Urs Schreiber

New York University, Abu Dhabi

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS

Electromagnetic Flux Quantization

IR-Completing Higher Gauge Fields

talk at:
ICMMP 2026
Hangzhou, China
(Oct 30 – Nov 3)

based in parts on joint work with:
D. Fiorenza, G. Giotopoulos, N. Golub, and H. Sati

materials at: ncatlab.org/schreiber/show/ICMMP26

IR-Completing Higher Gauge Fields

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Quantum Fields traditionally defined incrementally:

IR-Completing Higher Gauge Fields

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UV-completion: local interactions

IR-Completing Higher Gauge Fields

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widely considered, but not here

IR-Completing Higher Gauge Fields

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UV-completion: local interactions *widely considered, but not here*

IR-completion: global field topology

IR-Completing Higher Gauge Fields

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flux quantization

IR-Completing Higher Gauge Fields

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flux quantization

brane charges

IR-Completing Higher Gauge Fields

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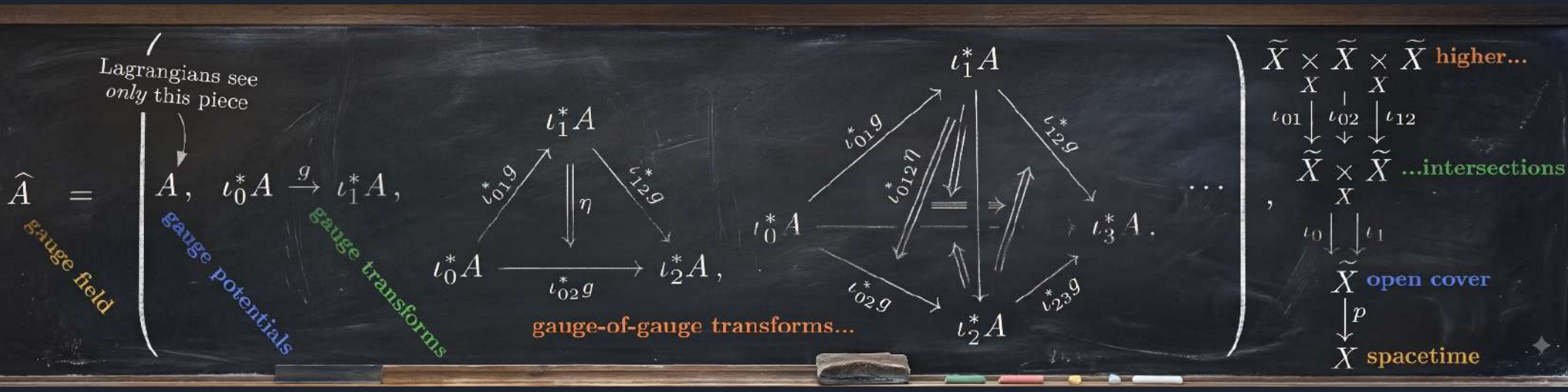
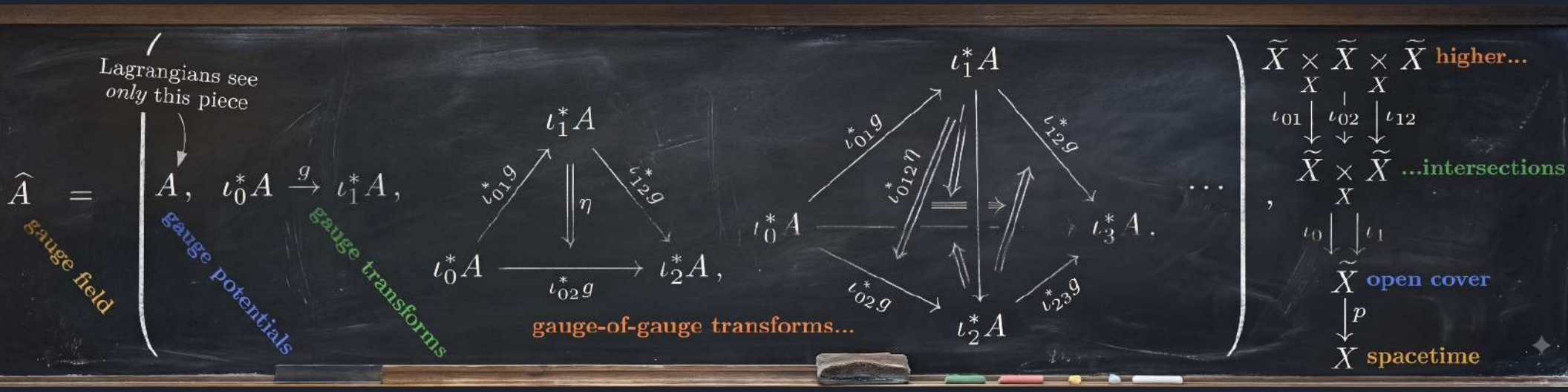


IR-Completing Higher Gauge Fields

Quantum Fields traditionally defined incrementally:

UV-completion: local interactions

IR-completion: global field topology $\Rightarrow$ TQFT sector
flux quantization
brane charges



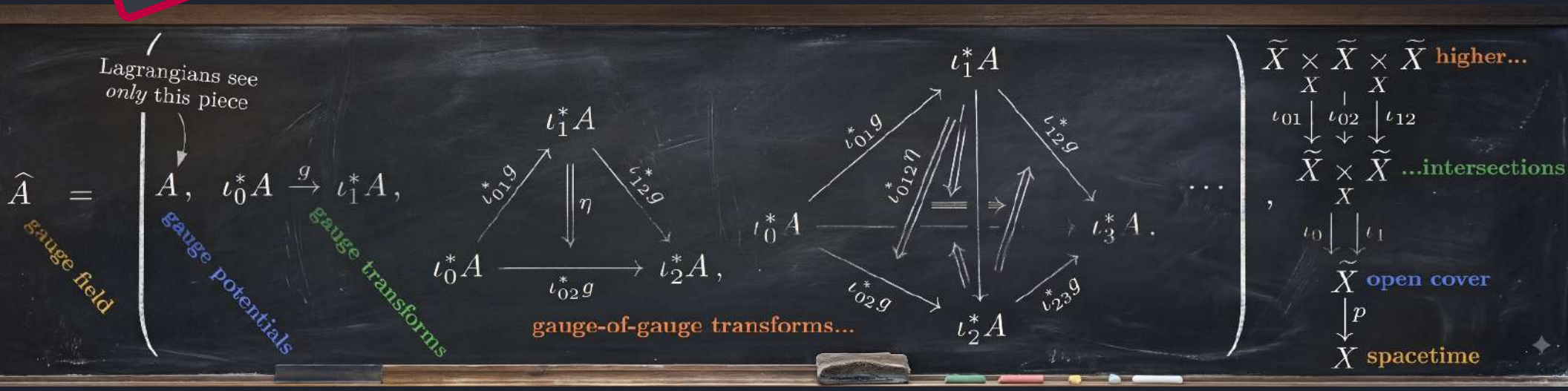
IR-Completing Higher Gauge Fields

Quantum Fields traditionally defined incrementally:

UV-completion: local interactions

IR-completion: global field topology ⇒ TQFT sector
flux quantization
brane charges

non-perturbative



For citations and pointers see:

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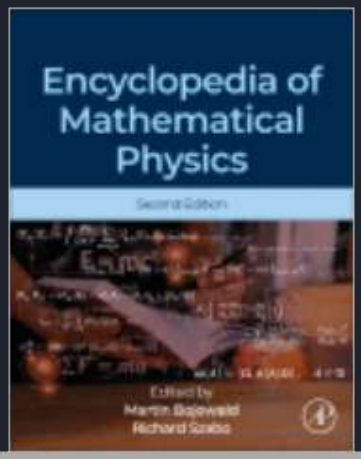

Chapter

Flux Quantization

Hisham Sati, Urs Schreiber

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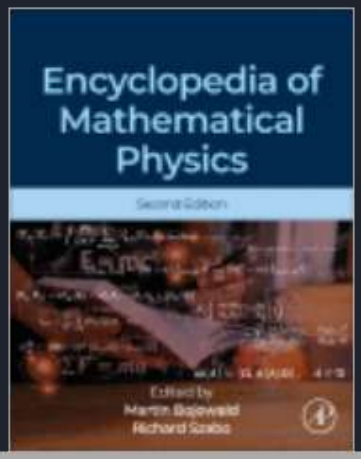

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High Energy Physics - Theory

[Submitted on 28 Sep 2026]

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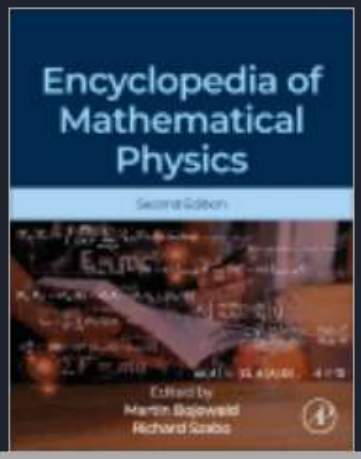

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Ordinary Magnetic Flux Quantization

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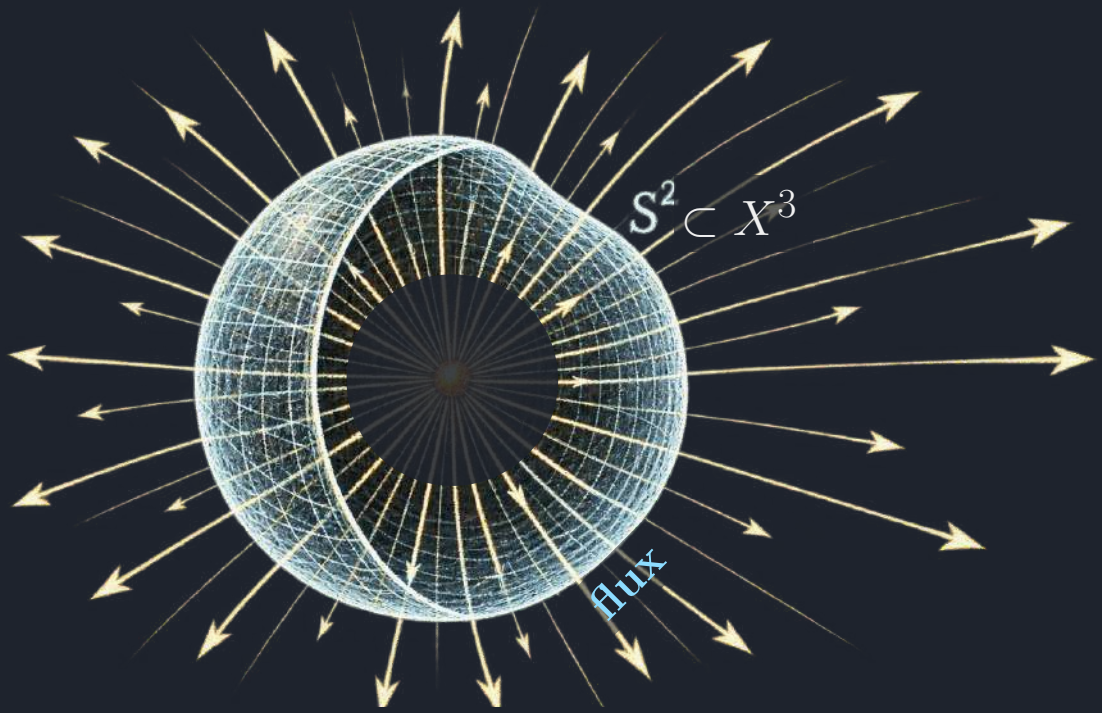
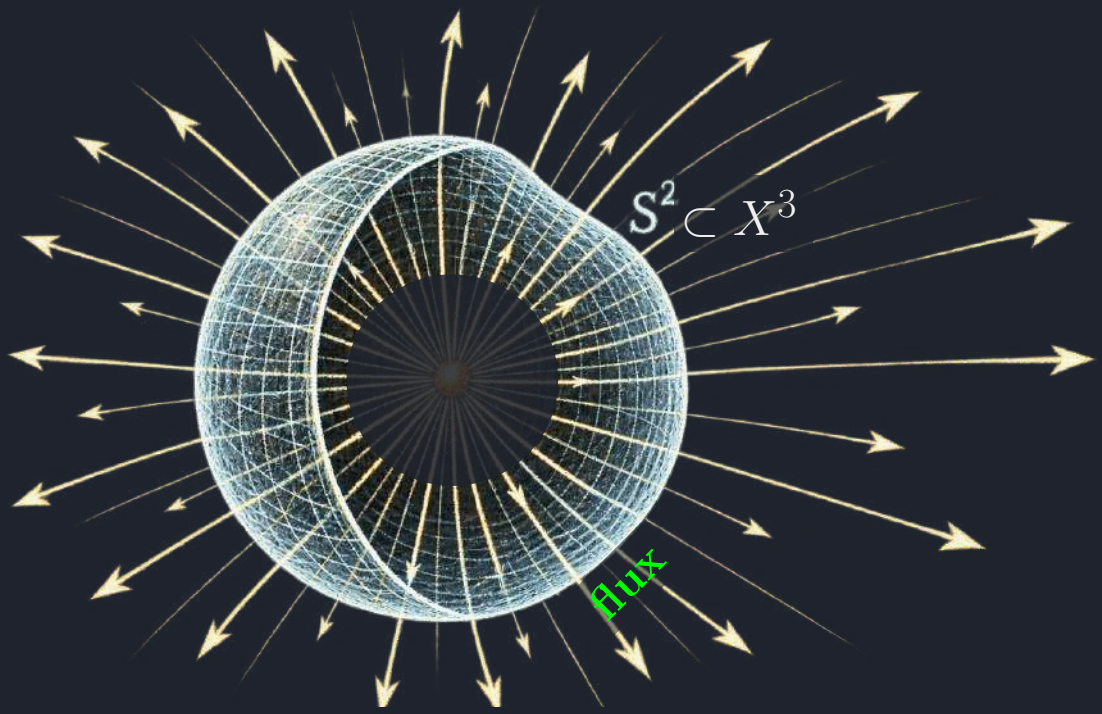


Ordinary Magnetic Flux Quantization

$$* \xrightarrow{B_2 \text{ flux}} \underbrace{\Omega^2_{\mathrm{cl}}(X^3)}_{\text{closed forms}}$$



Ordinary Magnetic Flux Quantization

$*$

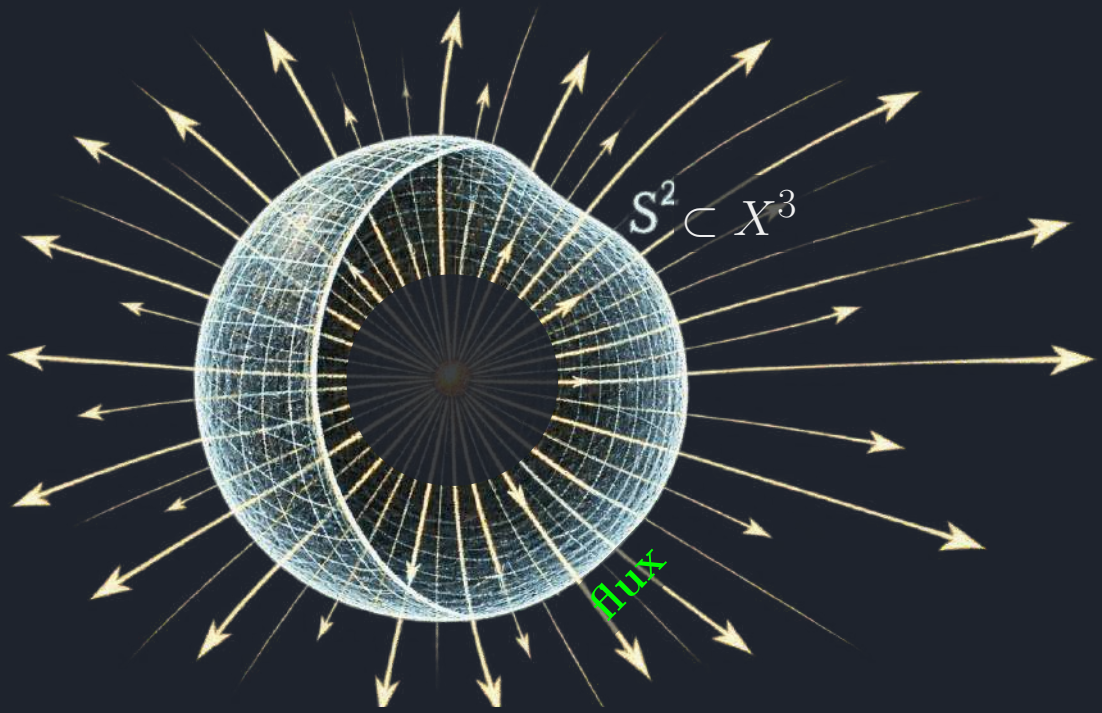
B_2 — flux

$$\Omega^2_{\mathrm{cl}}(X^3)$$

closed forms

$\Leftrightarrow$ **Gauss law:**

$$\mathrm{d}\, B_2 = 0$$



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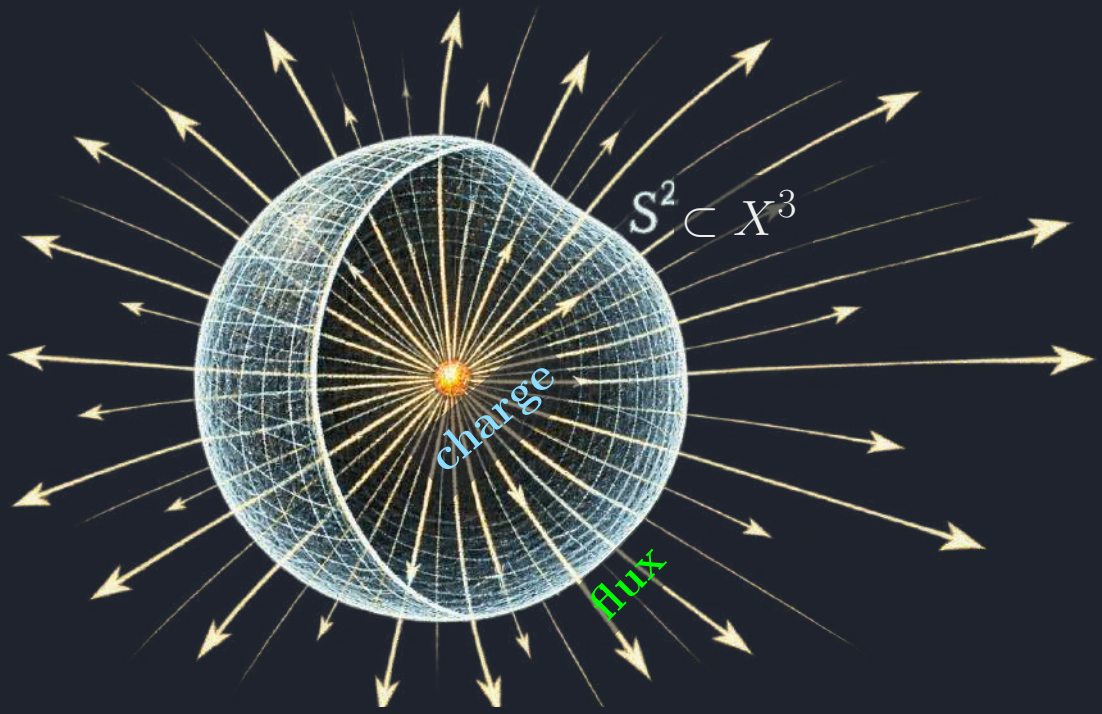

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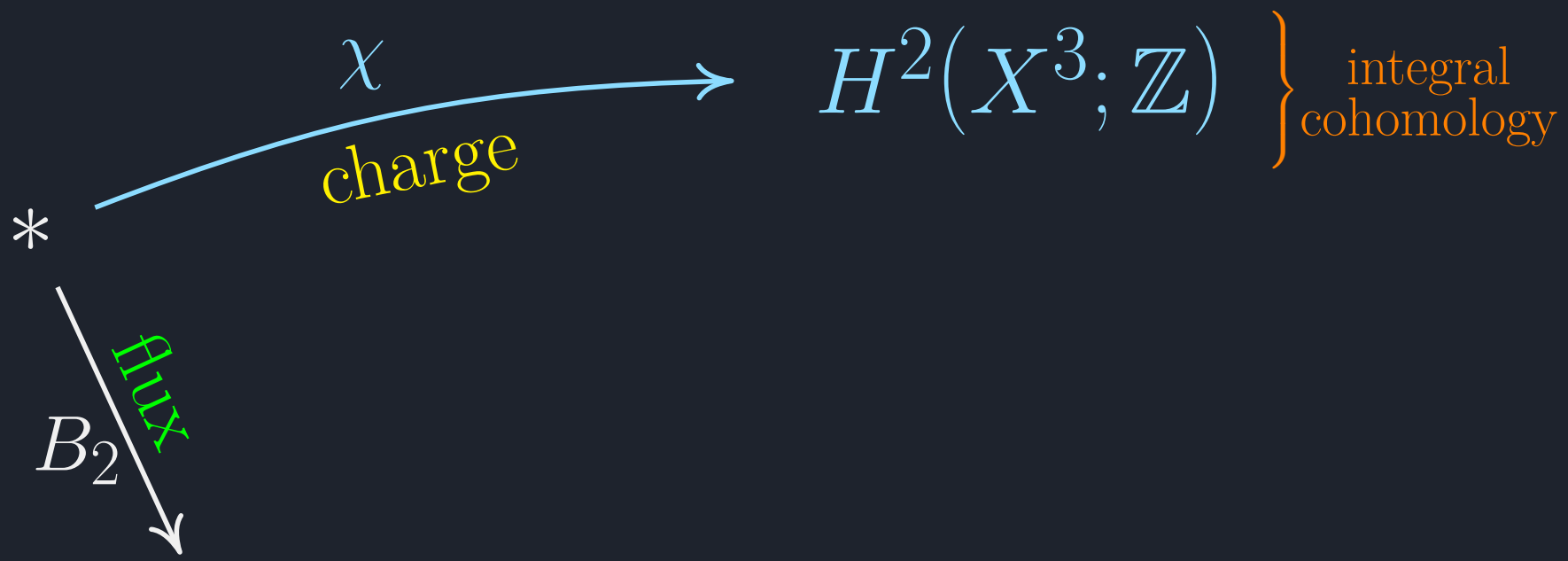

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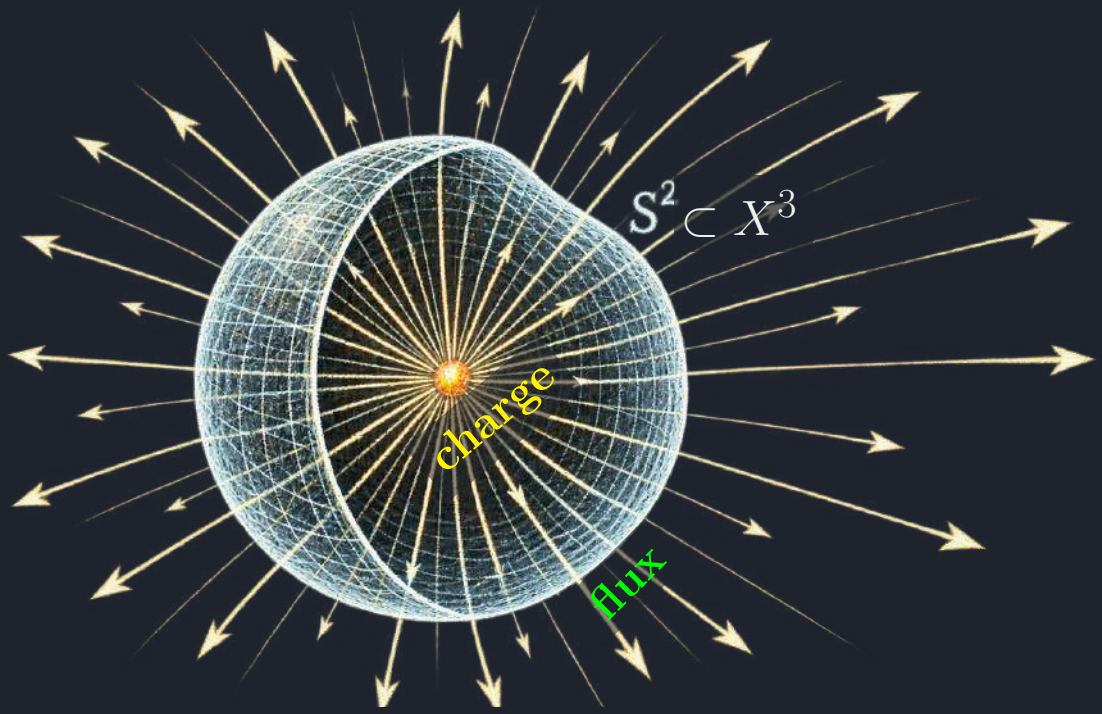

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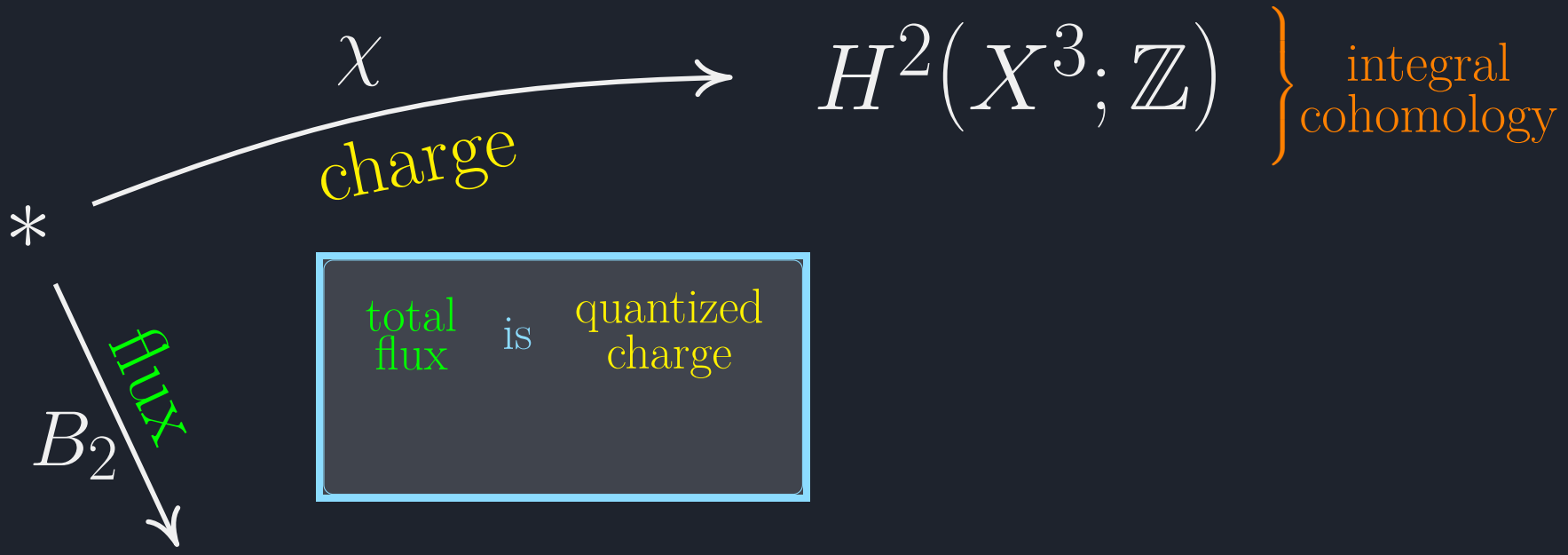

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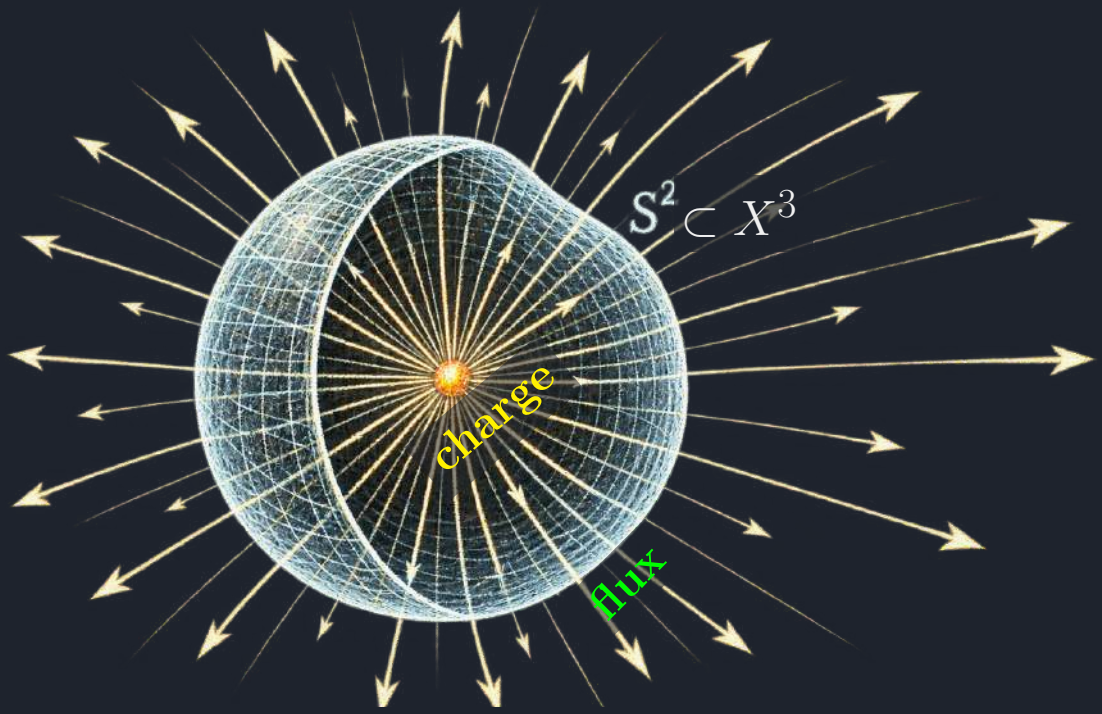

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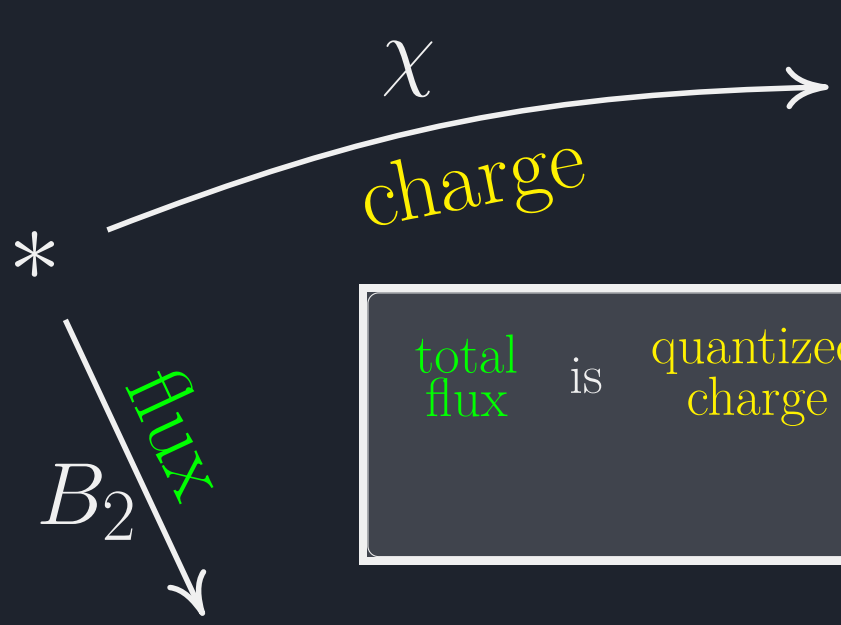

$H^2(X^3;\mathbb{Z})$ } integral cohomology

total flux	is	quantized charge

$\Omega^2_{\mathrm{cl}}(X^3)$

closed forms

$H^2(X^3;\mathbb{R})$ } real/de Rham cohomology

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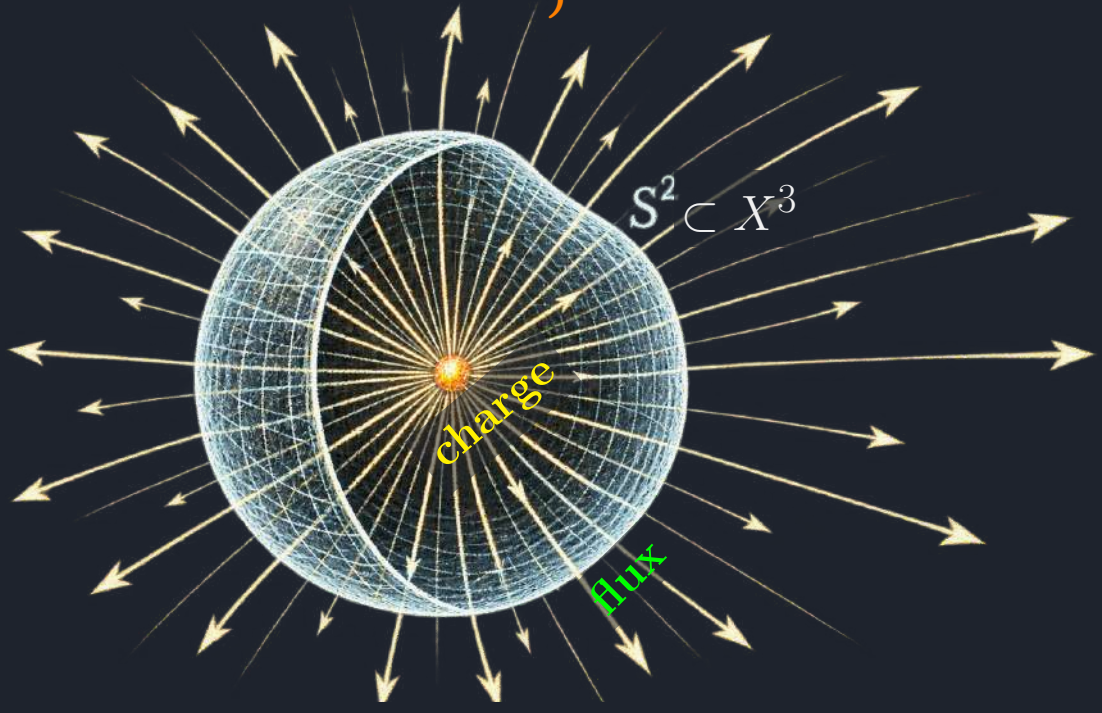

Ordinary Magnetic Flux Quantization

$$\ast \xrightarrow{\chi\ \text{charge}} H^2(X^3;\mathbb{Z}) \quad \Big\} \text{integral cohomology}$$

$$\ast \xrightarrow{B_2\ \text{flux}} \underbrace{\Omega^2_{\mathrm{cl}}(X^3)}_{\text{closed forms}} \xrightarrow[[-]]{\text{total}} H^2(X^3;\mathbb{R}) \quad \Big\} \text{real/de Rham cohomology}$$

total flux is quantized charge

$\Leftrightarrow$ **Gauss law:**

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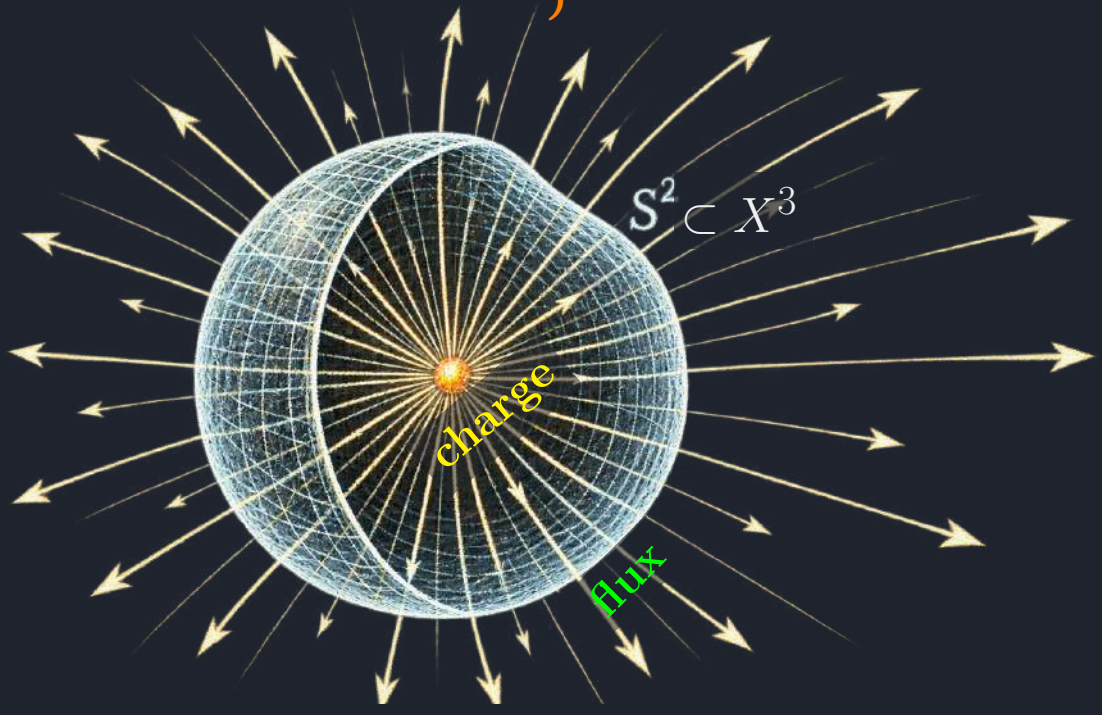
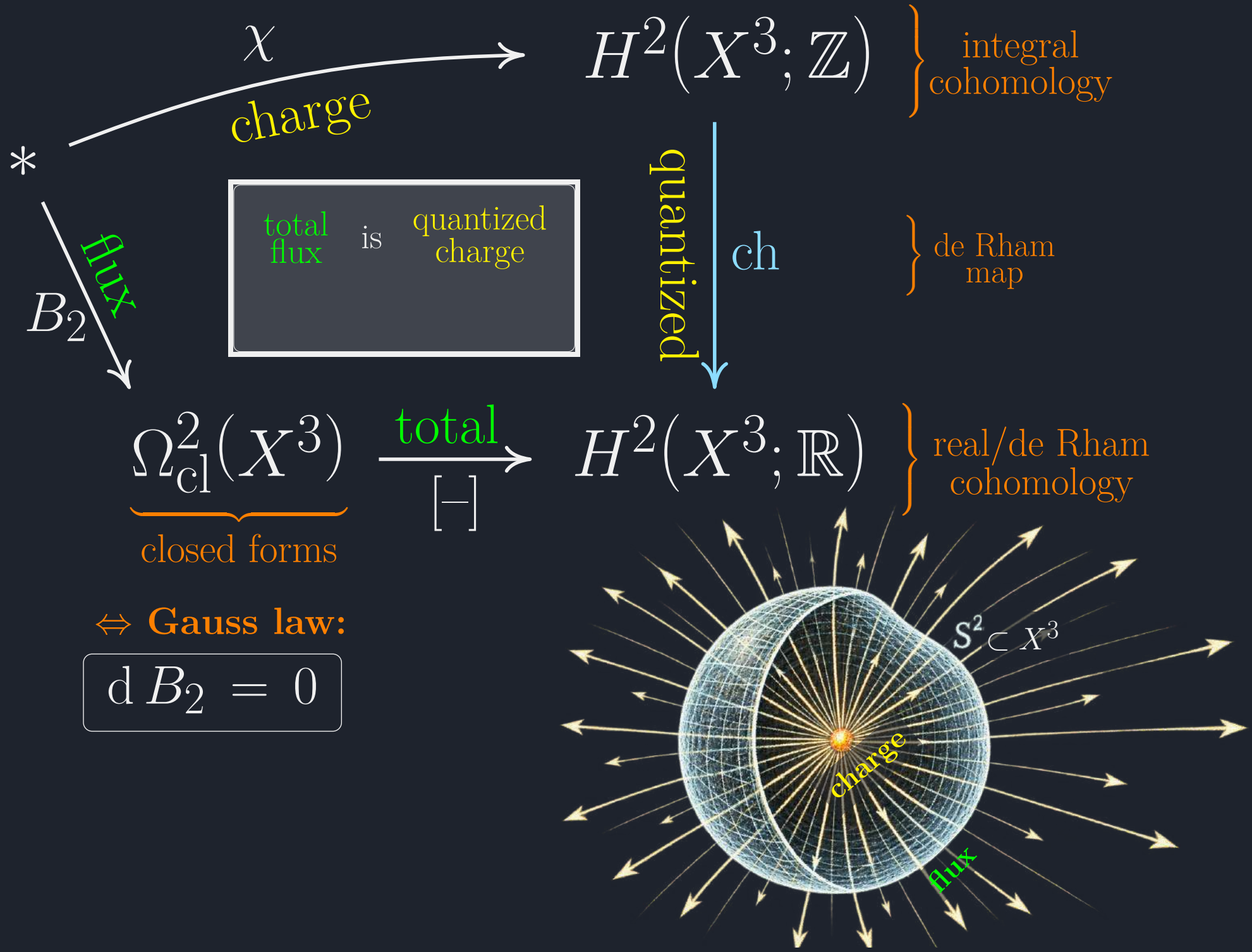


Ordinary Magnetic Flux Quantization



Ordinary Magnetic Flux Quantization

$$
\begin{tikzcd}
 & H^2(X^3;\mathbb{Z}) \arrow[d, "\text{ch}", "\text{quantized}"'] \\
* \arrow[ur, "\chi", "\text{charge}"'] \arrow[d, "B_2", "\text{flux}"'] & \\
\underbrace{\Omega^2_{\text{cl}}(X^3)}_{\text{closed forms}} \arrow[r, "\text{total}", "{[-]}"'] & H^2(X^3;\mathbb{R})
\end{tikzcd}
$$

integral cohomology

de Rham map

real/de Rham cohomology

total flux	is	quantized charge
$[B_2]$	$=$	$\text{ch}(\chi)$

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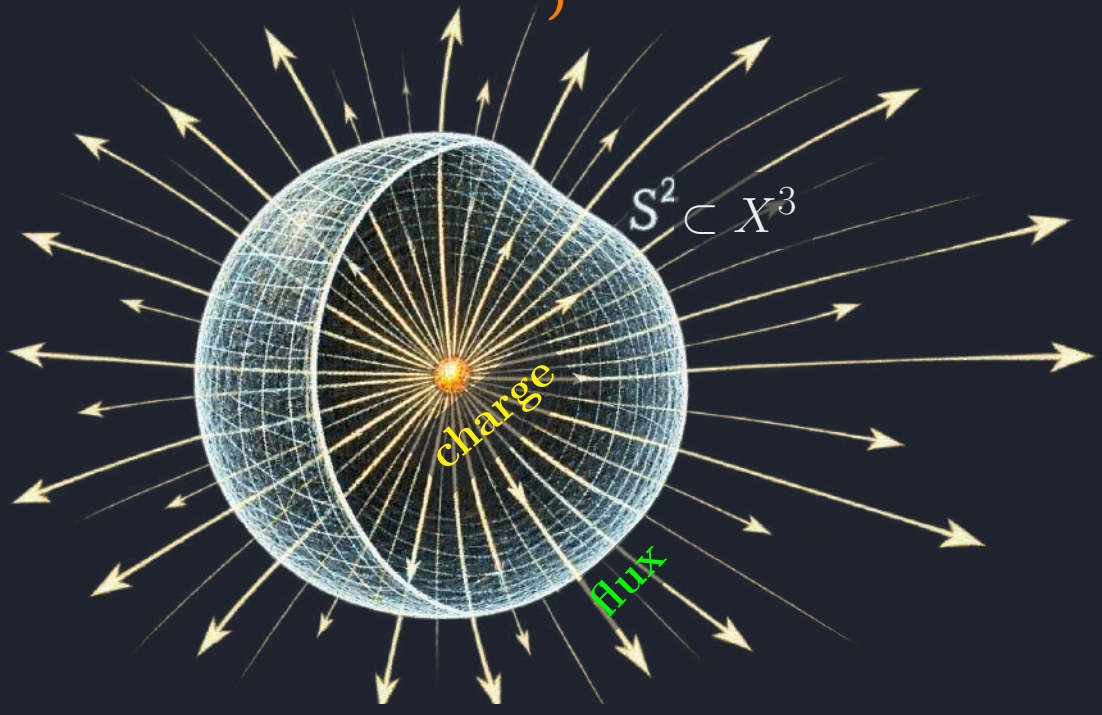

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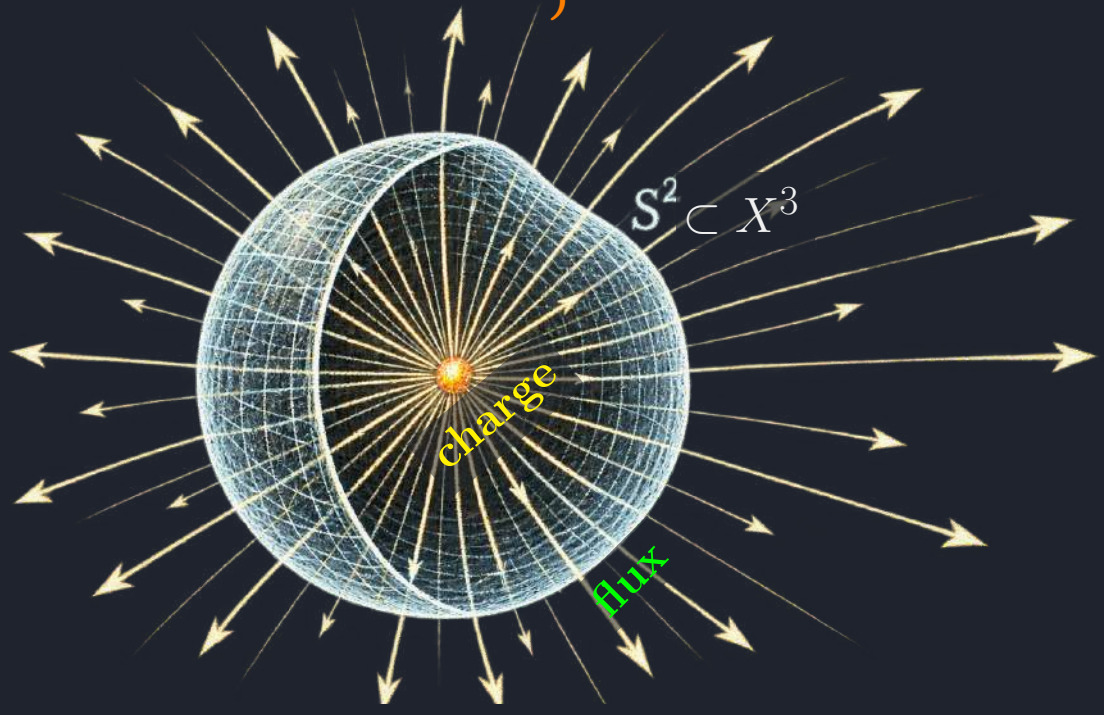

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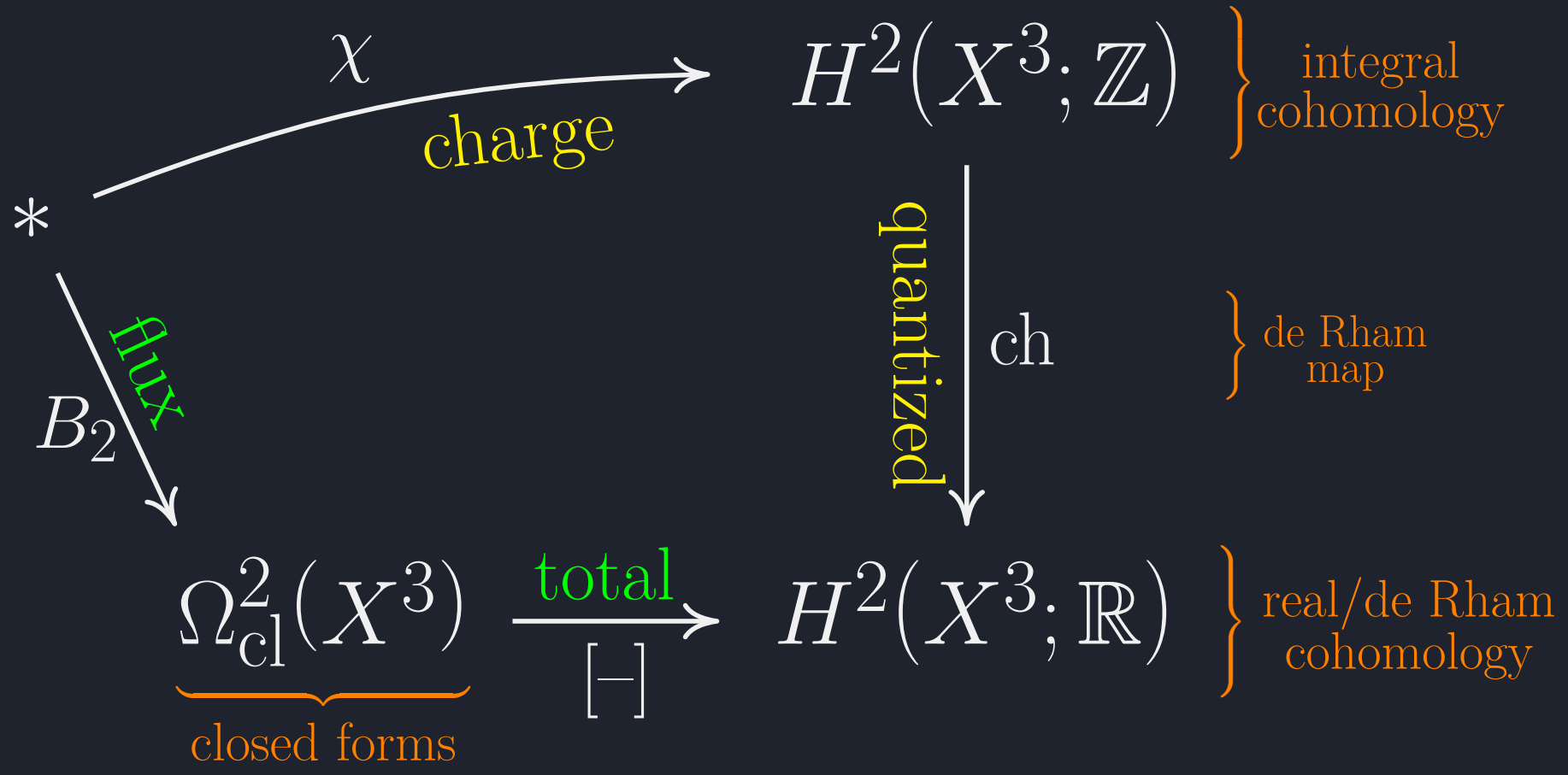

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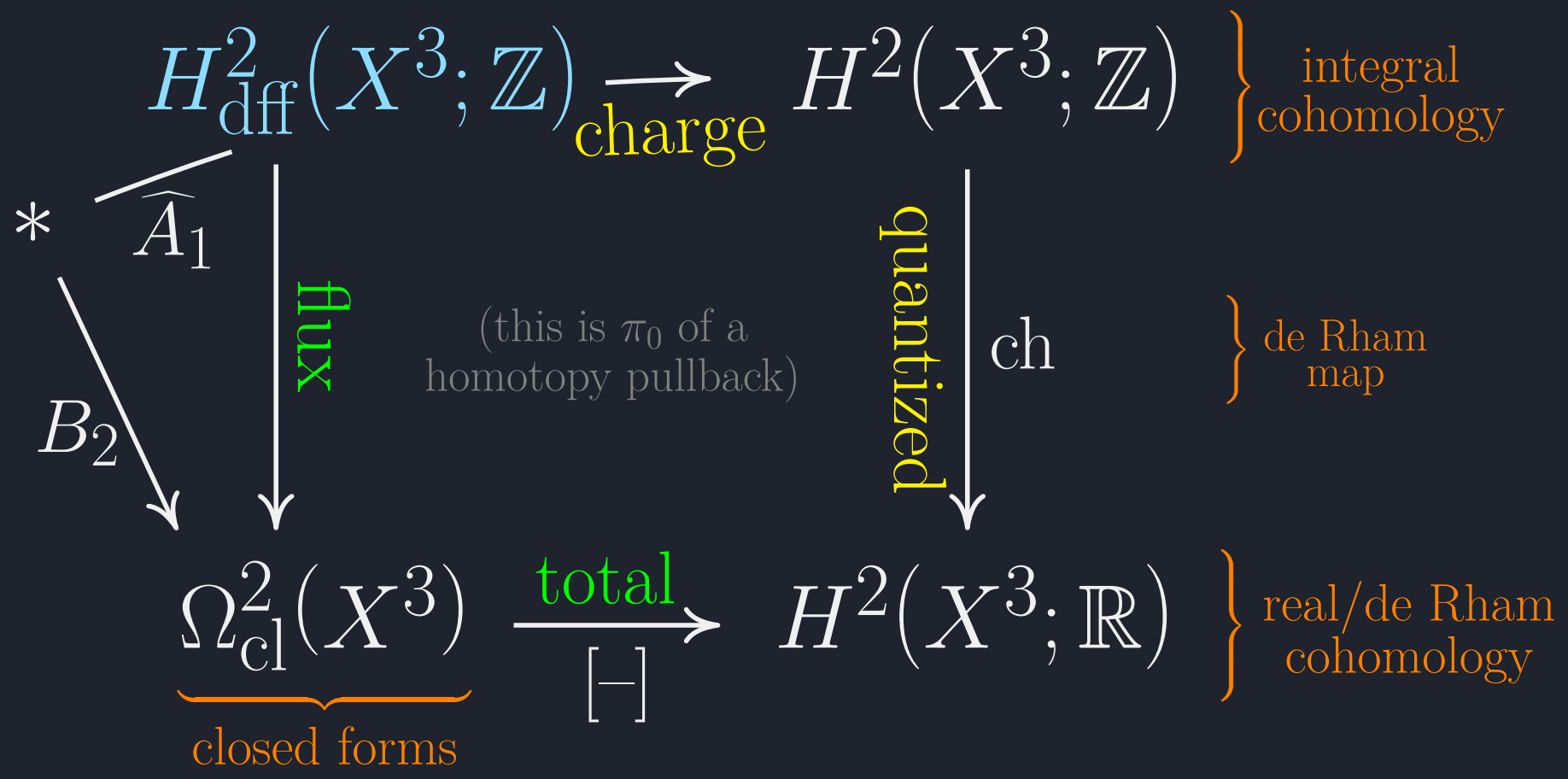
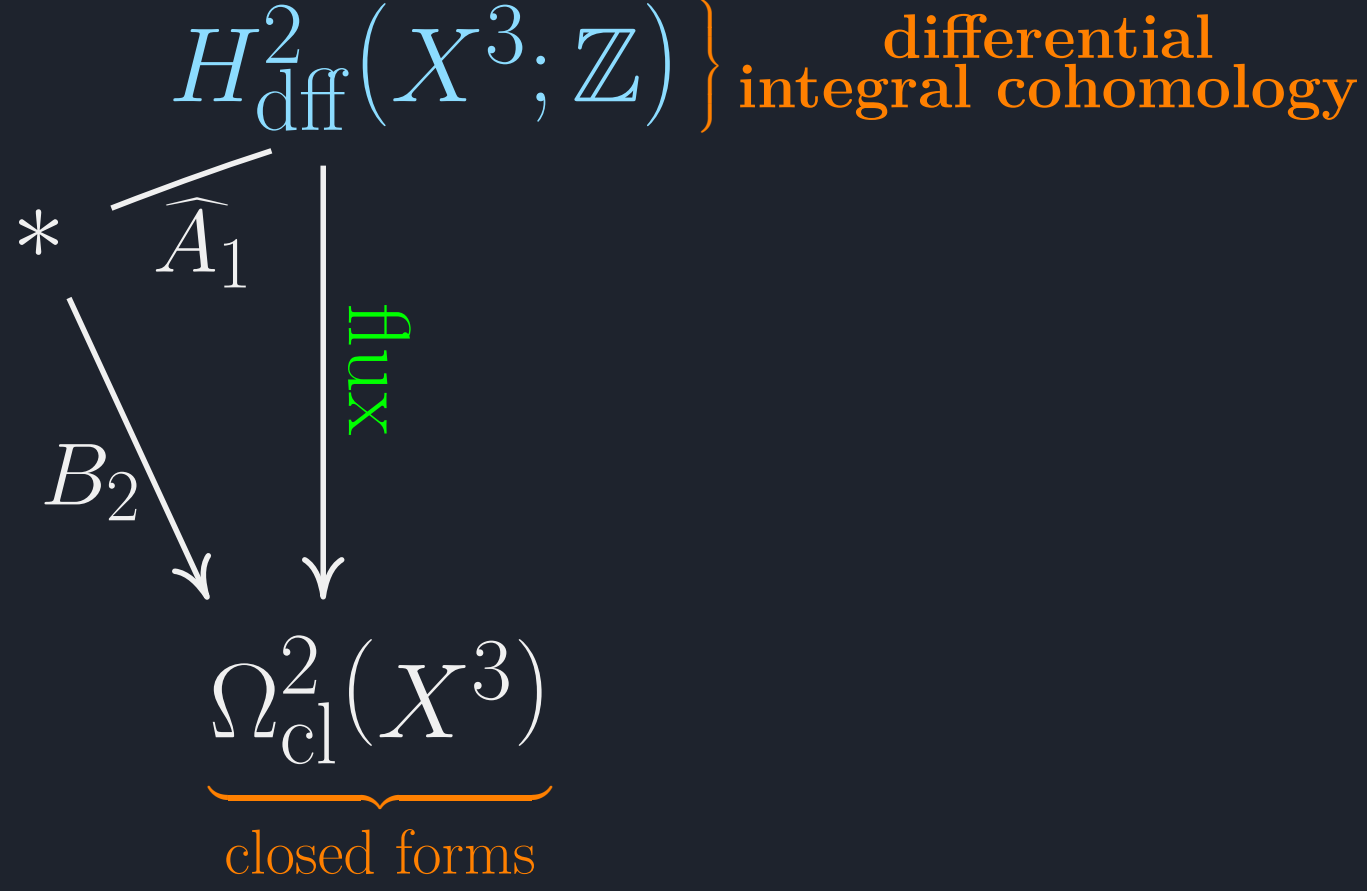


is Differential Ordinary Cohomology

Ordinary Magnetic Flux Quantization



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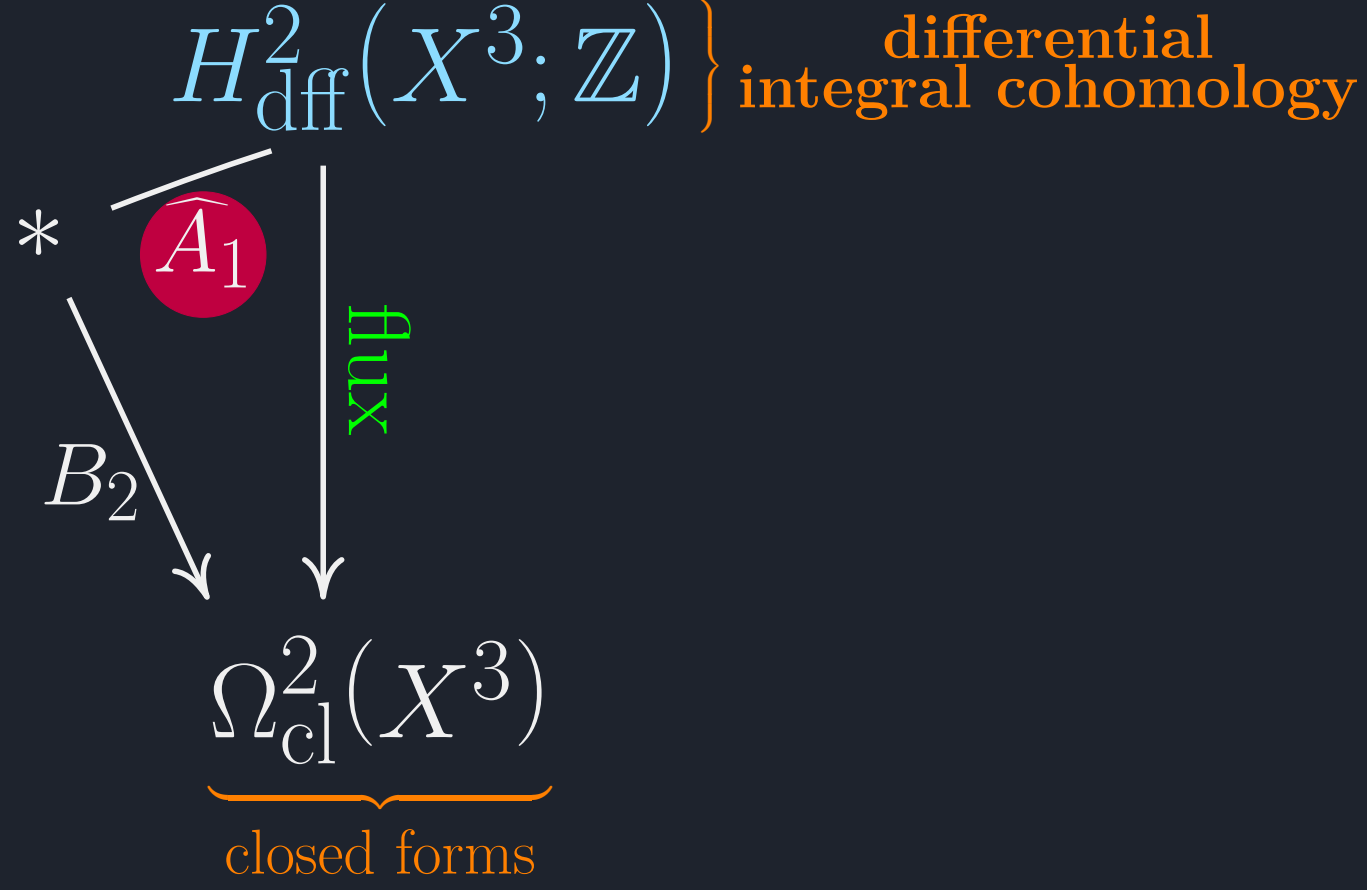
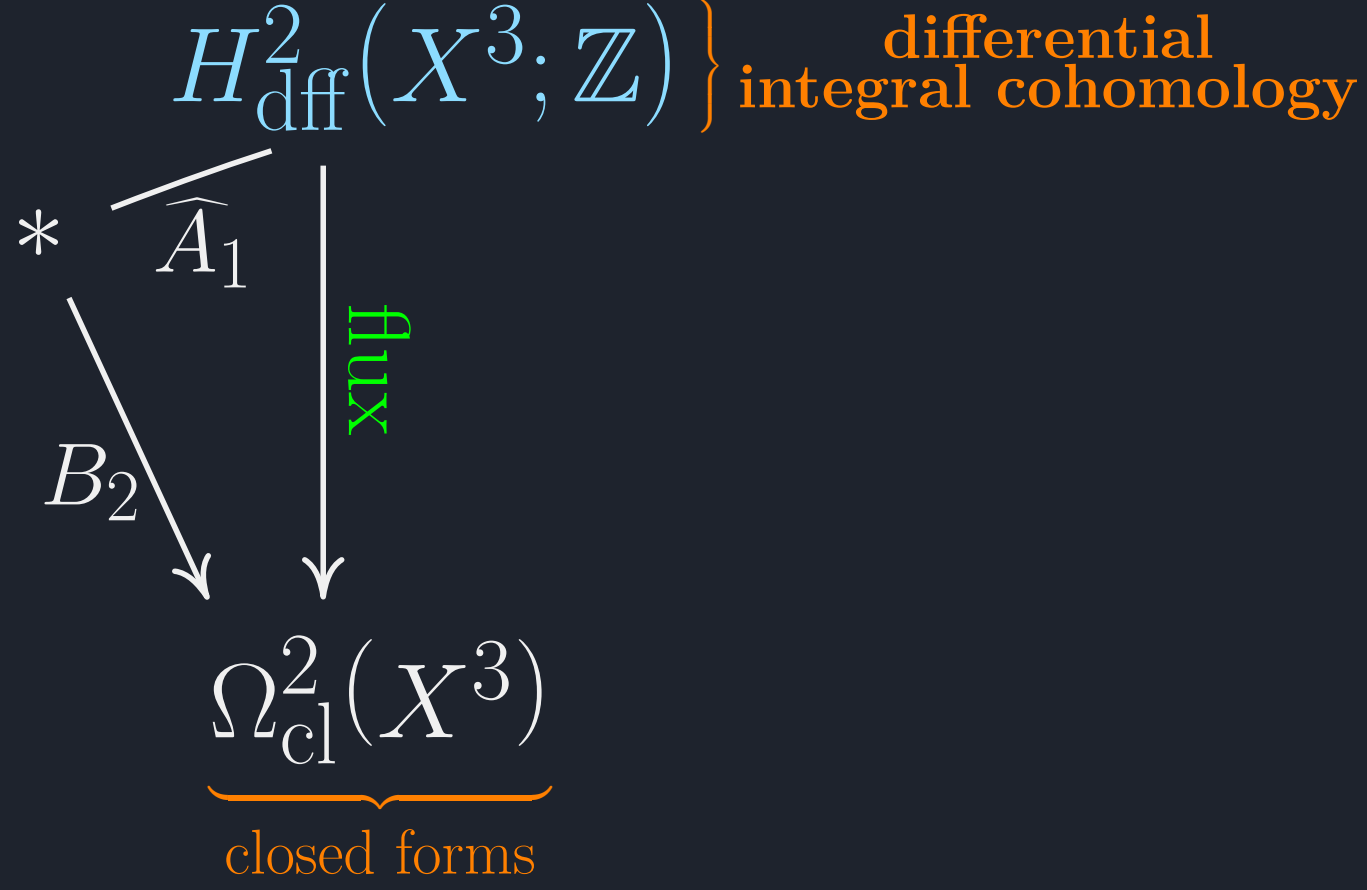


is Differential Ordinary Cohomology

$\simeq$ local gauge potentials & transition functions

Ordinary Magnetic Flux Quantization



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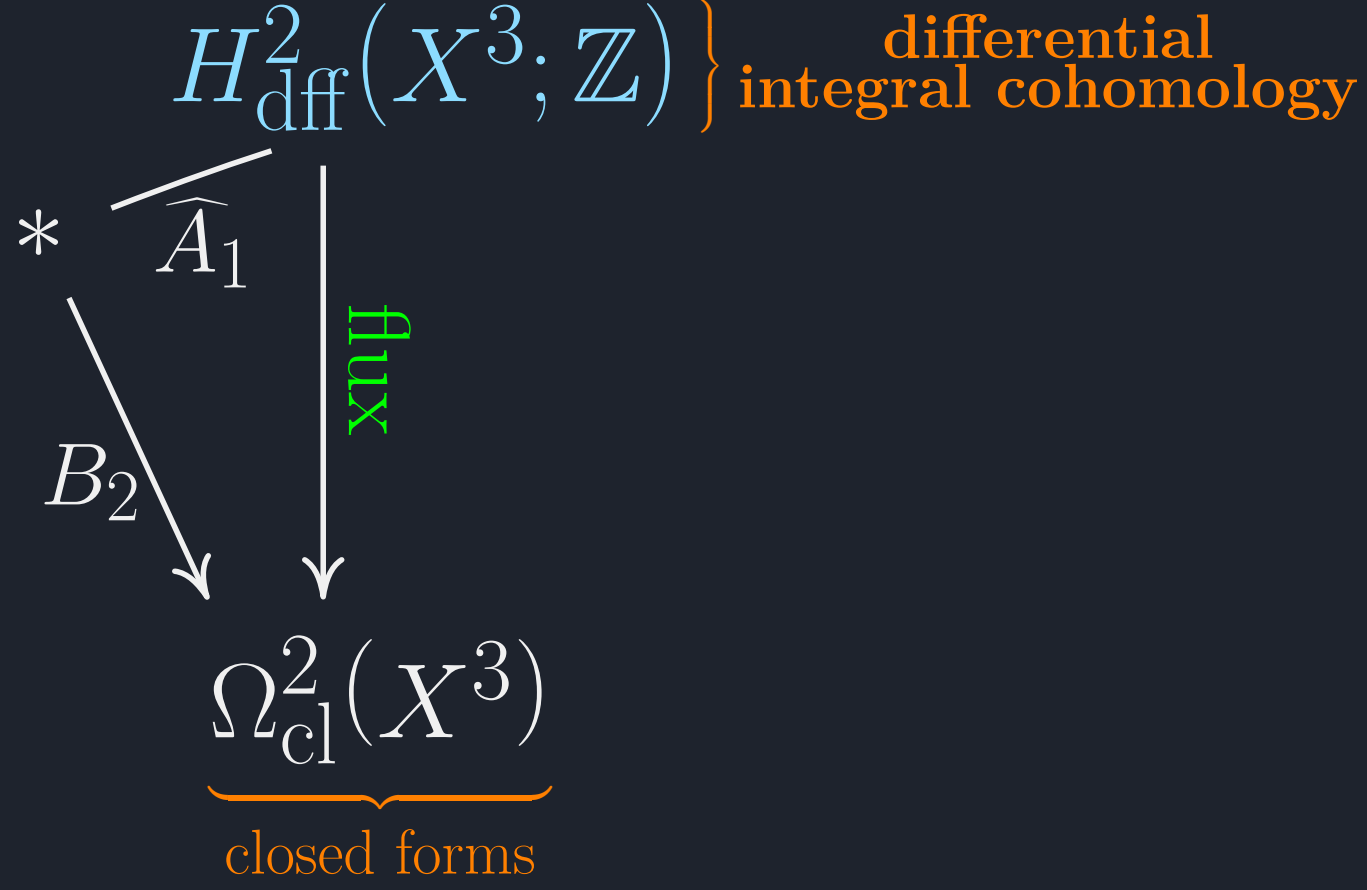
$\simeq$ local gauge potentials & transition functions

$\simeq$ U(1)-principal connections

Ordinary Magnetic Flux Quantization



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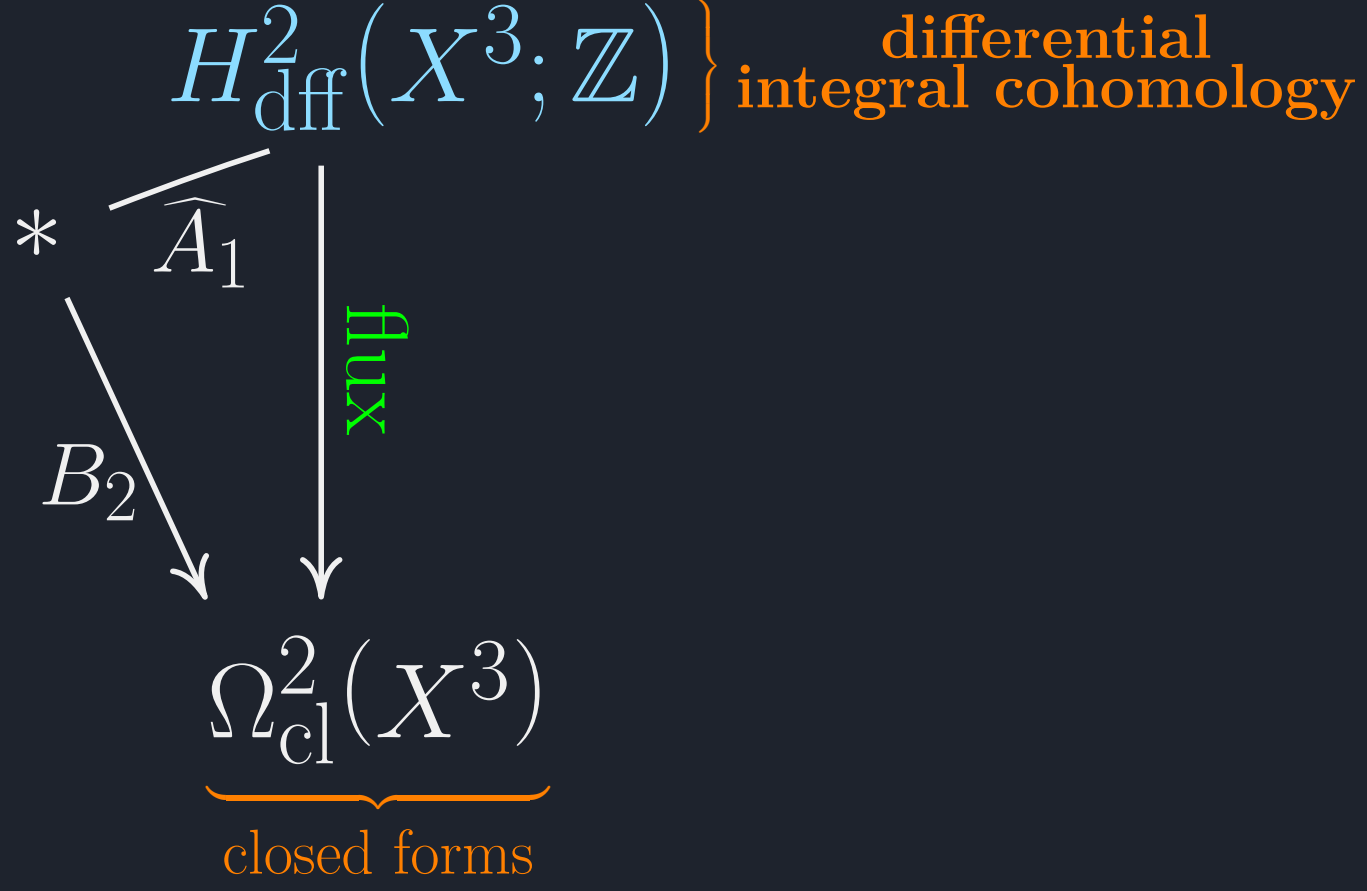

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- $\simeq$ local gauge potentials & transition functions
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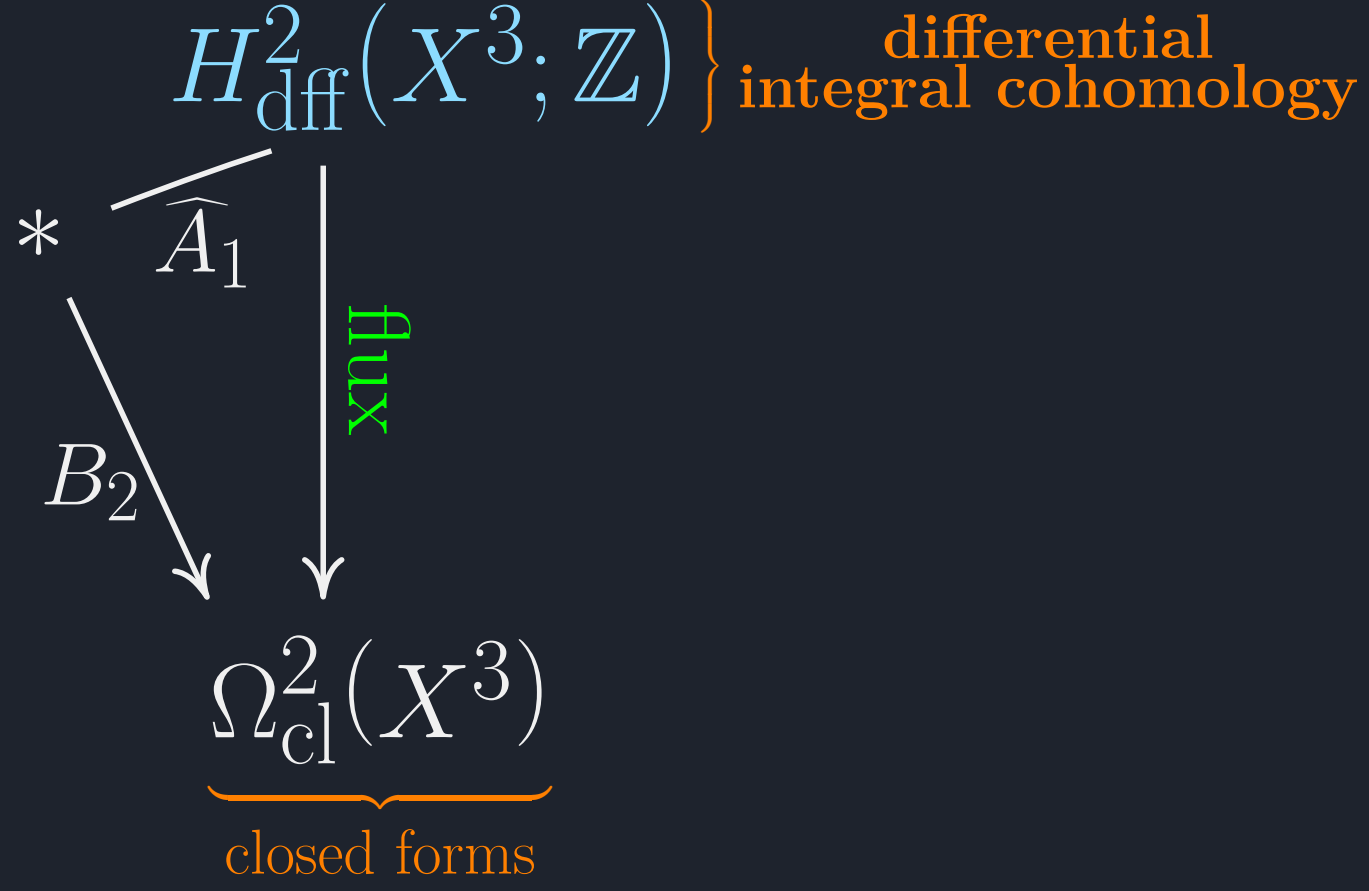
Ordinary Magnetic Flux Quantization



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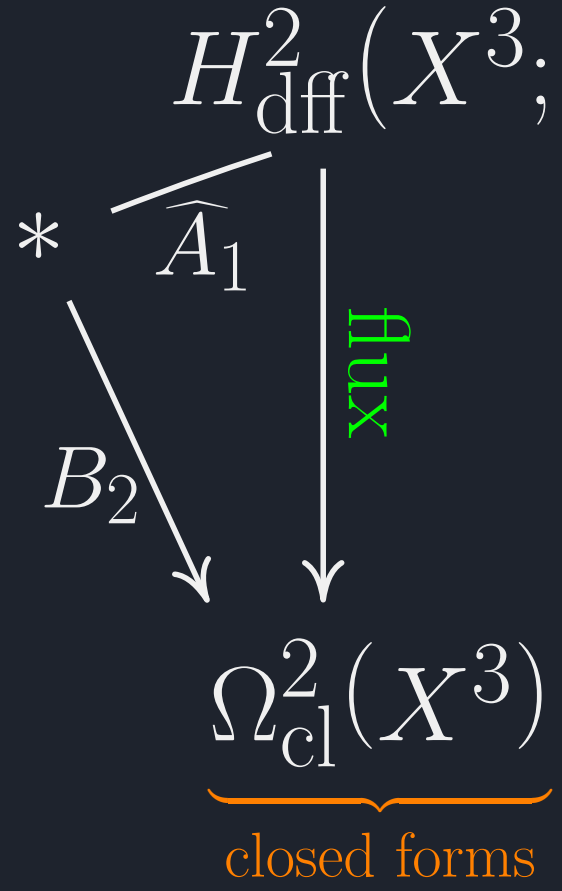

cancels anomaly in

$$\exp\left(\mathrm{ie}\int \widehat{A}_1\right)$$

[cf. Wen & Witten 1985 §5]

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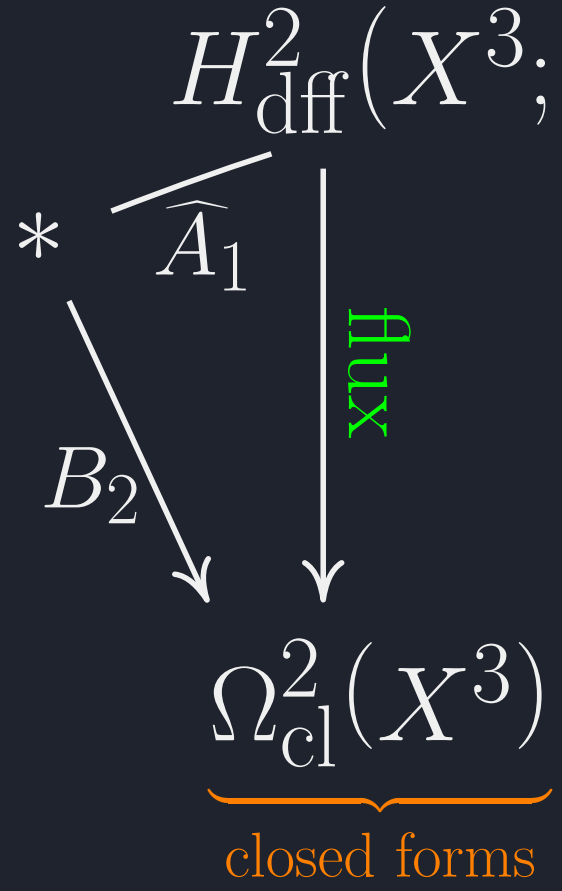

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action functional of electron in background field

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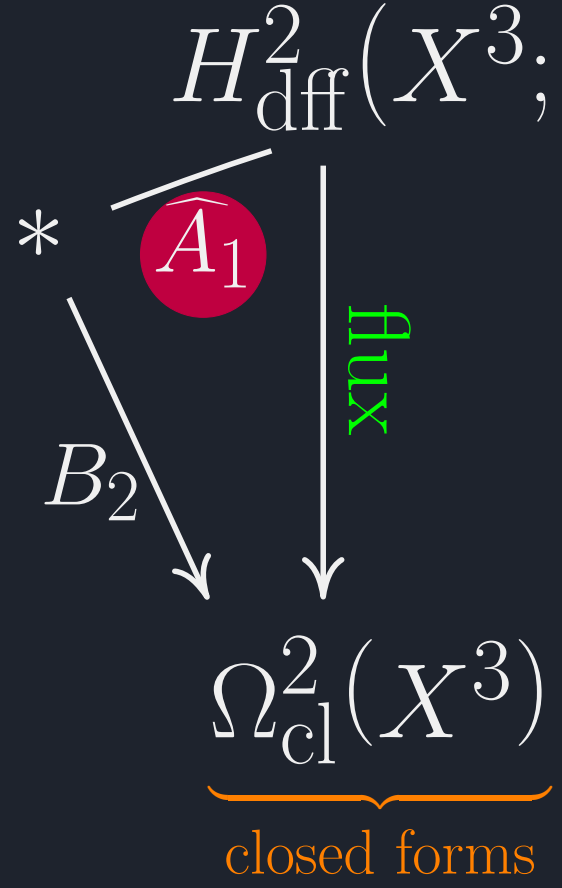

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Ordinary Magnetic Flux Quantization is Differential Ordinary Cohomology

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Higher Electromagnetic Flux Quantization

Ordinary Magnetic Flux Quantization
is Differential Ordinary Cohomology

Higher Electromagnetic Flux Quantization

is needed, e.g., for

making global sense of

the WZ action of an M5-brane

in 11D SuGra C-field background

Ordinary Magnetic Flux Quantization is Differential Ordinary Cohomology

Higher Electromagnetic Flux Quantization

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$$\exp\left(2\mathrm{i}T\int\left(\widehat{C}_6 - {}^1\!/_2\,\widehat{C}_3\wedge H_3\right)\right)$$

the WZ action of an M5-brane

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higher magnetic potential

the WZ action of an M5-brane

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higher electric potential — higher magnetic potential

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Ordinary Magnetic Flux Quantization
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Higher Electromagnetic Flux Quantization
is Differential Nonabelian Cohomology

making global sense of

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Differential Nonabelian Cohomology

Differential Nonabelian Cohomology

for general electromagnetic flux & charge quantization

Differential Nonabelian Cohomology

World Scientific Connect

Reviews in Mathematical Physics | to appear (2026)

Higher Gauge Theory via Differential Nonabelian Cohomology

Hisham Sati, Urs Schreiber

https://doi.org/10.48550/arXiv.2606.12534

Differential Nonabelian Cohomology

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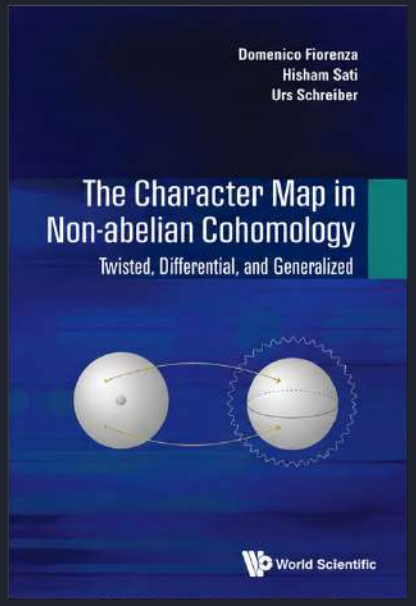

The Character Map in Non-abelian Cohomology

Twisted, Differential, and Generalized

Domenico Fiorenza • Hisham Sati • Urs Schreiber

https://doi.org/10.1142/13422 | September 2023

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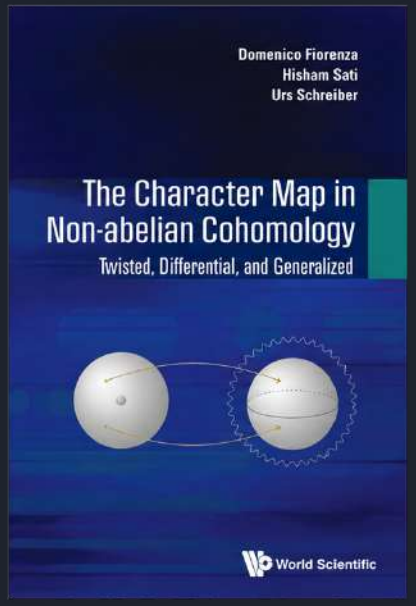

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Differential Nonabelian Cohomology

Differential Nonabelian Cohomology

nonabelian cohomology
classifying $\mathcal{G}$-principal bundles

$$H^1(X^d; \mathcal{G})$$

topological group

Differential Nonabelian Cohomology

nonabelian cohomology
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familiar case: $\mathcal{G} \equiv G$ a Lie group

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$$H^1\big(X^d;\mathcal{G}\big)$$

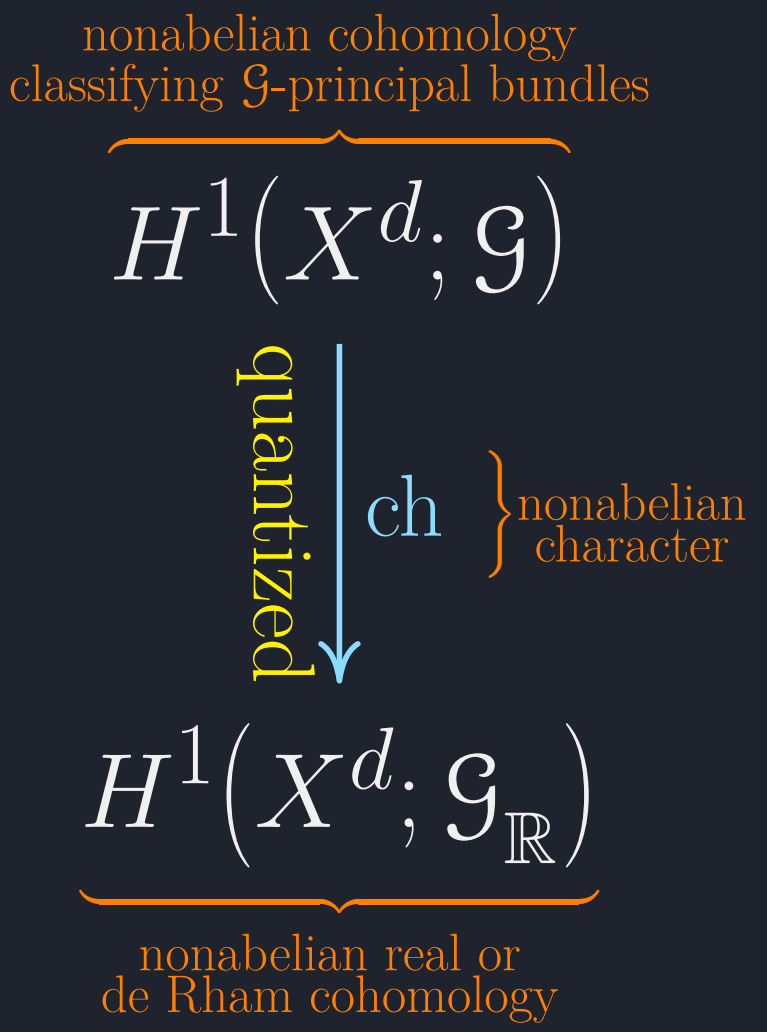
topological group

its real rationalization

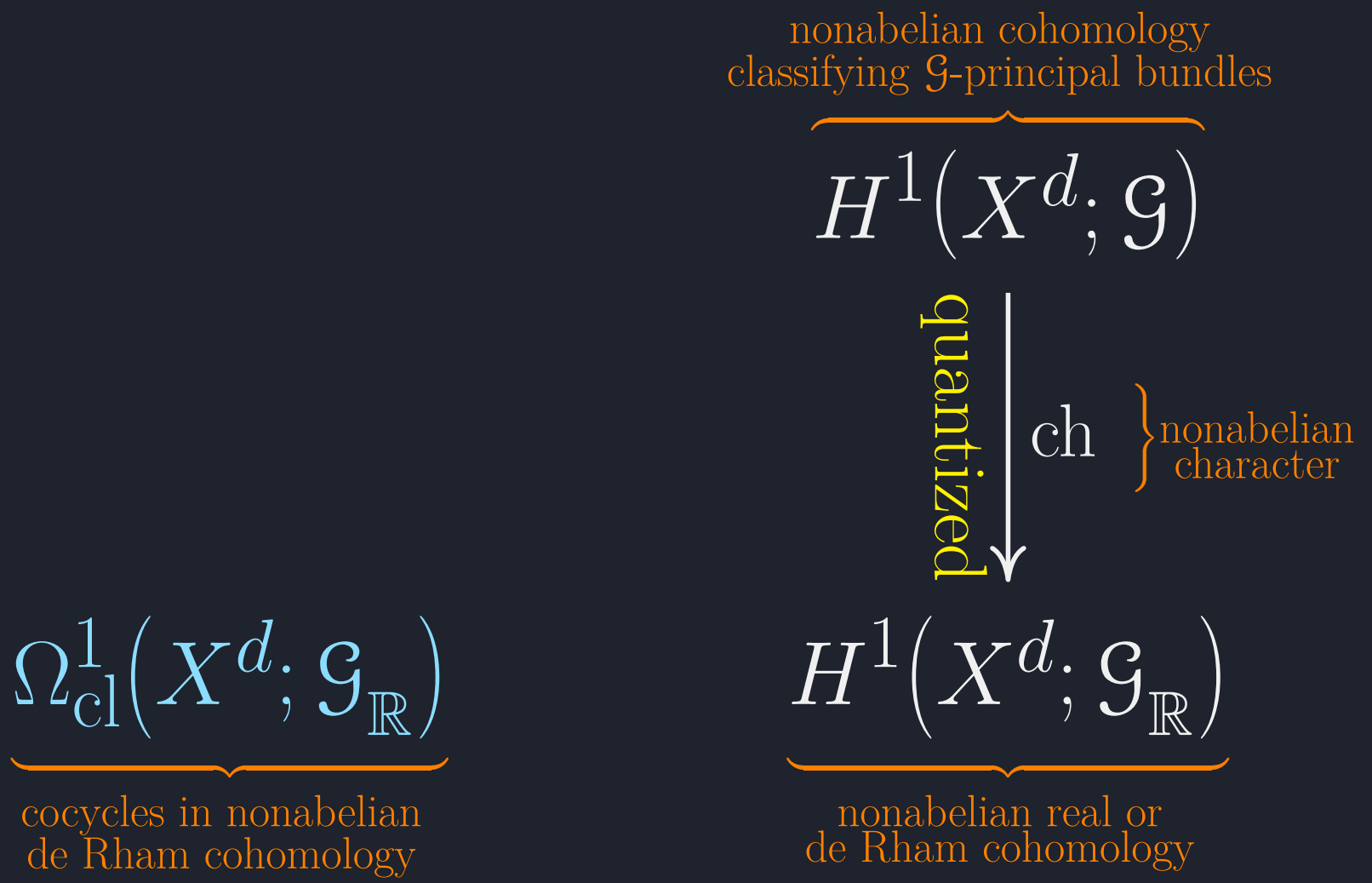
$$H^1\big(X^d;\mathcal{G}_{\mathbb{R}}\big)$$

nonabelian real or
de Rham cohomology

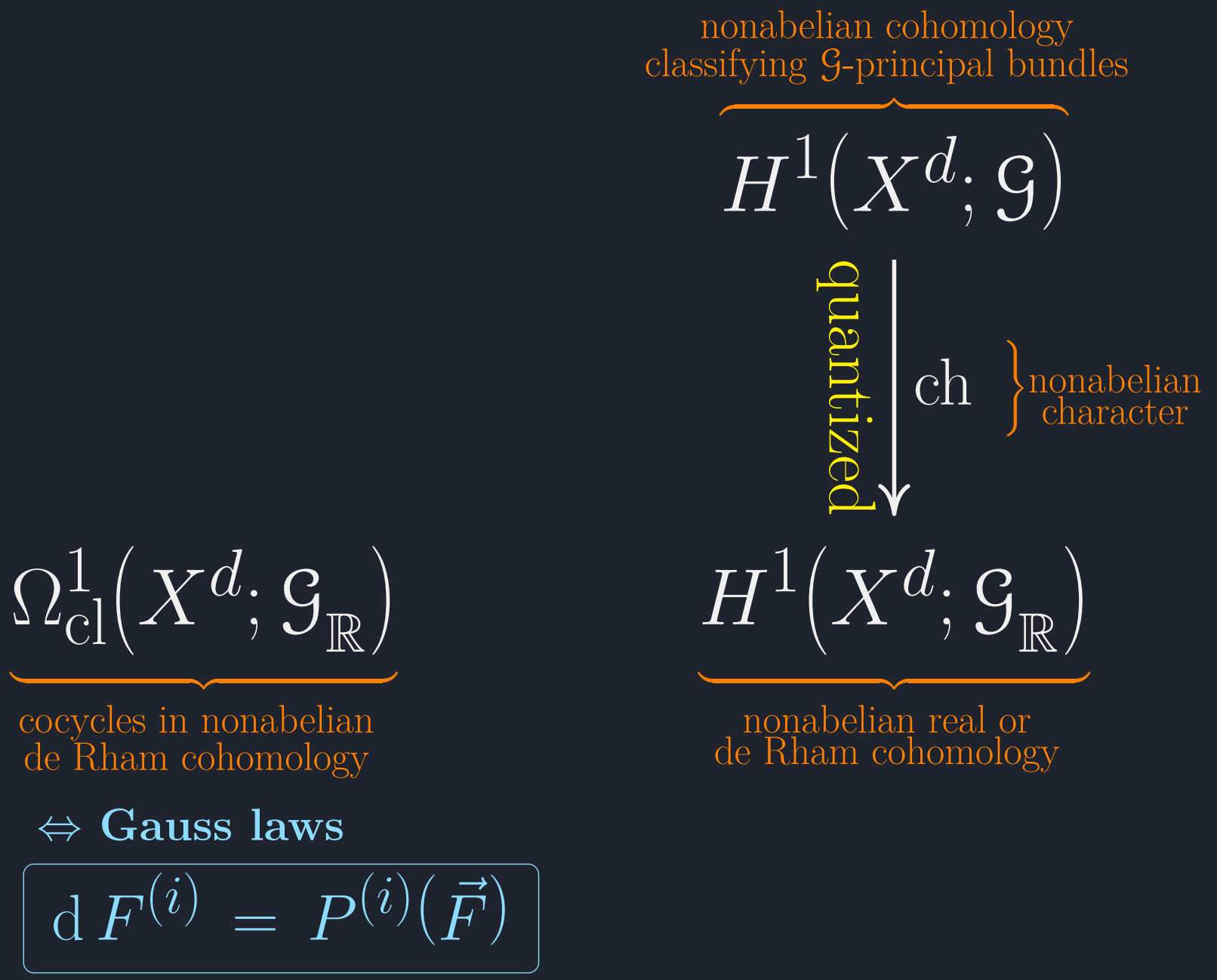
Differential Nonabelian Cohomology



Differential Nonabelian Cohomology



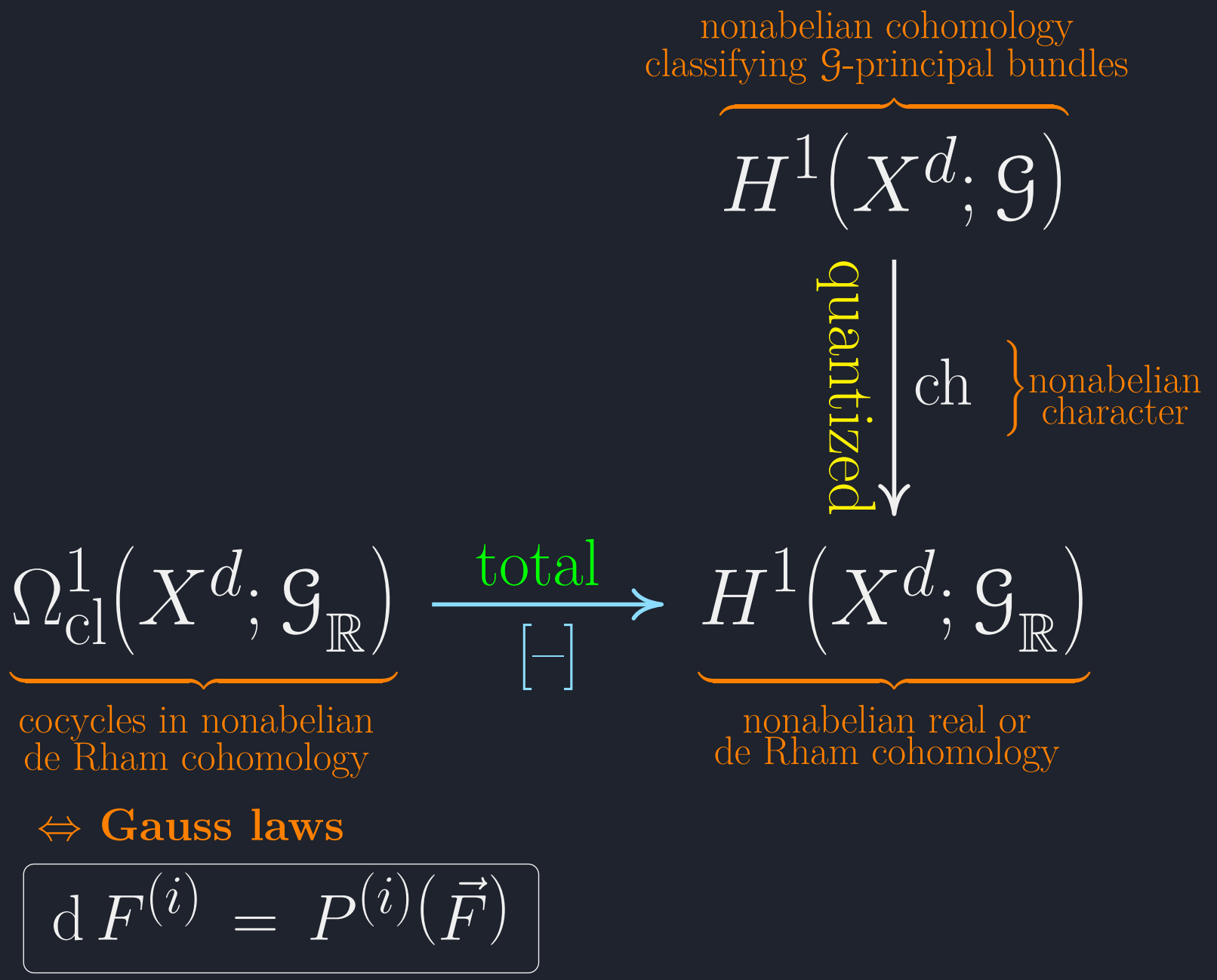
Differential Nonabelian Cohomology



$\Leftrightarrow$ **Gauss laws**

$$\mathrm{d}\,F^{(i)} \;=\; P^{(i)}\big(\vec{F}\big)$$

Differential Nonabelian Cohomology



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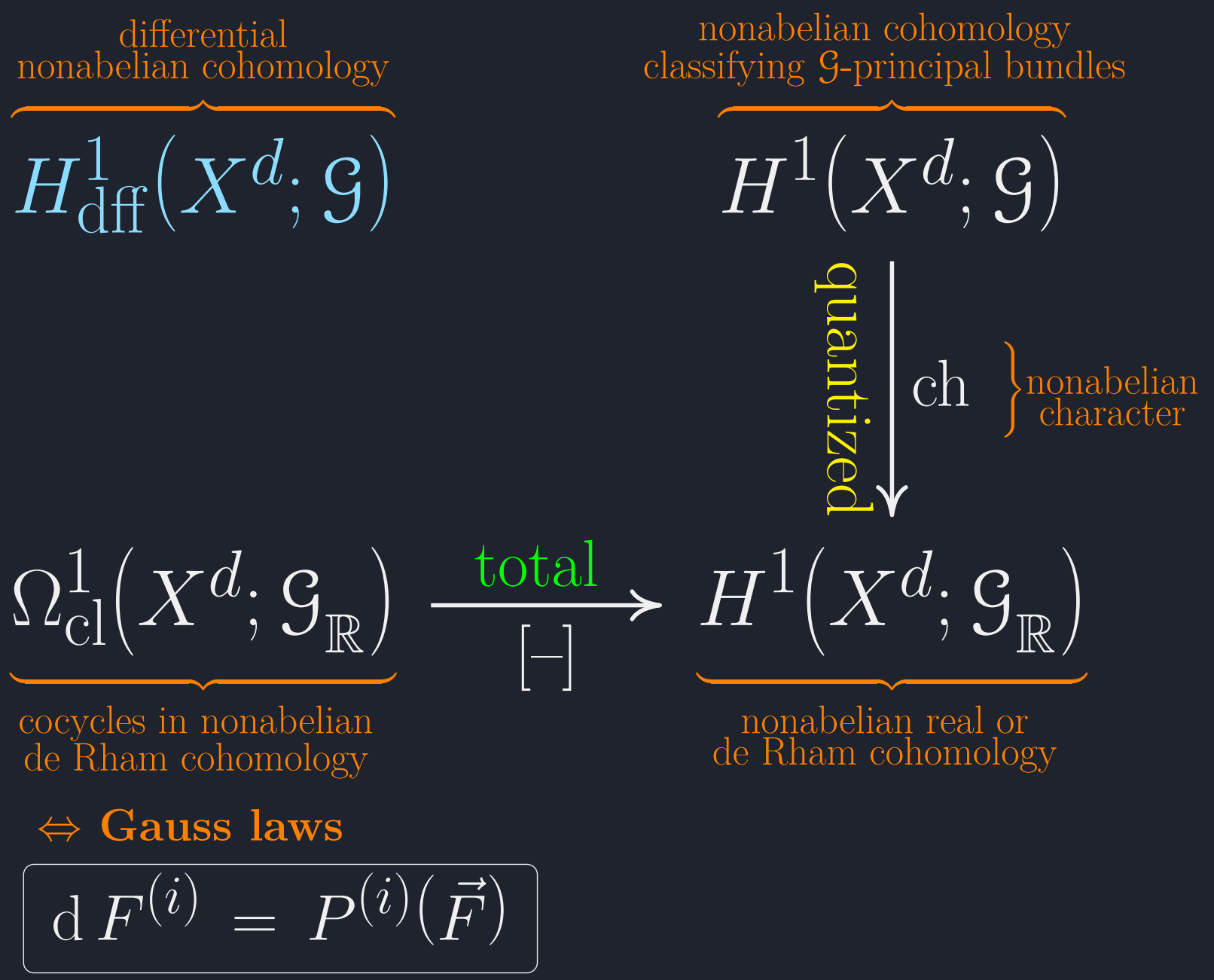

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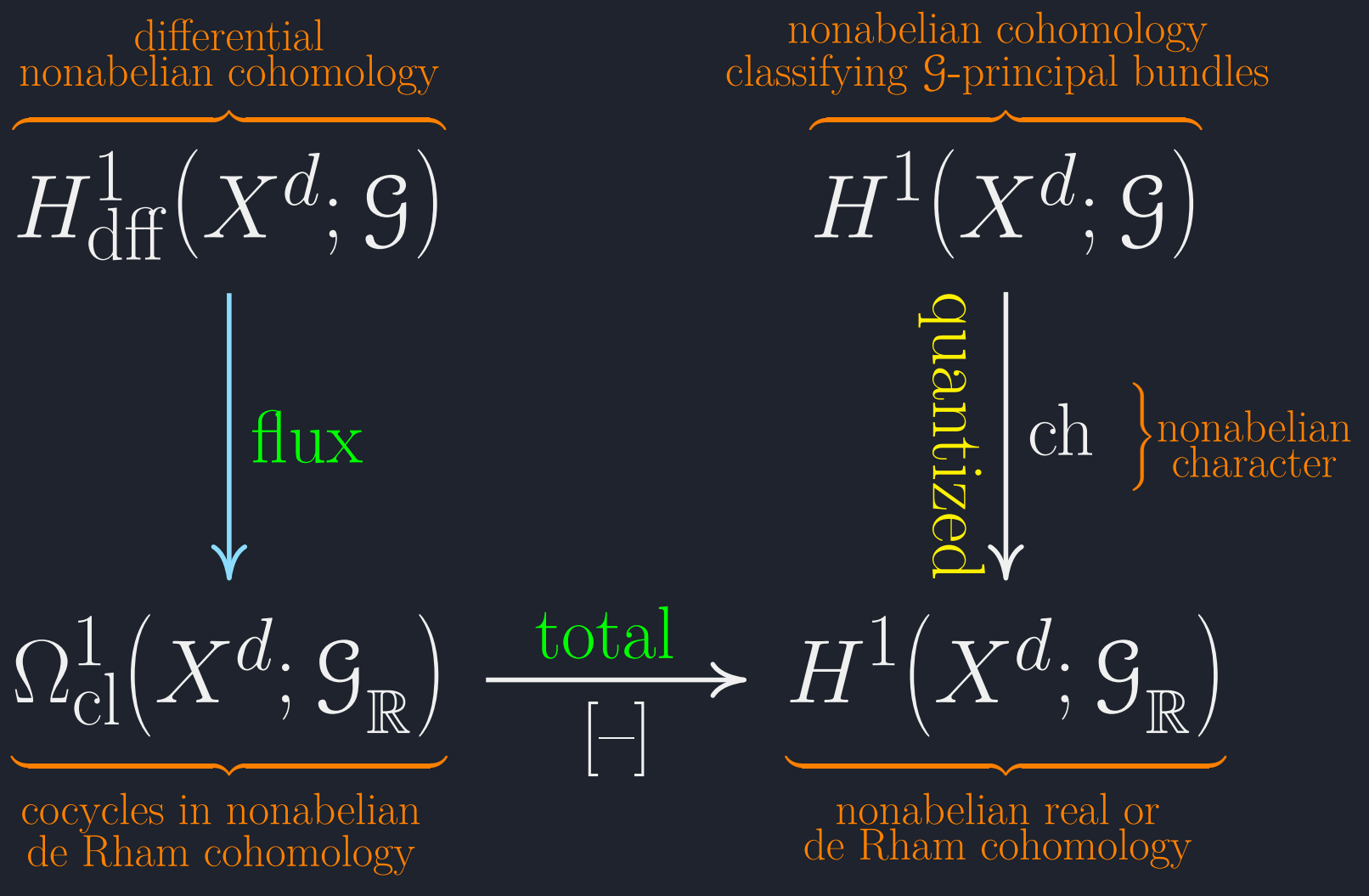

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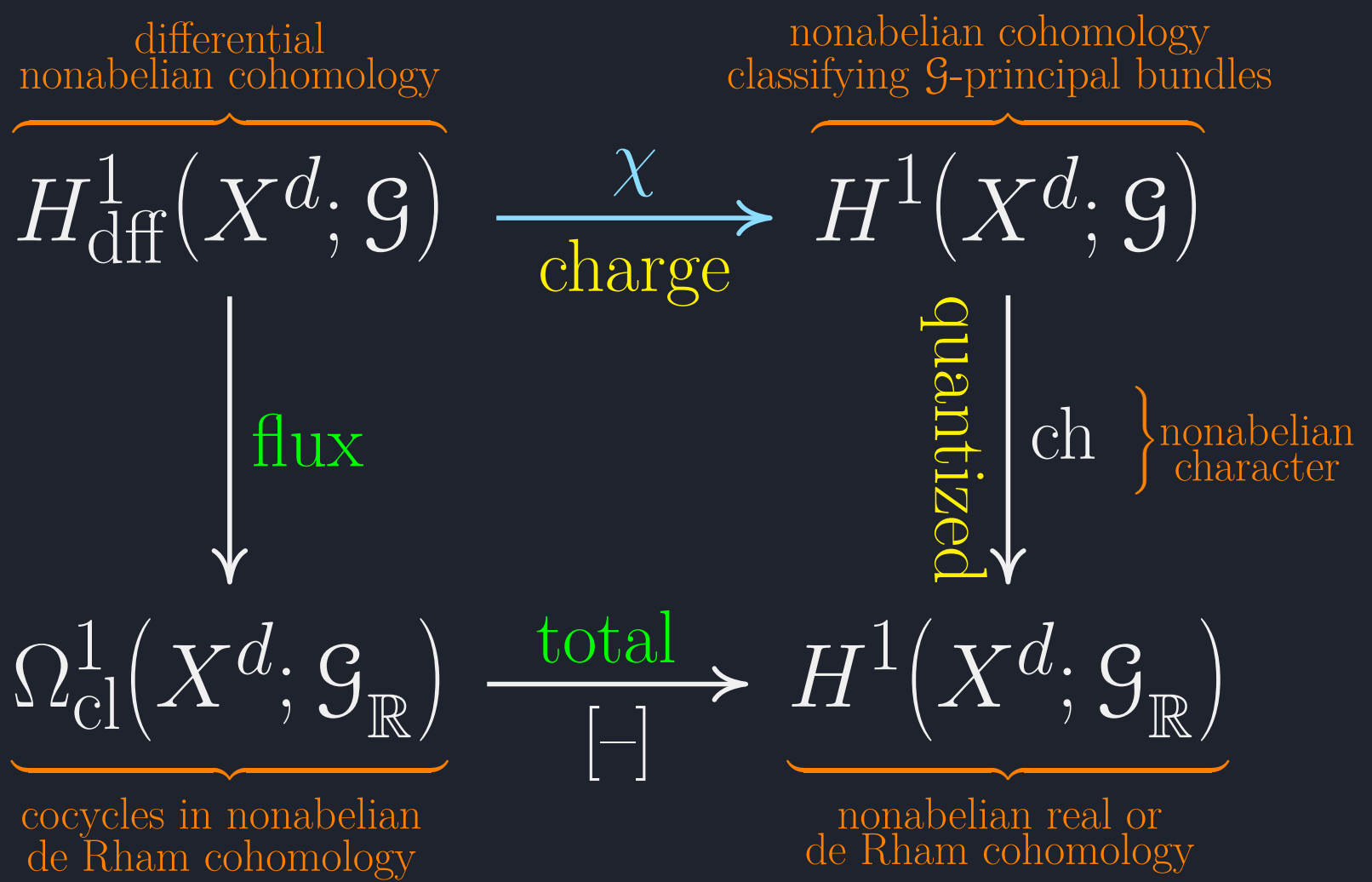

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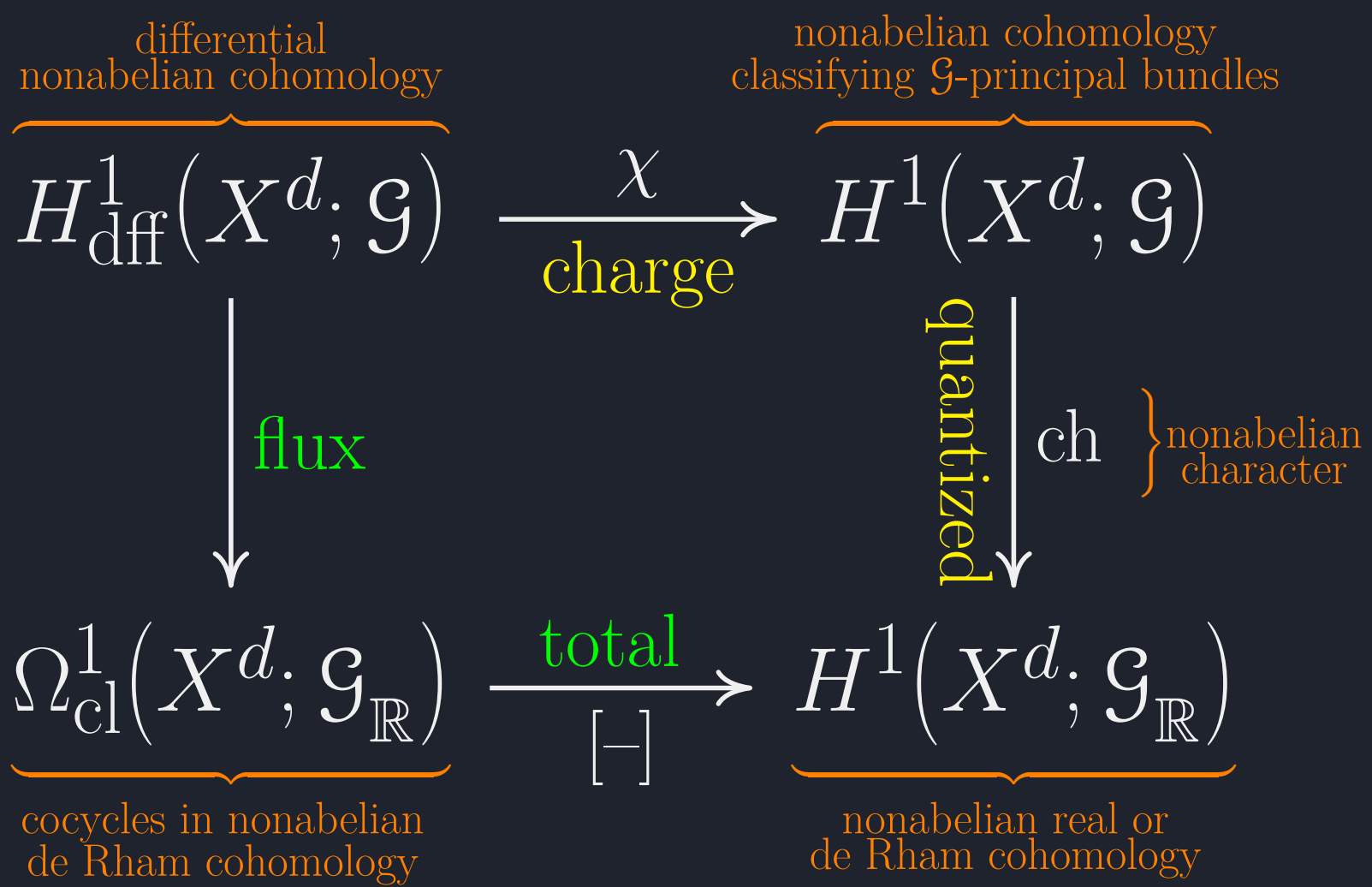

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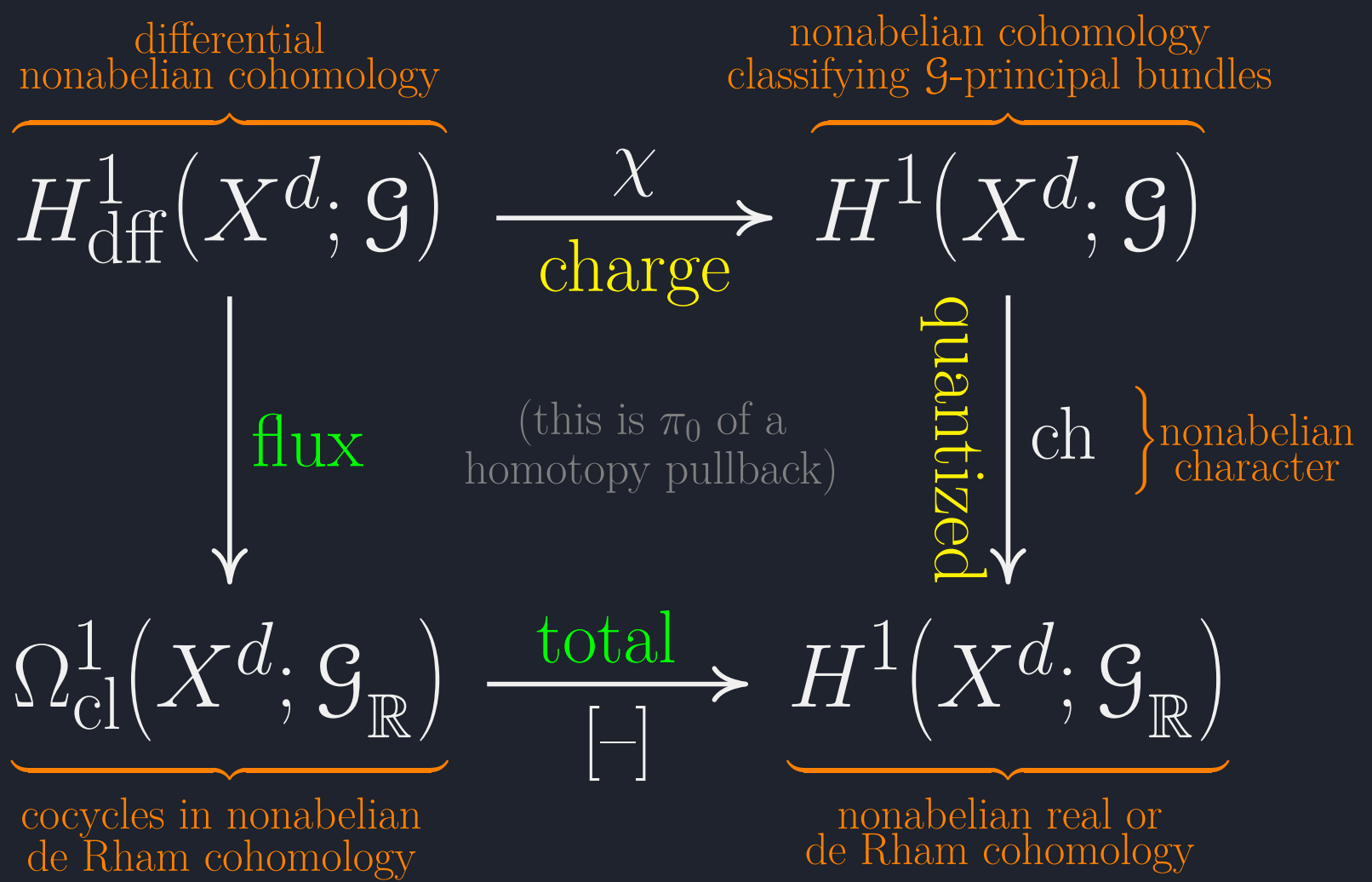

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Quantization of RR-Flux in K-Theory

Quantization of RR-Flux in K-Theory

Gauss laws

$$\mathrm{d}\,F_2 = 0 \qquad \mathrm{d}\,H_3 = 0$$

$$\mathrm{d}\,F_4 = H_3 \wedge F_2$$

magnetic

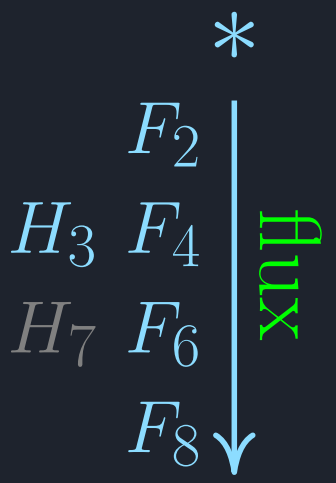
$\updownarrow$

electric

$$\mathrm{d}\,F_6 = H_3 \wedge F_4$$

$$\mathrm{d}\,F_8 = H_3 \wedge F_6 \qquad \mathrm{d}\,H_7 = {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6$$

Quantization of RR-Flux in K-Theory



$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^9;\quad (???)_{\,\mathbb{R}}\big)}$$

twisted de Rham cocycles

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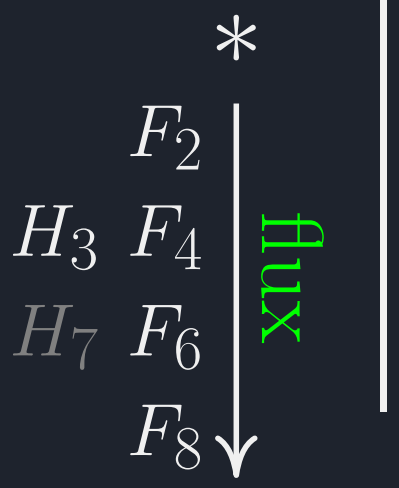
magnetic $\updownarrow$ electric

Quantization of RR-Flux in K-Theory

Fact: A natural candidate space exists



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twisted de Rham cocycles
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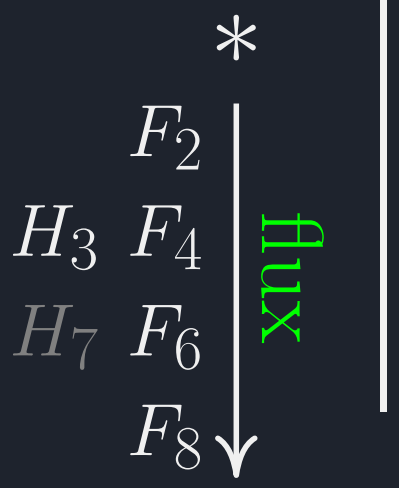

$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cocycles}}$$

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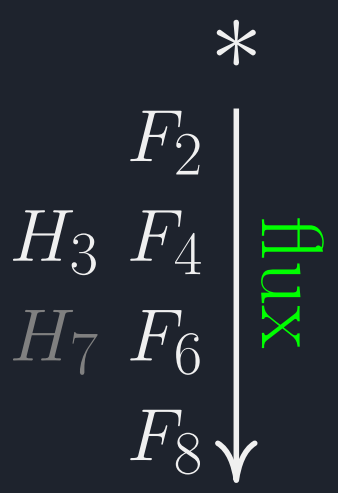
Quantization of RR-Flux in K-Theory



Fact: A natural candidate space exists:

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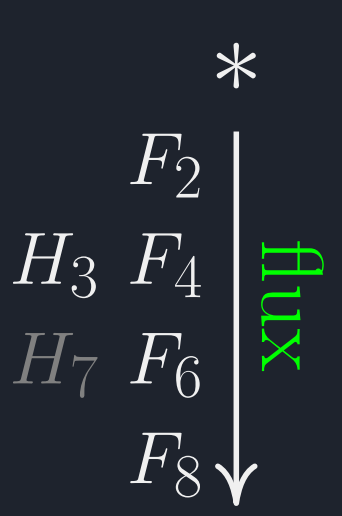
Quantization of RR-Flux in K-Theory



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$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cocycles}}$$

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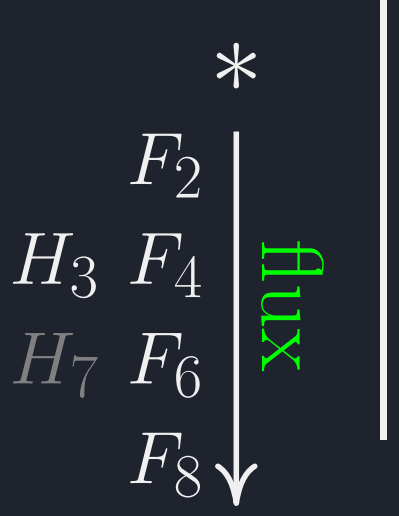

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twisted de Rham cocycles

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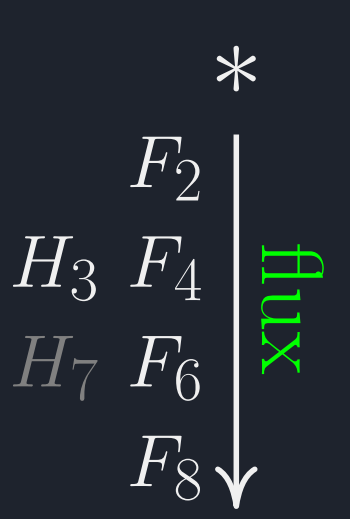
Quantization of RR-Flux in K-Theory



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twisted de Rham cocycles
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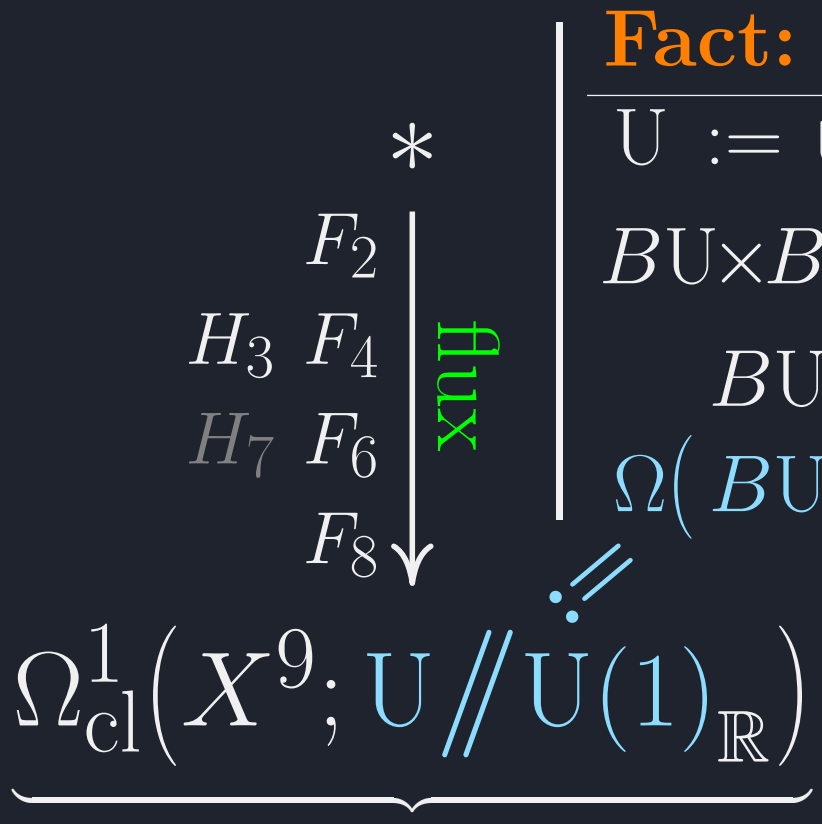
Quantization of RR-Flux in K-Theory



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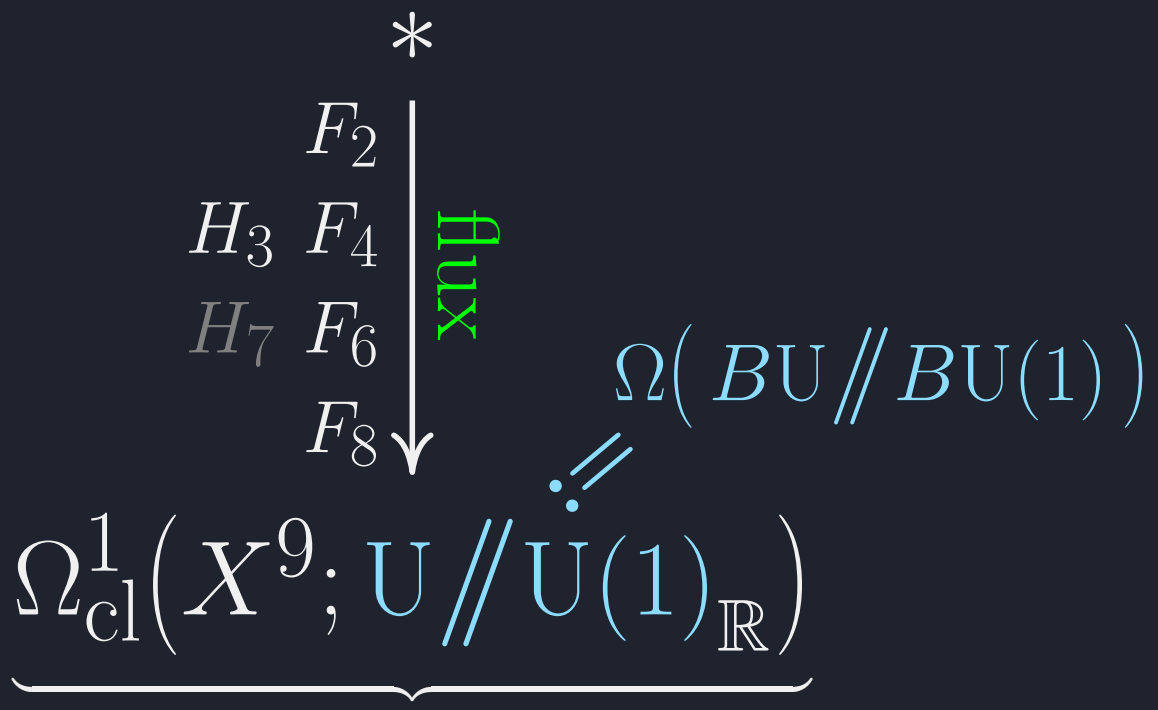
Quantization of RR-Flux in K-Theory



twisted de Rham cocycles

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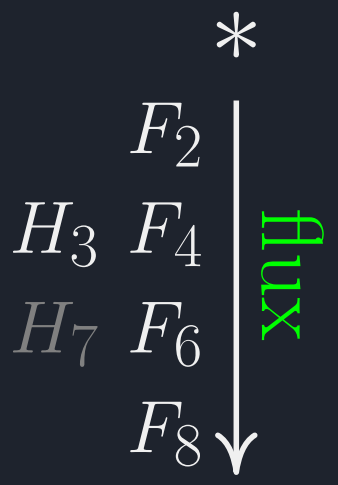
magnetic $\updownarrow$ electric

Quantization of RR-Flux in K-Theory

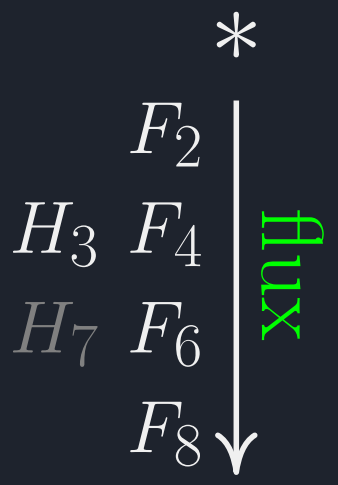


$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cocycles}} \qquad \underbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cohomology}}$$

$\Leftrightarrow$ **Gauss laws**

$$\begin{array}{ll} \mathrm{d}\,F_2 = 0 & \mathrm{d}\,H_3 = 0 \\ \mathrm{d}\,F_4 = H_3 \wedge F_2 & \text{magnetic} \\ \hline \mathrm{d}\,F_6 = H_3 \wedge F_4 & \updownarrow \\ \mathrm{d}\,F_8 = H_3 \wedge F_6 & \text{electric} \\ & \mathrm{d}\,H_7 = {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6 \end{array}$$

Quantization of RR-Flux in K-Theory



$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cocycles}} \xrightarrow[[-]]{\text{total}} \underbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)_{\mathbb{R}}\big)}_{\text{twisted de Rham cohomology}}$$

$\Leftrightarrow$ **Gauss laws**

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\mathrm{d}\,F_2 &= 0 & \mathrm{d}\,H_3 &= 0 \\
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\hline
\mathrm{d}\,F_6 &= H_3 \wedge F_4 & & \\
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\end{aligned}$$

magnetic $\updownarrow$ electric

Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9;\mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

$$\begin{array}{c} * \\ F_2 \\ H_3\ F_4 \\ H_7\ F_6 \\ F_8 \end{array} \Big\downarrow \text{flux}$$

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magnetic

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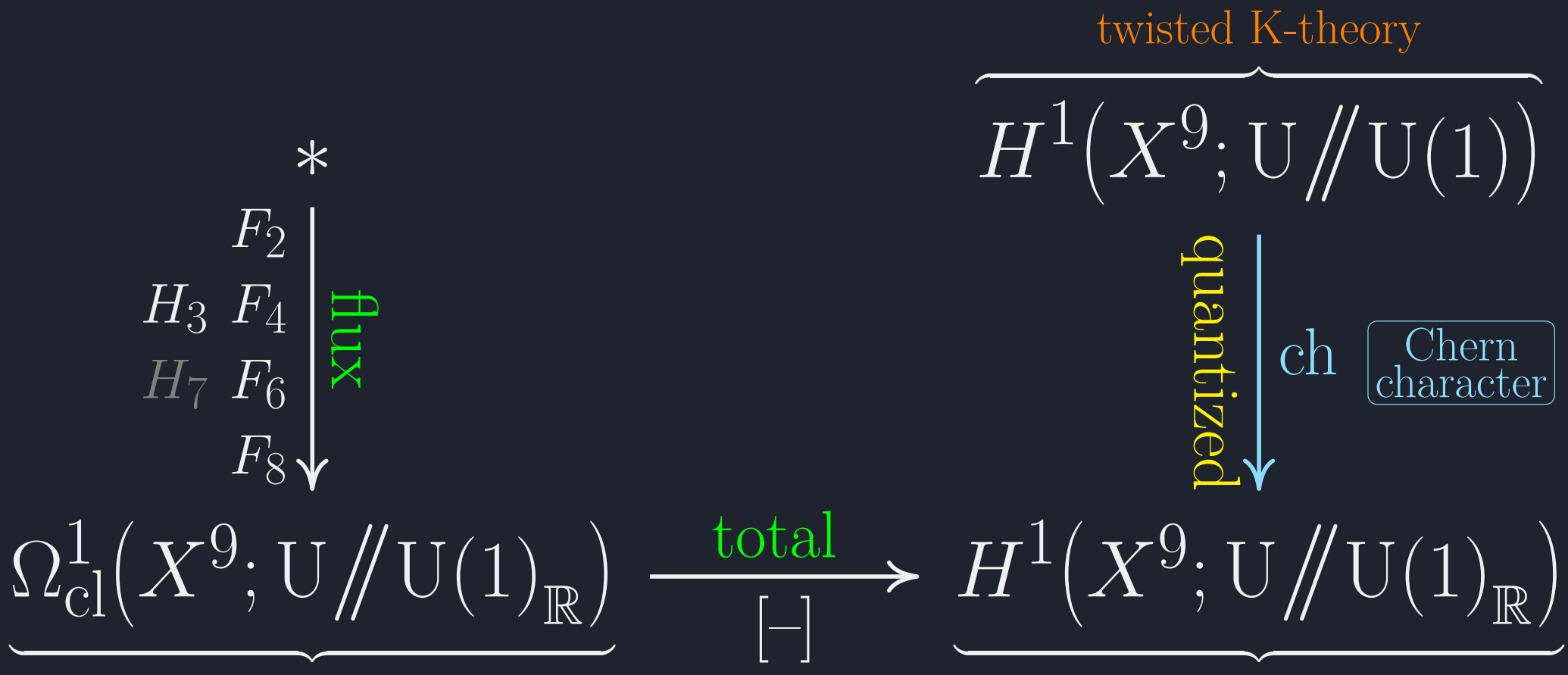
Quantization of RR-Flux in K-Theory



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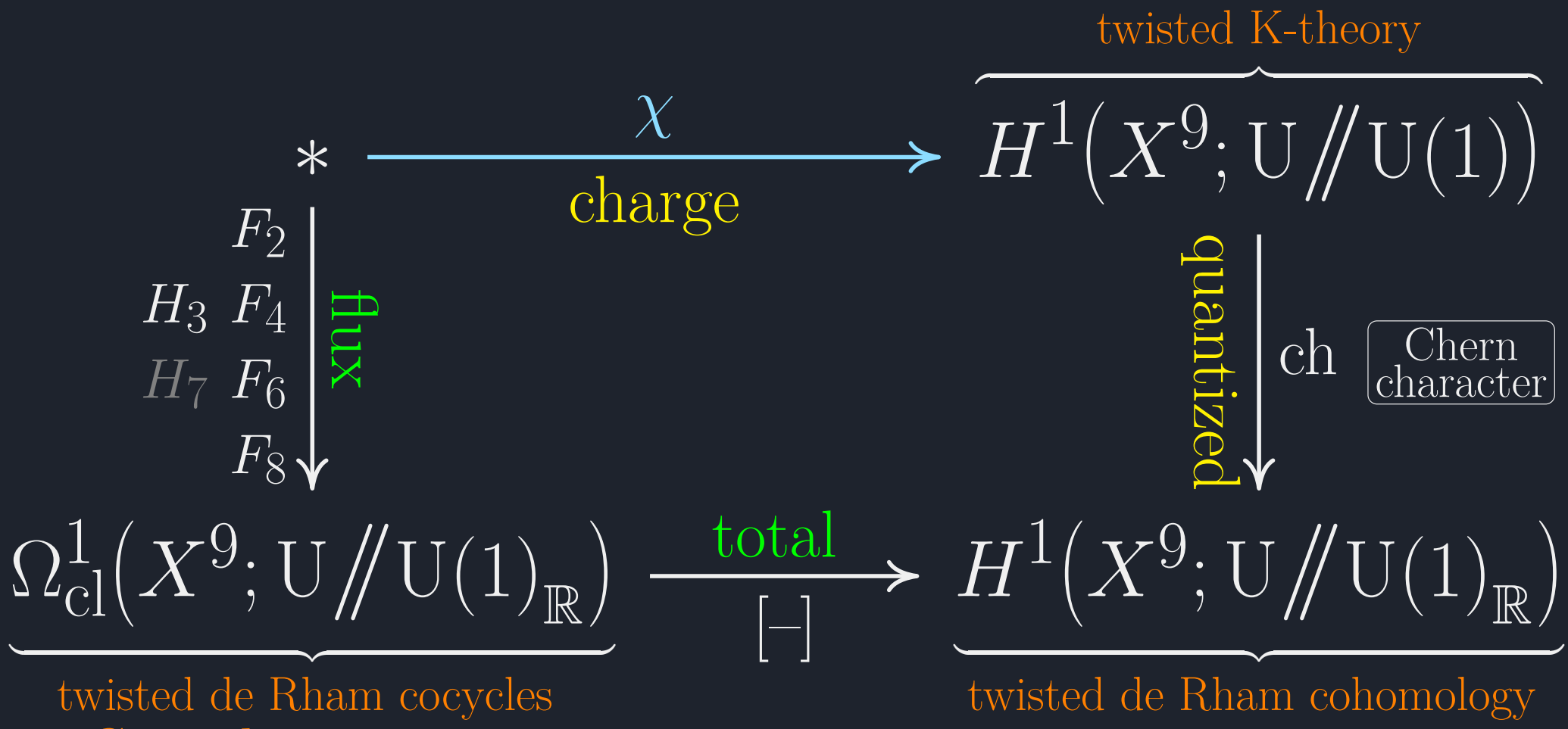
Quantization of RR-Flux in K-Theory



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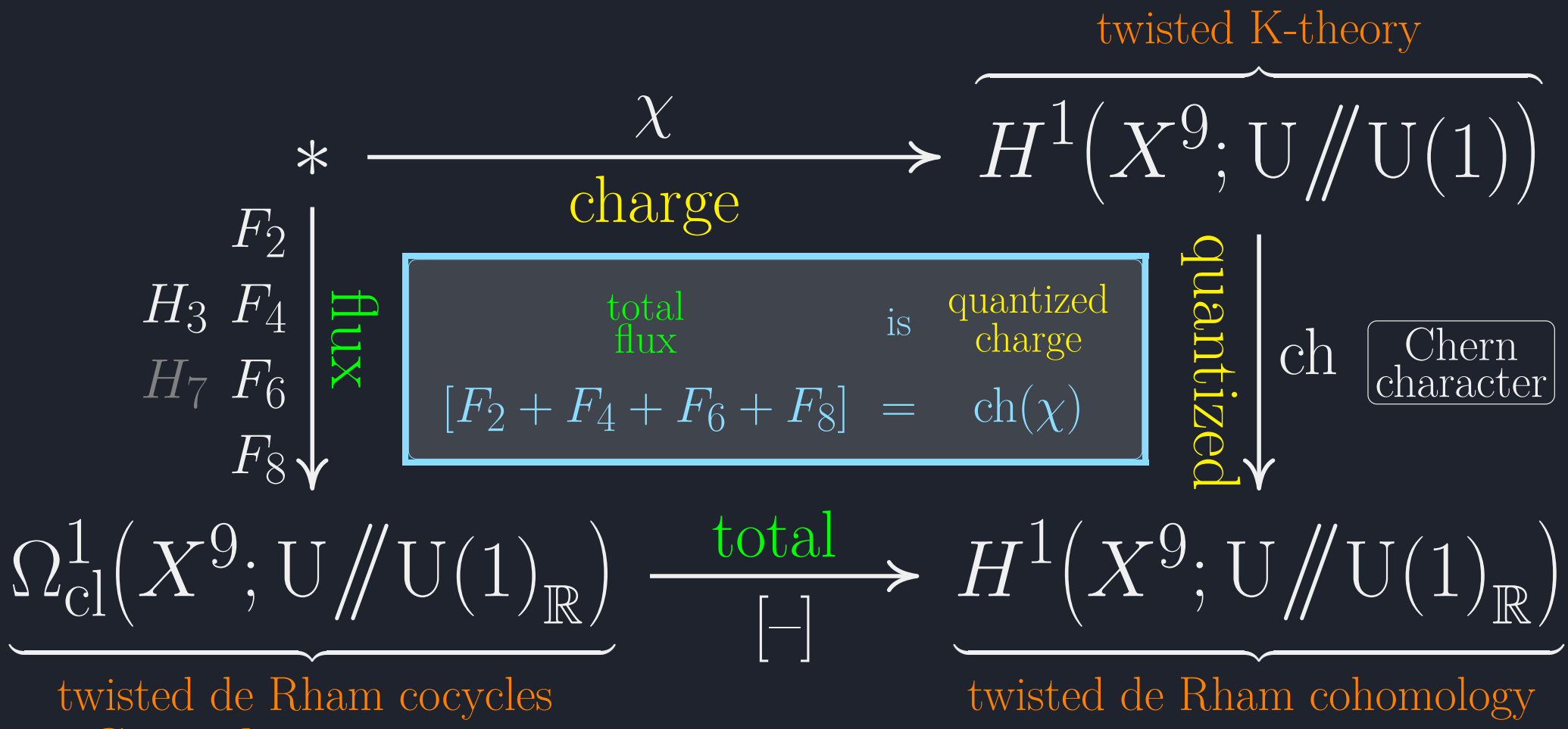
Quantization of RR-Flux in K-Theory



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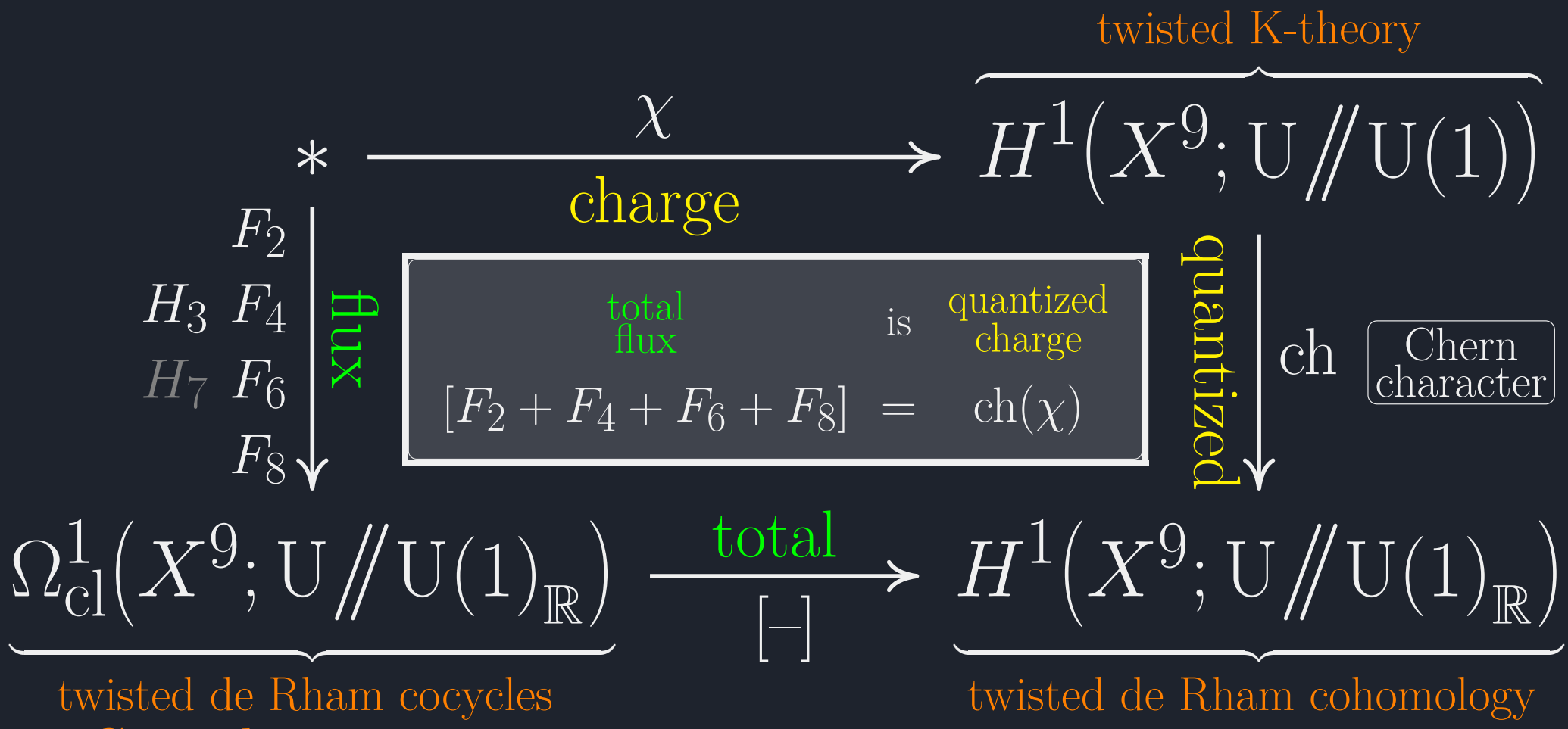

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Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

$\Rightarrow$ **Gauss laws**

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Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

Physics Problems:

$$\begin{array}{ll} \mathrm{d}\,F_2 = 0 & \mathrm{d}\,H_3 = 0 \\ \mathrm{d}\,F_4 = H_3 \wedge F_2 & \\ \hline \mathrm{d}\,F_6 = H_3 \wedge F_4 & \\ \mathrm{d}\,F_8 = H_3 \wedge F_6 & \mathrm{d}\,H_7 = {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6 \end{array}$$

magnetic ↕ electric

Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

Physics Problems:	Math Issues:

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\mathrm{d}\,F_4 &= H_3 \wedge F_2 & & \\
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\mathrm{d}\,F_6 &= H_3 \wedge F_4 & & \\
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magnetic ↕ electric

Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

Physics Problems:	**Math Issues:**
H_7-flux not accounted	

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\mathrm{d}\,F_6 &= H_3 \wedge F_4 & & \\
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Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

Physics Problems:	**Math Issues:**
H_7-flux not accounted	has non-linear Gauss law

$$\begin{aligned}
\mathrm{d}\,F_2 &= 0 & \mathrm{d}\,H_3 &= 0\\
\mathrm{d}\,F_4 &= H_3 \wedge F_2 & & \text{magnetic}\\
& & & \updownarrow\\
\mathrm{d}\,F_6 &= H_3 \wedge F_4 & & \text{electric}\\
\mathrm{d}\,F_8 &= H_3 \wedge F_6 & \mathrm{d}\,H_7 &= {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6
\end{aligned}$$

Quantization of RR-Flux in K-Theory

$$\overbrace{H^1\big(X^9; \mathrm{U}/\!\!/\mathrm{U}(1)\big)}^{\text{twisted K-theory}}$$

Physics Problems:	**Math Issues:**
H_7-flux not accounted	has non-linear Gauss law
$\Rightarrow$ NS1 not quantized	

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Quantization of RR-Flux in K-Theory

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Physics Problems:	**Math Issues:**
H_7-flux not accounted	has non-linear Gauss law
$\Rightarrow$ NS1 not quantized	needs nonabelian cohomology

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$\Rightarrow$ no lift to 11D	

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Physics Problems:	**Math Issues:**
H_7-flux not accounted	has non-linear Gauss law
$\Rightarrow$ NS1 not quantized	needs nonabelian cohomology
$\Rightarrow$ no lift to 11D	
$\Rightarrow$ no S-duality	

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Next!

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\hline
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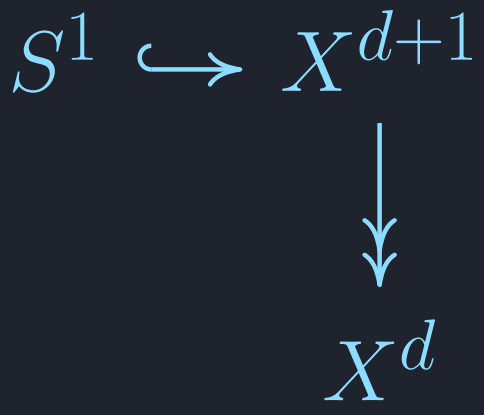
\end{aligned}$$

magnetic $\updownarrow$ electric

Dim-Reduction in Nonabelian Cohomology

Dim-Reduction in Nonabelian Cohomology

for principal
circle fibration



Dim-Reduction in Nonabelian Cohomology

for principal
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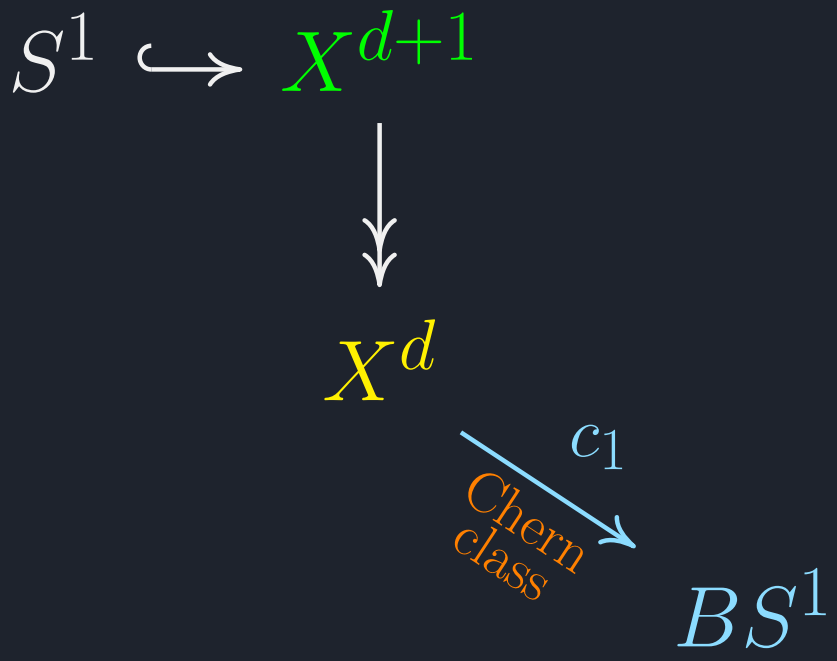

Dim-Reduction in Nonabelian Cohomology

for principal circle fibration

$$S^1 \hookrightarrow X^{d+1} \twoheadrightarrow X^d \xrightarrow{c_1} BS^1$$

(Chern class)

we have

$$H^1(X^{d+1}; \mathcal{G}) \cong H^1_{c_1}(X^d; \; ? \; ? \; ? \;)$$

Dim-Reduction in Nonabelian Cohomology

for principal circle fibration

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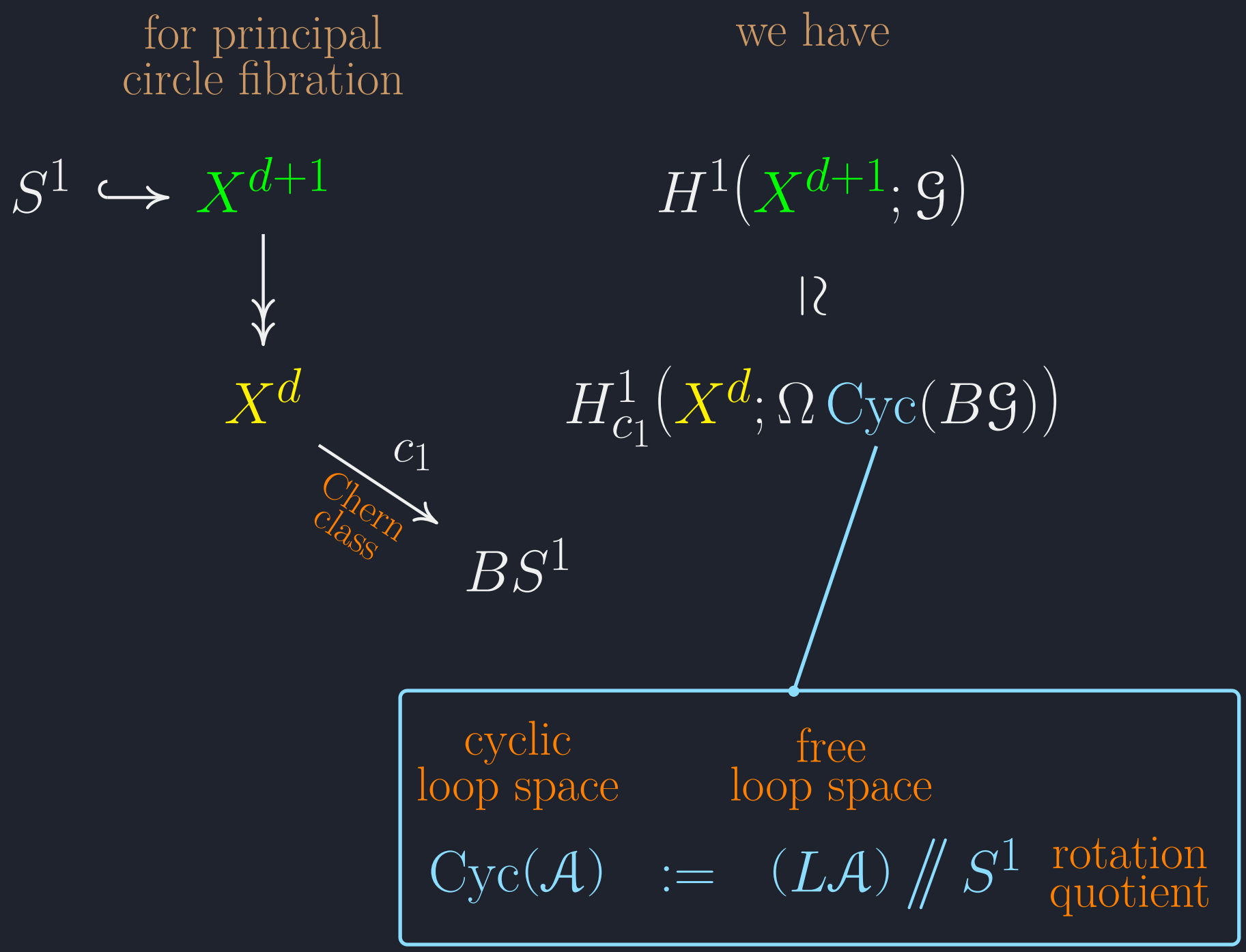
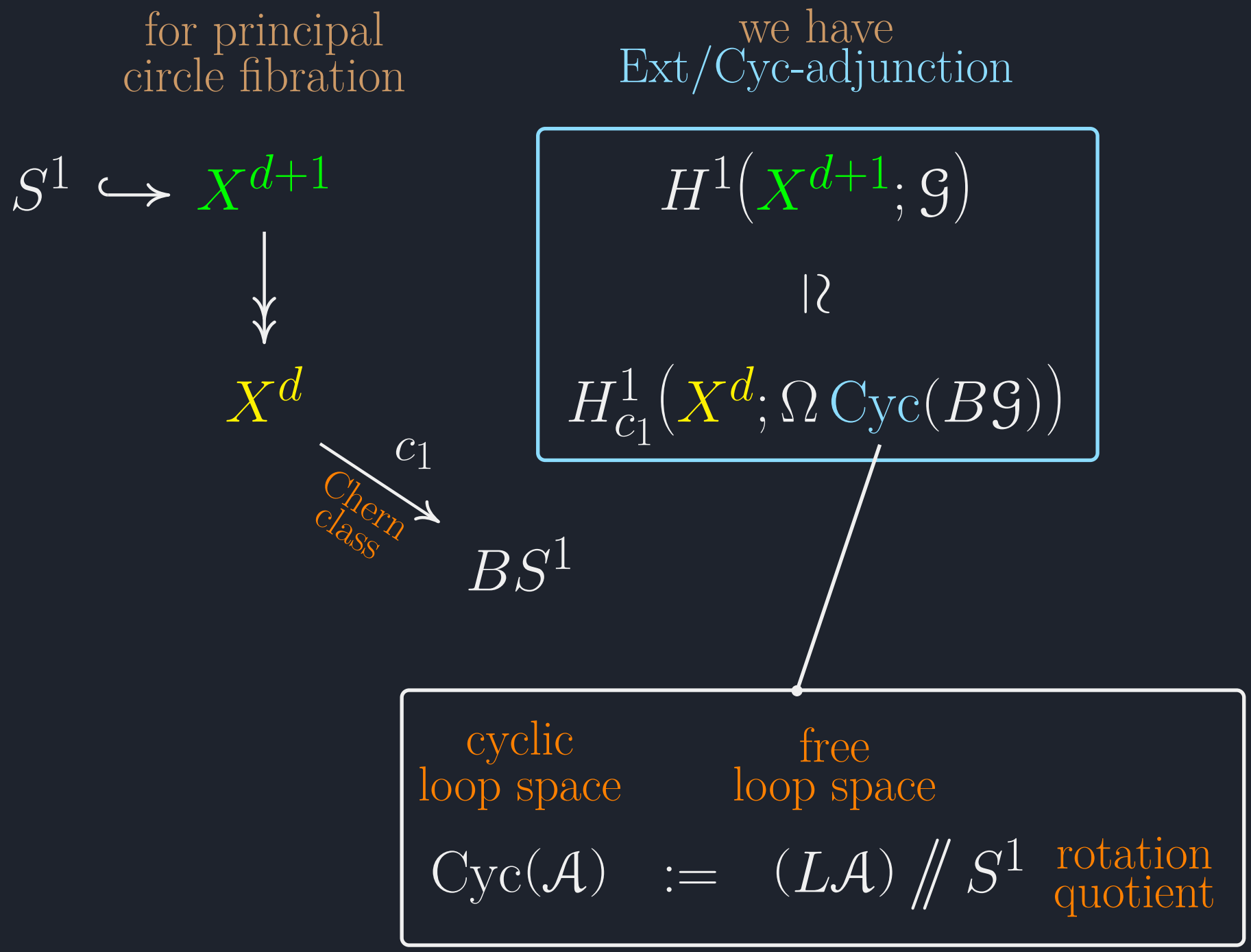
$$H^1\big(X^{d+1}; \mathcal{G}\big) \cong H^1_{c_1}\big(X^d; \Omega\,\mathrm{Cyc}(B\mathcal{G})\big)$$

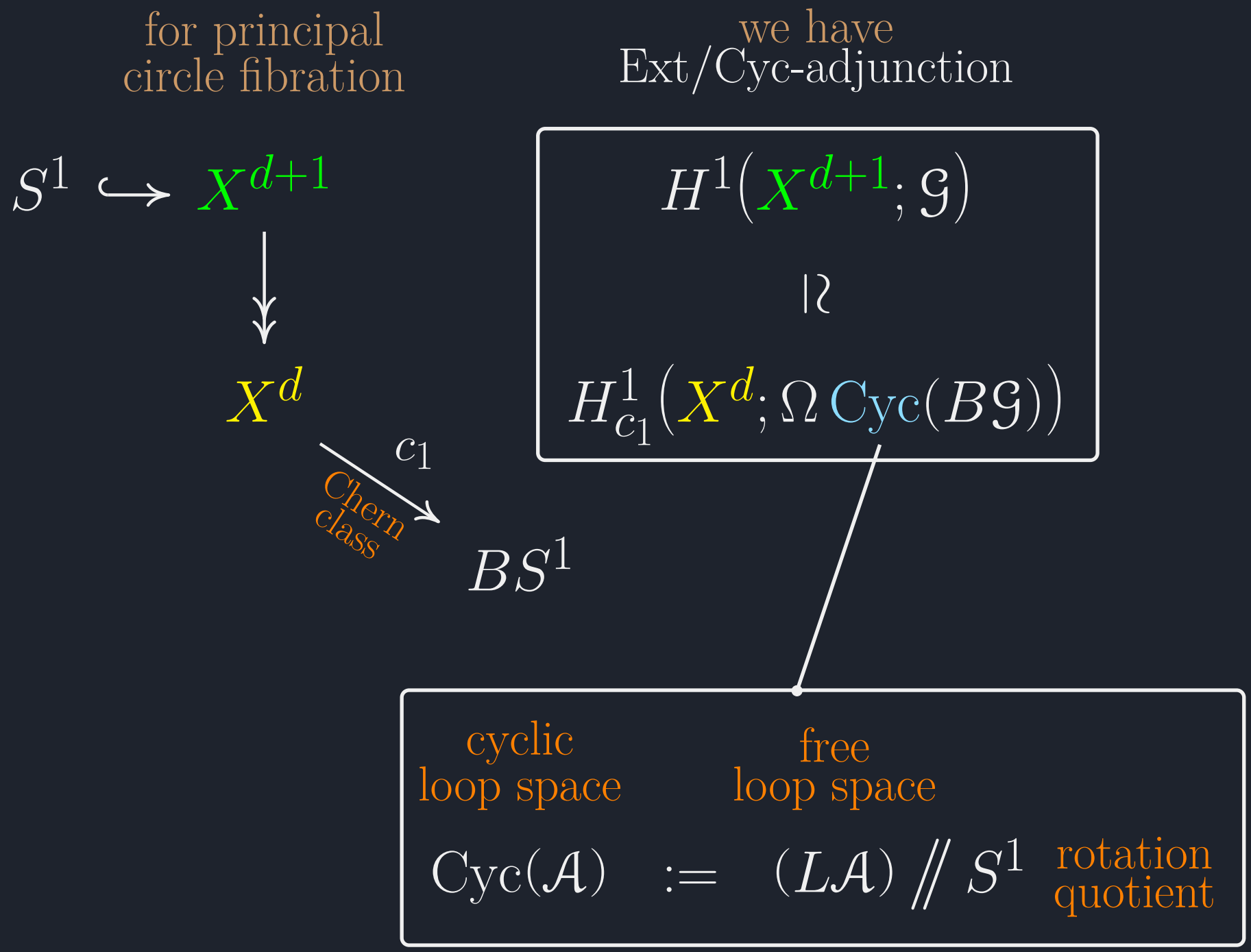
Dim-Reduction in Nonabelian Cohomology



Dim-Reduction in Nonabelian Cohomology



Dim-Reduction in Nonabelian Cohomology



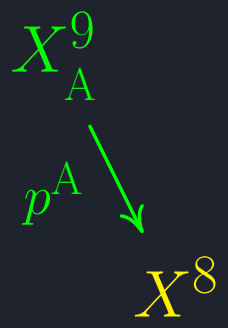
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

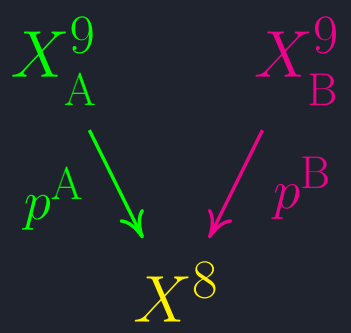
type II A/B
circle bundles



Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B
circle bundles



Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B
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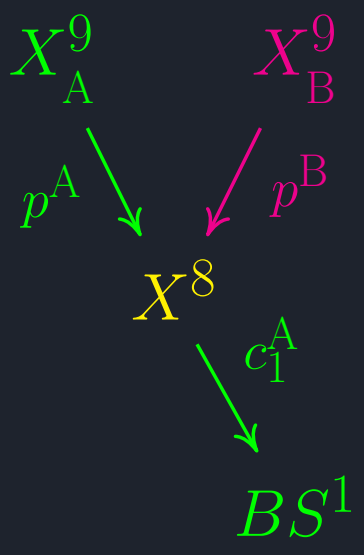

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B
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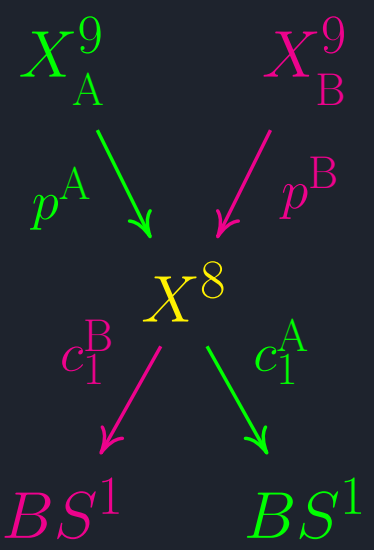
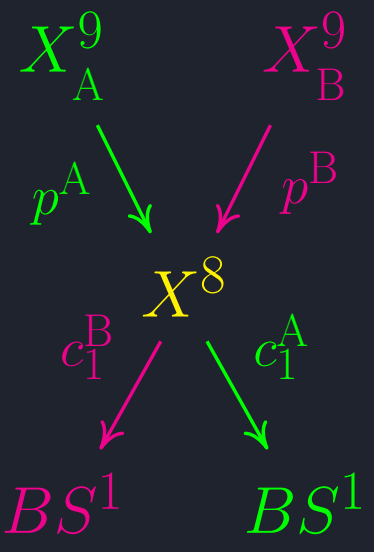


Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B
circle bundles



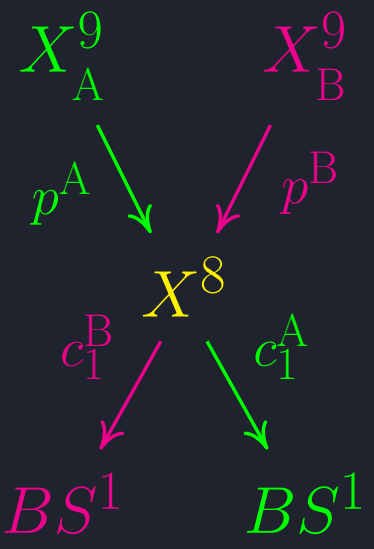
NS5-D0-D2-D4-D6 charges
in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

$$H^1\Big(X^9_{\mathrm{A}}; \Omega\big(B\mathrm{U} /\!\!/ B\mathrm{U}(1)\big)\Big)$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

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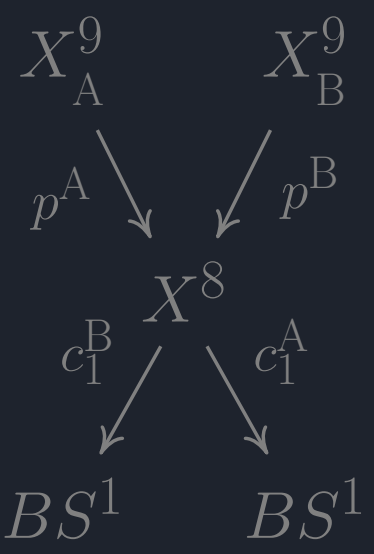
$$\wr \quad \text{Ext/Cyc-adjunction}$$

$$H^1_{c_1^{\mathrm{A}}}\Big(X^8; \Omega\,\mathrm{Cyc}\big(B\mathrm{U} /\!\!/ B\mathrm{U}(1)\big)\Big)$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

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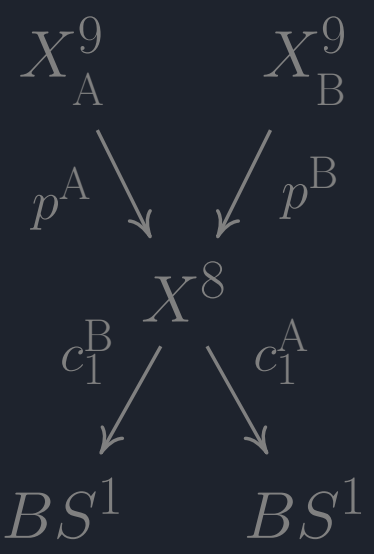
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

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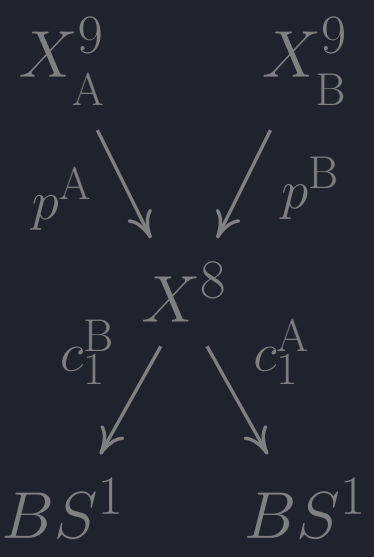
$$H^1_{c_1^{\mathrm{A}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \xrightarrow{\ \sim\ } \mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

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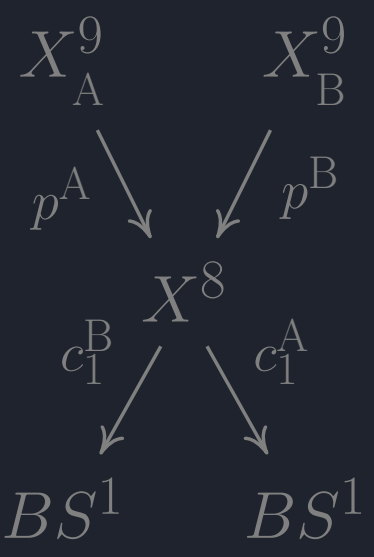
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) \xrightarrow[c_1^{\mathrm{A}} \,\leftrightarrow\, c_1^{\mathrm{B}}]{\ \sim\ } \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

$$H^1\big(X^9_{\mathrm{A}};\, \Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big)$$

$\wr$ —— Ext/Cyc-adjunction

$$H^1_{c_1^{\mathrm{A}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \simeq H^1_{c_1^{\mathrm{B}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\big)$$

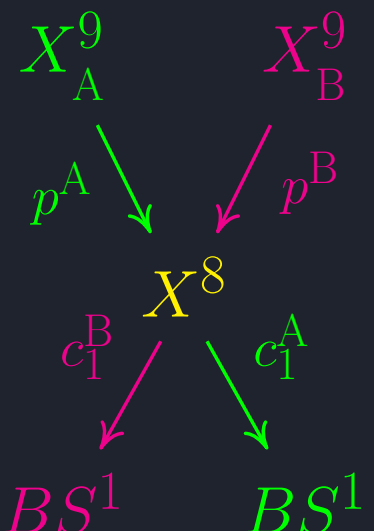
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) \xrightarrow[c_1^{\mathrm{A}} \;\leftrightarrow\; c_1^{\mathrm{B}}]{\sim} \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

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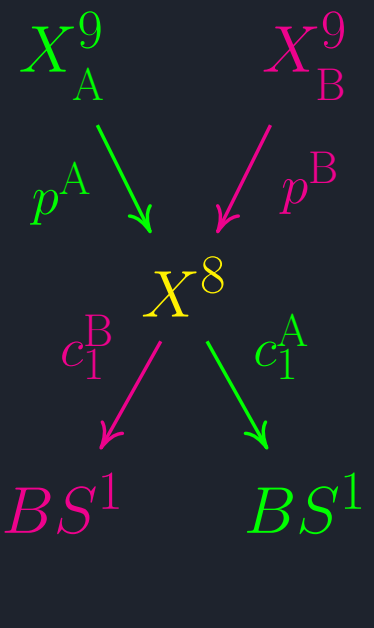
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

$$H^1\big(X^9_{\mathrm{A}};\, \Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \qquad H^1\big(X^9_{\mathrm{B}};\, \Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big)_{p^{\mathrm{B}}_* f_1 = 0}$$

$$\wr \quad \text{Ext/Cyc-adjunction} \quad \wr$$

$$H^1_{c_1^{\mathrm{A}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \simeq H^1_{c_1^{\mathrm{B}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\big)$$

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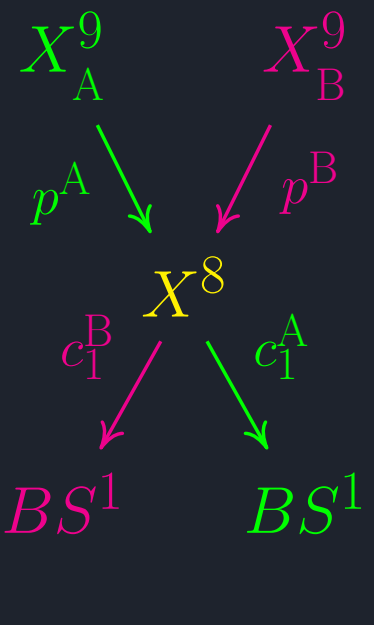
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

NS5-D1-D3-D5-D7$_{/S^1_{\mathrm{B}}}$ charges in twisted $\mathrm{KU}^1_{\mathrm{wd}=0}$ of IIB

$$H^1\Big(X^9_{\mathrm{A}};\, \Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big) \qquad H^1\Big(X^9_{\mathrm{B}};\, \Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big)_{p^{\mathrm{B}}_* f_1 = 0}$$

$$\wr \qquad\qquad \wr$$

$$H^1_{c_1^{\mathrm{A}}}\Big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big) \simeq H^1_{c_1^{\mathrm{B}}}\Big(X^8;\, \Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\Big)$$

$$\downarrow \qquad\qquad \downarrow$$

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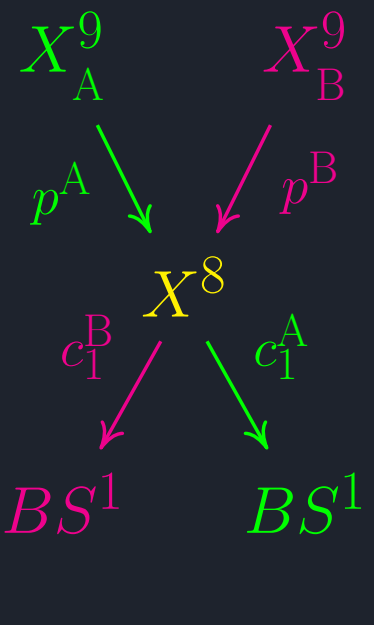
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA

NS5-D1-D3-D5-D7$_{/S^1_{\mathrm{B}}}$ charges in twisted $\mathrm{KU}^1_{\mathrm{wd}=0}$ of IIB

$$H^1\big(X^9_{\mathrm{A}};\Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \overset{\approx}{\simeq} H^1\big(X^9_{\mathrm{B}};\Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big)_{p^{\mathrm{B}}_* f_1 = 0}$$

$$\wr \qquad\qquad \wr$$

$$H^1_{c_1^{\mathrm{A}}}\big(X^8;\Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \simeq H^1_{c_1^{\mathrm{B}}}\big(X^8;\Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\big)$$

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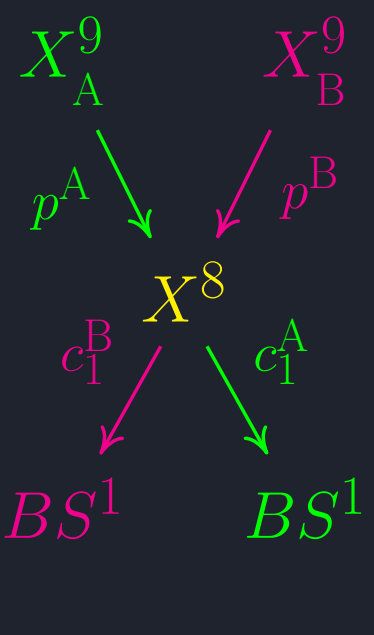
Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA



NS5-D1-D3-D5-D7$_{/S^1_{\mathrm{B}}}$ charges in twisted $\mathrm{KU}^1_{\mathrm{wd}=0}$ of IIB

$$H^1\big(X^9_{\mathrm{A}};\Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \simeq H^1\big(X^9_{\mathrm{B}};\Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big)_{p^{\mathrm{B}}_* f_1 = 0}$$

$$\wr \qquad\qquad \wr$$

$$H^1_{c_1^{\mathrm{A}}}\big(X^8;\Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) \simeq H^1_{c_1^{\mathrm{B}}}\big(X^8;\Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\big)$$

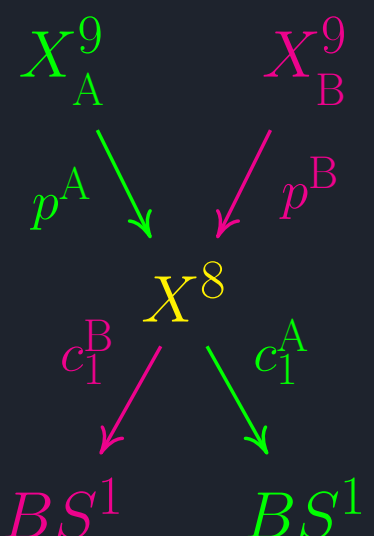
$$\downarrow \qquad\qquad \downarrow$$

$$\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) \xrightarrow[c_1^{\mathrm{A}} \leftrightarrow c_1^{\mathrm{B}}]{\sim} \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)$$

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA $\quad\leftrightarrow\quad$ NS5-D1-D3-D5-D7$_{/S^1_{\mathrm{B}}}$ charges in twisted $\mathrm{KU}^1_{\mathrm{wd}=0}$ of IIB

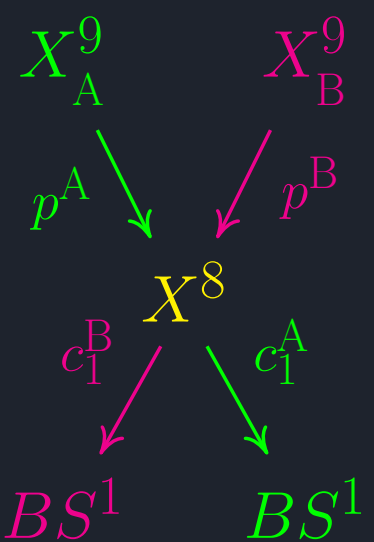
$$
\begin{array}{ccc}
H^1\Big(X^9_{\mathrm{A}};\, \Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big) & \simeq & H^1\Big(X^9_{\mathrm{B}};\, \Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big)_{p^{\mathrm{B}}_* f_1 = 0} \\
\wr & & \wr \\
H^1_{c_1^{\mathrm{A}}}\Big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\Big) & \simeq & H^1_{c_1^{\mathrm{B}}}\Big(X^8;\, \Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\Big) \\
\downarrow & & \downarrow \\
\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) & \xrightarrow[c_1^{\mathrm{A}} \;\leftrightarrow\; c_1^{\mathrm{B}}]{\sim} & \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)
\end{array}
$$

- actual massless Type II theory

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



NS5-D0-D2-D4-D6 charges in twisted $\mathrm{KU}^0_{\mathrm{rk}=0}$ of IIA $\quad\leftrightarrow\quad$ NS5-D1-D3-D5-D7$_{/S^1_{\mathrm{B}}}$ charges in twisted $\mathrm{KU}^1_{\mathrm{wd}=0}$ of IIB

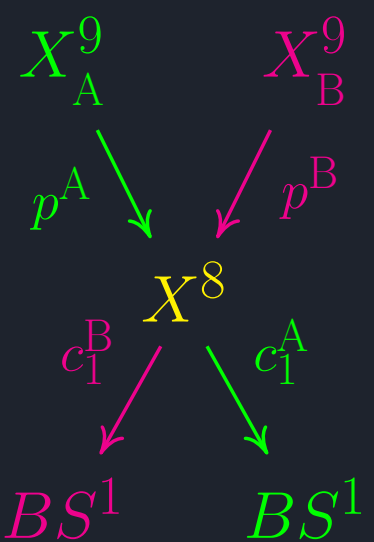
$$
\begin{array}{ccc}
H^1\big(X^9_{\mathrm{A}};\, \Omega\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) & \simeq & H^1\big(X^9_{\mathrm{B}};\, \Omega\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big)_{p^{\mathrm{B}}_* f_1 = 0} \\
\wr & & \wr \\
H^1_{c_1^{\mathrm{A}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(B\mathrm{U}/\!\!/B\mathrm{U}(1)\big)\big) & \simeq & H^1_{c_1^{\mathrm{B}}}\big(X^8;\, \Omega\,\mathrm{Cyc}\big(\mathrm{U}/\!\!/B\mathrm{U}(1)\big)_0\big) \\
\downarrow & & \downarrow \\
\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) & \xrightarrow[c_1^{\mathrm{A}} \,\leftrightarrow\, c_1^{\mathrm{B}}]{\sim} & \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)
\end{array}
$$

- actual massless Type II theory, but
- NS1 charge (still) not accounted for

Dim-Reduction in Nonabelian Cohomology

Example: Topological T-duality

type II A/B circle bundles



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$$
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\downarrow & & \downarrow \\
\mathrm{Cyc}\big(B^2\mathrm{U}(1)\big) & \xrightarrow[c_1^{\mathrm{A}} \;\leftrightarrow\; c_1^{\mathrm{B}}]{\sim} & \mathrm{Cyc}\big(B^2\mathrm{U}(1)\big)
\end{array}
$$

- actual massless Type II theory, but
- NS1 charge (still) not accounted for

Dim-Reduction in Nonabelian Cohomology

Problem:

- actual massless Type II theory, but
- NS1 charge (still) not accounted for

Dim-Reduction in Nonabelian Cohomology

Problem:

- actual massless Type II theory, but
- NS1 charge (still) not accounted for

Solution strategy:

Dim-Reduction in Nonabelian Cohomology

Problem:

- actual massless Type II theory, but
- NS1 charge (still) not accounted for

Solution strategy:

quantize M5 & M2 charge in 11D

Dim-Reduction in Nonabelian Cohomology

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Dim-Reduction in Nonabelian Cohomology

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Solution strategy:

quantize M5 & M2 charge in 11D

then dim-reduce on a small circle!

Dim-Reduction in Nonabelian Cohomology

Problem:

- actual massless Type II theory, but
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Solution strategy:

quantize M5 & M2 charge in 11D

then dim-reduce on a small circle!

Quantizing M5 & M2 charge – Hypothesis H

quantize M5 & M2 charge in 11D

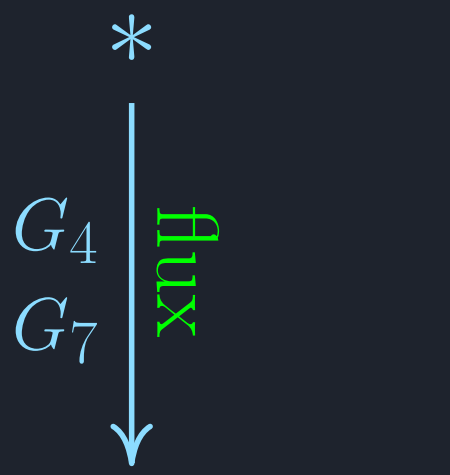
Quantizing M5 & M2 charge – Hypothesis H

Gauss laws of 11D SuGra

$$\begin{array}{l} \mathrm{d}\, G_4 \;=\; 0 \\ \mathrm{d}\, G_7 \;=\; {}^1\!/_2\, G_4 \wedge G_4 \end{array} \quad \begin{array}{c} \text{magnetic} \\ \updownarrow \\ \text{electric} \end{array} \quad \text{fluxes sourced by} \quad \begin{array}{c} \text{M5} \\ \\ \text{M2} \end{array}$$

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



Gauss laws of 11D SuGra

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4$		M2

fluxes sourced by

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



Candidate coefficients exist,

$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10};\ \mathcal{G}_{\ \mathbb{R}}\big)}_{\textbf{Gauss laws of 11D SuGra}}$$

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



Candidate coefficients exist,
the minimal/initial choice is
ΩS^4

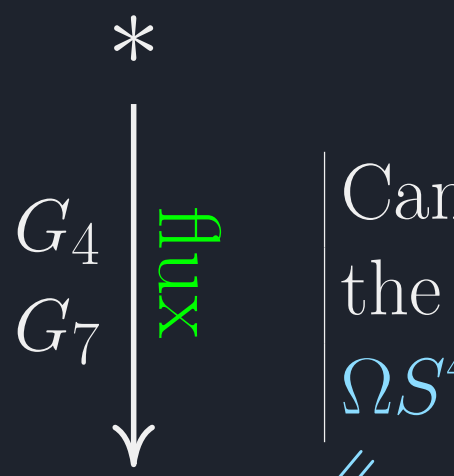
$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10};\ \mathcal{G}_{\ \mathbb{R}}\big)}$$

Gauss laws of 11D SuGra

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



Candidate coefficients exist,
the minimal/initial choice is
ΩS^4: loop group of 4-sphere

$$\Omega^1_{\mathrm{cl}}\big(X^{10};\ \mathcal{G}_{\mathbb{R}}\big)$$

Gauss laws of 11D SuGra

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10}; \Omega S^4_{\mathbb{R}}\big)}$$

Hypothesis H: *This is the choice relevant for completing 11D SuGra to "M-theory".*

Gauss laws of 11D SuGra

$\mathrm{d}\, G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

$$\overbrace{H^1\big(X^{10};\Omega S^4\big)}^{\text{4-Cohomotopy}}$$

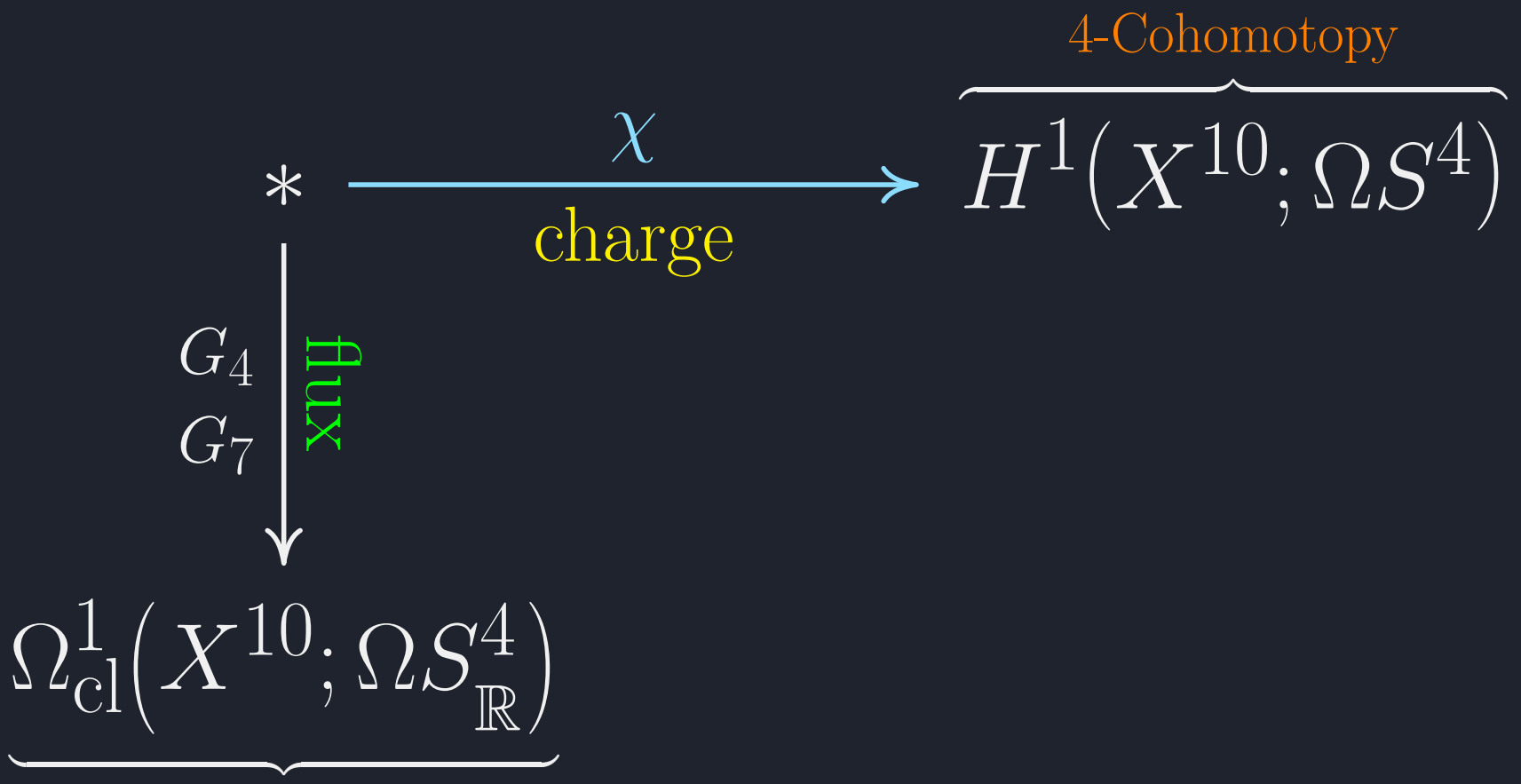


$$\underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10};\Omega S^4_{\mathbb{R}}\big)}_{\text{Gauss laws of 11D SuGra}}$$

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

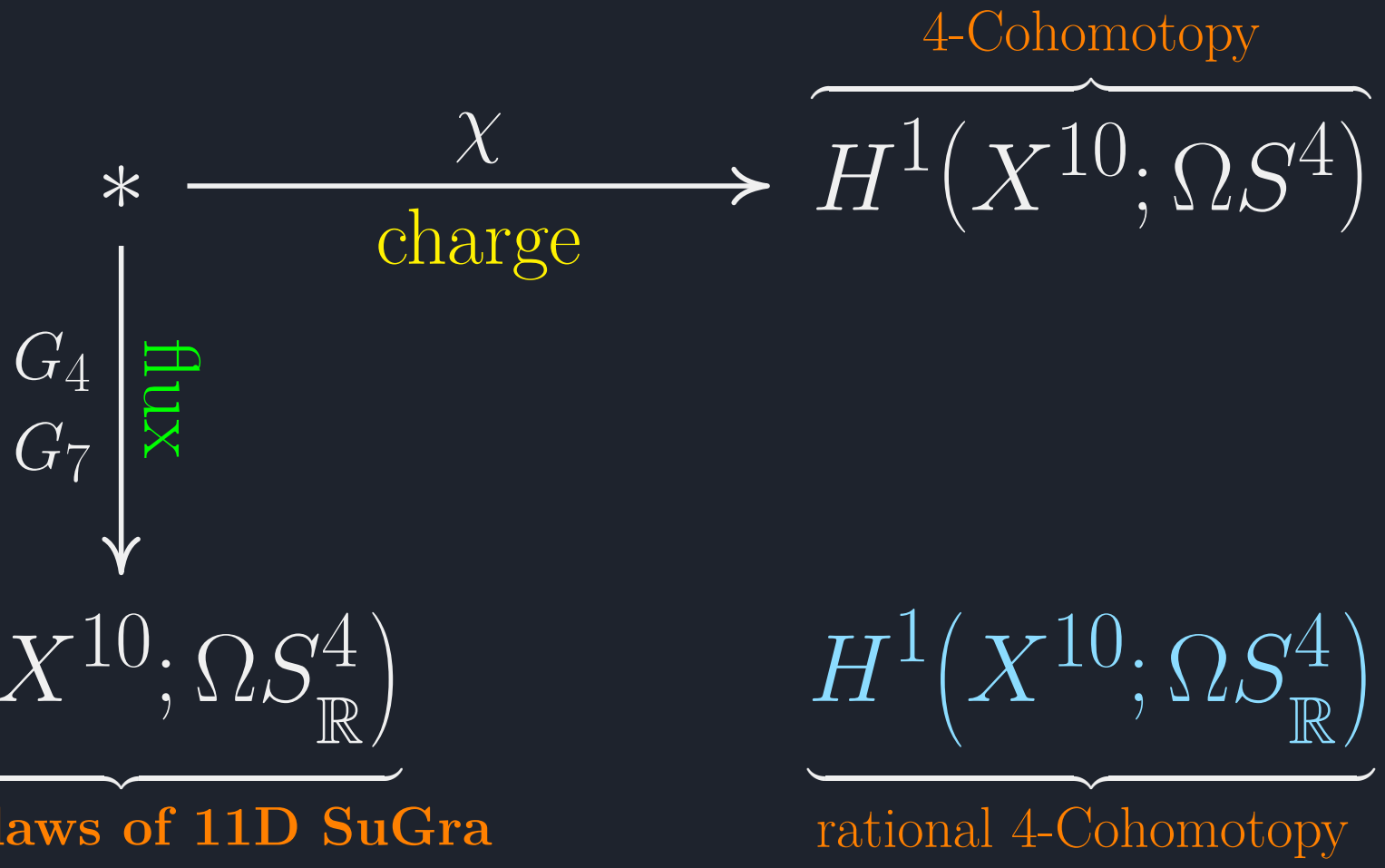
Quantizing M5 & M2 charge – Hypothesis H



$\mathrm{d}\, G_4 = 0$	magnetic ↕ electric	fluxes sourced by	M5
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$			M2

quantize M5 & M2 charge in 11D

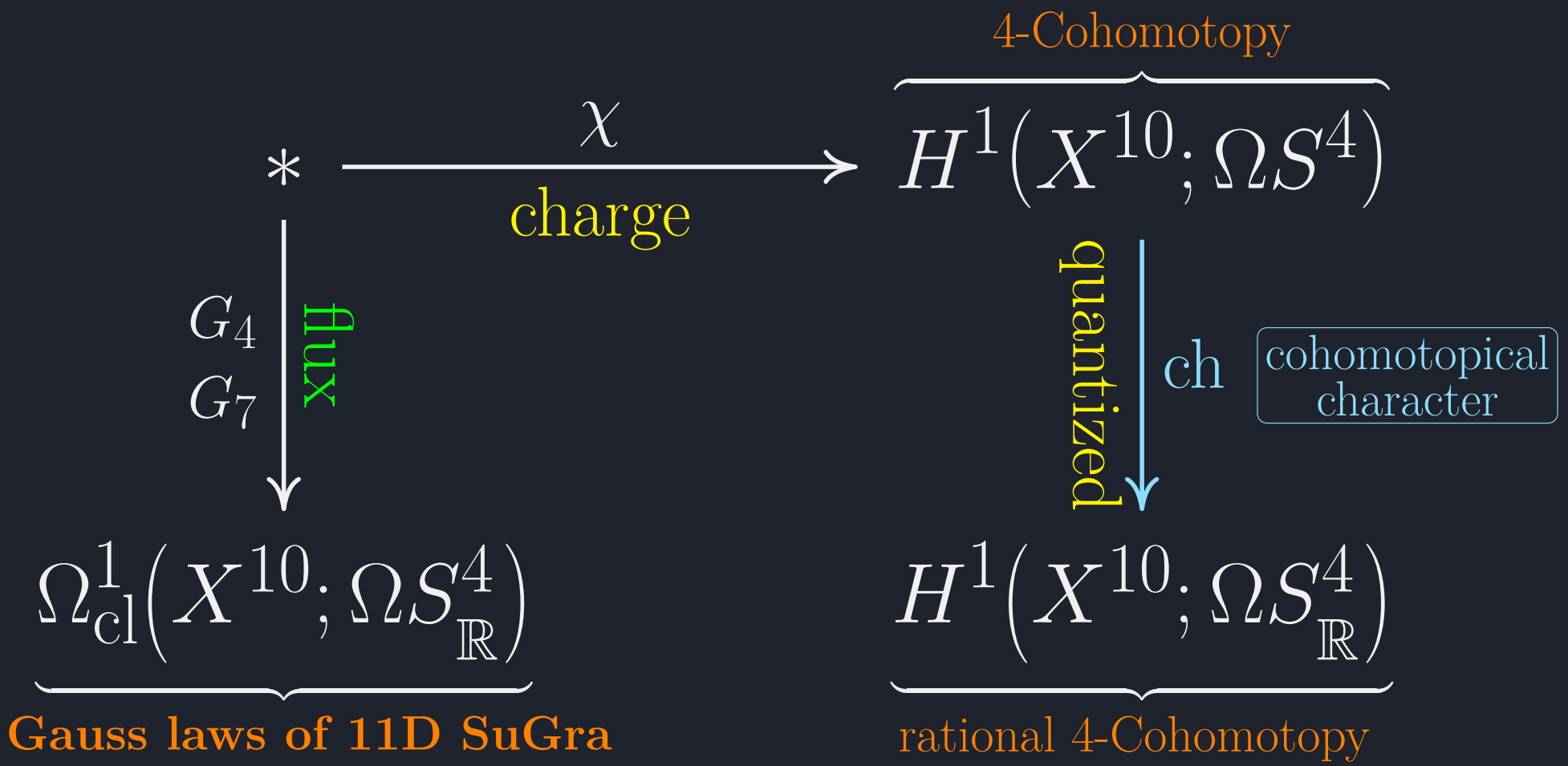
Quantizing M5 & M2 charge – Hypothesis H



$\mathrm{d}\, G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

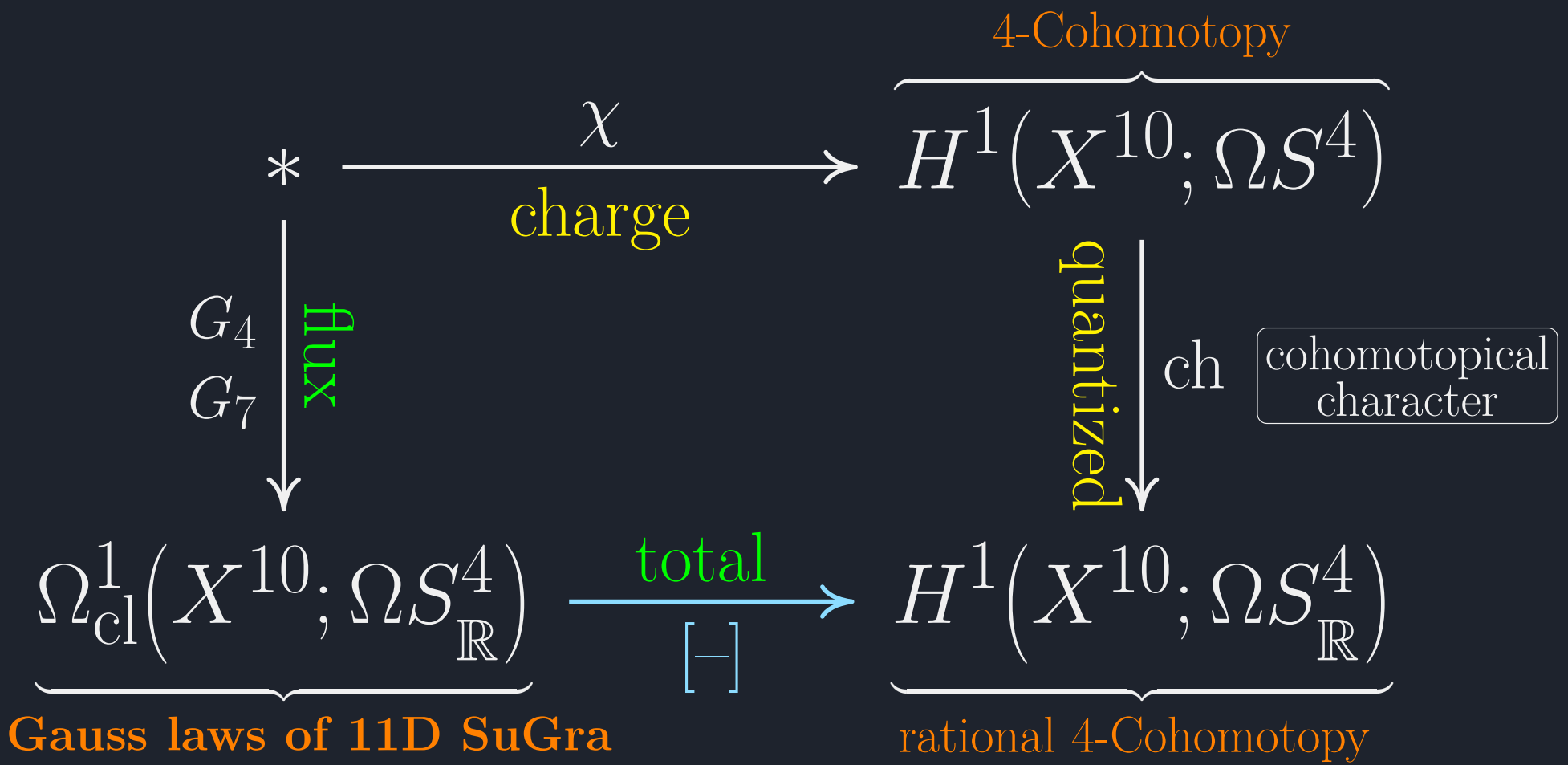
Quantizing M5 & M2 charge – Hypothesis H



$$\begin{array}{l} \mathrm{d}\, G_4 \;=\; 0 \\ \mathrm{d}\, G_7 \;=\; {}^1\!/_2\, G_4 \wedge G_4 \end{array} \quad \begin{array}{c} \text{magnetic} \\ \updownarrow \\ \text{electric} \end{array} \quad \text{fluxes sourced by} \quad \begin{array}{c} \text{M5} \\ \text{M2} \end{array}$$

quantize M5 & M2 charge in 11D

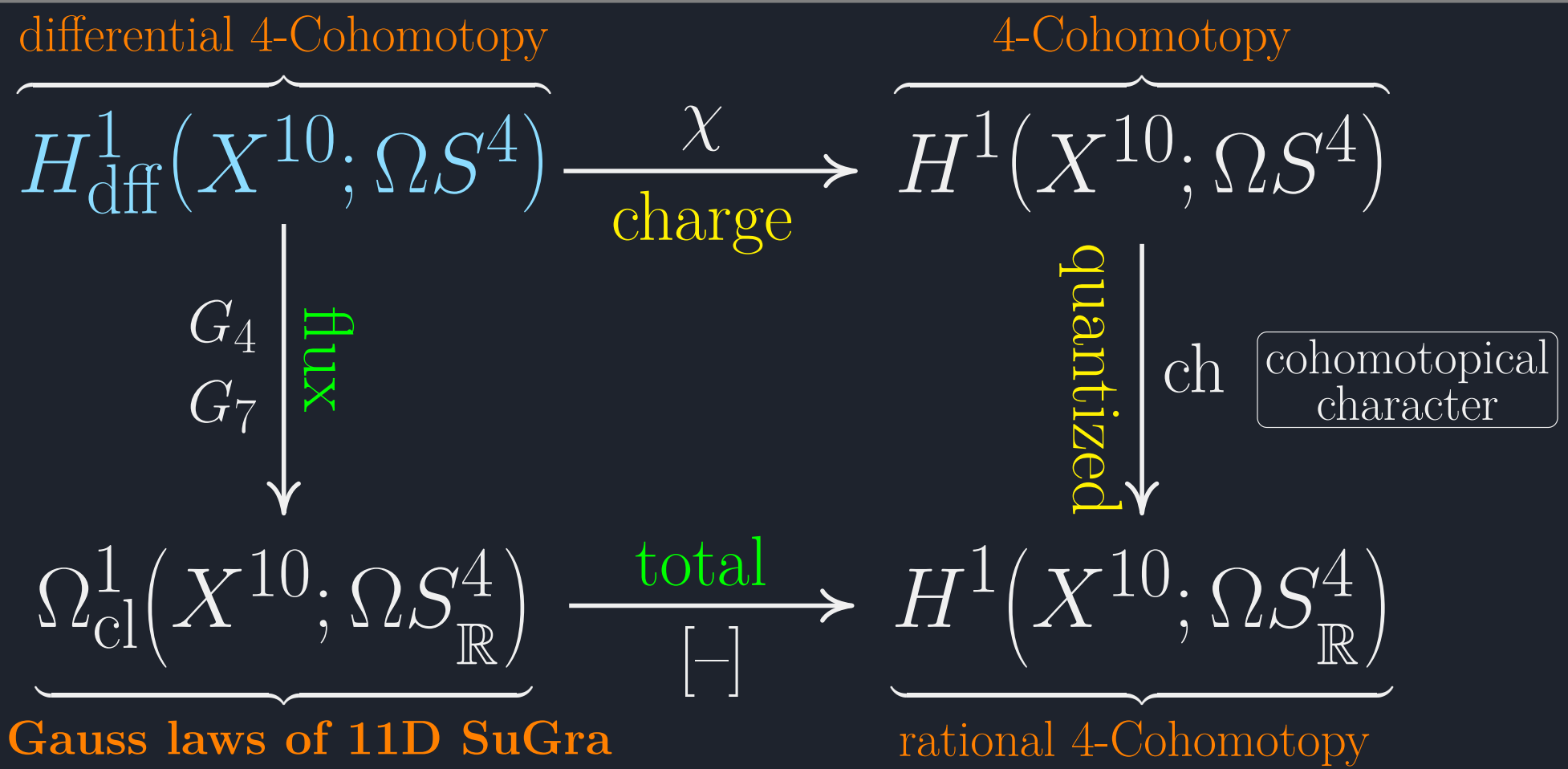
Quantizing M5 & M2 charge – Hypothesis H



$\mathrm{d}\, G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

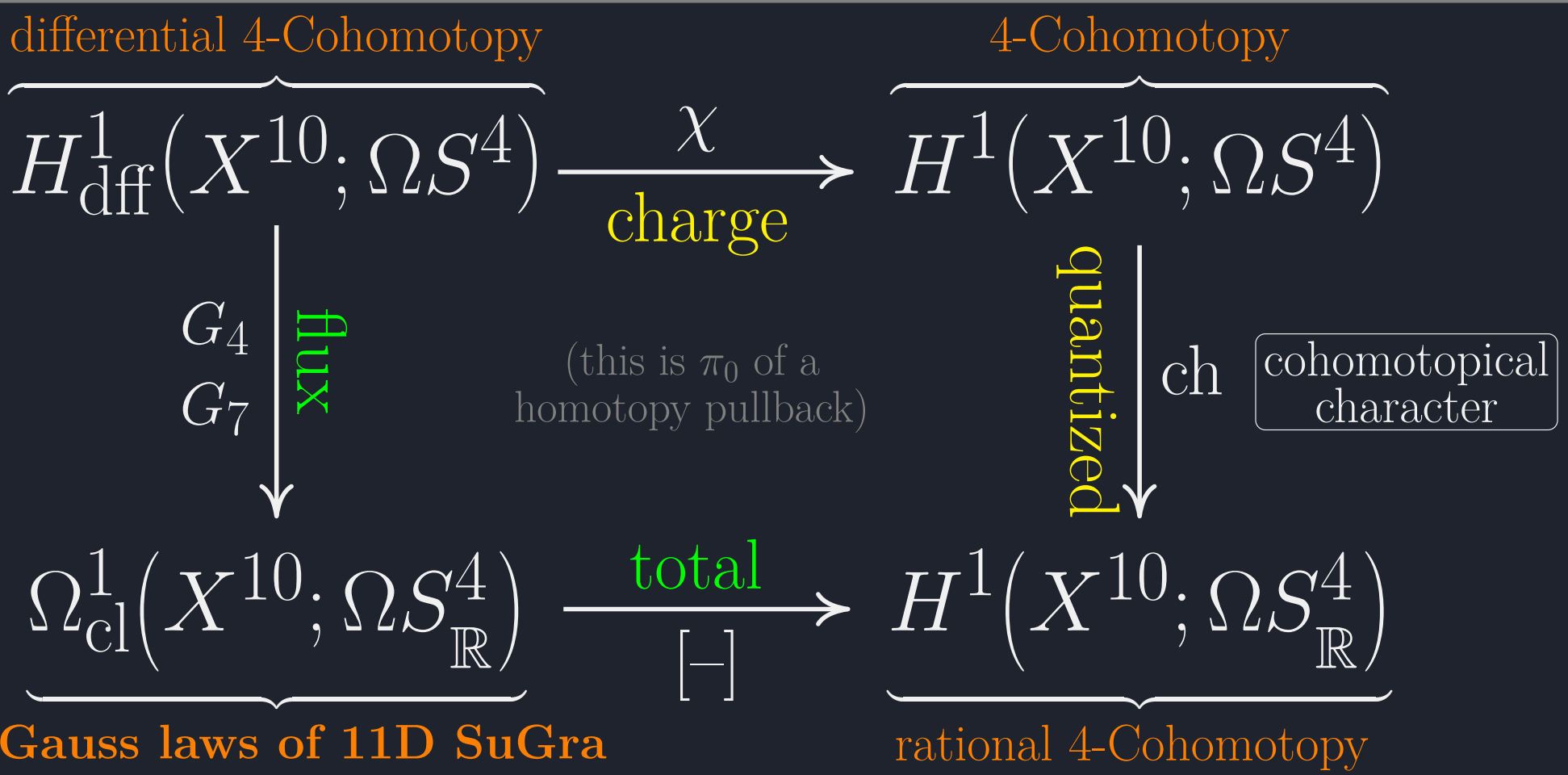
Quantizing M5 & M2 charge – Hypothesis H



$$\begin{array}{l} \mathrm{d}\, G_4 \;=\; 0 \\ \hline \mathrm{d}\, G_7 \;=\; {}^1\!/_2\, G_4 \wedge G_4 \end{array} \quad \begin{array}{c} \text{magnetic} \\ \updownarrow \\ \text{electric} \end{array} \quad \text{fluxes sourced by} \quad \begin{array}{c} \text{M5} \\ \hline \text{M2} \end{array}$$

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H



$\mathrm{d}\, G_4 = 0$	magnetic	M5
	↕ fluxes sourced by	
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$	electric	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

differential 4-Cohomotopy

A *model of the C-field*

$$H^1_{\mathrm{dff}}(X^{10}; \Omega S^4)$$

$$\Big\downarrow \begin{matrix} G_4 \\ G_7 \end{matrix} \text{ flux}$$

$$\Omega^1_{\mathrm{cl}}\big(X^{10}; \Omega S^4_{\mathbb{R}}\big)$$

Gauss laws of 11D SuGra

$\mathrm{d}\, G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\, G_7 = {}^1\!/_2\, G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

$$\overbrace{H^1_{\mathrm{dff}}(X^{10};\Omega S^4)}^{\text{differential 4-Cohomotopy}}$$

$$\Big\downarrow \begin{matrix} G_4 \\ G_7 \end{matrix} \ \text{flux}$$

$$\underbrace{\Omega^1_{\mathrm{cl}}(X^{10};\Omega S^4_{\mathbb{R}})}_{\textbf{Gauss laws of 11D SuGra}}$$

A *model of the C-field* which quantizes not only M5-charge but also M2-charge (Page charge).

$$\begin{array}{lcl} \mathrm{d}\,G_4 = 0 & \text{magnetic} & \text{M5} \\ & \updownarrow \ \text{fluxes sourced by} & \\ \mathrm{d}\,G_7 = {}^1\!/_2\, G_4 \wedge G_4 & \text{electric} & \text{M2} \end{array}$$

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

$$\overbrace{H^1_{\mathrm{dff}}(X^{10}; \Omega S^4)}^{\text{differential 4-Cohomotopy}} \xrightarrow[G_7]{G_4 \;\#\; \text{flux}} \underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10}; \Omega S^4_{\mathbb{R}}\big)}_{\textbf{Gauss laws of 11D SuGra}}$$

A *model of the C-field* which quantizes not only M5-charge but also M2-charge (Page charge).

cancels anomaly in

$$\exp\big(2\mathrm{i}T\textstyle\int(\widehat{C}_6 - {}^1\!/_2\,\widehat{C}_3 \wedge H_3)\big)$$

action functional of probe M5-brane

[FSS 2021 Thm. 4.8]

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\,G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

$$\overbrace{H^1_{\mathrm{dff}}(X^{10}; \Omega S^4)}^{\text{differential 4-Cohomotopy}} \xrightarrow{\begin{matrix} G_4 \\ G_7 \end{matrix} \;\; \text{flux}} \underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10}; \Omega S^4_{\mathbb{R}}\big)}_{\textbf{Gauss laws of 11D SuGra}}$$

A *model of the C-field*
which quantizes not only M5-charge
but also M2-charge (Page charge).

cancels anomaly in

$$\exp\left(2\mathrm{i}T\int(\widehat{C}_6 - {}^1\!/_2\,\widehat{C}_3 \wedge H_3)\right)$$

action functional
of probe M5-brane in
background bulk C-field
with self-dual B-field
on its worldvolume

[FSS 2021 Thm. 4.8]

$$\begin{array}{l} \mathrm{d}\,G_4 \;=\; 0 \\ \hline \mathrm{d}\,G_7 \;=\; {}^1\!/_2\,G_4 \wedge G_4 \end{array} \quad \begin{matrix} \text{magnetic} \\ \updownarrow \\ \text{electric} \end{matrix} \quad \text{fluxes sourced by} \quad \begin{array}{l} \text{M5} \\ \hline \text{M2} \end{array}$$

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

$$\overbrace{H^1_{\mathrm{dff}}(X^{10}; \Omega S^4)}^{\text{differential 4-Cohomotopy}} \xrightarrow[G_7]{G_4 \;\#\; \text{flux}} \underbrace{\Omega^1_{\mathrm{cl}}\big(X^{10}; \Omega S^4_{\mathbb{R}}\big)}_{\textbf{Gauss laws of 11D SuGra}}$$

A *model of the C-field* which quantizes not only M5-charge but also M2-charge (Page charge).

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$$\exp\big(2\mathrm{i}T\textstyle\int(\widehat{C}_6 - {}^1\!/_2\,\widehat{C}_3 \wedge H_3)\big)$$

action functional of probe M5-brane

[FSS 2021 Thm. 4.8]

$\mathrm{d}\,G_4 = 0$	magnetic ↕ electric	M5
$\mathrm{d}\,G_7 = {}^1\!/_2\,G_4 \wedge G_4$	fluxes sourced by	M2

quantize M5 & M2 charge in 11D

Quantizing M5 & M2 charge – Hypothesis H

differential 4-Cohomotopy

$$H^1_{\mathrm{dff}}(X^{10}; \Omega S^4)$$

G_4, G_7 — flux

$$\Omega^1_{\mathrm{cl}}\left(X^{10}; \Omega S^4_{\mathbb{R}}\right)$$

Gauss laws of 11D SuGra

A *model of the C-field* which quantizes not only M5-charge but also M2-charge (Page charge).

cancels anomaly in

$$\exp\left(2\mathrm{i}T\int(\widehat{C}_6 - {}^1\!/_2\,\widehat{C}_3 \wedge H_3)\right)$$

action functional of probe M5-brane

[FSS 2021 Thm. 4.8]

	magnetic ↕ electric	fluxes sourced by
$\mathrm{d}\,G_4 = 0$		M5
$\mathrm{d}\,G_7 = {}^1\!/_2\,G_4 \wedge G_4$		M2

quantize M5 & M2 charge in 11D

then dim-reduce on a small circle!

Dimensional Reduction on M-Circle

quantize M5 & M2 charge in 11D

then dim-reduce on a small circle!

Dimensional Reduction on Small M-Circle

$$\overbrace{H^1_{\text{dff}}\big(X^9; \Omega\, \text{Cyc}\;\; S^4\big)}^{\substack{\text{differential} \\ \text{cyclified 4-Cohomotopy}}}$$

the IR-completed C-fields
according to Hypothesis H,
seen after circle-reduction

Dimensional Reduction on Small M-Circle

$$\overbrace{H^1_{\mathrm{dff}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4\big)}^{\substack{\text{differential} \\ \text{cyclified 4-Cohomotopy}}}$$

Dimensional Reduction on Small M-Circle

differential cyclified 4-Cohomotopy

$$H^1_{\mathrm{dff}}(X^9; \Omega\,\mathrm{Cyc}\;\; S^4)$$

cyclification of 4-sphere

$\mathrm{Cyc}\, S^4$

space of M-circle maps to 4-sphere



Dimensional Reduction on Small M-Circle

differential cyclified 4-Cohomotopy

$$H^1_{\mathrm{dff}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4\big)$$

$$\begin{array}{cc} F_2 & \\ H_3 & F_4 \\ H_7 & F_6 \\ & F_8 \end{array} \Big\downarrow \text{flux}$$

$$\Omega^1_{\mathrm{cl}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4_{\mathbb{R}}\big)$$

cyclification of 4-sphere

$\mathrm{Cyc}\, S^4$

space of M-circle maps to 4-sphere



Dimensional Reduction on Small M-Circle

differential cyclified 4-Cohomotopy

$$H^1_{\mathrm{dff}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4\big)$$

$$\begin{array}{c} F_2 \\ H_3\;\; F_4 \\ H_7\;\; F_6 \\ F_8 \end{array} \Big\downarrow \text{flux}$$

$$\Omega^1_{\mathrm{cl}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4_{\mathbb{R}}\big)$$

Gauss laws

cyclification of 4-sphere

$\mathrm{Cyc}\, S^4$

space of M-circle maps to 4-sphere



$$\begin{array}{ll} \mathrm{d}\, F_2 = 0 & \mathrm{d}\, H_3 = 0 \\ \mathrm{d}\, F_4 = H_3 \wedge F_2 & \\ \hline \mathrm{d}\, F_6 = H_3 \wedge F_4 & \\ \mathrm{d}\, F_8 = H_3 \wedge F_6 & \mathrm{d}\, H_7 = {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6 \end{array}$$

magnetic ↕ electric

Dimensional Reduction on Small M-Circle

differential
cyclified 4-Cohomotopy

$$H^1_{\mathrm{dff}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4\big)$$

F_2

H_3 F_4

H_7 F_6

F_8

flux

$$\Omega^1_{\mathrm{cl}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4_{\mathbb{R}}\big)$$

Gauss laws

cyclification
of 4-sphere

small
cyclification

$$\mathrm{Cyc}\, S^4 \hookleftarrow \mathrm{Cyc}_1 S^4$$

space of M-circle
maps to 4-sphere



$$\mathrm{d}\,F_2 = 0 \qquad \mathrm{d}\,H_3 = 0$$

$$\mathrm{d}\,F_4 = H_3 \wedge F_2$$

magnetic

$\updownarrow$

electric

$$\mathrm{d}\,F_6 = H_3 \wedge F_4$$

$$\mathrm{d}\,F_8 = H_3 \wedge F_6 \qquad \mathrm{d}\,H_7 = {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6$$

Dimensional Reduction on Small M-Circle

differential cyclified 4-Cohomotopy

$$H^1_{\mathrm{dff}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4\big)$$

$$\Big\downarrow \begin{matrix} F_2 \\ H_3\;\; F_4 \\ H_7\;\; F_6 \\ F_8 \end{matrix} \text{ flux}$$

$$\Omega^1_{\mathrm{cl}}\big(X^9; \Omega\,\mathrm{Cyc}\;\; S^4_{\mathbb{R}}\big)$$

Gauss laws

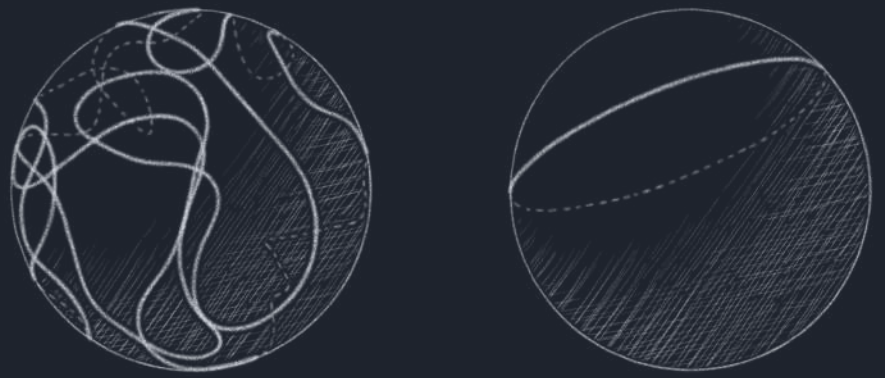
cyclification of 4-sphere

small cyclification

$$\mathrm{Cyc}\, S^4 \hookleftarrow \mathrm{Cyc}_1 S^4$$

space of M-circle maps to 4-sphere

maps that go once around a round circle



$$\begin{aligned}
\mathrm{d}\,F_2 &= 0 & \mathrm{d}\,H_3 &= 0 \\
\mathrm{d}\,F_4 &= H_3 \wedge F_2 & & \text{magnetic} \\
\hline
\mathrm{d}\,F_6 &= H_3 \wedge F_4 & & \updownarrow \\
\mathrm{d}\,F_8 &= H_3 \wedge F_6 & \mathrm{d}\,H_7 &= {}^1\!/_2\, F_4 \wedge F_4 - F_2 \wedge F_6
\end{aligned}$$

electric

Dimensional Reduction on Small M-Circle

differential small-cyclified 4-Cohomotopy

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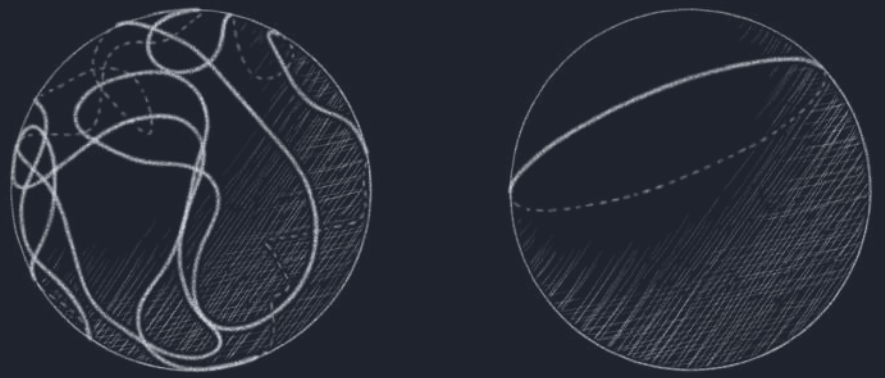

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flux: F_2, H_3 F_4, H_7 F_6, F_8

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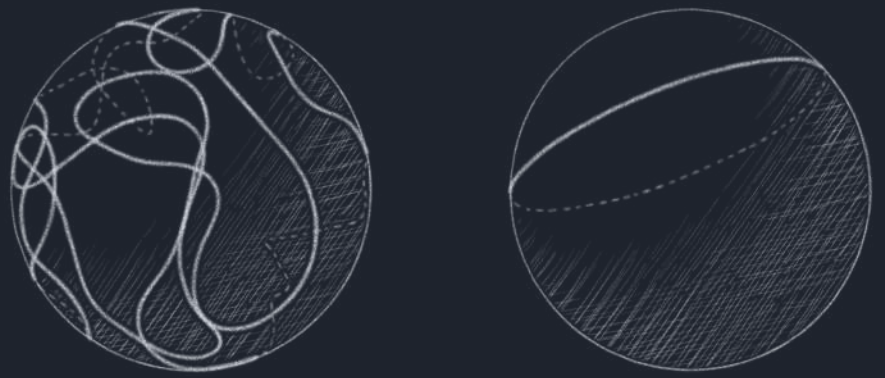

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Gauss laws

First complete
flux/charge quantization
of all NS/RR-fluxes &
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in 10D IIA

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Dimensional Reduction on Small M-Circle

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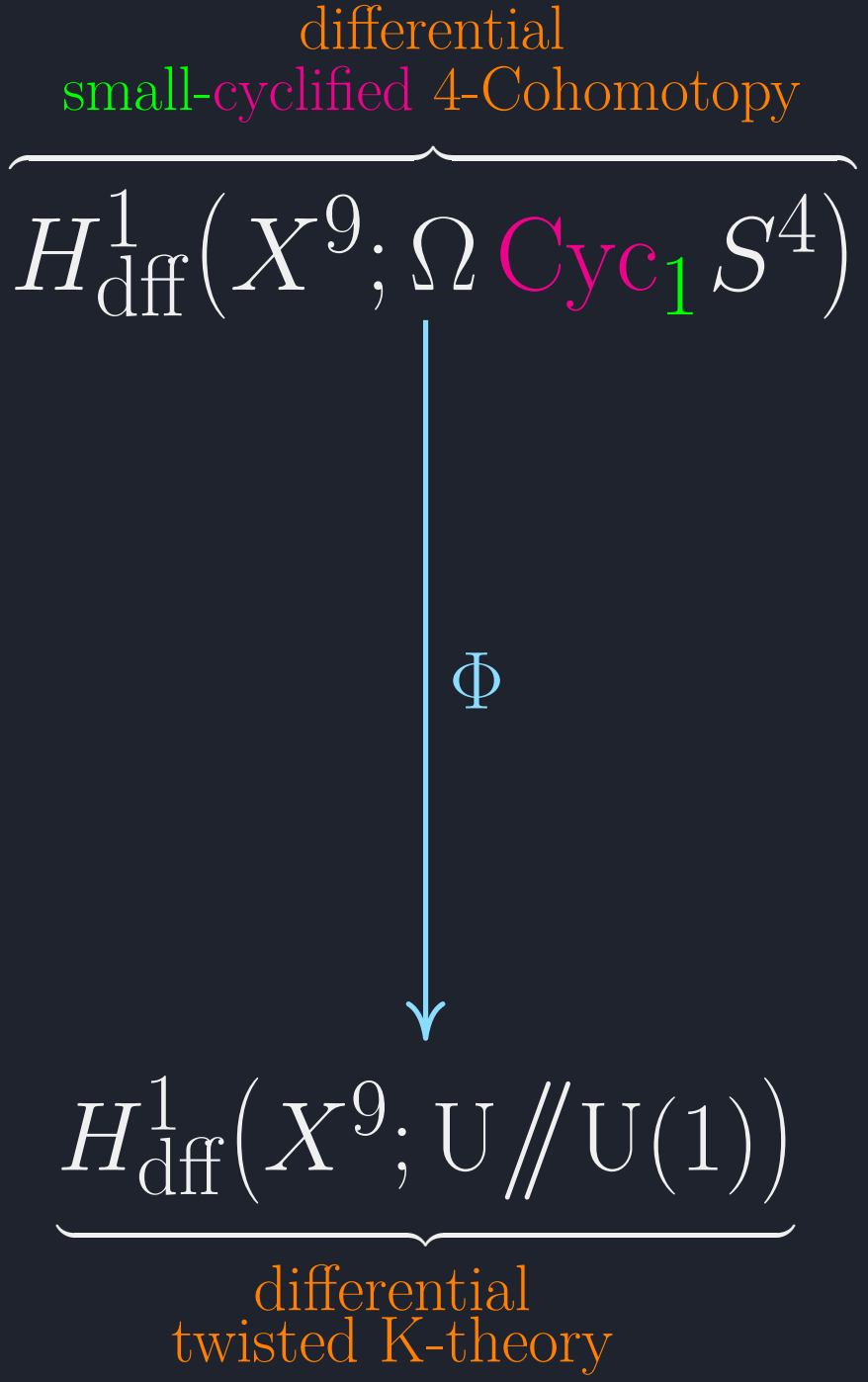

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[GSS 2026]

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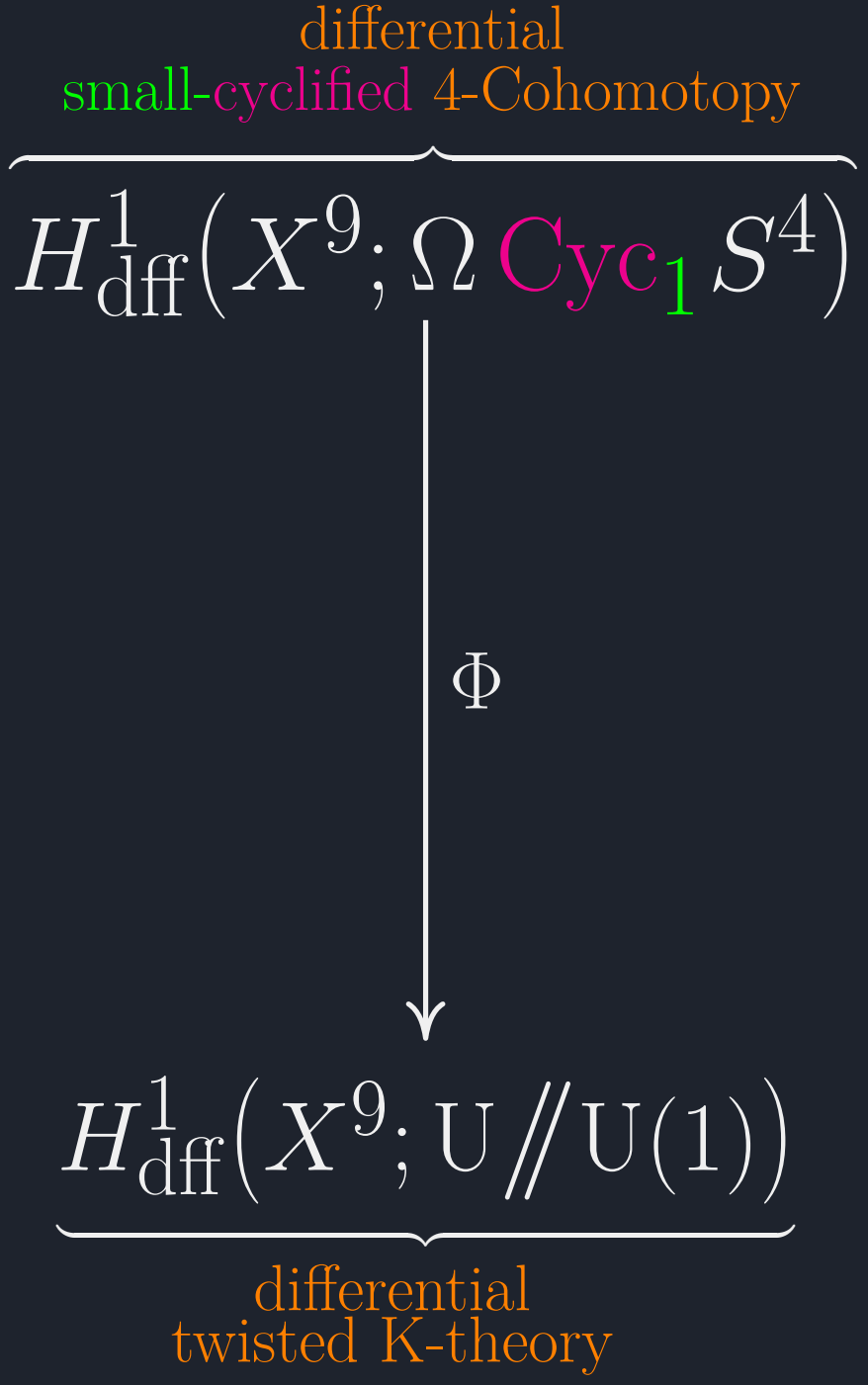

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Urs Schreiber

New York University, Abu Dhabi

CENTER FOR QUANTUM & TOPOLOGICAL SYSTEMS



talk at:
ICMMP 2026
Hangzhou, China
(Oct 30 – Nov 3)

based in parts on joint work with:
D. Fiorenza, G. Giotopoulos, N. Golub, and H. Sati

materials at: ncatlab.org/schreiber/show/ICMMP26