The key open question of fundamental quantum physics is not primarily the lack of coherent quantum gravity theory as such, as often portrayed, but the general lack of *non-perturbative* quantum theory of almost any sort, due to which exotic quantum states of matter – such as topologically ordered solid states thought to be needed for topological quantum computation – but even mundane phenomena – such as room-temperature matter, namely "confined" quarks in hadron bound states, reflected (just as are topological phases!) in a "mass gap" – remain theoretically ill-understood, to the extent that one speaks of an open *Millennium Problem*[1].

The role of string theory. String theory originates as a model for these elusive hadron bound states, specifically for the string-like "flux tubes" between pairs of quarks, conceptually explaining both their confinement and their scattering behavior. The unexpected discovery that subtle quantum effects make these *hadronic* strings propagate in an effectively higher dimensional space – with only their endpoint quarks attached to observed 3+1 dimensional spacetime (now: the "brane") or else carrying gravitons into an otherwise unobserved higher dimensional "bulk" – came to be appreciated as a "holographic" description of non-perturbative quantum physics.[2]

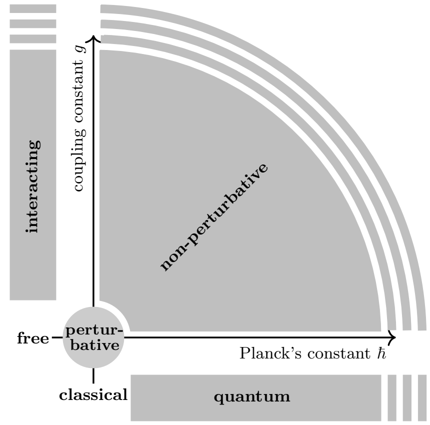
The role of M-theory. Ironically, string dynamics is itself primarily understood only perturbatively, which makes holography require the unrealistic assumption of a large (in fact: humongous) number N of coincident branes, to be tractable. But understanding branes as physical objects yields a web of hints as to what non-perturbative string theory should be like, enough so that it famously has a working title (since 1995): "M-theory".

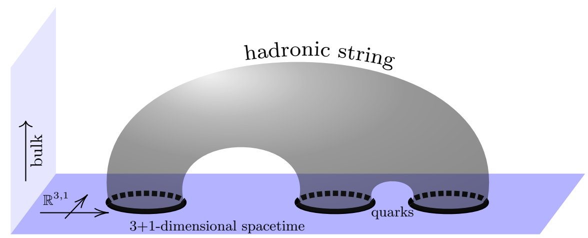
To highlight, in conclusion: One strategy for addressing the "Millennium Problem" of formulating non-perturbative QFT is to mathematically formulate M-theory:[3] With this it ought to be possible to define and investigate, with precision, individual quantum branes whose intersections should exhibit non-perturbative quantum dynamics such as anyonic topological order (which we discuss in §2.4) and eventually confined hadrodynamics.

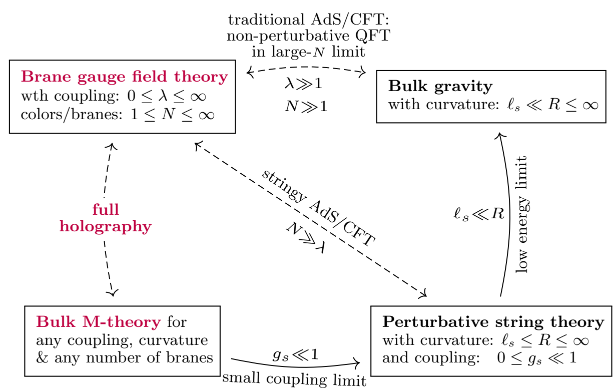
The role of Algebraic Topology. After initial excitement, progress on actually formulating M-theory had stagnated and efforts had been largely abandoned[4], arguably due to a lack of appropriate mathematical tools: Where famous examples of physical theories were formulated within a fairly well-understood framework of mathematical principles (e.g. general relativity in differential geometry or quantum physics in functional analysis), the real problem with formulating M-theory is (or was) that even its underlying mathematical principles remained unclear. It was the vision of [Sa10] (review in [FSS19]) that M-theory ought to find its formulation in *algebraic topology*; initiating a program of looking for algebro-topological patterns in the available information on M-brane physics, deducing clues as to their fundamental mathematical meaning.

The role of Hypothesis H. This analysis eventually culminated in a formulation of a hypothesis – *Hypothesis H* – of what M-theory really is about [FSS20], namely about the generalized non-abelian cohomology theory called *CoHomotopy Theory*. This we explain below in §1.3.

It is noteworthy here that algebraic topology is not a field of mathematics as any other, but has recently been understood to serve, in its guise of *homotopy theory*, as an alternative *foundation* for mathematics itself (`HoTT`[5]). Moreover, within algebraic topology, cohomotopy is not a (multiplicative) cohomology theory as any other, but is *initial* among all of them. This may be more than a coincidence given that M-theory is meant to be not just a theory of physics as any other, but the initial foundation of all of them.







Bare gauge coupling	g_{YM}
Number of colors/branes	N
Actual (t'Hooft-)coupling	$\lambda = g_s\,N = g_{YM}^2 N$
String coupling	$g_s = g_{YM}^2 = \lambda/N$
Spacetime curvature	$1/R$
String length	$\ell_s = R/\sqrt[4]{\lambda}$

	Physics	**Mathematics**
Principles	Gauge Principle[6]	Homotopy Th.
Foundations	M-Theory	Alg. Topology
Basic Notions	Fluxes/Charges	Cohomology
Initial Notions	M-Brane Charge	Cohomotopy
Derived Notions	D-brane charge	K-Theory