Urs Schreiber on joint work with Hisham Sati:

surveying our pre-print: [ncatlab.org/schreiber/show/Rethinking+FQH+Anyons]

Rethinking FQH Anyons

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS @ NYU ABU DHABI

(May 2025) find these slides at: [ncatlab.org/schreiber/show/Rethinking+FQH+Anyons -- talk]

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Rethinking FQH Anyons

via Algebraic Topology of exotic Flux Quantization

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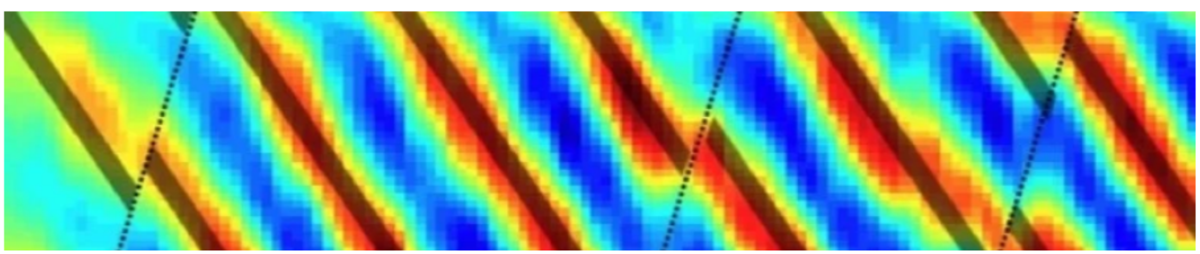
nature

NEWS | 03 July 2020

Welcome anyons! Physicists find best evidence yet for long-sought 2D structures

The 'quasiparticles' defy the categories of ordinary particles and herald a potential way to build quantum computers.

By Davide Castelvecchi



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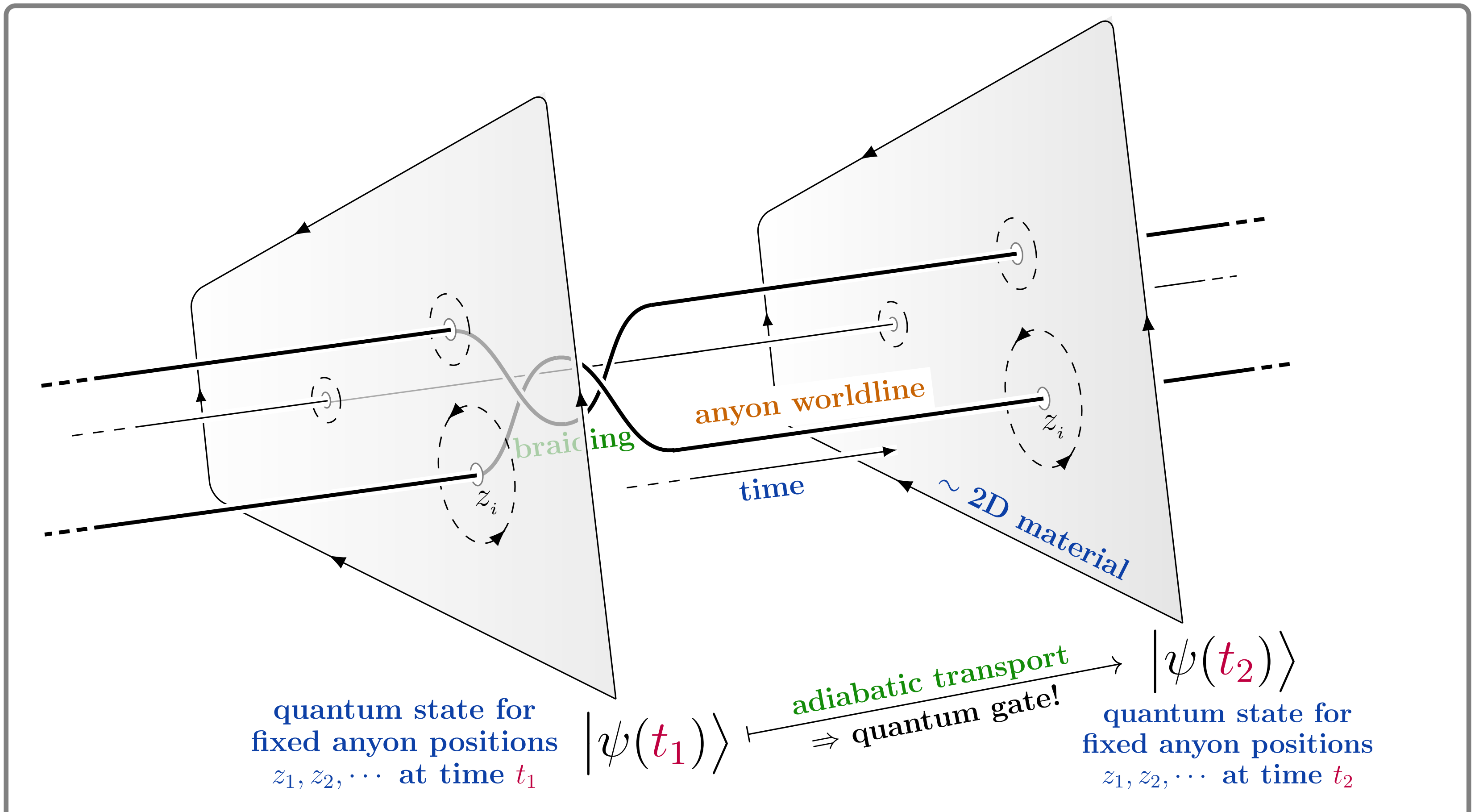

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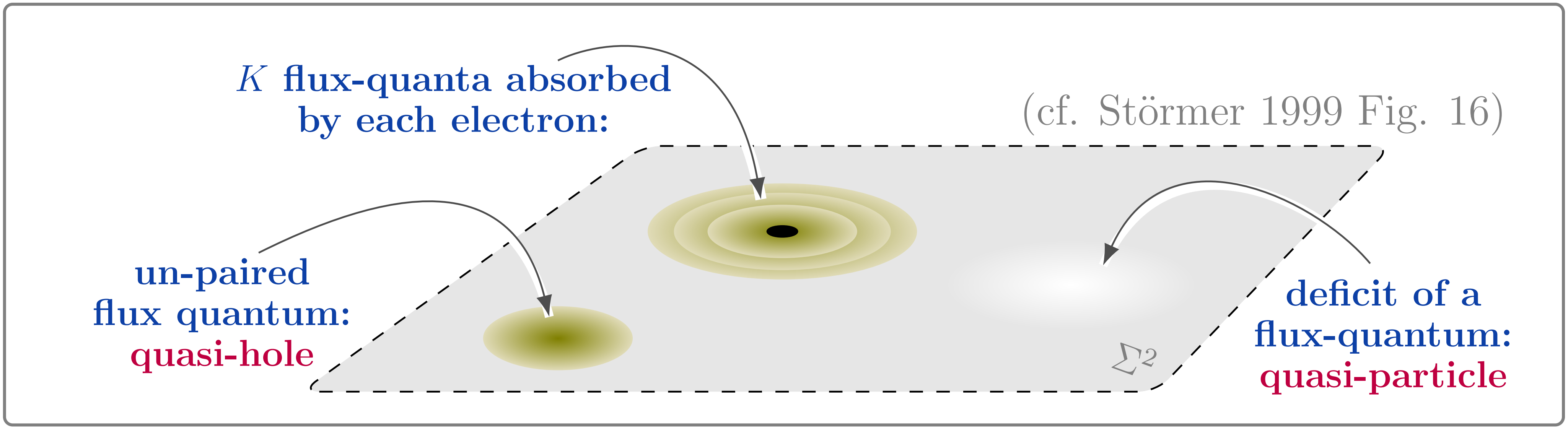

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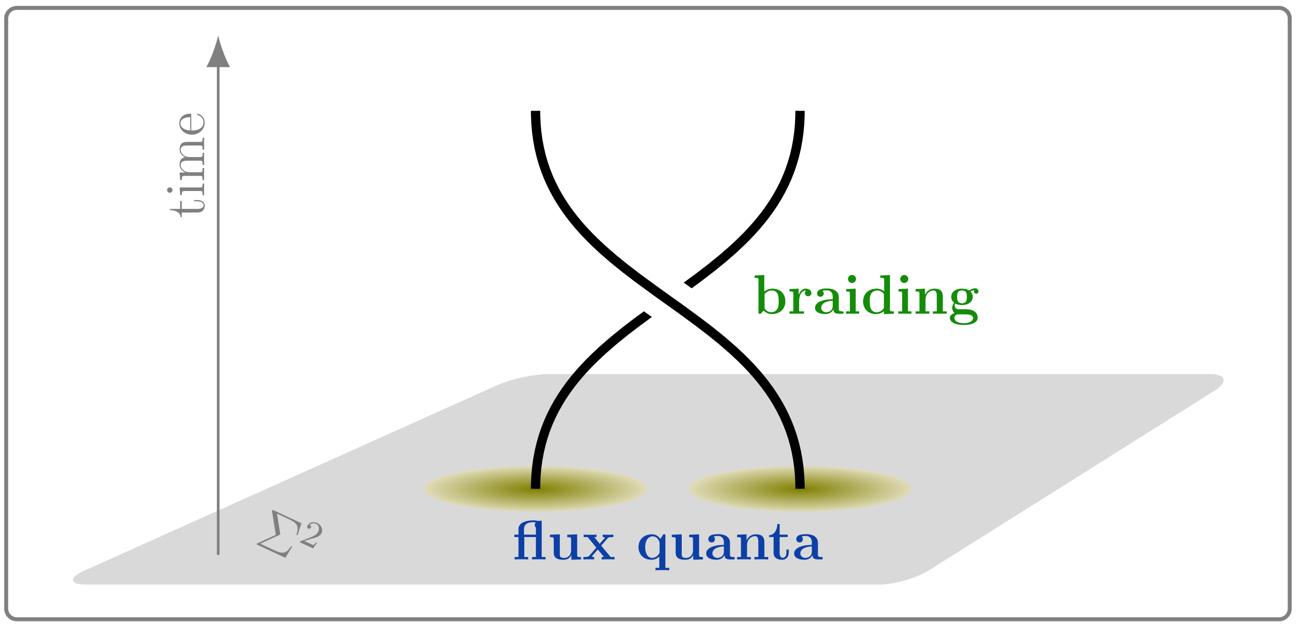

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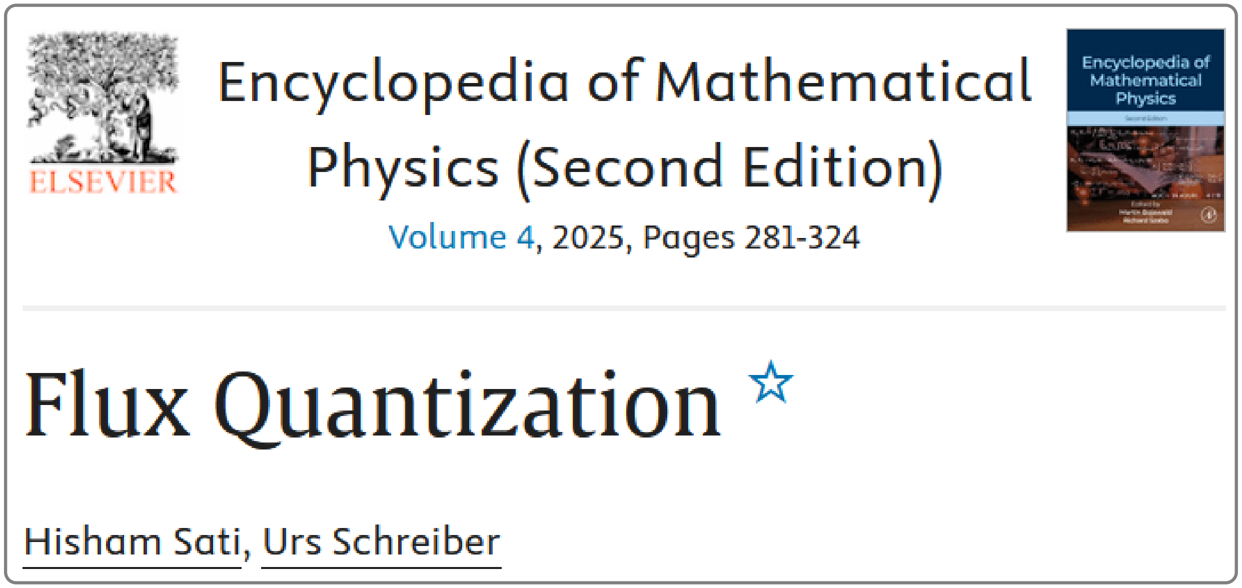

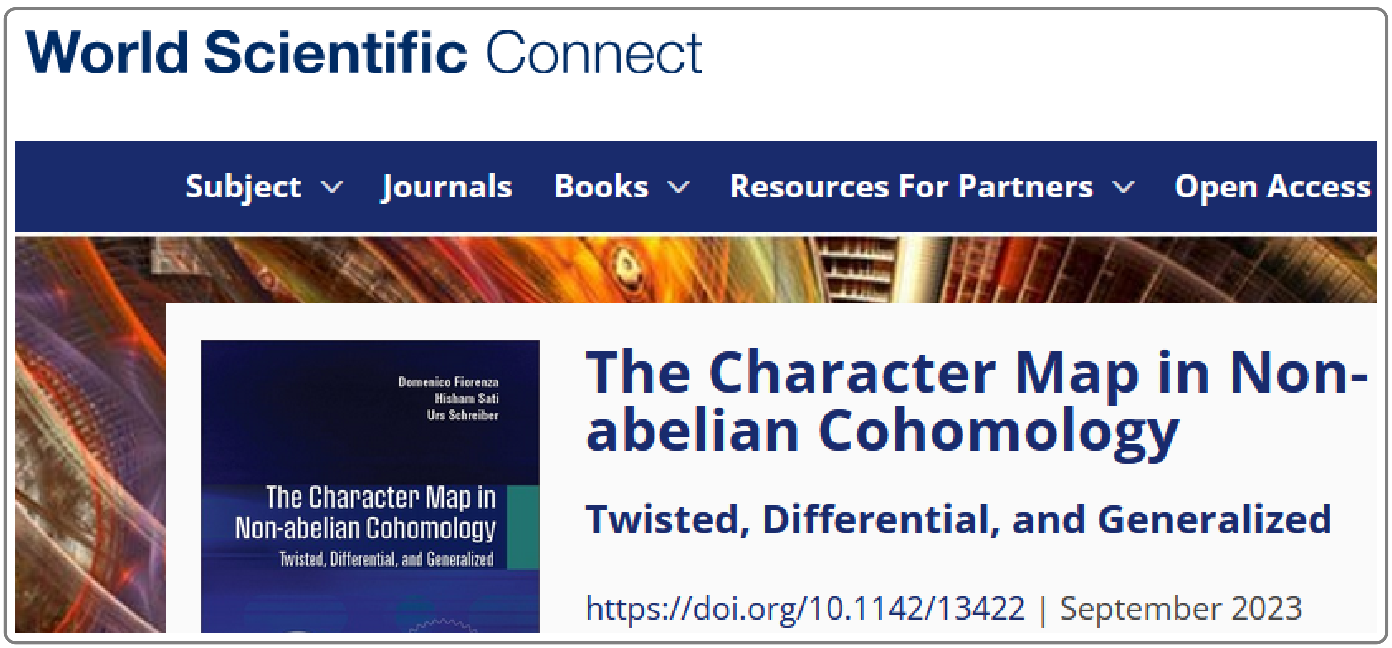

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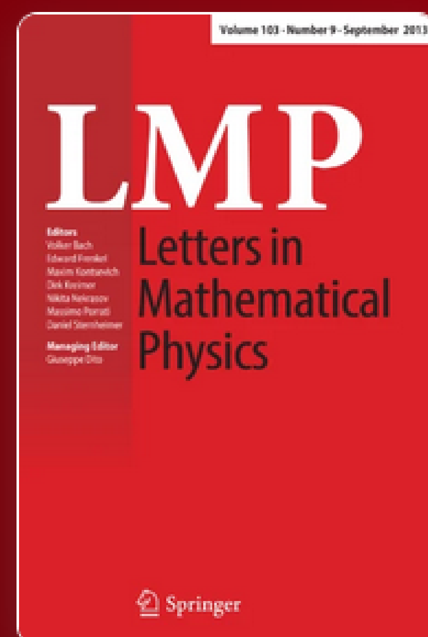

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An article that we are finalizing at CQTS:

- Hisham Sati and Urs Schreiber:

  Rethinking Fractional Quantum Hall Anyons via the Algebraic Topology of exotic Flux Quanta

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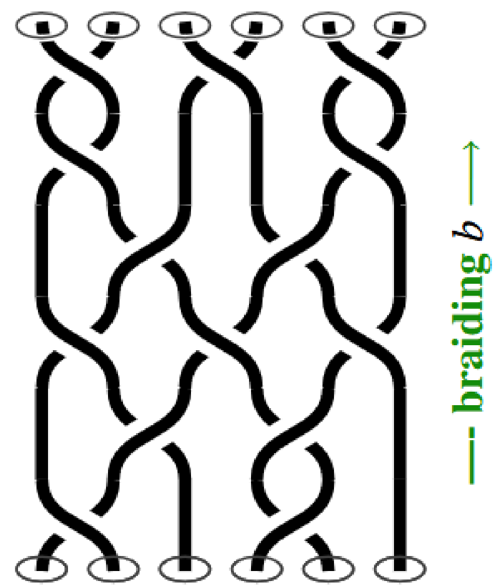

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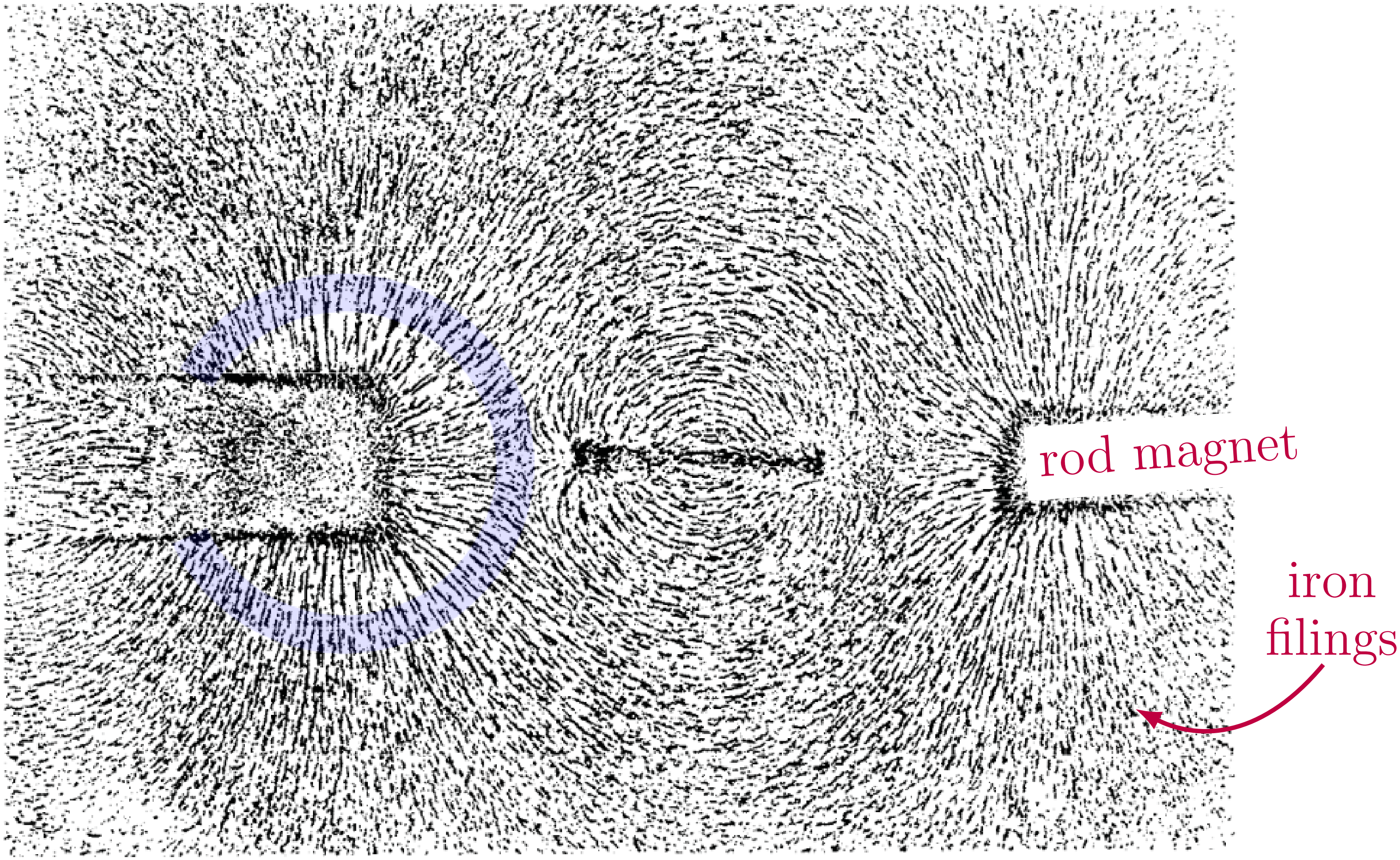

From Faraday's *Diary of experimental investigation*, vol VI, entry from 11th Dec. 1851, as reproduced in [Martin09]; the colored arc is our addition, for ease of comparison with the next graphics.

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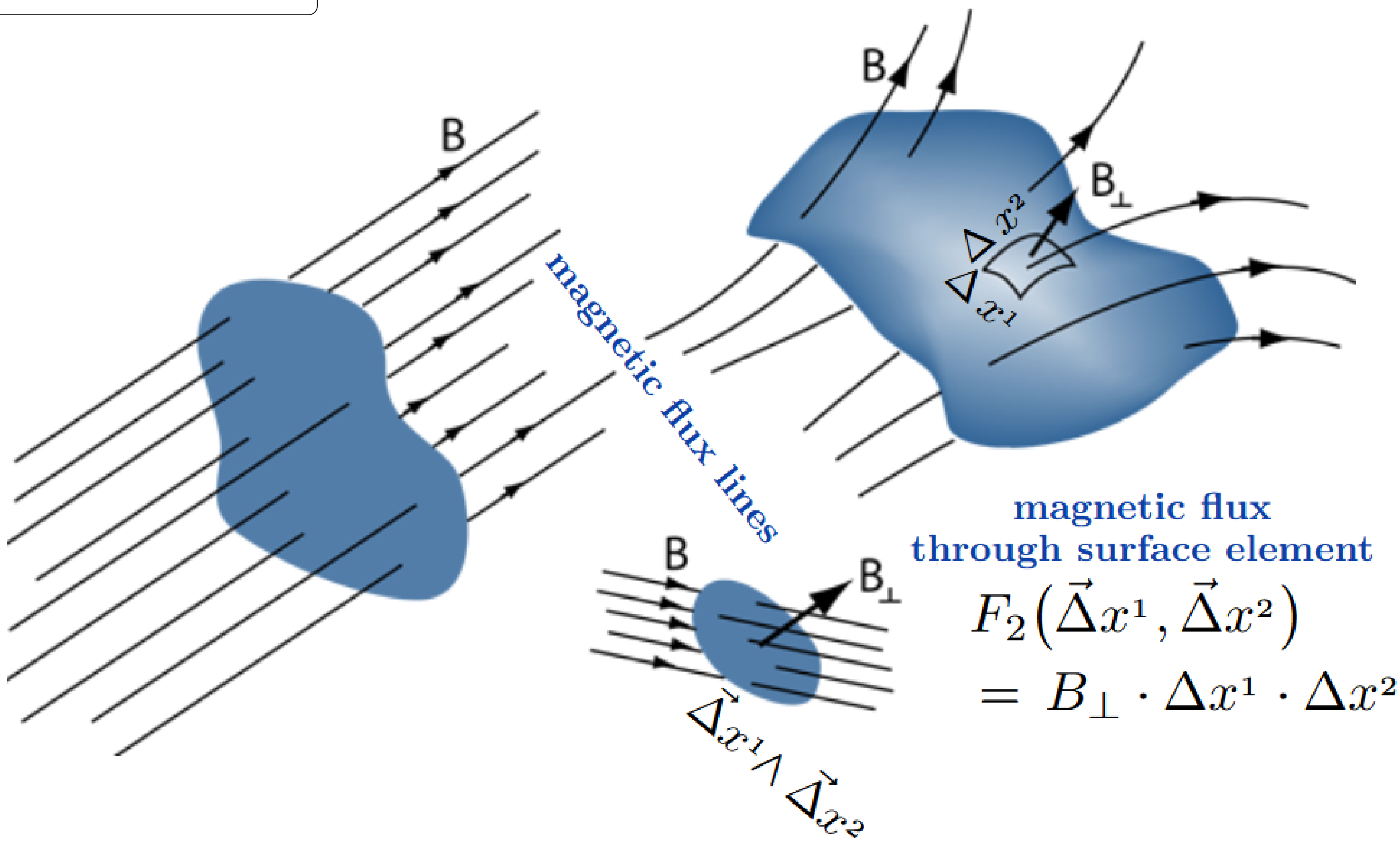

The density and orientation of magnetic field flux lines are encoded in a differential 2-form F_2 whose integral over a given surface is proportional to the total magnetic flux through that surface. (Graphics adapted from [Hyperphysics].)

Recap flux.

recall ordinary magnetic flux quantization:

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1985

Topological quantization and cohomology

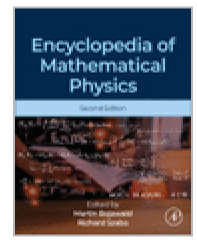
Orlando Alvarez

Comm. Math. Phys. 100(2): 279-309 (1985).

ELSEVIER

Encyclopedia of Mathematical Physics (Second Edition)

Volume 4, 2025, Pages 281-324



Flux Quantization ☆

Hisham Sati, Urs Schreiber

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total flux = charge character

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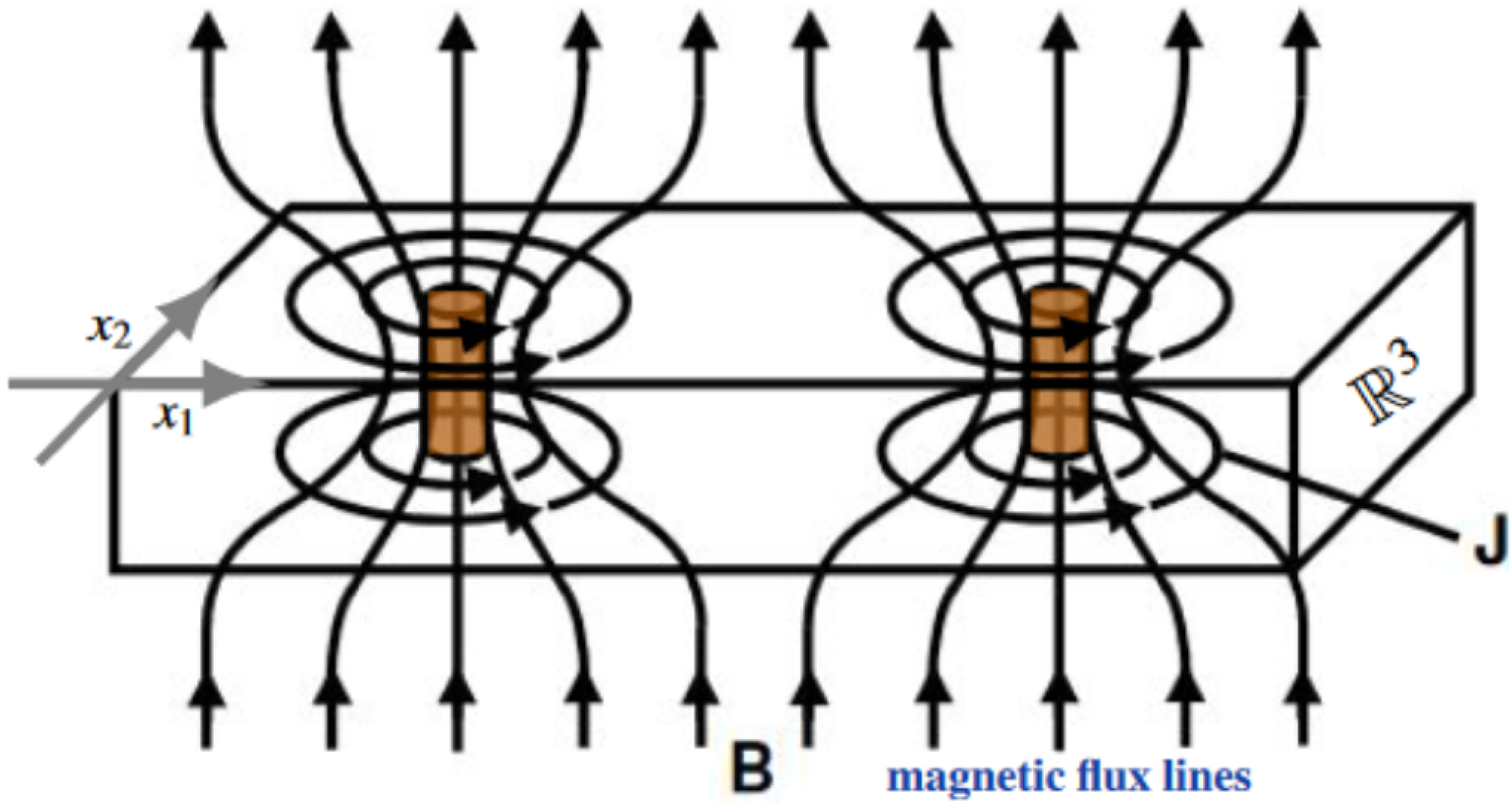

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pointed mapping space
makes flux *vanish-at-infinity*
(the soliton condition)

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fundamental group (monodromy of flux)

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group algebra
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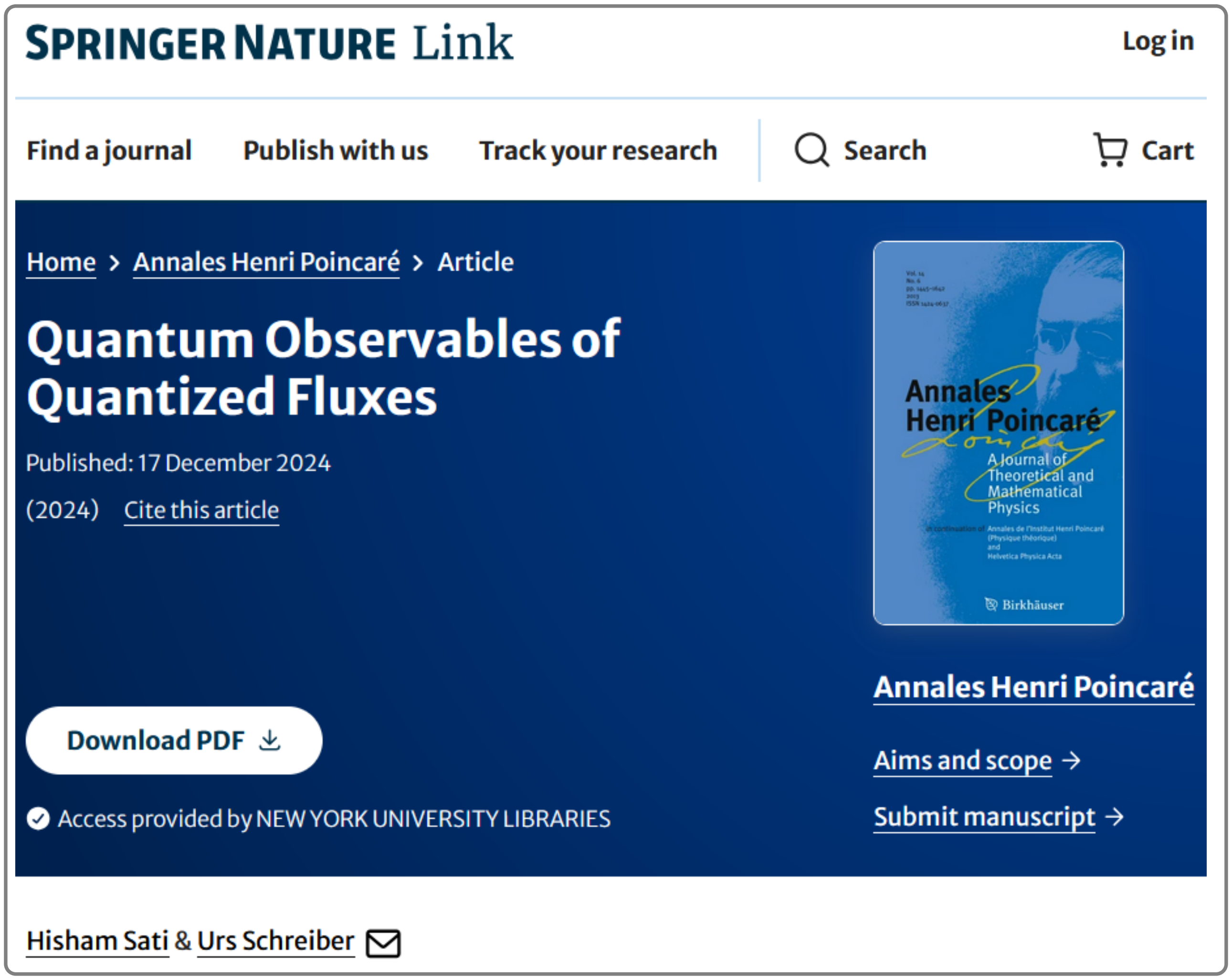

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What gives?

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Exotic Flux Quantization:

use another classifying space!

Exotic flux. Exotic Flux Quantization:

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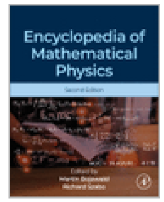
Encyclopedia of Mathematical Physics (Second Edition)

Volume 4, 2025, Pages 281-324



Flux Quantization ☆

Hisham Sati, Urs Schreiber

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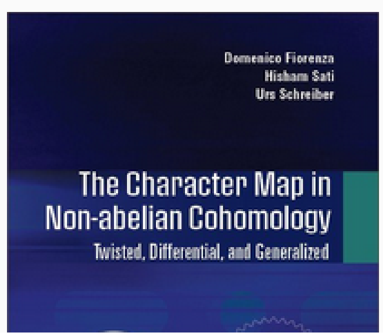

The Character Map in Non-abelian Cohomology

Twisted, Differential, and Generalized

https://doi.org/10.1142/13422 | September 2023

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use another classifying space!

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Fact. This quantizes EM-flux *with* Chern-Simons flux:

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Reviews in Mathematical Physics | Vol. 34, No. 05, 2250013 (2022) | Research

Twistorial cohomotopy implies Green–Schwarz anomaly cancellation

Domenico Fiorenza, Hisham Sati, and Urs Schreiber

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Hypothesis h:

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Volume 59, Issue 6
June 2018

RESEARCH ARTICLE | JUNE 19 2018

Framed M-branes, corners, and topological invariants

Hisham Sati

Check for updates

+ Author & Article Information

J. Math. Phys. 59, 062304 (2018)

https://doi.org/10.1063/1.5007185 **Article history**

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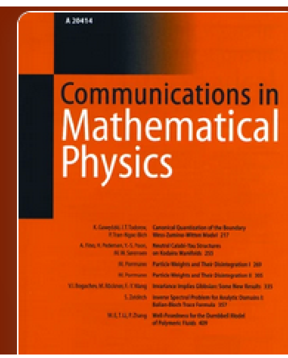
Twisted Cohomotopy Implies M-Theory Anomaly Cancellation on 8-Manifolds

Published: 06 April 2020

Volume 377, pages 1961–2025, (2020) Cite this article



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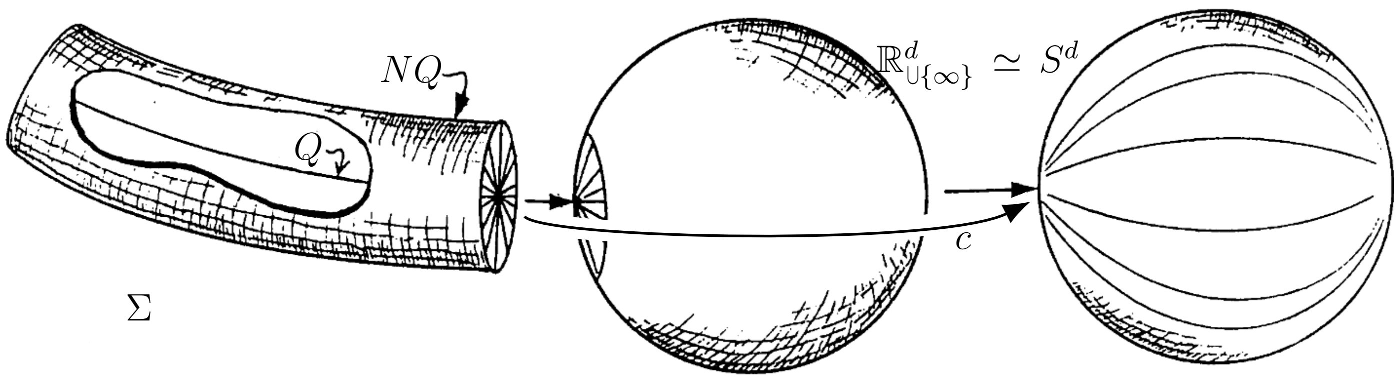

figure adapted
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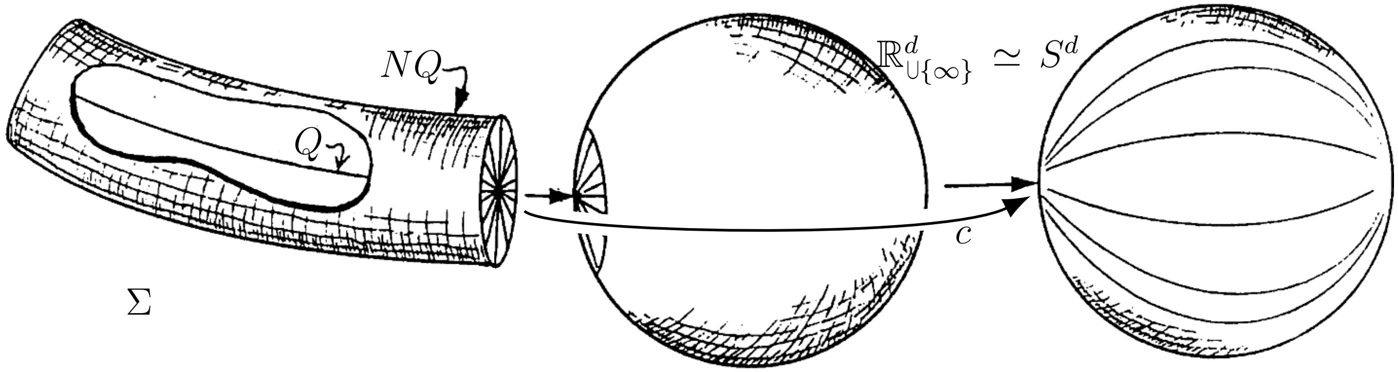

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Smooth manifolds and their applications in homotopy theory

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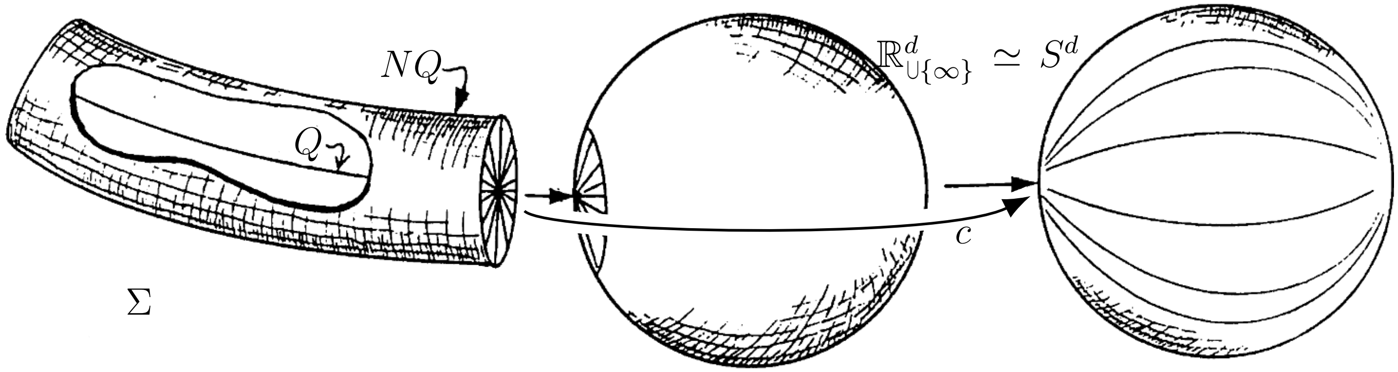

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Journal of Geometry and Physics
Volume 156, October 2020, 103775

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Hisham Sati, Urs Schreiber [1]

Reviews in Mathematical Physics | Vol. 35, No. 10, 2350028 (2023)

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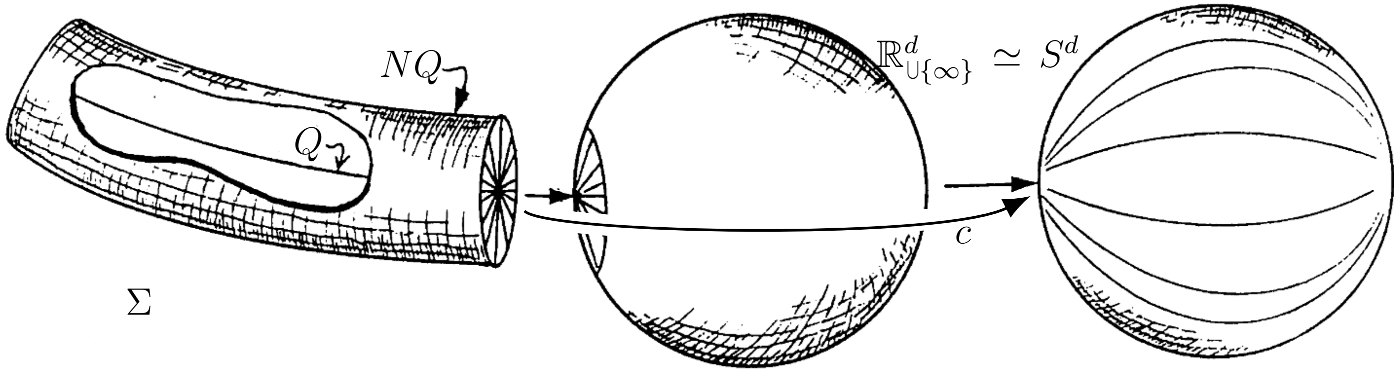

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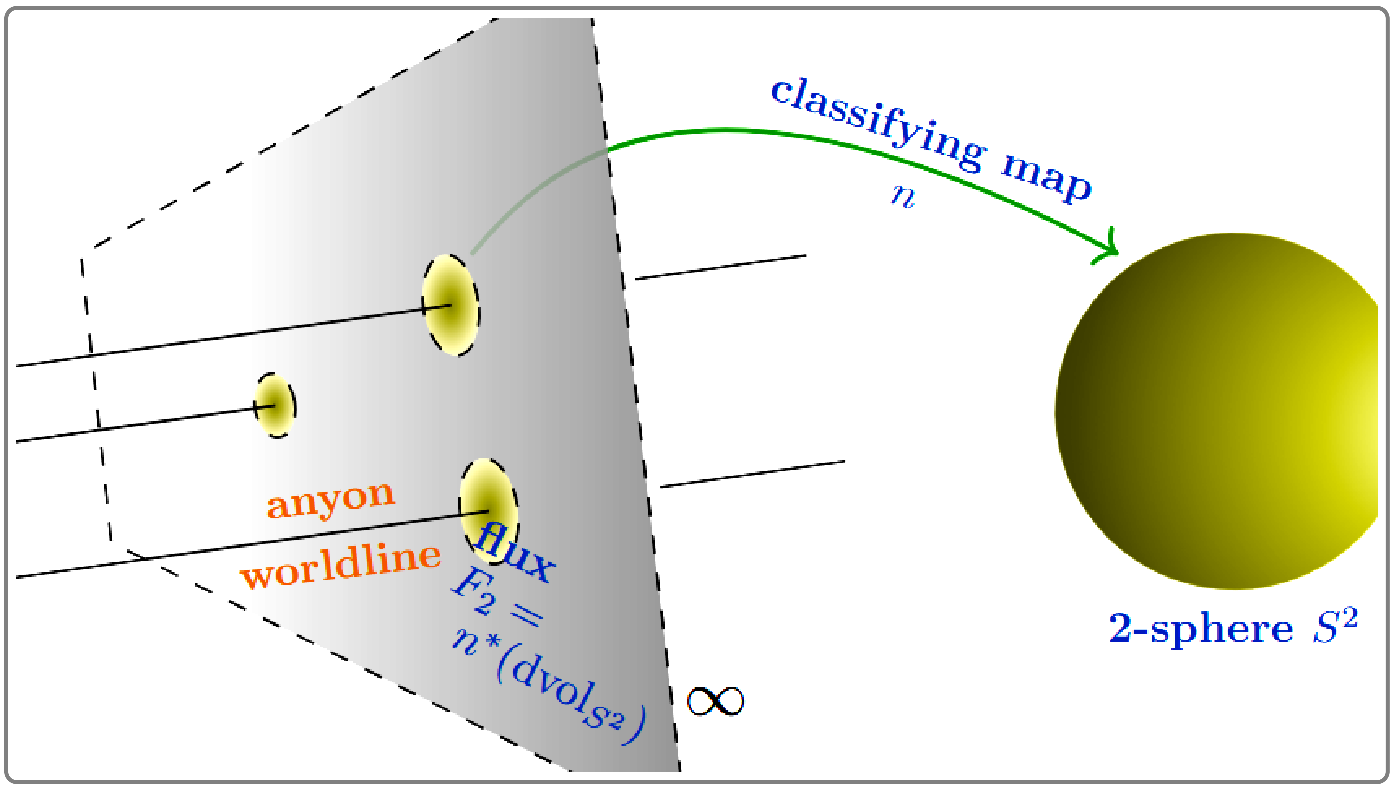

First case: $\Sigma^2 \equiv \mathbb{R}^2$ the plane — fractional statistics

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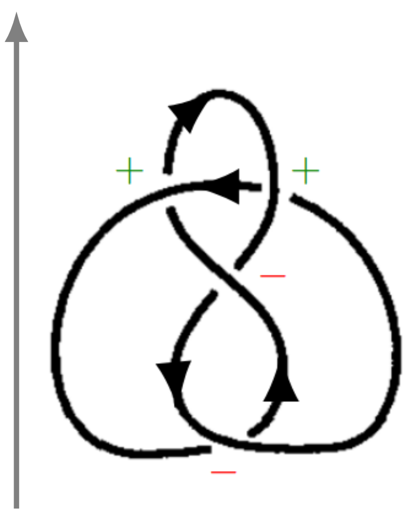

vacuum-to-vacuum process
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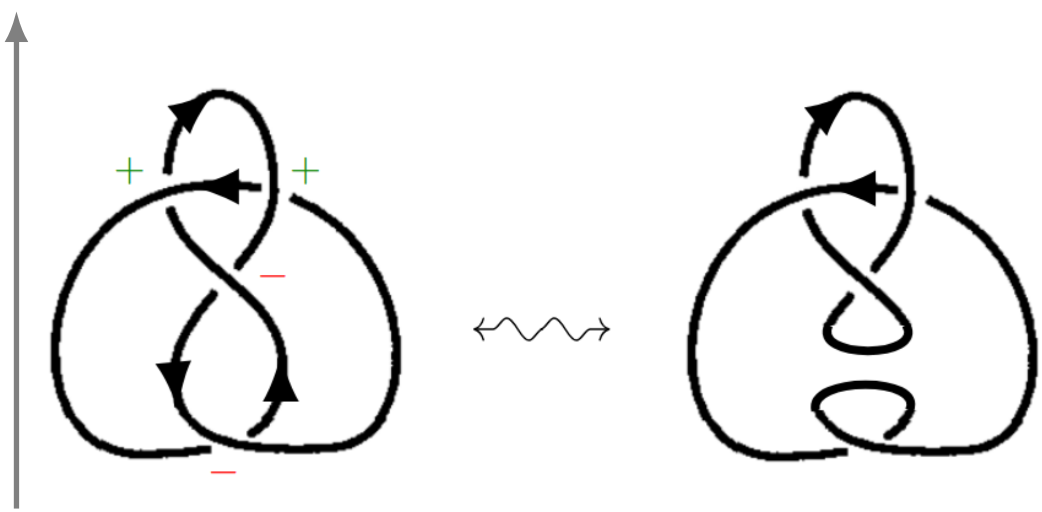

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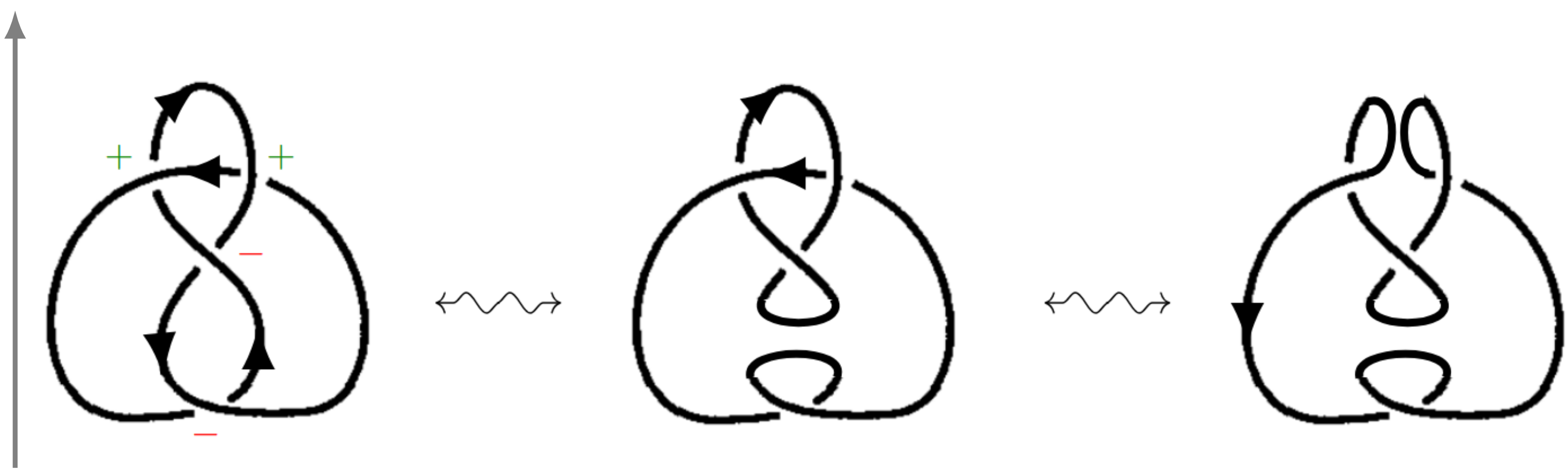

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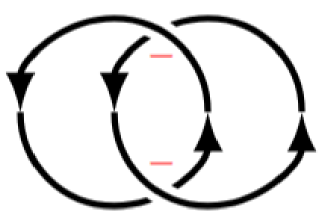
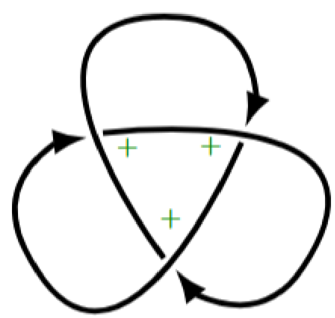
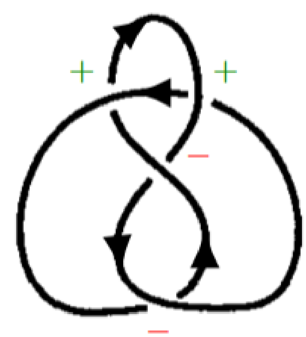

$\xmapsto{\text{writhe}} -2\,,$


$\xmapsto{\text{writhe}} +3\,,$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Pure GNS states $|\zeta\rangle \in \mathcal{H}^{\mathrm{h}}_{\mathbb{R}^2}$ labeled by phases $\zeta \in \mathrm{U}(1)$

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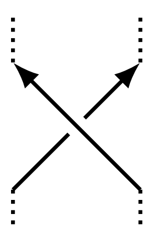
$\langle \zeta |$  $| \zeta \rangle \;=\; \zeta \cdots,$

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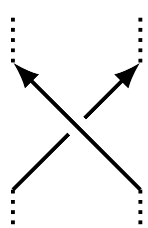
$\langle \zeta |$  $| \zeta \rangle \;=\; \zeta \cdots ,$ flux quanta braiding phase!

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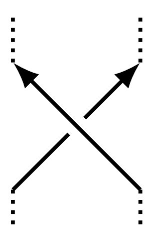
$\langle\zeta|$  $|\zeta\rangle \;=\; \zeta \cdots,$ flux quanta braiding phase!

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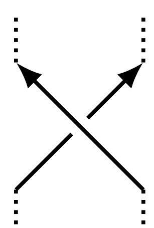
$\langle \zeta |$  $| \zeta \rangle \;=\; \zeta \cdots,$ flux quanta braiding phase!

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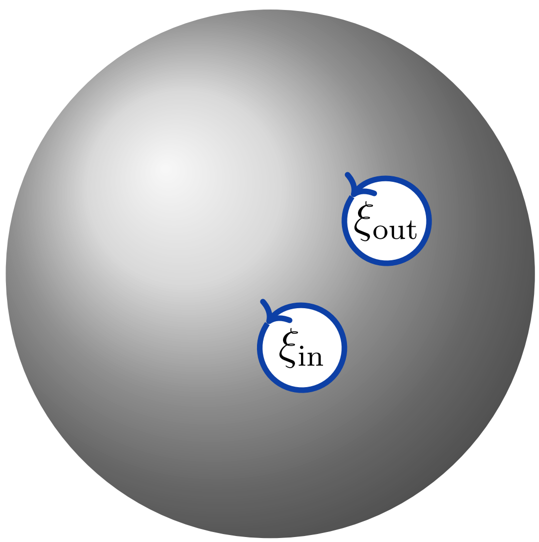

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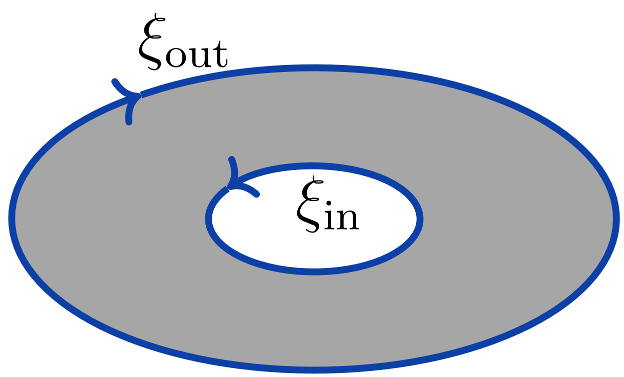

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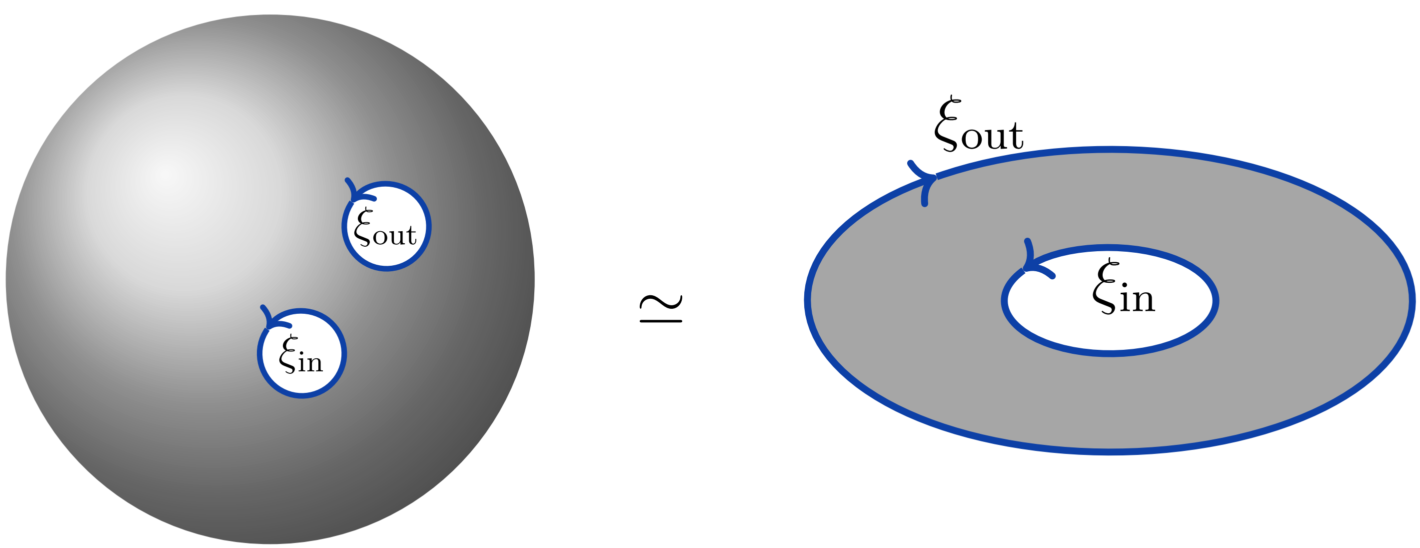

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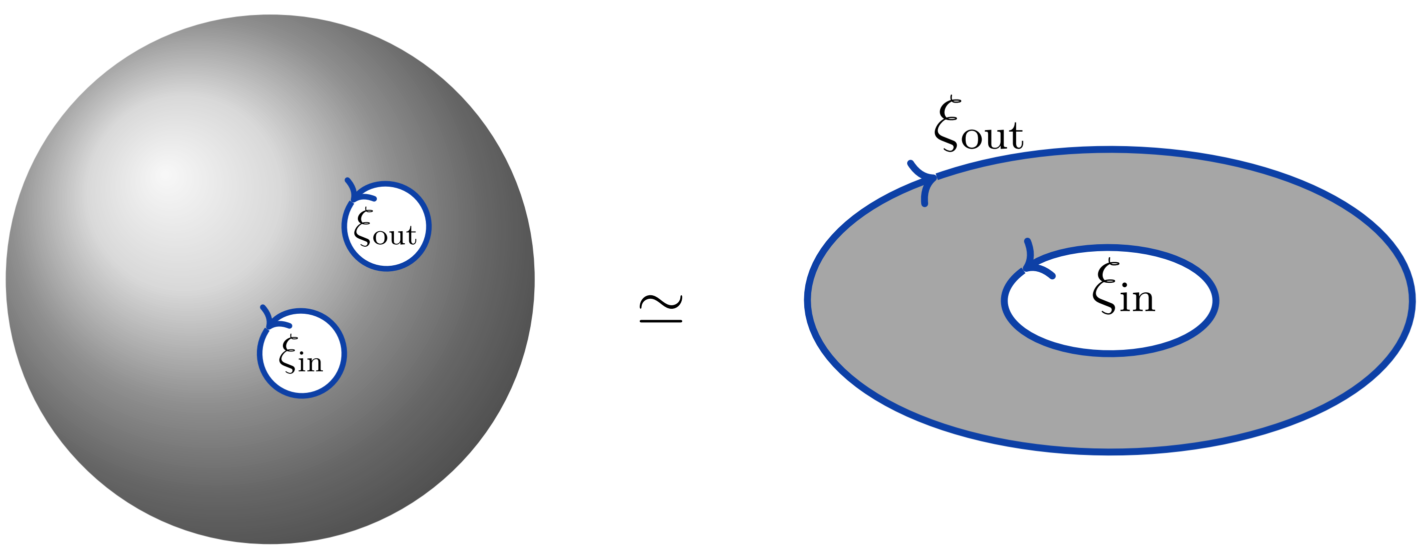

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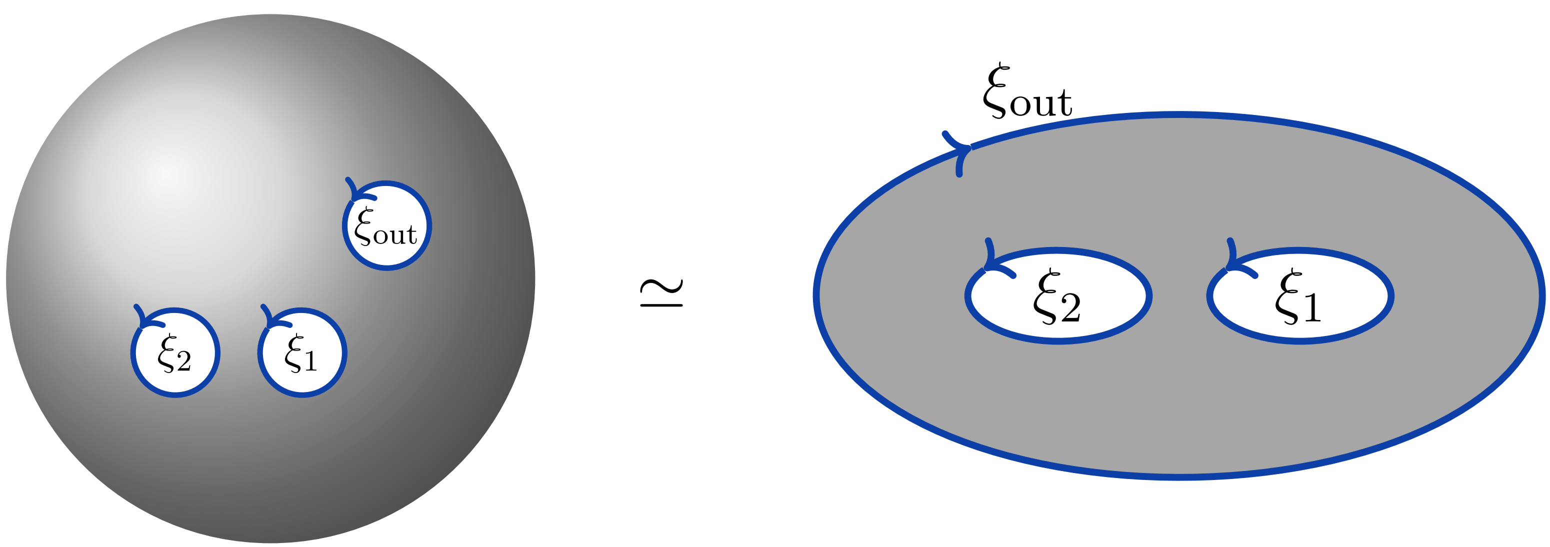

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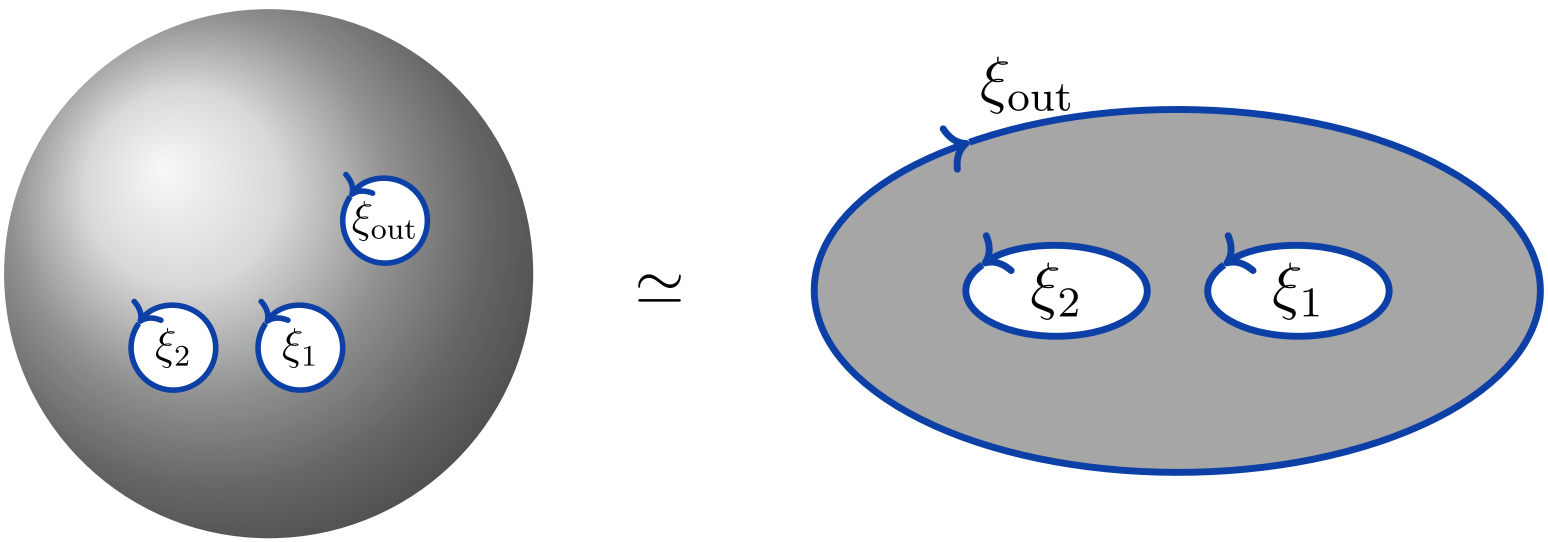

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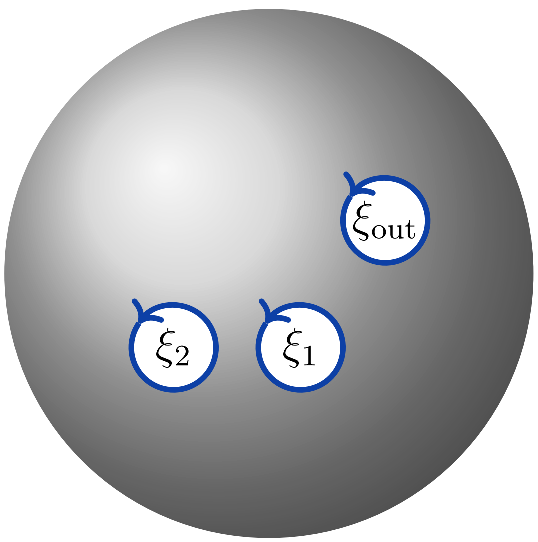

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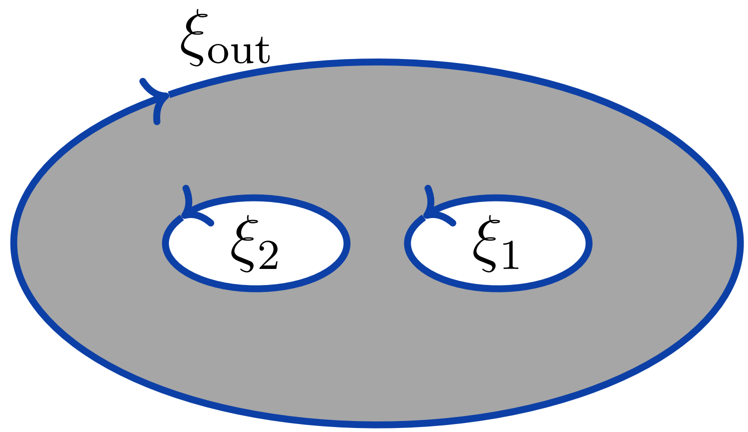

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