:40–18:00**

Relativistic Luttinger Fermions as models for rigorous QFT in four spacetime dimensions

I will introduce the recently developed theories of relativistic Luttinger Fermions. Based on the Abrikosov rather than the Clifford algebra, these Fermions have a number of peculiar properties: in four spacetime dimensions, their canonical scaling dimension is one, and thus quartic self-interactions are marginal; moreover, these theories exhibit asymptotic freedom in the ultraviolet. This makes relativistic Luttinger Fermions ideal models for rigorous QFT in four spacetime dimensions.

ONLINE: *Léonard Ferdinand* (Berkeley) **18:00–18:20**

Renormalisation group flow approach to singular stochastic PDEs

I will try to give a brief overview of some recent developments in the study of singular stochastic PDEs, which are inspired by the renormalisation group. This trend was initiated by Kupianen in 2014, and has gained attention since the work of Duch (2021), who introduced a systematic recursive construction of the enhancement of the noise present in any pathwise solution to singular PDEs/ODEs.