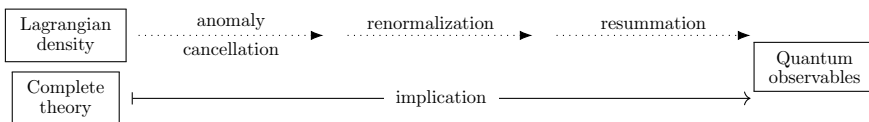


Renormalization and Complete QFTs

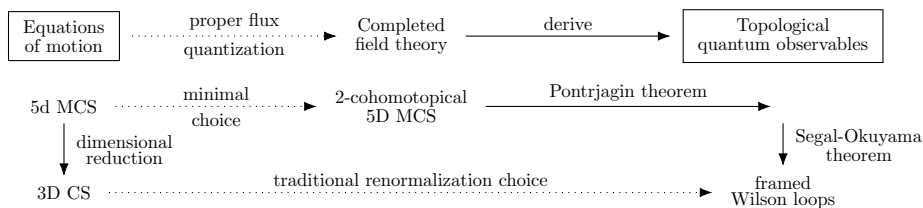
URS SCHREIBER

(joint work with Hisham Sati)

In the practice of physics model building, the process of *renormalization*, *resummation*, and *anomaly cancellation* is to incrementally repair initially ill-defined *Lagrangian* quantum field theories by a successive choice of partial fixes. Impressive as this is, one would rather have concisely defined complete theories to begin with, and understand these choices as emergent from fundamental principles:



As an instructive example, we recall renormalization choices for Wilson loop observables in abelian Chern-Simons theory. Then we show that these emerge in a novel non-Lagrangian topological completion of 5D Maxwell-Chern-Simons QFT, by means of proper flux quantization in 2-Cohomotopy [1, 2, 3]:



Here it is the classical *Pontrjagin theorem* (refined by Segal '73 and, underappreciatedly, by Okuyama '05) — identifying n -Cohomotopy cocycles of a manifold with *normally framed* $\text{codim}=n$ submanifolds — which makes emerge [4] the *writhe* of *framed Wilson loops* that is traditionally their *ad hoc* renormalization choice:

$$\left\{ \begin{array}{l} \text{Quantum observables} \\ \text{on 2-cohomotopical} \\ \text{solitonic flux} \end{array} \right\} \xrightarrow[\text{[1, 2, 3]}]{\sim} \text{Map}^*(\mathbb{R}_{\cup\{\infty\}}^3, S^2) \xrightarrow[\text{[4, §2]}]{\sim} \left\{ \begin{array}{l} \text{Framed} \\ \text{oriented} \\ \text{links} \end{array} \right\} \xrightarrow[\text{[4, §3]}]{e^{\frac{\pi i}{K} \text{wrth}(-)}} \mathbb{C}.$$

This result is a modest cousin, with applications to topological quantum materials, of a completion of 11D supergravity by proper flux-quantization in 4-Cohomotopy (“Hypothesis H”). Details are at: ncatlab.org/schreiber/show/MF02539b.

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