Urs Schreiber

(New York University, Abu Dhabi & Czech Academy of Science, Prague)

Microscopic brane physics from Cohomotopy

talk at

`M-Theory and Mathematics`

NYU AD 2020

based on joint work with

H. Sati

0) Introduction

1) Microscopic Brane Charge

2) Orientifold Tadpole Cancellation

3) $D6 \perp D8$-Brane Intersections

4) Hanany-Witten Theory

5) Chan-Paton Data

6) BMN Matrix Model States

7) M2/M5 Brane Bound States

(0)

Introduction

Open Problem M and Hypothesis H

back to top

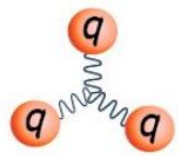
Open problem QCD

Confined QCD

“Millennium problem”

- QCD-cosmology
- nucleosynthesis
- form factors
- . . .

baryon



Flavored QCD

“Flavor problem”

Higgs sector:
- cosm. constant
- EW hierarchy
- vacuum stability

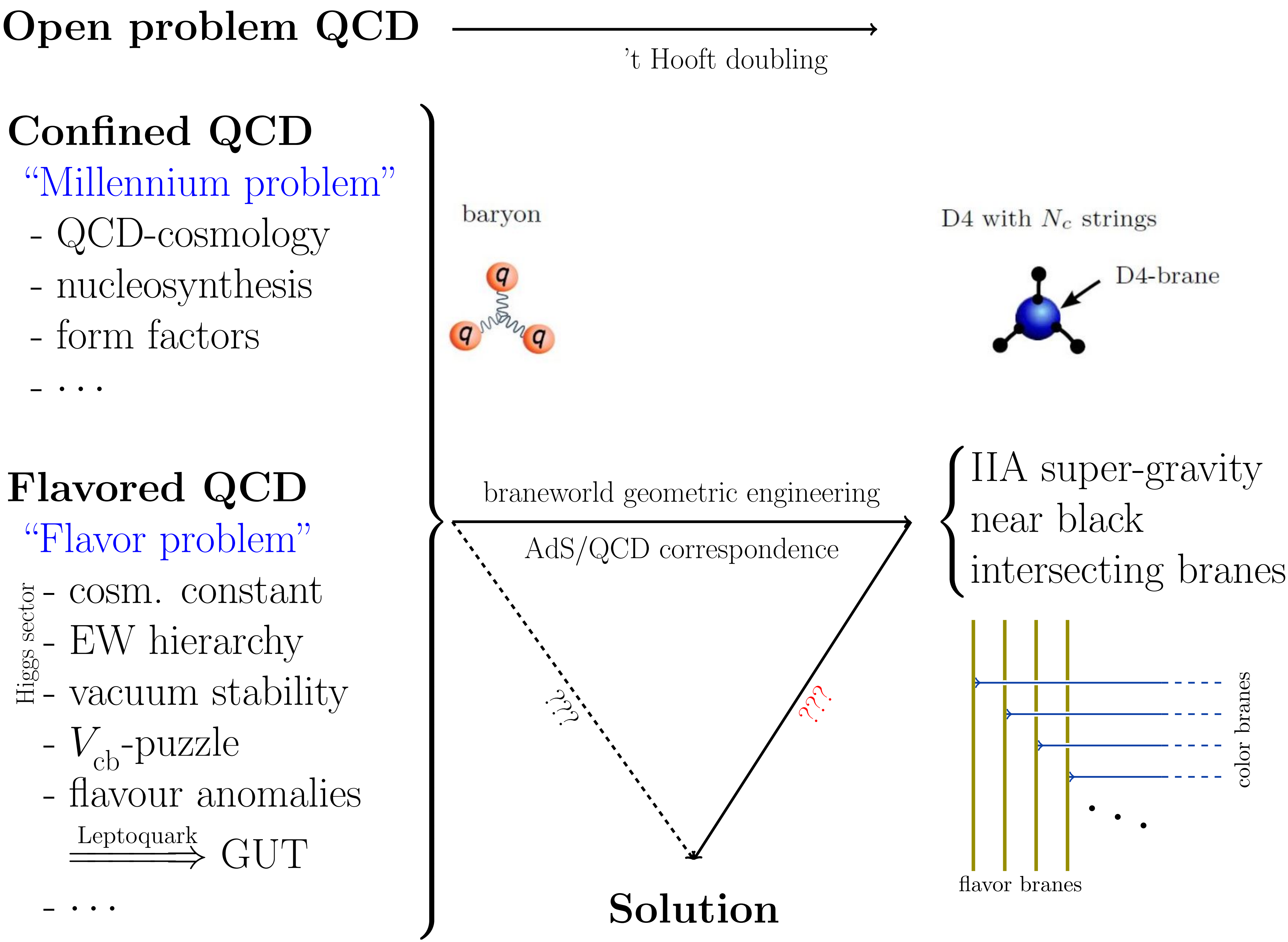
- V_{cb}-puzzle
- flavour anomalies

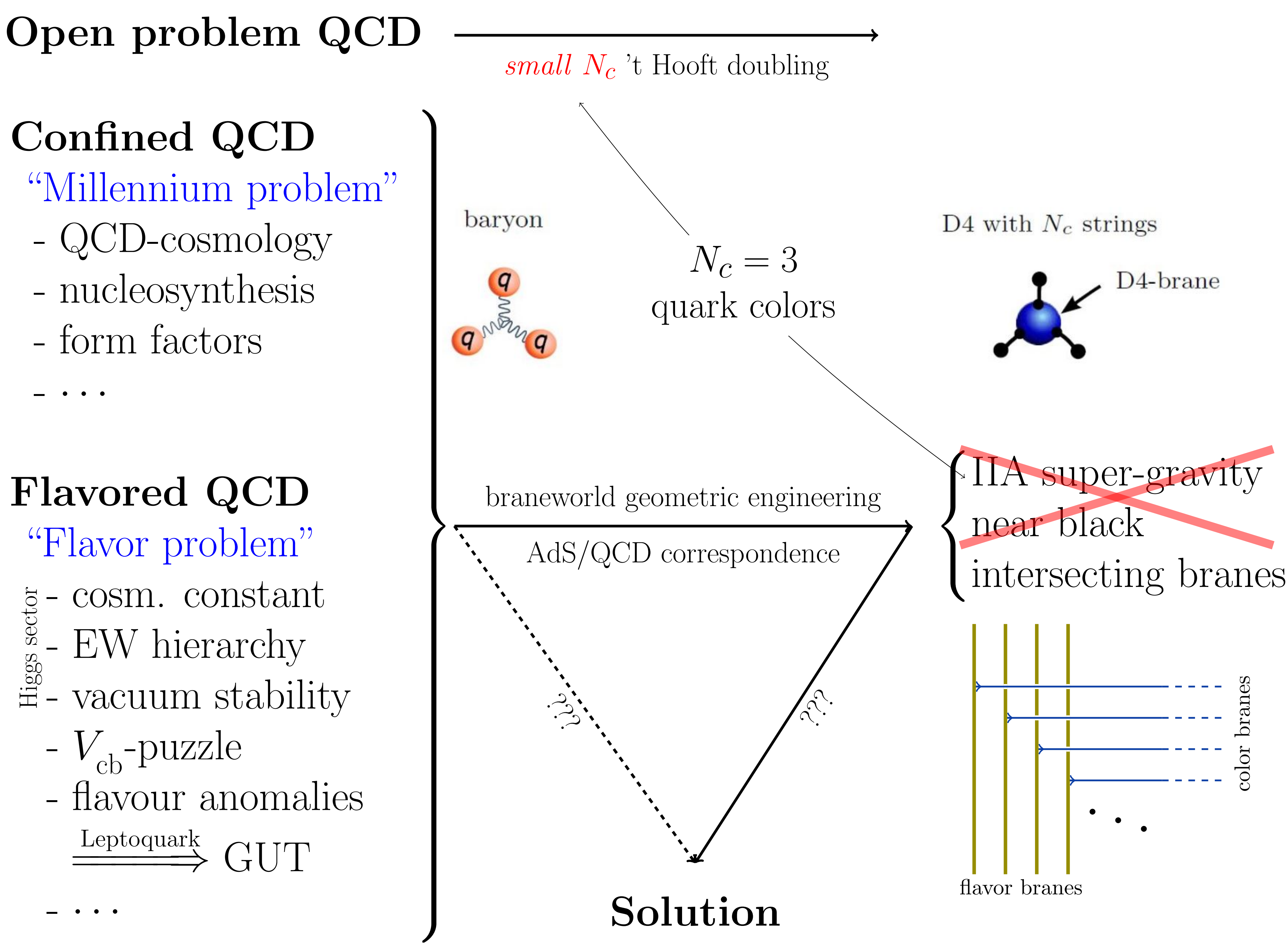
$\xrightarrow{\text{Leptoquark}}$ GUT

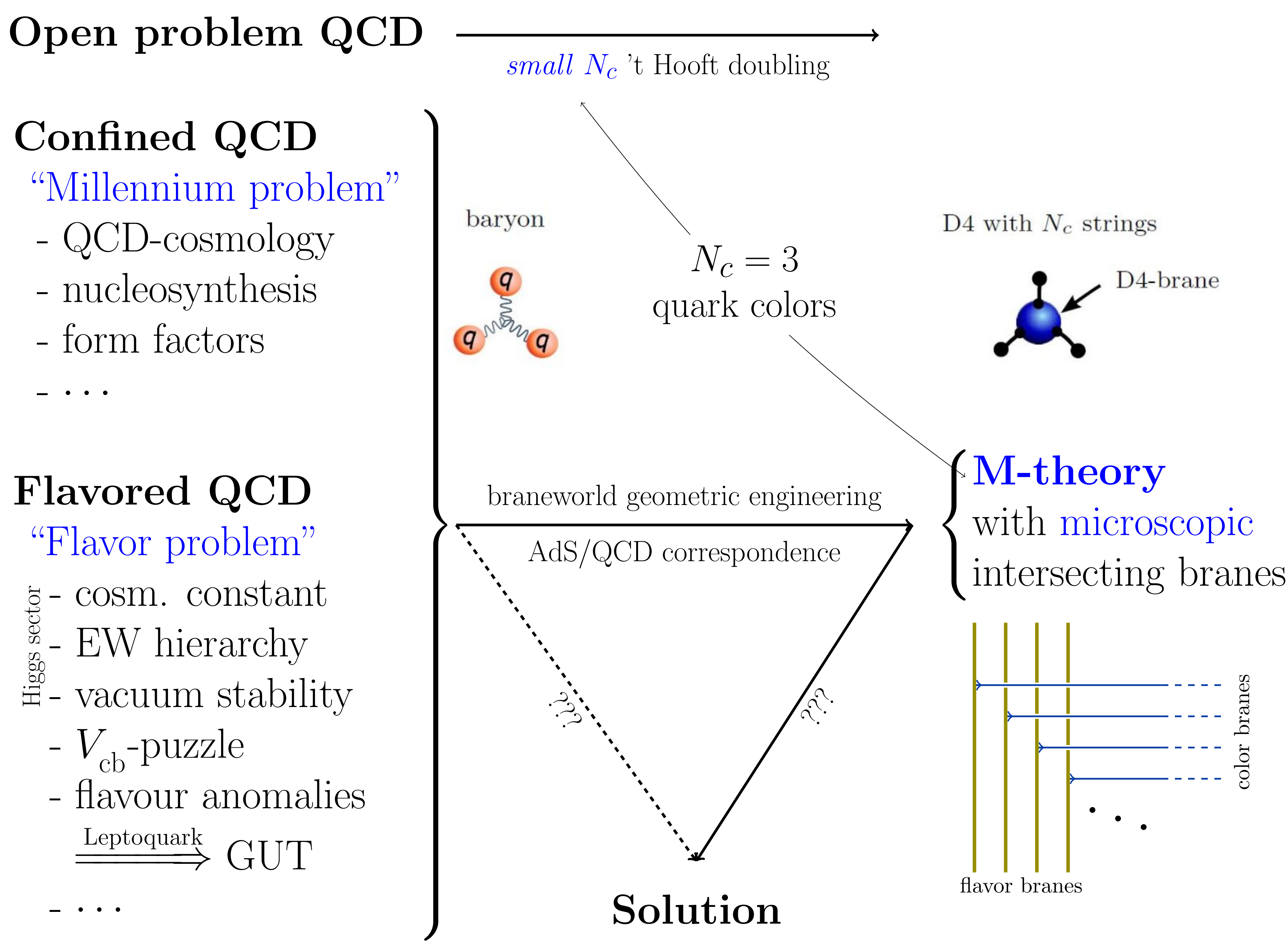
- . . .

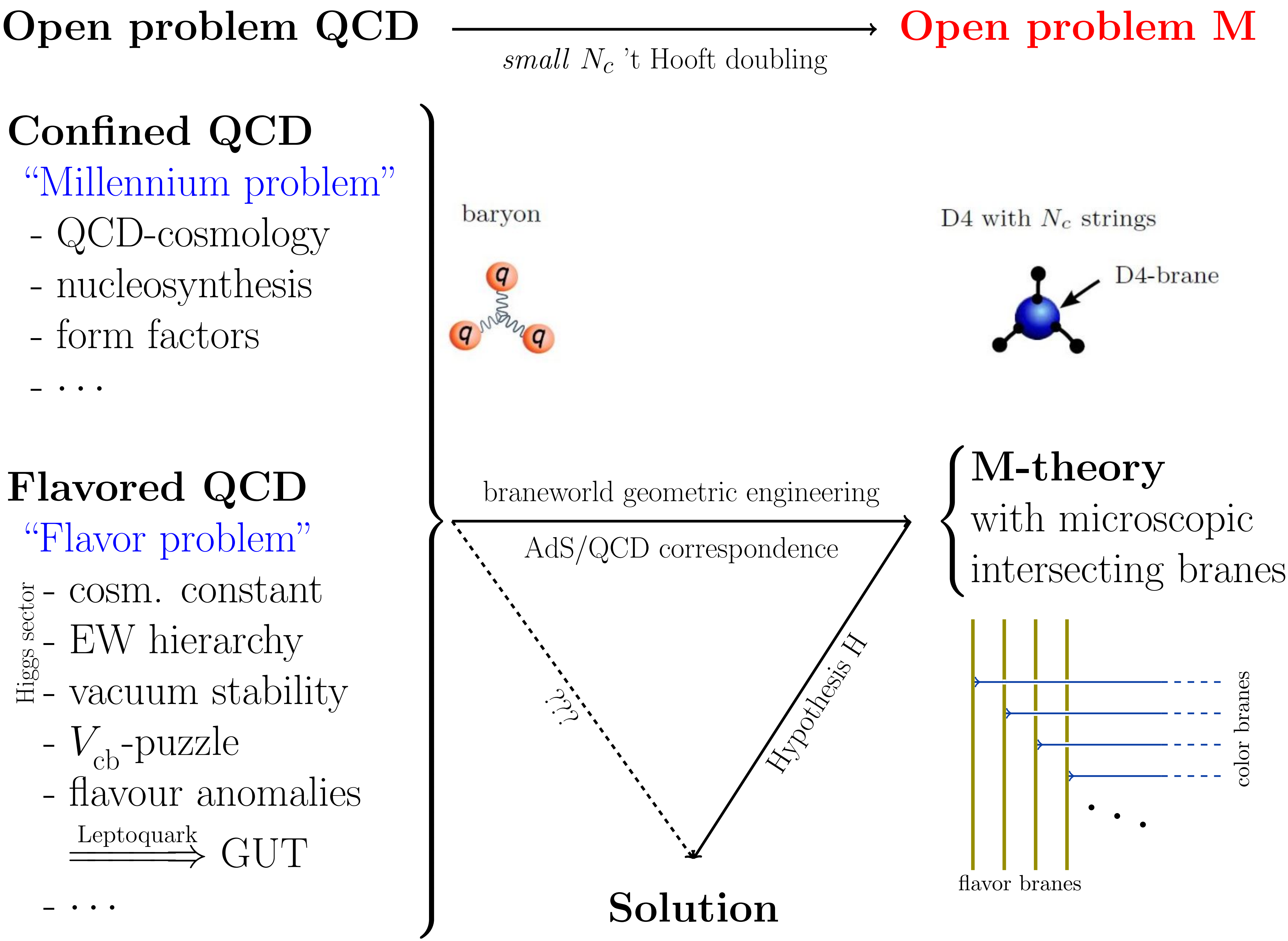
???

Solution


Open problem QCD
't Hooft doubling
Confined QCD
"Millennium problem"
- QCD-cosmology
- nucleosynthesis
- form factors
- . . .
Flavored QCD
"Flavor problem"
Higgs sector
- cosm. constant
- EW hierarchy
- vacuum stability
- V_{cb}-puzzle
- flavour anomalies
Leptoquark
GUT
- . . .
baryon
q
q
q
D4 with N_c strings
D4-brane
braneworld geometric engineering
AdS/QCD correspondence
¿¿¿
???
IIA super-gravity
near black
intersecting branes
color branes
flavor branes
Solution


Open problem QCD
small N_c 't Hooft doubling
Confined QCD
"Millennium problem"
- QCD-cosmology
- nucleosynthesis
- form factors
- ...
Flavored QCD
"Flavor problem"
Higgs sector
- cosm. constant
- EW hierarchy
- vacuum stability
- V_{cb}-puzzle
- flavour anomalies
Leptoquark
GUT
- ...
baryon
q
q
q
$N_c = 3$
quark colors
D4 with N_c strings
D4-brane
braneworld geometric engineering
AdS/QCD correspondence
IIA super-gravity
near black
intersecting branes
???
???
color branes
flavor branes
Solution


Open problem QCD
small N_c 't Hooft doubling
Confined QCD
"Millennium problem"
- QCD-cosmology
- nucleosynthesis
- form factors
- ...
baryon
q
q
q
$N_c = 3$
quark colors
D4 with N_c strings
D4-brane
Flavored QCD
"Flavor problem"
Higgs sector
- cosm. constant
- EW hierarchy
- vacuum stability
- V_{cb}-puzzle
- flavour anomalies
Leptoquark
GUT
- ...
braneworld geometric engineering
AdS/QCD correspondence
???
???
M-theory
with microscopic
intersecting branes
color branes
flavor branes
Solution


Open problem QCD
small N_c 't Hooft doubling
Open problem M
Confined QCD
"Millennium problem"
- QCD-cosmology
- nucleosynthesis
- form factors
- ...
Flavored QCD
"Flavor problem"
Higgs sector
- cosm. constant
- EW hierarchy
- vacuum stability
- V_{cb}-puzzle
- flavour anomalies
Leptoquark
GUT
- ...
baryon
q
D4 with N_c strings
D4-brane
braneworld geometric engineering
AdS/QCD correspondence
Hypothesis H
Solution
M-theory
with microscopic
intersecting branes
color branes
flavor branes

Open problem QCD $\xrightarrow[\text{small } N_c \text{ 't Hooft doubling}]{}$ **Open problem M**

Open problem QCD $\longrightarrow$ **Open problem M**

small N_c 't Hooft doubling

first consider *large* $g^2_{\mathrm{YM}} N_c$ $\Longrightarrow$ **11d supergravity**

Open problem QCD $\longrightarrow$ **Open problem M**

small N_c 't Hooft doubling

first consider *large* $g^2_{\mathrm{YM}} N_c$ $\Longrightarrow$ **11d supergravity**

$$\mathbf{CovariantPhaseSpace}_{\text{11d SuGra}} =$$

spacetimes

formalized as:	G_{ADE}–orbi $\mathbb{R}^{10,1\vert\mathbf{32}}$–folds ($\mathcal{X}$) super-orbifold	
equipped with:	**0) gravity**	**1) C-field**
formalized as:	Pin$^+$-structure (E, Ψ) super-vielbein	differential forms (G_4, G_7) flux densities
subject to:	**Einstein equations** $G = T(\Psi, G_4, G_7)$	**Page equation** $dG_7 + \frac{1}{2} G_4 \wedge G_4 = 0$

Open problem QCD $\xrightarrow[\text{small } N_c \text{ 't Hooft doubling}]{}$ **Open problem M**

first consider *large* $g^2_{\mathrm{YM}} N_c$ $\Longrightarrow$ **11d supergravity**

$$\mathbf{CovariantPhaseSpace}_{\text{11d SuGra}} =$$

spacetimes

formalized as:	G_{ADE}–orbi $\mathbb{R}^{10,1\vert\mathbf{32}}$–folds $(\mathcal{X})$ (super-orbifold)	
equipped with:	**0) gravity**	**1) C-field**
formalized as:	Pin^+-structure (E, Ψ) (super-vielbein)	differential forms (G_4, G_7) (flux densities)
subject to:	**Einstein equations**	**Page equation**
equivalent to:	super-torsion $= 0$ Candiello-Lechner 93, Howe 97	flux is in rationalized J-twisted Cohomotopy Sati 13, Fiorenza-Sati-S. 19a

Open problem QCD $\xrightarrow{\textit{small } N_c \text{ 't Hooft doubling}}$ **Open problem M**

& *large* $g^2_{\mathrm{YM}} N_c$ $\Longrightarrow$ **11d supergravity charge-quantized in Cohomotopy**

$$\mathbf{CovariantPhaseSpace}_{\text{11d SuGra}} =$$

spacetimes

formalized as:	G_{ADE}–orbi $\mathbb{R}^{10,1\vert\mathbf{32}}$–folds $\overset{\text{super-orbifold}}{(\mathcal{X})}$	
equipped with:	**0) gravity**	**1) C-field**
formalized as:	Pin$^+$-structure $\overset{\text{super-vielbein}}{(E, \Psi)}$	differential forms $\overset{\text{flux densities}}{(G_4, G_7)}$
subject to:	**Einstein equations**	**Page equation**
equivalent to:	super-torsion = 0 Candiello-Lechner 93, Howe 97	flux is in ~~rationalized~~ J-twisted Cohomotopy Sati 13, Fiorenza-Sati-S. 19a

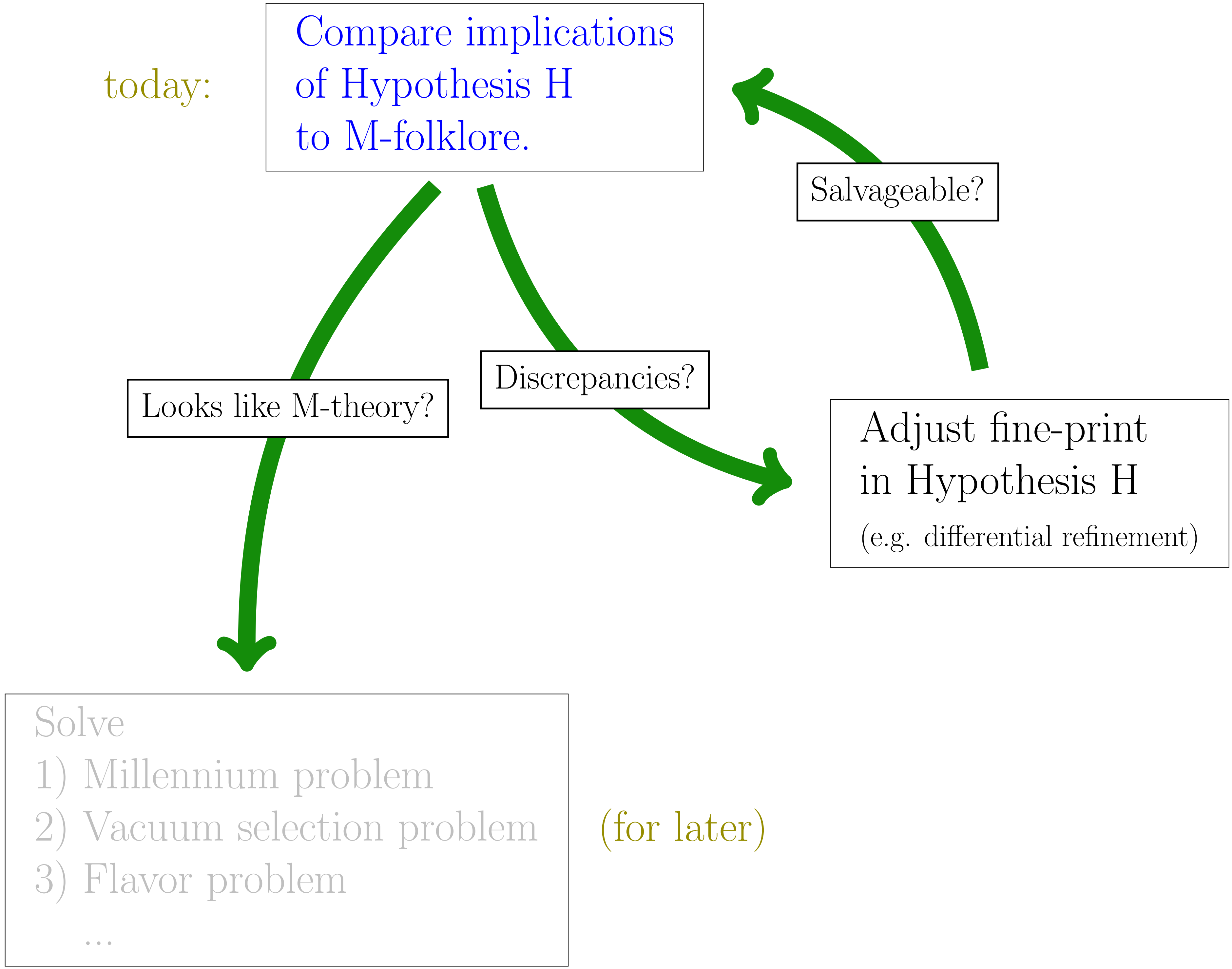
Hypothesis H

Sati 13, Fiorenza-Sati-S. 19b,19c

$\mathbf{CovariantPhaseSpace}_{\text{M-Theory}} =$

spacetimes

formalized as:	super-orbifold G_{ADE}–orbi $\mathbb{R}^{10,1\vert\mathbf{32}}$–folds $(\mathcal{X})$	
equipped with:	**0) gravity**	**1) C-field**
formalized as:	super-vielbein Pin^+-structure (E, Ψ)	flux densities differential forms (G_4, G_7)
subject to:	**Einstein equations**	**Page equation**
formalized as:	super-torsion $= 0$	flux is in ~~rationalized~~ J-twisted Cohomotopy FSS 19b,19c, SS 19a,19b,19c


today:
Compare implications of Hypothesis H to M-folklore.
Salvageable?
Discrepancies?
Looks like M-theory?
Adjust fine-print in Hypothesis H
(e.g. differential refinement)
Solve
1) Millennium problem
2) Vacuum selection problem
3) Flavor problem
...
(for later)

Implications of Hypothesis H

on **curved but smooth** spacetimes	on **flat but orbi-singular** spacetimes	on spacetimes **with horizons**
FSS 19b, FSS 19c, GS20	BSS 18, SS 19a	SS 19c
topological anomaly cancellation:	**equivariant anomaly cancellation**	D$p \perp$ D$(p+2)$ **worldvolume QFT**
- shifted C-field flux quantization - C-field tadpole cancellation - M5 Hopf-WZ level quantization - DMW anomaly cancellation - C-field integral eom ...	- M5/MO5 anomaly cancellation - RR-field tadpole cancellation - no irractional D-brane charge	- fuzzy funnels - BLG 3-algebras - BMN matrix model - M2/M5 bound states - AdS3-holography - Coulomb branch indices - Hanany-Witten rules - ...

H. Sati's talk	
D. Fiorenza's talk	my talk

at **M-Theory and Mathematics 2020**

(1)

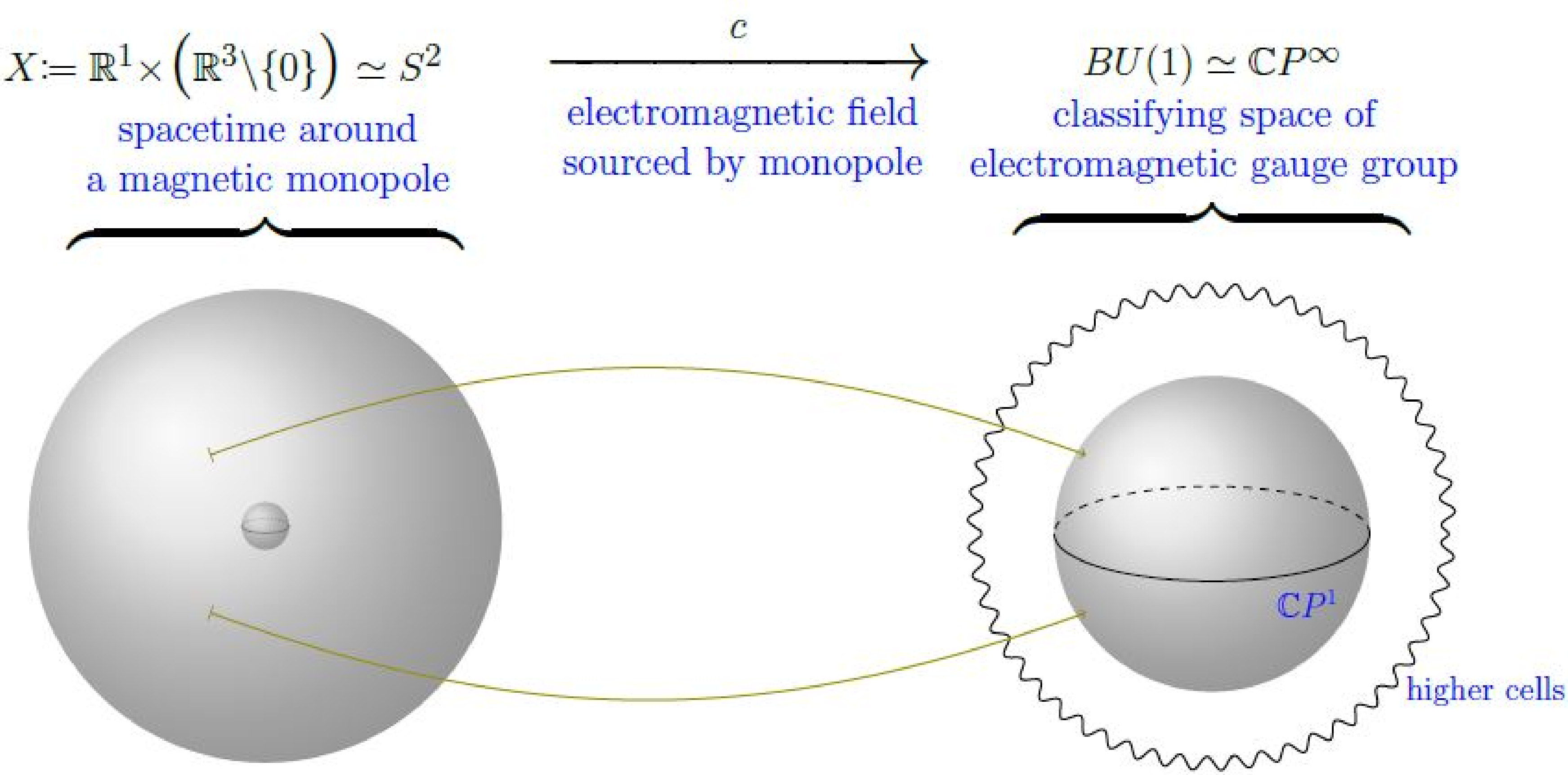
Microscopic Brane Charge

implied by

Hypothesis H with Pontrjagin-Thom Theorem

Sati-Schreiber 19a [arXiv:1909.12277]

back to top

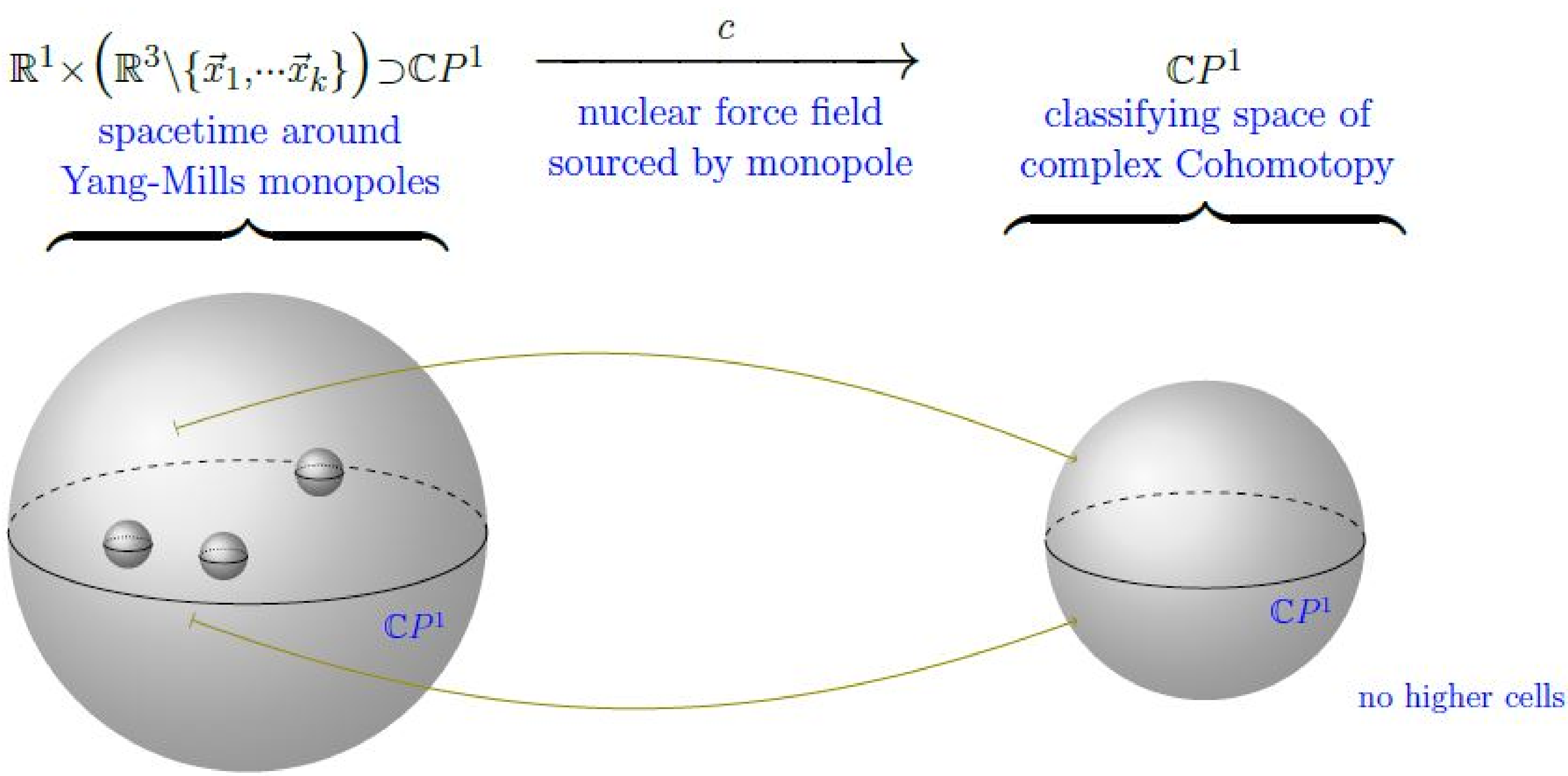


$$[c] \in \left\{ X \longrightarrow BU(1) \right\} \Big/_{\sim_{\text{homotopy}}} \simeq \mathbb{Z}$$

charge
lattice

charge = homotopy class

Dirac charge quantization – The topological sector of the electromagnetic field is a cocycle in degree-2 ordinary cohomology, with classifying/coefficient space $BU(1)$.



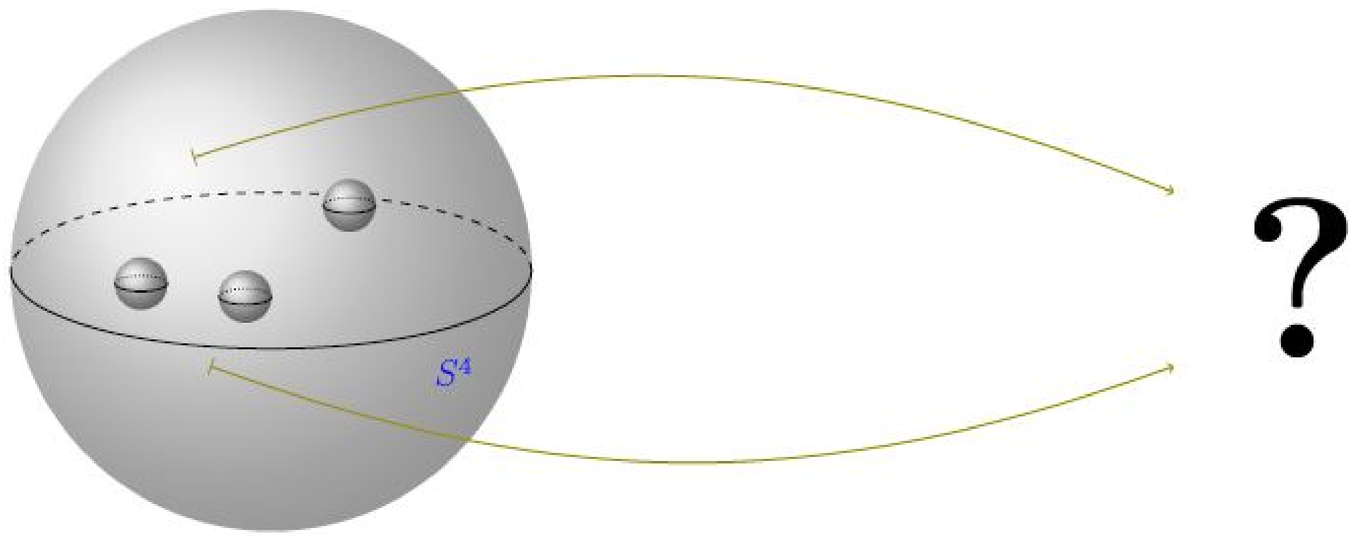
$$[c] \;\in\; \left\{ \mathbb{C}P^1 \longrightarrow \mathbb{C}P^1 \right\} \Big/_{\sim_{\text{homotopy}}} \;\simeq\; \mathbb{Z}$$

charge lattice

charge = homotopy class

Atiyah-Hitchin charge quantization – The moduli space of SU(2) Yang-Mills monopoles is the cocycle space of complex-rational Cohomotopy of any sphere enclosing them.

$$\underbrace{\mathbb{R}^{5,1} \times \left(\mathbb{R}^4 \setminus \{\vec{x}_1, \cdots \vec{x}_k\}\right) \times \mathbb{R}^1_{/\mathbb{Z}_2^{\mathrm{HW}}}}_{\text{spacetime around M5-branes}} \xrightarrow[\text{C-field sourced by M-branes}]{c} \underbrace{??}_{\text{classifying space of which cohomology theory??}}$$



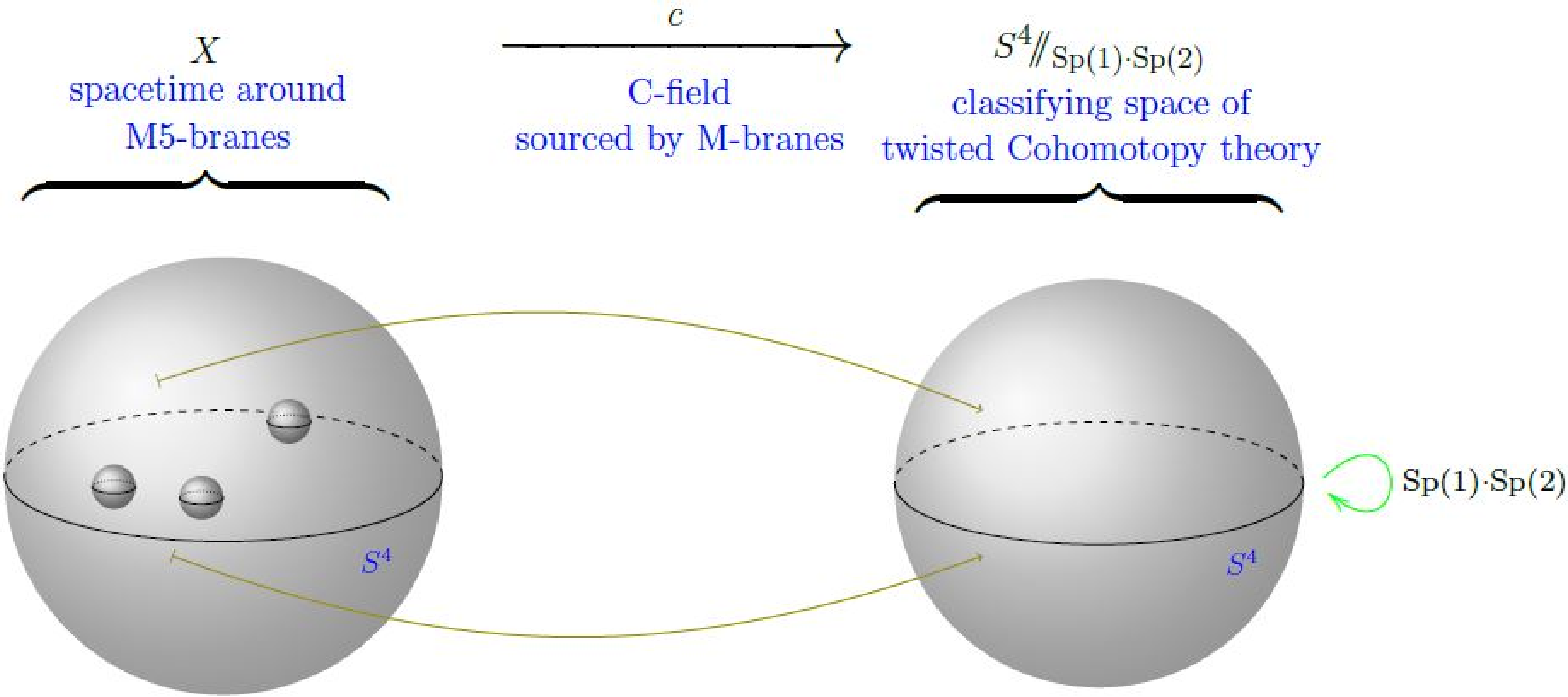
$$[c] \in \left\{ X \longrightarrow ?? \right\}_{/\sim_{\text{homotopy}}} \simeq \underset{\text{charge lattice}}{??}$$

charge = homotopy class

Strominger-Witten: Monopoles are wrapped M5-branes

and the elusive non-perturbative Yang-Mills theory is in M-theory.

$\rightsquigarrow$ Open problem: *Wherein is M5-brane charge quantization?*



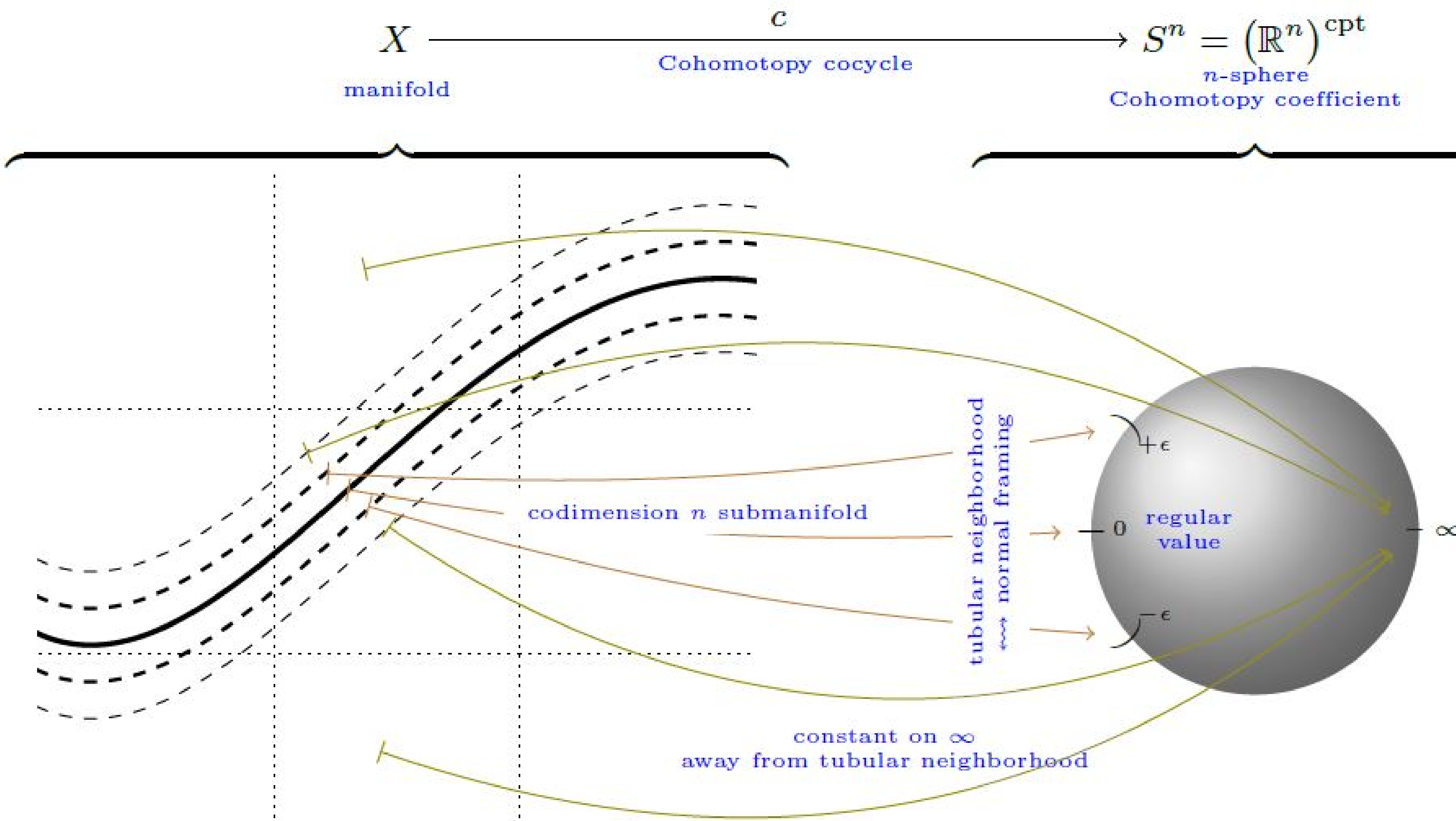
$$[c] \in \left\{ X \xrightarrow{\quad} S^4 /\!\!/_{\mathrm{Sp}(1)\cdot\mathrm{Sp}(2)} \ \text{ over } \ X \xrightarrow{TX} B(\mathrm{Sp}(1)\cdot\mathrm{Sp}(2)) \leftarrow S^4 /\!\!/_{\mathrm{Sp}(1)\cdot\mathrm{Sp}(2)} \right\} \Big/ \sim_{\text{homotopy}} \simeq \mathrm{Cob}^{TX}_{\mathrm{fr}}(X)$$

charge = homotopy class

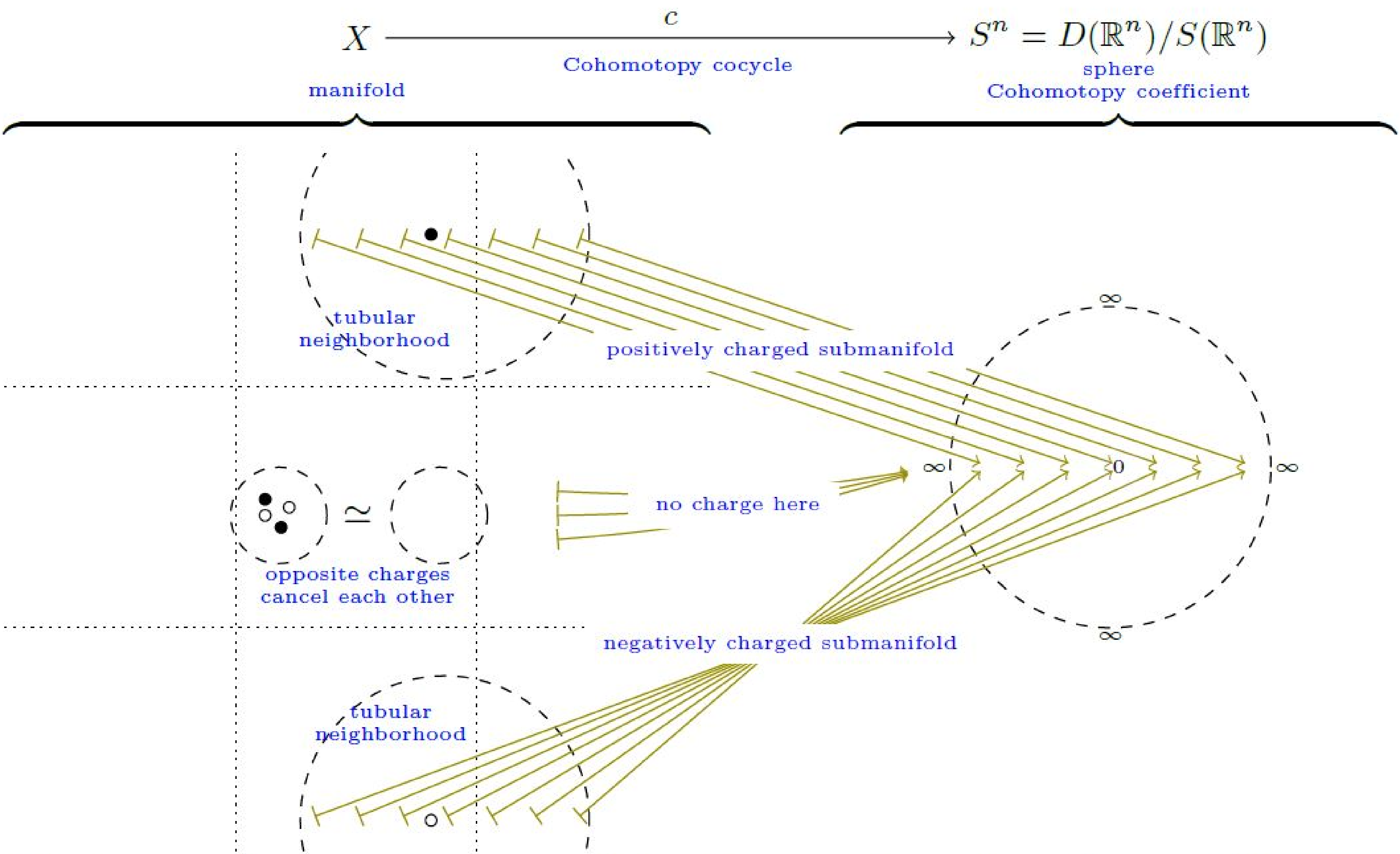
charge "lattice"

Hypothesis H (Fiorenza-Sati-Schreiber 19):

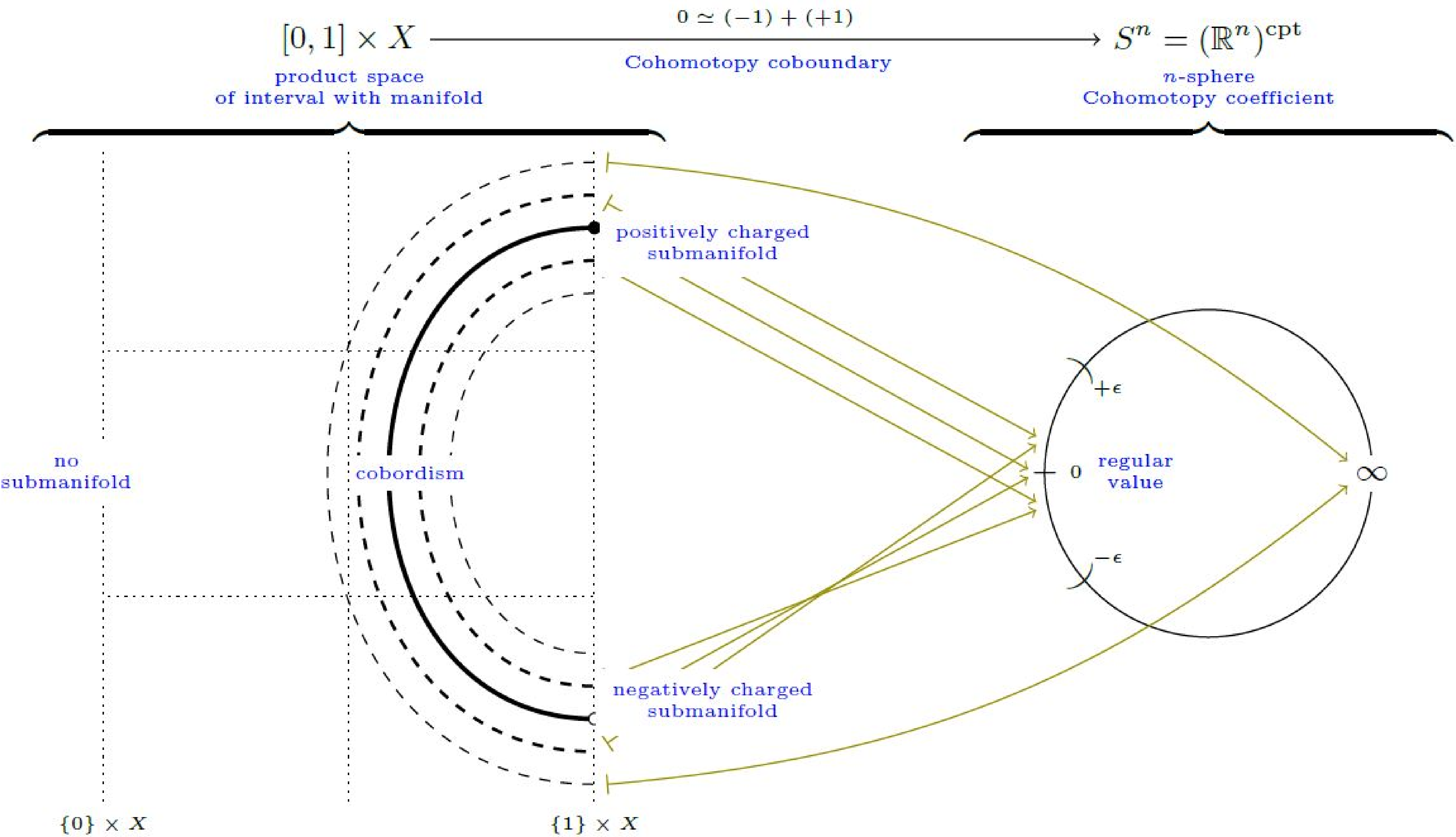
C-field is charge-quantized in J-twisted Cohomotopy theory.



Cohomotopy charge of normally framed submanifolds
is represented by the submanifold's *asymptotic distance function*, traditionally known as the *Pontrjagin-Thom collapse*.

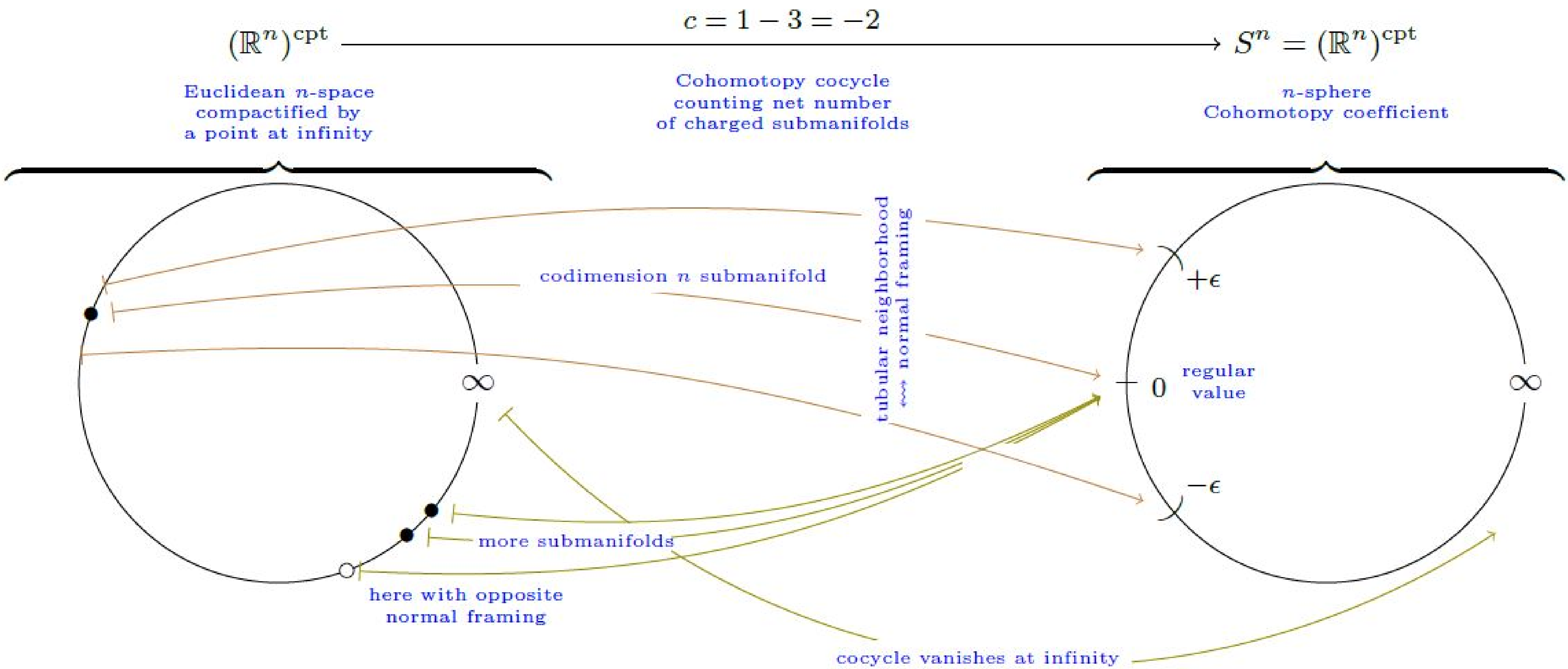


Cohomotopy charge of 0-dimensional submanifolds
(traditionally known as "electric field map" or *scanning map*)
exhibits net brane/anti-brane charge in $\mathbb{Z}$.



Brane/anti-brane pair creation & annihilation

is exhibited, under Hypothesis H, by normally framed cobordism.



Cohomotopy charge vanishing at ∞ on Euclidean n-space
is equivalently the Cohomotopy charge of the n-sphere
and hence takes values in homotopy groups of spheres.

(2)

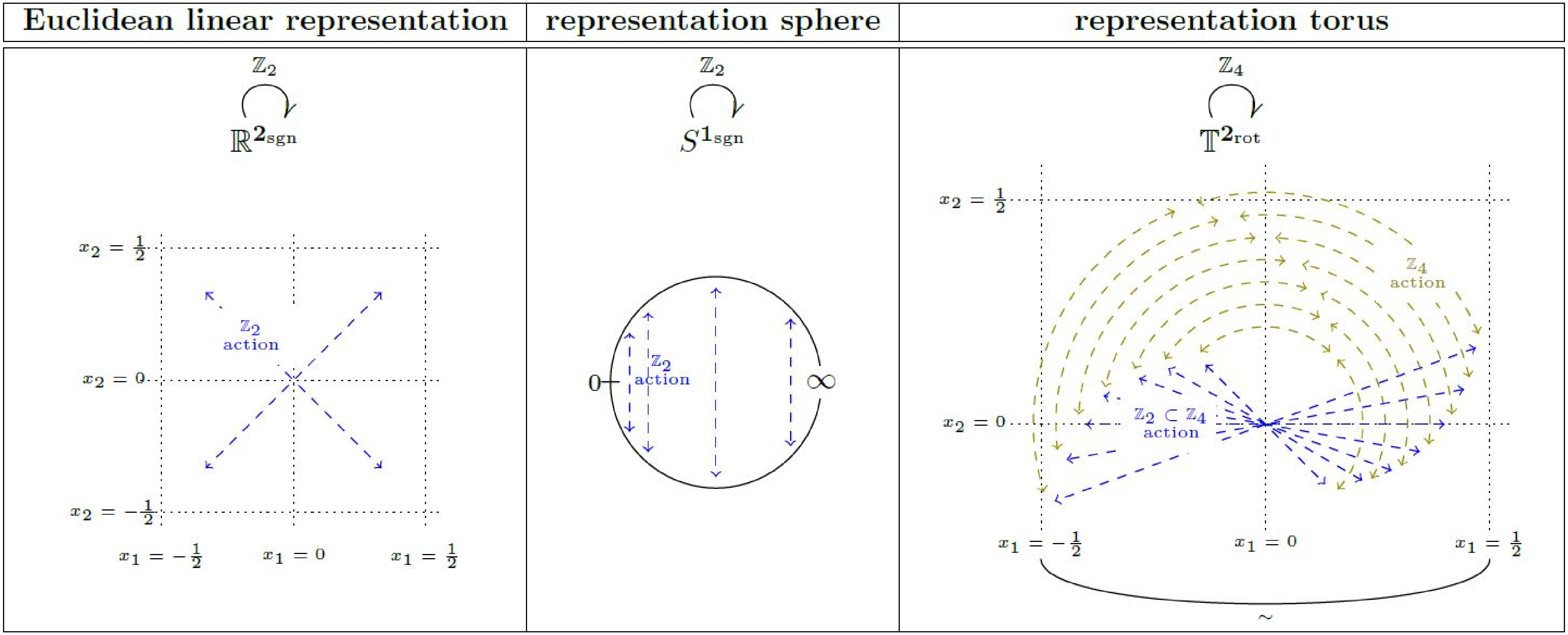
Orientifold Tadpole Cancellation

implied by

Hypothesis H with Equivariant Hopf Degree Theorem

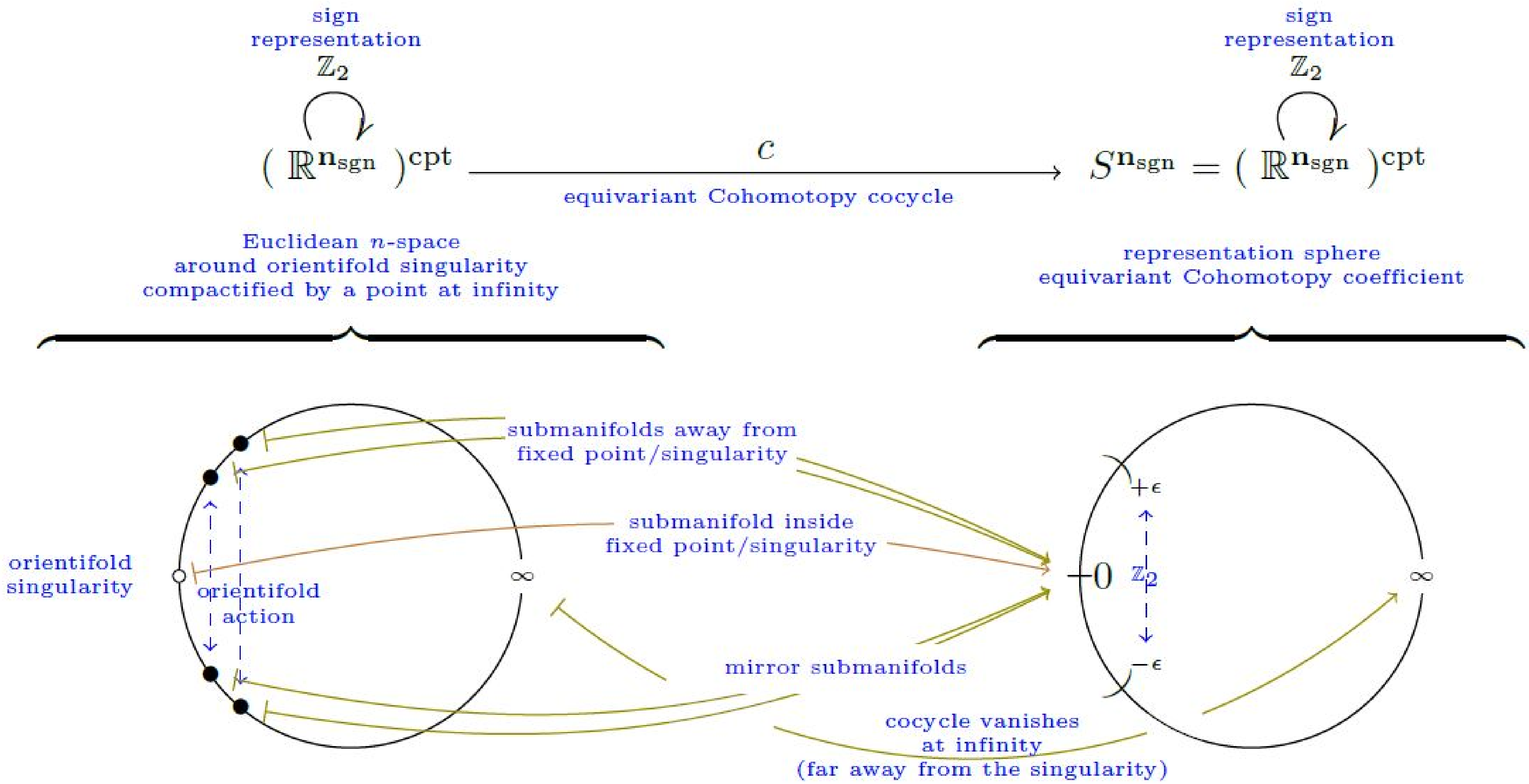
Sati-Schreiber 19a [arXiv:1909.12277]

back to top



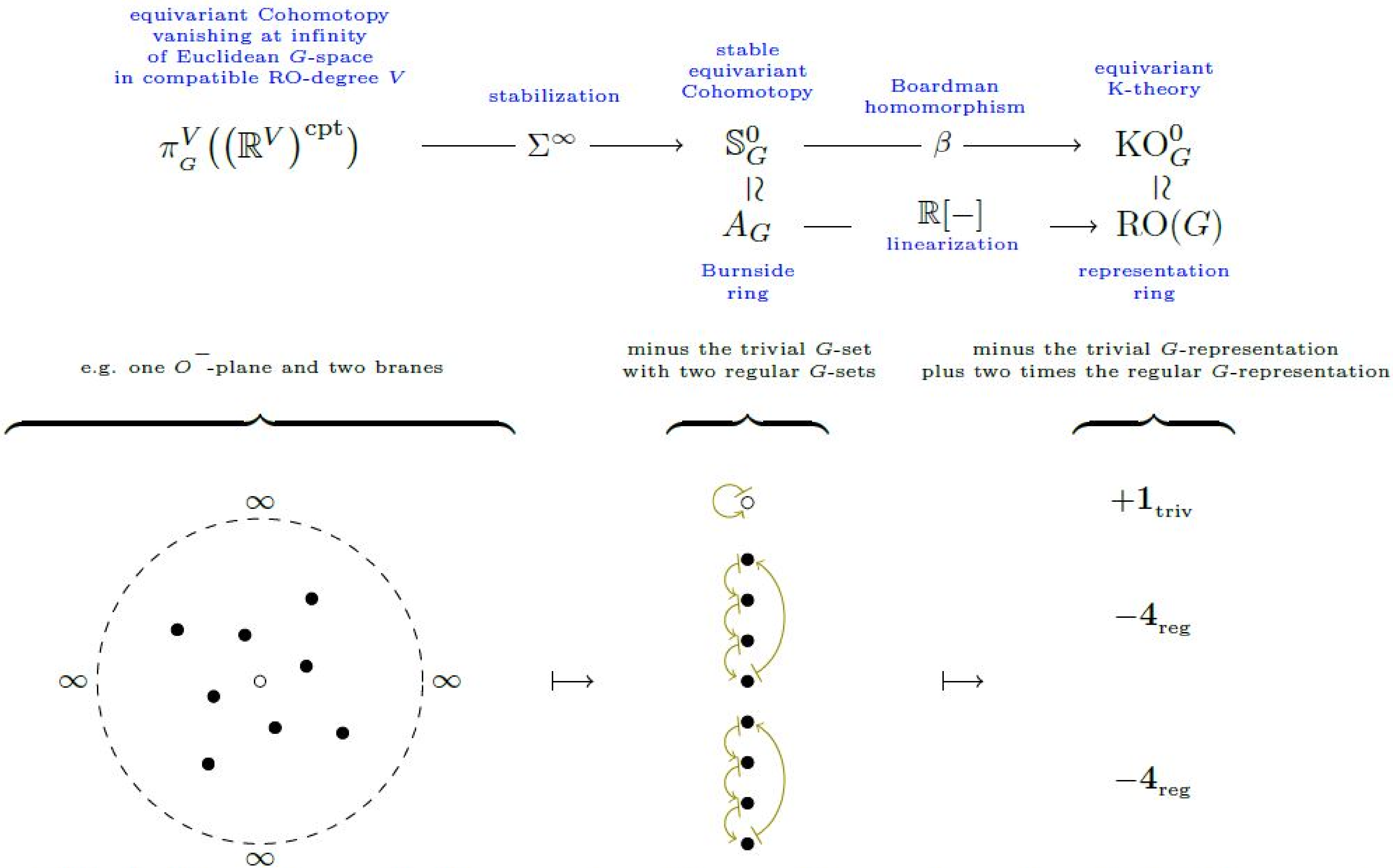
Examples of linear representations and induced G-spaces

Figure H – Hopf degree in the unstable range takes values in the set $\{0, 1\}$ (14), corresponding to the binary choice of there being or not being a unit charge at the single point.

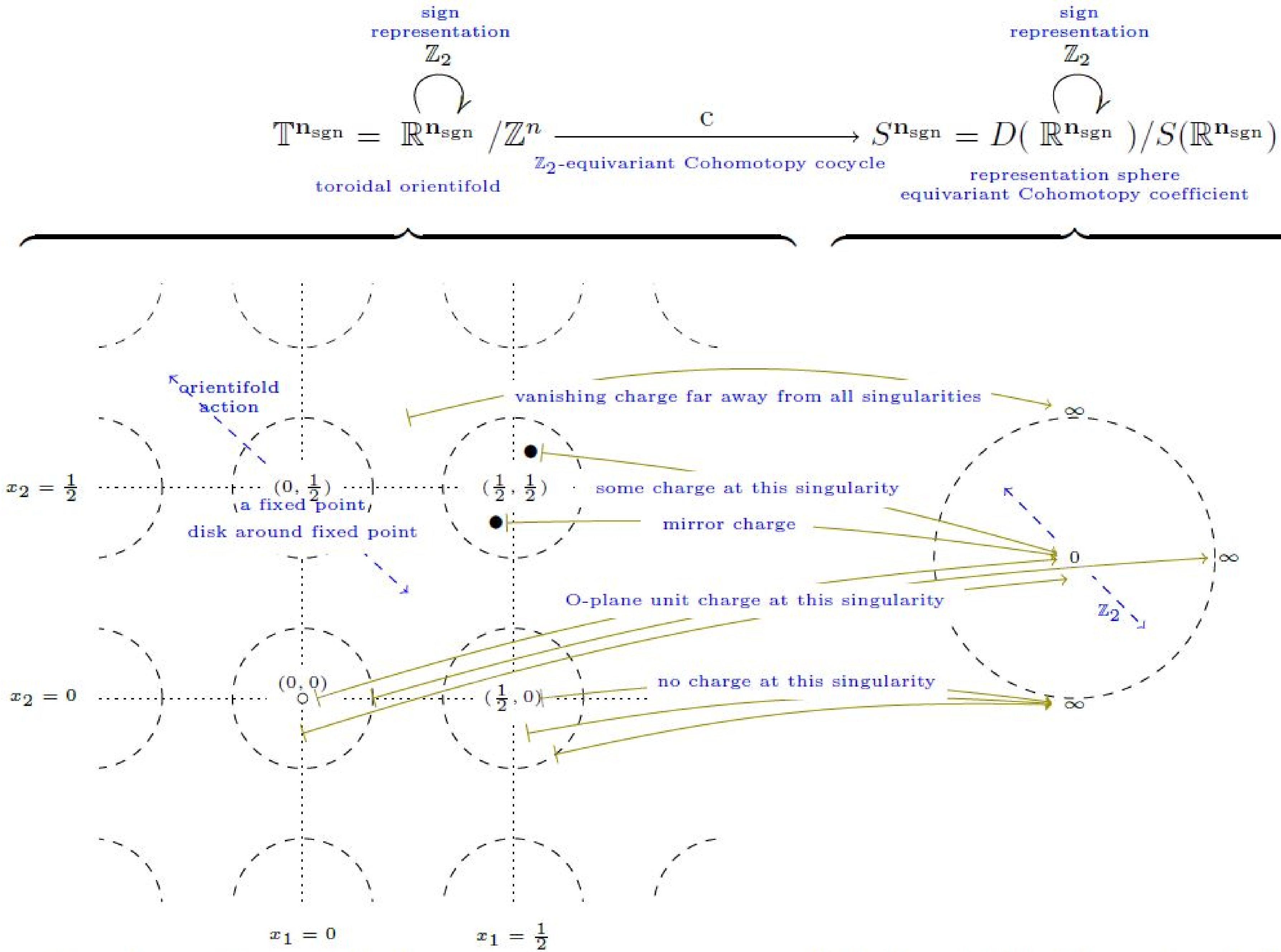


The equivariant Hopf degree theorem

says that $\mathbb{Z}_2$-equivariant Cohomotopy charges near singularities are sourced by, possibly, a charge attached to the singularity and any integer number of twice this charge located nearby.



Stabilization & linearization of equivariant Cohomotopy lands in equivariant K-theory. In this approximation virtual G-sets of (anti-)branes map to virtual permutation representations.



Equivariant Cohomotopy on toroidal orbifolds glued from local cocycles in the vicinity of singularities. By the equivariant Hopf degree theorem, all global cocucles are obtained this way.

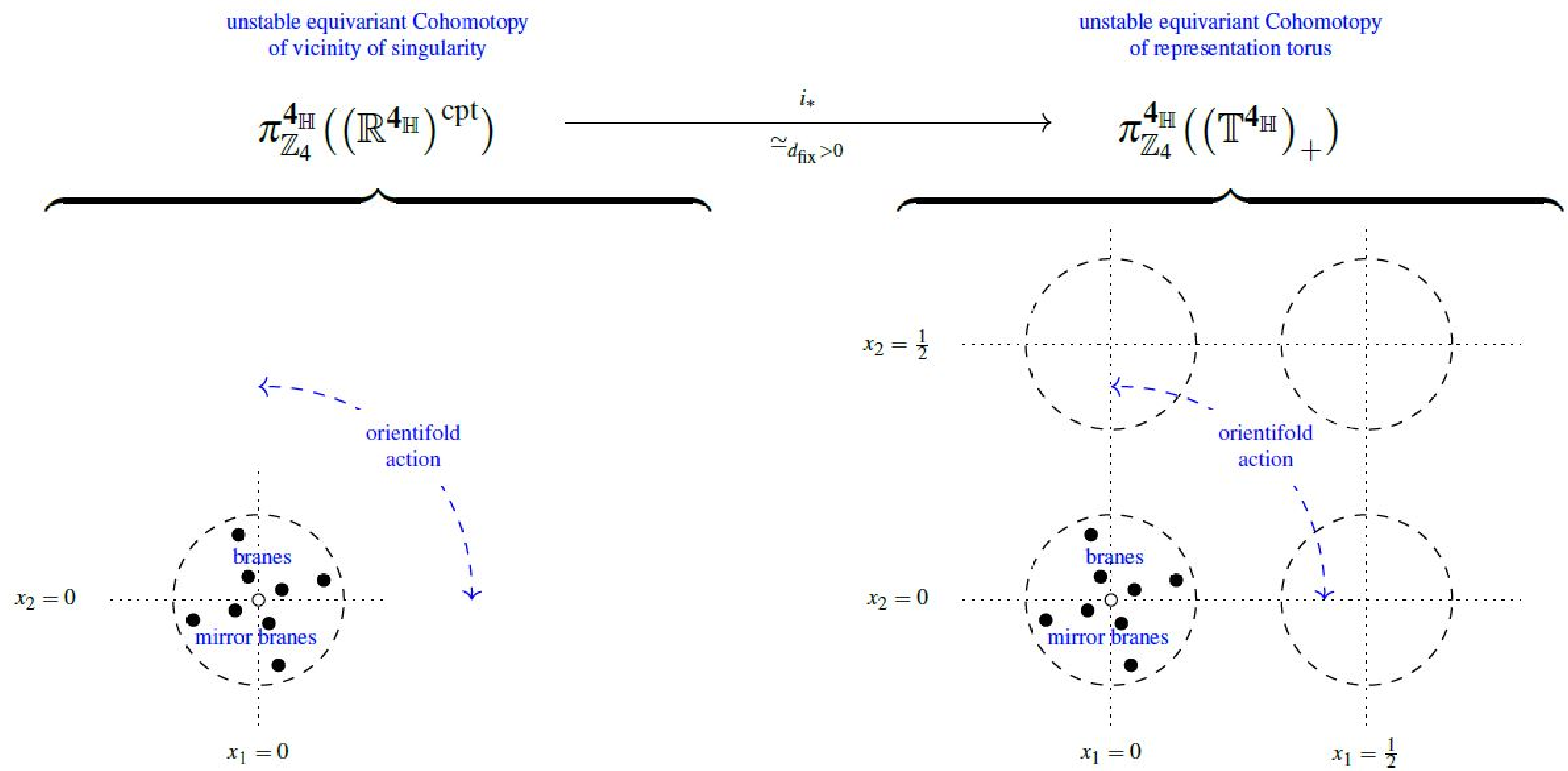
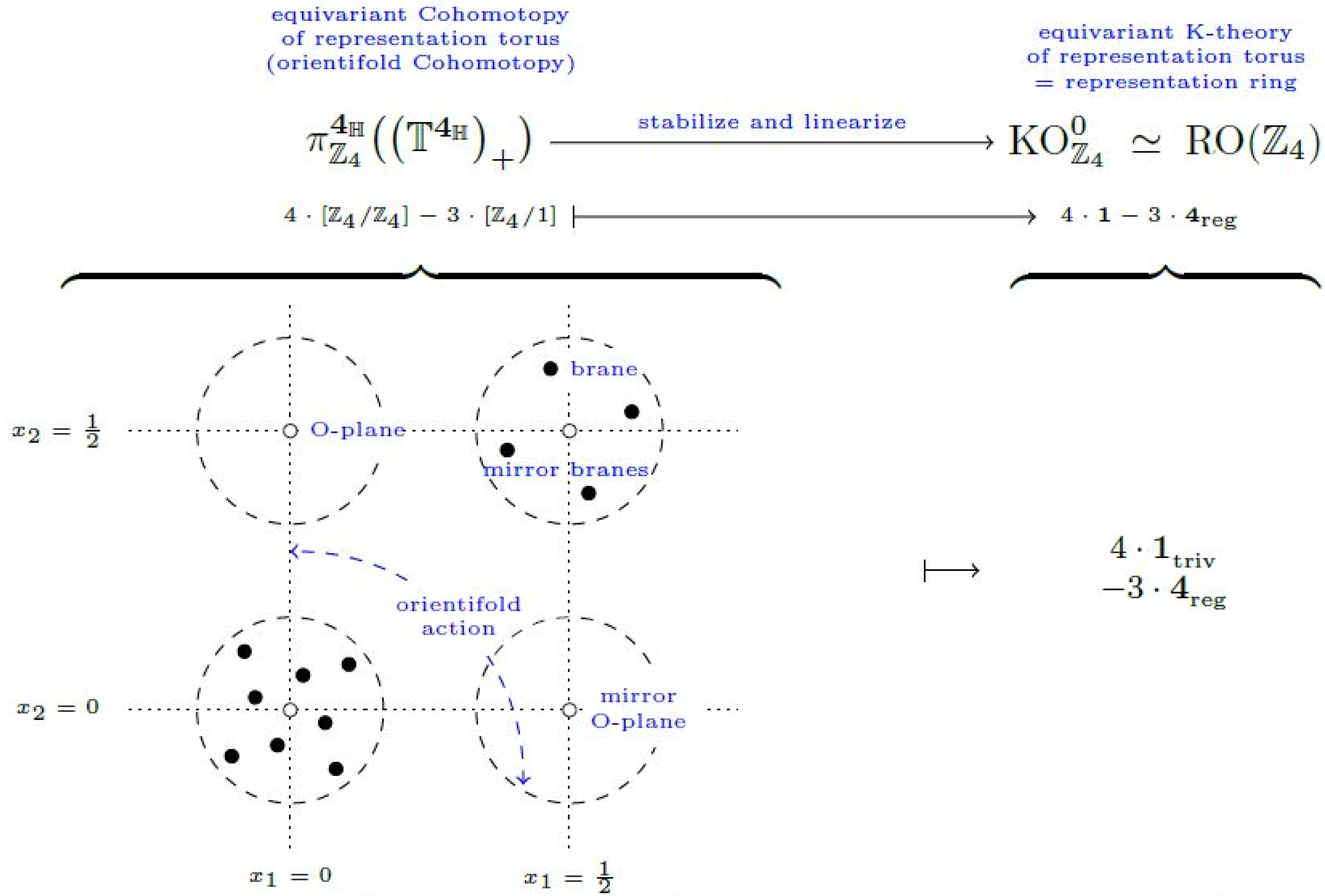
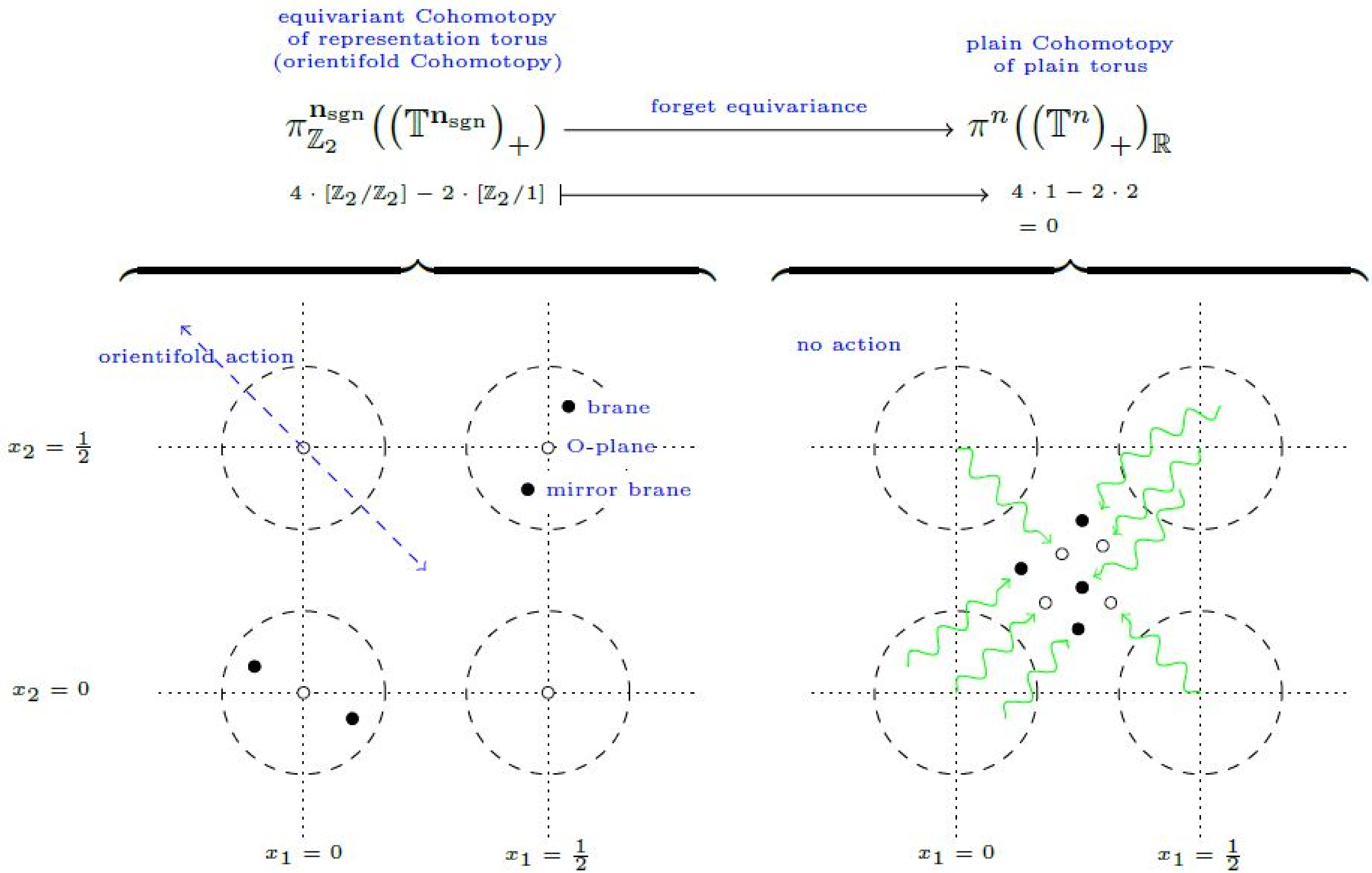


Figure O – Pushforward in equivariant Cohomotopy from the vicinity of a singularity to the full toroidal orientifold is an isomorphism on brane charges and an injection on O-plane charges, by Prop. 3.18. Shown is a case with $G = \mathbb{Z}_4$, as in *Figure M*.



Equivariant Cohomotopy implies local tadpole cancellation

by the combined unstable and stable version of the equivariant Hopf degree theorem.



Super-differential equivariant Cohomotopy implies global tadpole cancellation by forcing the charge to vanish at global Elmendorf stage, and only there.

The following four slides show technical detail of the realization of this mechanism for MO5-planes at ADE-singularities in heterotic M-theory

Skip over technical detail ahead to section (3).

Orientifold		MO5	$\frac{1}{2}$M5
Global quotient group	$G =$	$\mathbb{Z}_2$	$\mathbb{Z}_2^{\mathrm{HW}} \times G^{\mathrm{ADE}}$
Global quotient group action	$G \curvearrowright \mathbb{T}^V =$	$\mathbb{Z}_2 \curvearrowright \mathbb{T}^{\mathbf{5}\mathrm{sgn}}$	$\mathbb{Z}_2^{\mathrm{HW}} \curvearrowright \mathbb{T}^{\mathbf{1}\mathrm{sgn}} \times G^{\mathrm{ADE}} \curvearrowright \mathbb{T}^{\mathbf{4}\mathrm{sgn}}$
Fixed/singular points	$(T^V)^G =$	$\{0, \frac{1}{2}\}^5 = \overline{32}$	
Far horizon-limit of M5 SuGra solution?		no	yes

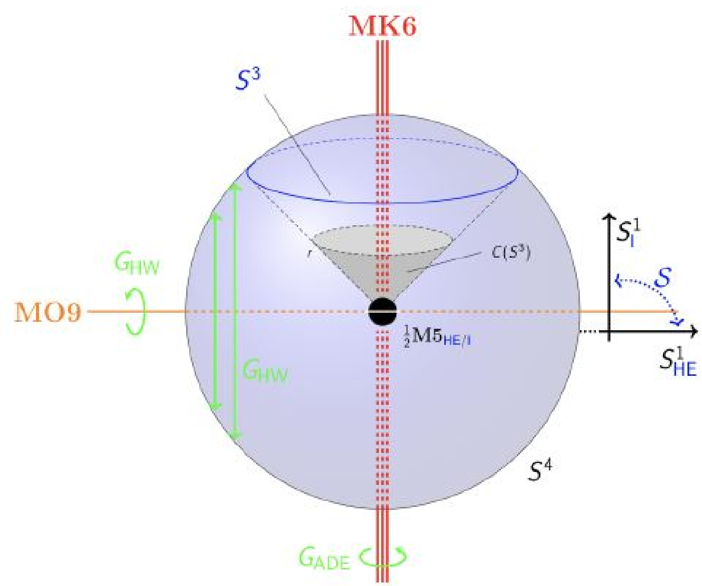


Figure S – Singularity structure of heterotic M-theory on ADE-singularities, as in Figure R, [HSS18, 2.2.2, 2.2.7]. The corresponding toroidal orbifolds (as per *Table 5*) are illustrated in *Figure V* and *Table 8*.

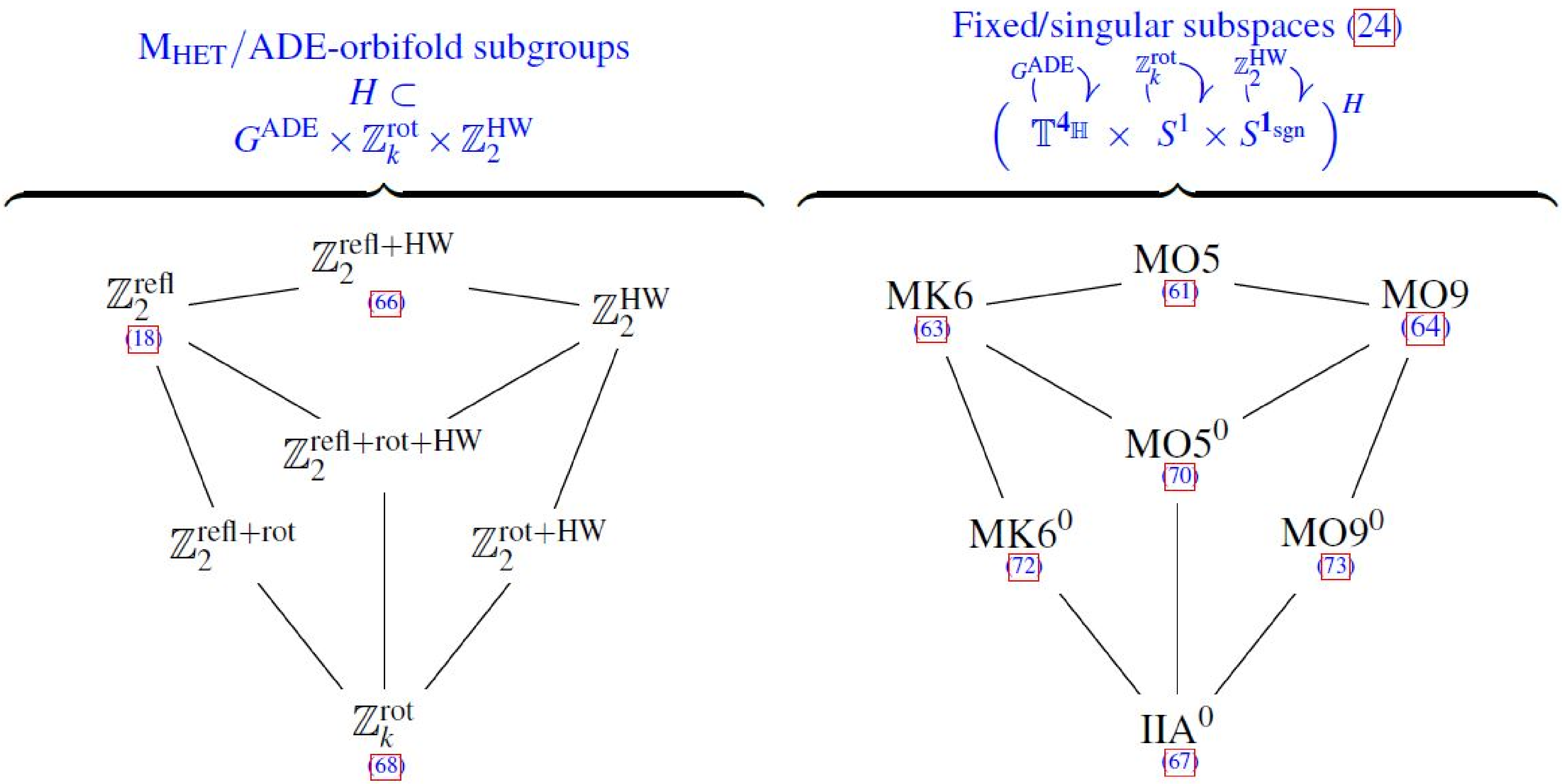


Figure T – Subgroup lattice and fixed/singular subspaces in the singularity structure from *Table 7*. On the left, groups associated to the middle of a sub-simplex are diagonal subgroups inside the direct product of subgroups associated to the vertices, as indicated by the superscripts. On the right, all fixed loci with superscript $(-)^0$ are actually empty, but appear as superficially non-empty (un-charged) singularities after M/IIA KK-reduction (68), e.g. $\mathrm{O4}^0$ (71), $\mathrm{O8}^0$ (74), as on the right of *Figure OP*. The numbered subscripts (xx) indicate the corresponding expression in the text.

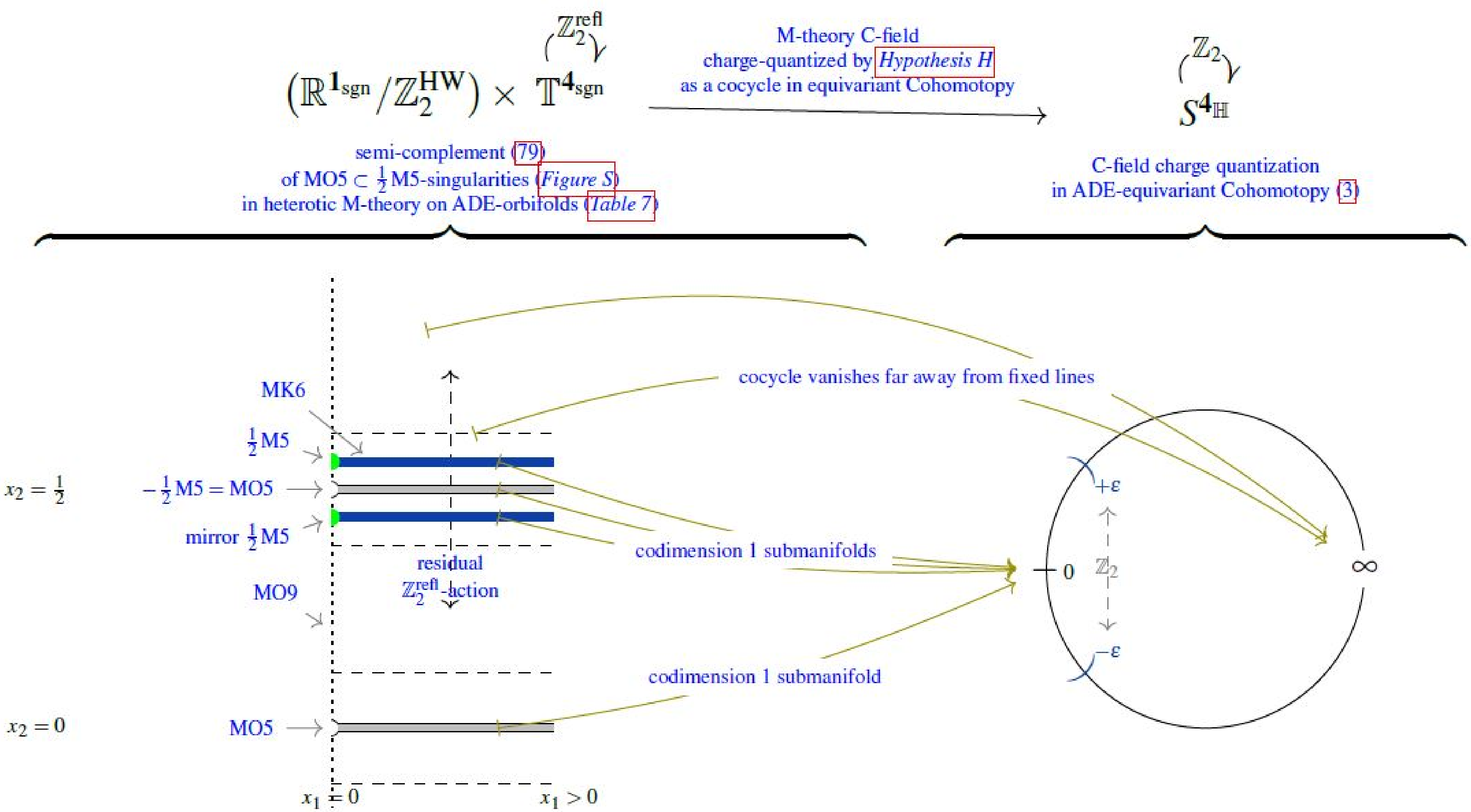
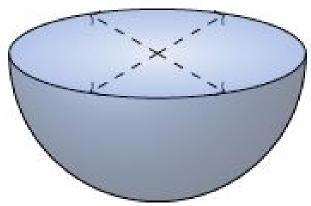
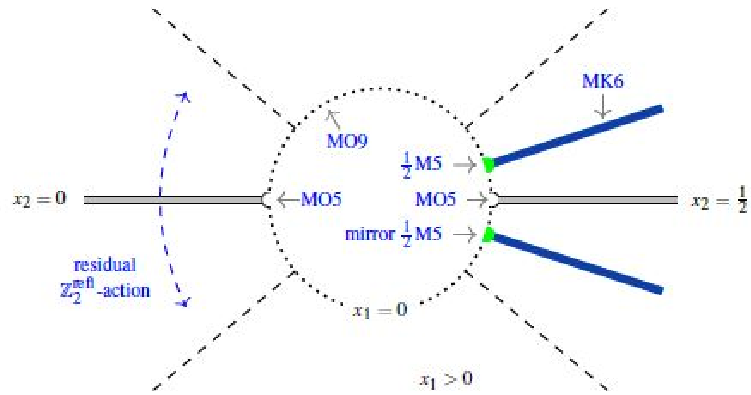
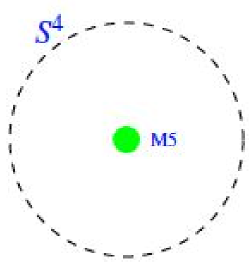
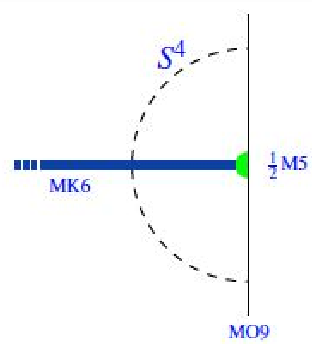


Figure V – Equivariant Cohomotopy of ADE-orbifolds in heterotic M-theory with singularity structure as in *Figure S*. The resulting charge classification (Cor. 4.4) implies, via the unstable PT isomorphism (§2.1), the $\frac{1}{2}$M5 = MO9 ∩ MK6-brane configurations (65) similarly shown in [FLO99, Fig. 1][KSTY99, p. 7][FLO00a, Fig. 1][FLO00b, Fig. 2][FLO00c, Fig. 1][GKST01, p. 4, 68, 71]. This is as in *Figure L* but with points (M5s) extended to half-line (MK6s), see Remark 4.7 and *Table 8*.

Spacetimes on which to measure flux sourced by M5/MO5-charge		
Definition	$X_{\mathrm{MO5}} \simeq_{\mathrm{htpy}} S(\mathbb{R}^{\mathbf{1}_{\mathrm{sgn}}+\mathbf{4}_{\mathrm{sgn}}})/\mathbb{Z}_2^{\mathrm{het}+\mathrm{HW}}$ (75)	$X_{1/2\mathrm{M5}} \simeq_{\mathrm{htpy}} S(\mathbb{R}^{\mathbf{1}_{\mathrm{sgn}}})/\mathbb{Z}_2^{\mathrm{HW}} \times \mathbb{T}^{\mathbf{4}_{\mathbb{H}}} /\!\!/ \mathbb{Z}_2^{\mathrm{refl}}$ (79)
Illustration		
Geometry	smooth but curved	singular but flat

Cohomological charge quantization by *Hypothesis H*		
Cohomology theory (by *Table 4*)	J-twisted Cohomotopy $\pi^{TX}(X)$ [FSS19b][FSS19c]	equivariant Cohomotopy $\pi^{V}(\mathbb{T}^{V})$ §3
Illustration (Remark 4.7)		
Charge classification	$c_{\mathrm{tot}} = 1 - N \cdot 2$ (77)	$c_{\mathrm{tot}} = N_{\mathrm{MO5}} \cdot \mathbf{1}_{\mathrm{triv}} - N_{\mathrm{M5}} \cdot \mathbf{2}_{\mathrm{reg}}$ (Cor. 4.4) $\Leftrightarrow$ $\lvert Q_{\mathrm{tot}} \rvert = 0$, $N_{\mathrm{M5}} = 8$ (Cor. 4.6)

(3)

$\mathrm{D6} \perp \mathrm{D8}$ -brane intersections

implied by

Hypothesis H with May-Segal Theorem

Sati-Schreiber 19c [arXiv:1912.10425]

back to top

Cohomotopy cocycle space pointed mapping space

$$\boldsymbol{\pi}^4(X) \;:=\; \mathrm{Maps}^{*/}(X, S^4)$$

boldface!

$$\pi_0\big(\boldsymbol{\pi}^4(X)\big) \;=\; \left\{ \begin{array}{c} \text{Cohomotopy} \\ \text{cohomology} \\ \text{classes} \end{array} \right\} \;=\; \pi^4(X)$$

not boldface

Cohomotopy set

$$\pi_1\big(\boldsymbol{\pi}^4(X)\big) \;=\; \left\{ \begin{array}{c} \text{Cohomotopy} \\ \text{gauge} \\ \text{transformations} \end{array} \right\}$$

$$\pi_2\big(\boldsymbol{\pi}^4(X)\big) \;=\; \left\{ \begin{array}{c} \text{Cohomotopy} \\ \text{gauge of gauge} \\ \text{transformations} \end{array} \right\}$$

$\vdots$

Cohomotopy cocycle space
vanishing at ∞ on Euclidean 3-space

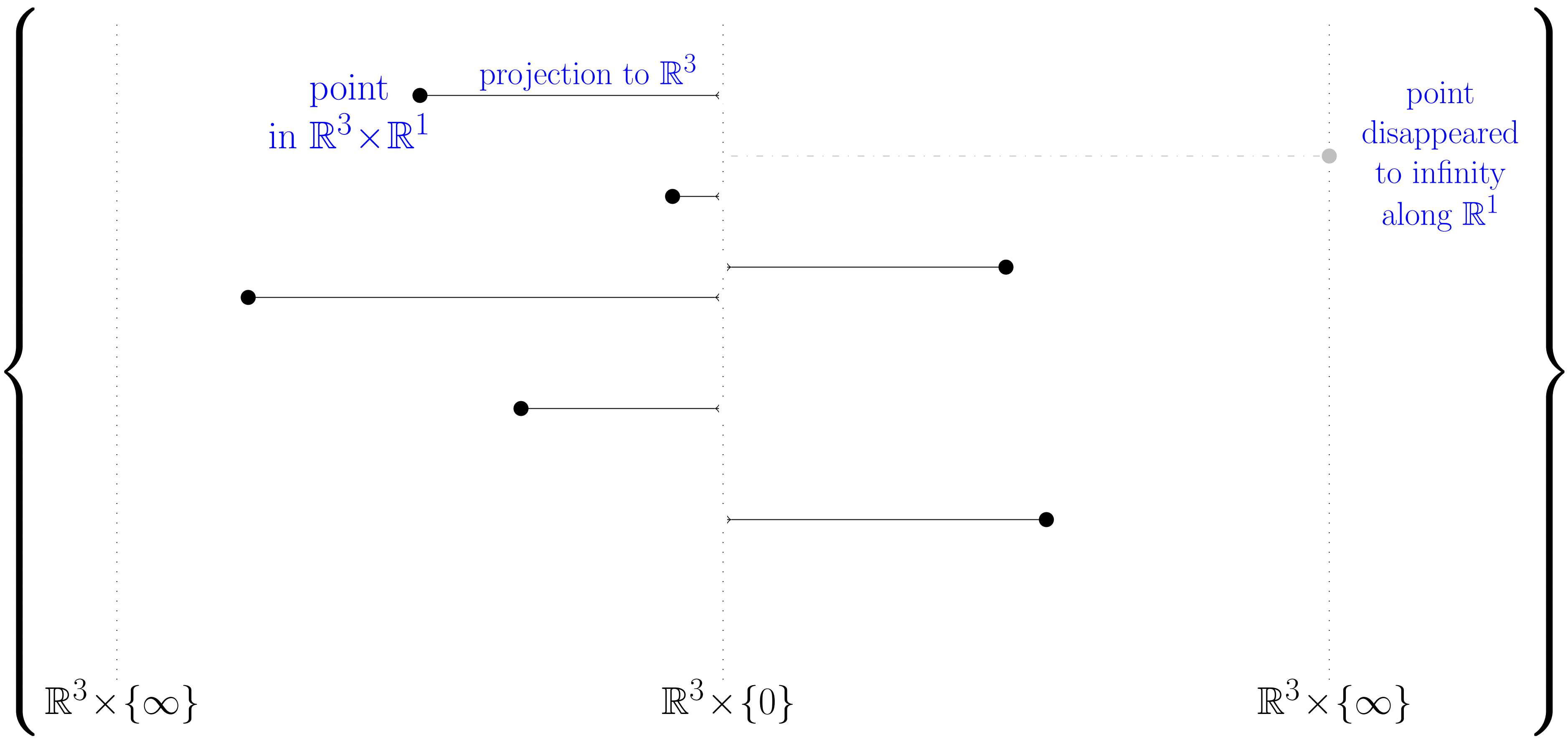
May-Segal theorem

$$\boldsymbol{\pi}^4\Big((\mathbb{R}^3)^{\mathrm{cpt}}\Big) \overset{\text{hmtpy}\,\simeq}{\longleftarrow} \mathrm{Conf}\big(\mathbb{R}^3, \mathbb{D}^1\big)$$

assign unit charge
in Cohomotopy
to each point

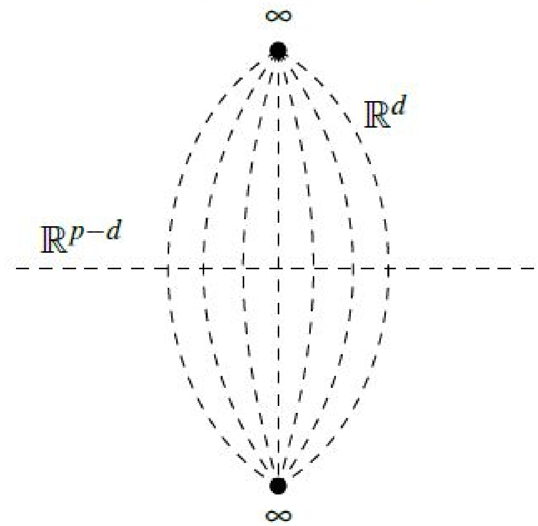
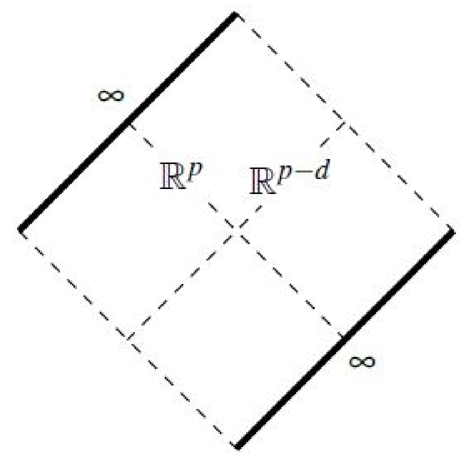
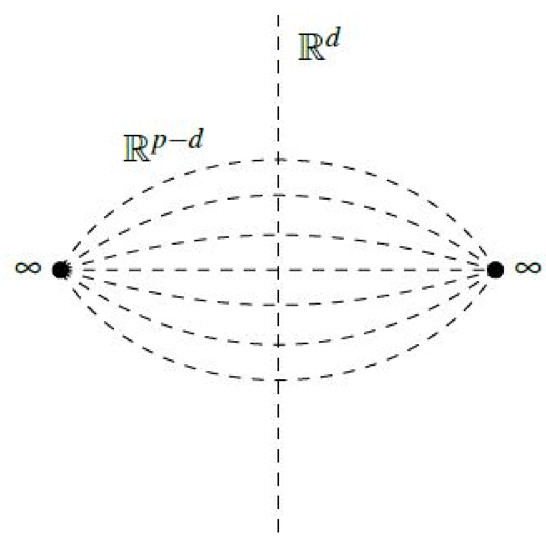
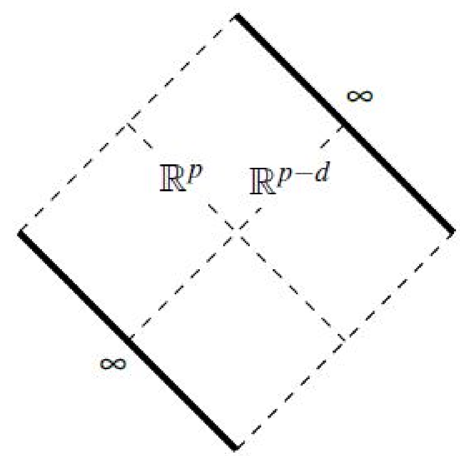
configuration space of points
in $\mathbb{R}^3 \times \mathbb{R}^1$
which are:
1) unordered
2) distinct after projection to $\mathbb{R}^3$
3) allowed to vanish to ∞ along $\mathbb{R}^1$

$=$



hence: a form of differential Cohomotopy assigns configuration spaces:

$$\boldsymbol{\pi}^4\Big((\mathbb{R}^d)^{\mathrm{cpt}} \wedge (\mathbb{R}^{4-d})_+\Big) \xleftarrow[\;]{\mathrm{hmtpy}\simeq} \boldsymbol{\pi}^4_{\mathrm{diff}}\Big((\mathbb{R}^d)^{\mathrm{cpt}} \wedge (\mathbb{R}^{4-d})_+\Big) := \mathrm{Conf}\big(\mathbb{R}^d, \mathbb{D}^{4-d}\big)$$

Smash product of pointed topological spaces	**Visualization** with point at infinity	as Penrose diagram
cocycles vanish at infinity along these direction $(\mathbb{R}^d)^{\mathrm{cpt}} \wedge (\mathbb{R}^{p-d})_+$ ...but not necessarily along these		
cocycles vanish at infinity along these direction $(\mathbb{R}^d)_+ \wedge (\mathbb{R}^{p-d})^{\mathrm{cpt}}$ ...but not necessarily along these		

Lemma:

$$\underset{n\in\mathbb{N}}{\bigsqcup} \underset{\{1,\cdots,n\}}{\mathrm{Conf}}\big(\mathbb{R}^D\big) \underset{hmtpy}{\simeq} \mathrm{Conf}\big(\mathbb{R}^D, \mathbb{D}^1\big) \underset{\mathrm{Conf}(\mathbb{D}^{D+1})}{\times} \mathrm{Conf}\big(\mathbb{R}^1, \mathbb{D}^D\big)$$

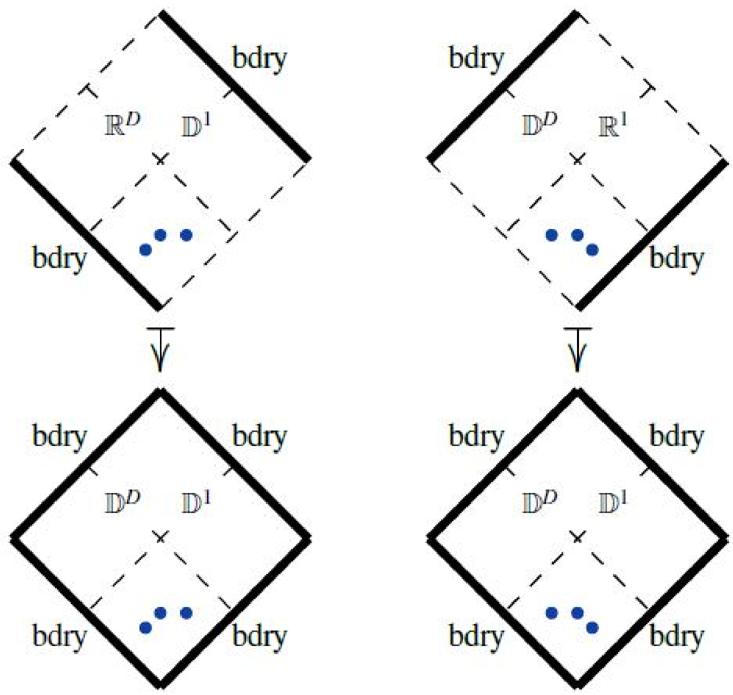
Ordered configurations of points in $\mathbb{R}^D$

Un-ordered configurations of points in $\mathbb{R}^D$ with labels in $\mathbb{D}^1$

Un-ordered configurations of points in $\mathbb{D}^{D+1}$

Un-ordered configurations of points in $\mathbb{R}^1$ with labels in $\mathbb{R}^D$

$$\left(\mathrm{Conf}\big(\mathbb{R}^D, \mathbb{D}^1\big) \xrightarrow{((i_L)^*)_*} \mathrm{Conf}\big(\mathbb{D}^{D+1}\big) \xleftarrow{((i_R)^*)_*} \mathrm{Conf}\big(\mathbb{R}^1, \mathbb{D}^D\big) \right)$$



Lemma:

Un-ordered configurations of points in $\mathbb{R}^D$ with labels in $\mathbb{D}^1$

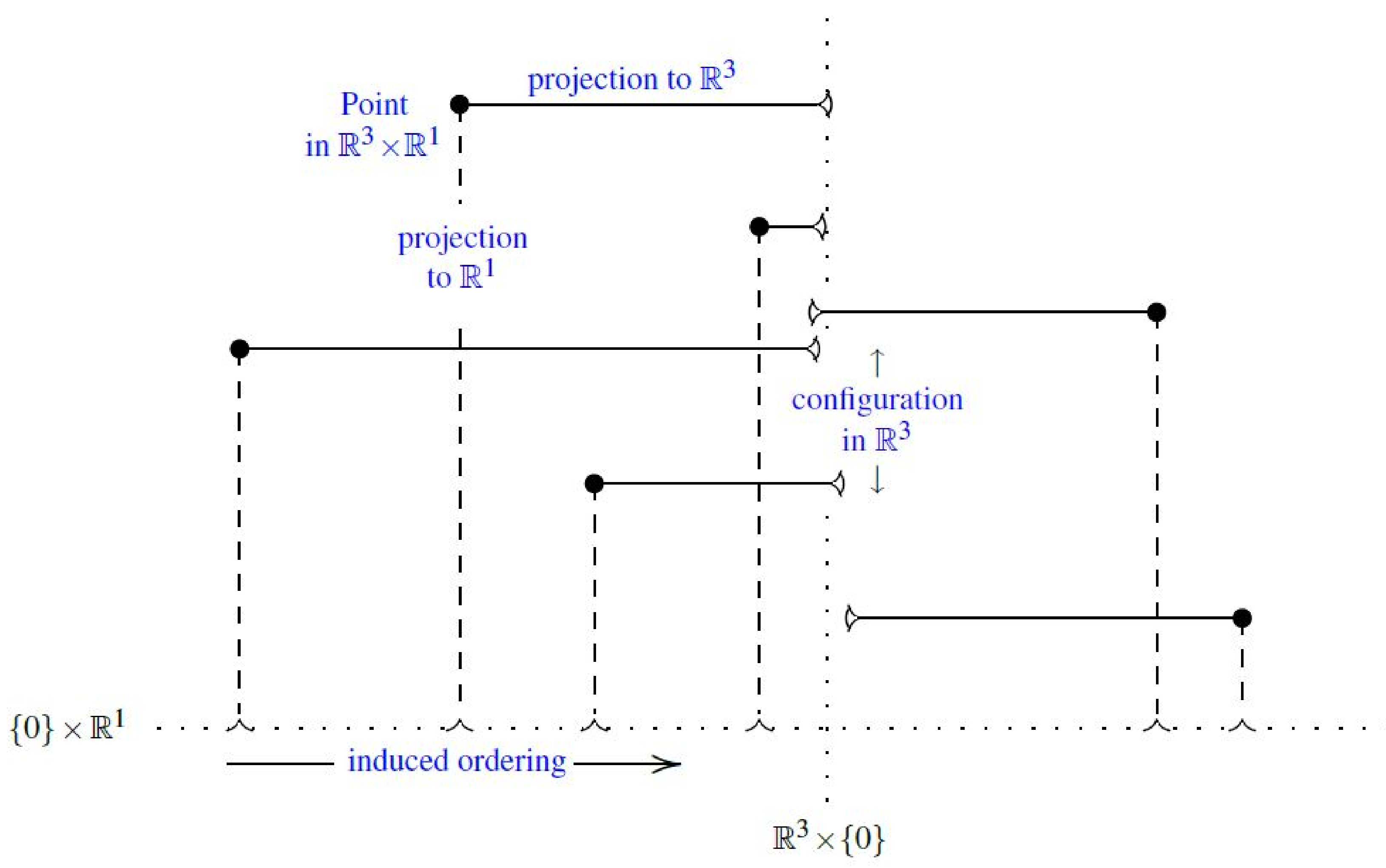
Un-ordered configurations of points in $\mathbb{R}^1$ with labels in $\mathbb{R}^D$

$$\underset{n\in\mathbb{N}\{1,\cdots,n\}}{\bigsqcup} \mathrm{Conf}\big(\mathbb{R}^D\big) \underset{hmtpy}{\simeq} \mathrm{Conf}\big(\mathbb{R}^D,\mathbb{D}^1\big) \underset{\mathrm{Conf}(\mathbb{D}^{D+1})}{\times} \mathrm{Conf}\big(\mathbb{R}^1,\mathbb{D}^D\big)$$

Ordered configurations of points in $\mathbb{R}^D$

Un-ordered configurations of points in $\mathbb{D}^{D+1}$

$$= \left\{ \vphantom{\rule{0pt}{3em}} \right.$$

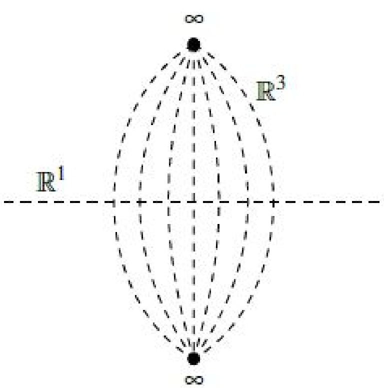


$$\left. \vphantom{\rule{0pt}{3em}} \right\}$$

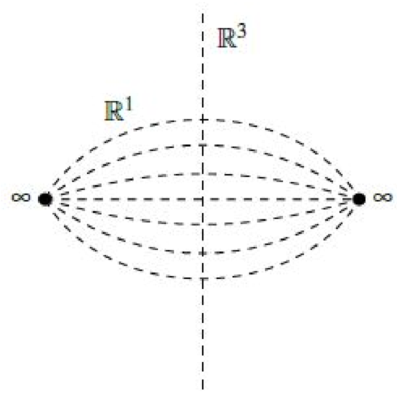
Consequence:

assuming
Hypothesis H:

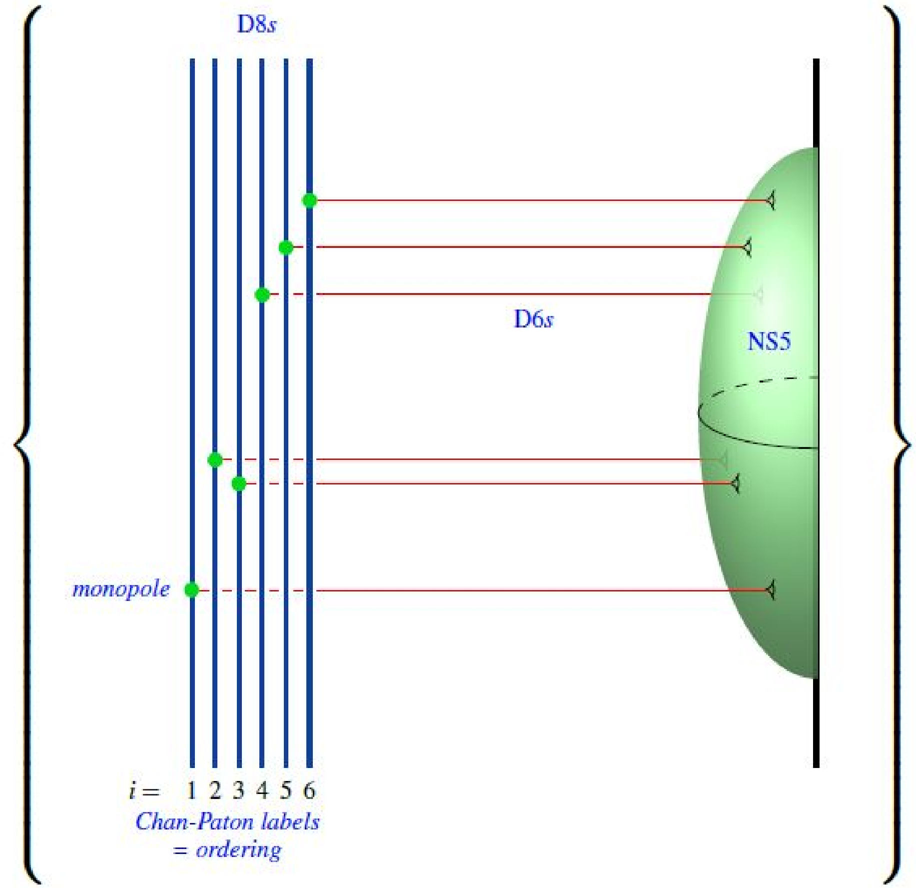
Transversal space to 3-codim branes hence to D6-branes		*Transversal space to 1-codim branes hence to D8-branes*	*Differential Cohomotopy*	differential Cohomotopy cocycle space reflecting D6 ⊥ D8 -charges
$(\mathbb{R}^3)^{\mathrm{cpt}} \wedge (\mathbb{R}^1)_+$	$\cup$	$(\mathbb{R}^3)_+ \wedge (\mathbb{R}^1)^{\mathrm{cpt}}$	$\xmapsto{\ \boldsymbol{\pi}^4_{\mathrm{diff}}\ }$	$\underset{n\in\mathbb{N}}{\coprod} \underset{\{1,\cdots,n\}}{\mathrm{Conf}}(\mathbb{R}^3)$



$\cup$



$\longmapsto$



(4)

Hanany-Witten Theory

implied by

Hypothesis H with Fadell-Husseini Theorem

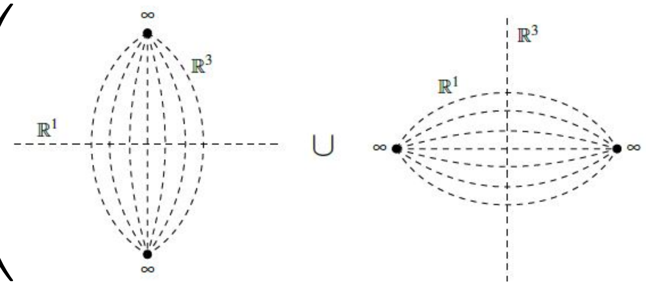
Sati-Schreiber 19c [arXiv:1912.10425]

back to top

higher co-observables on D6 ⊥ D8 -intersections

$$H_\bullet\left(\underset{[c]}{\bigsqcup} \Omega_c \, \boldsymbol{\pi}^4_{\mathrm{diff}} \right.$$

topological phase space

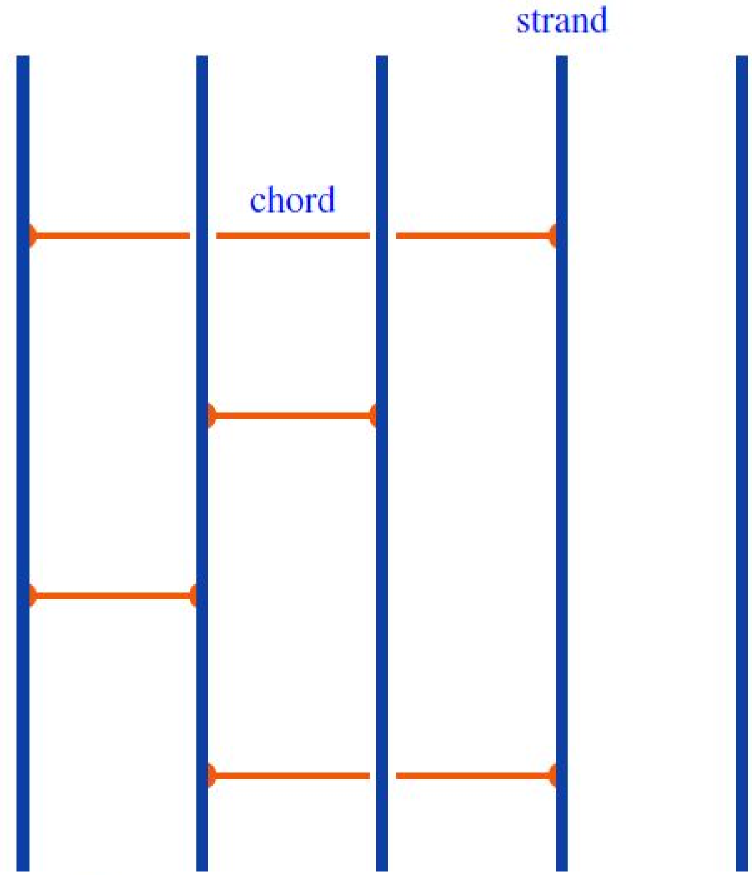


$$\simeq H_\bullet\left(\underset{N_{\mathrm{f}} \in \mathbb{N}}{\bigsqcup} \mathrm{Conf}_{\{1,\cdots,N_{\mathrm{f}}\}}\big(\mathbb{R}^3\big) \right)$$ (by the above)

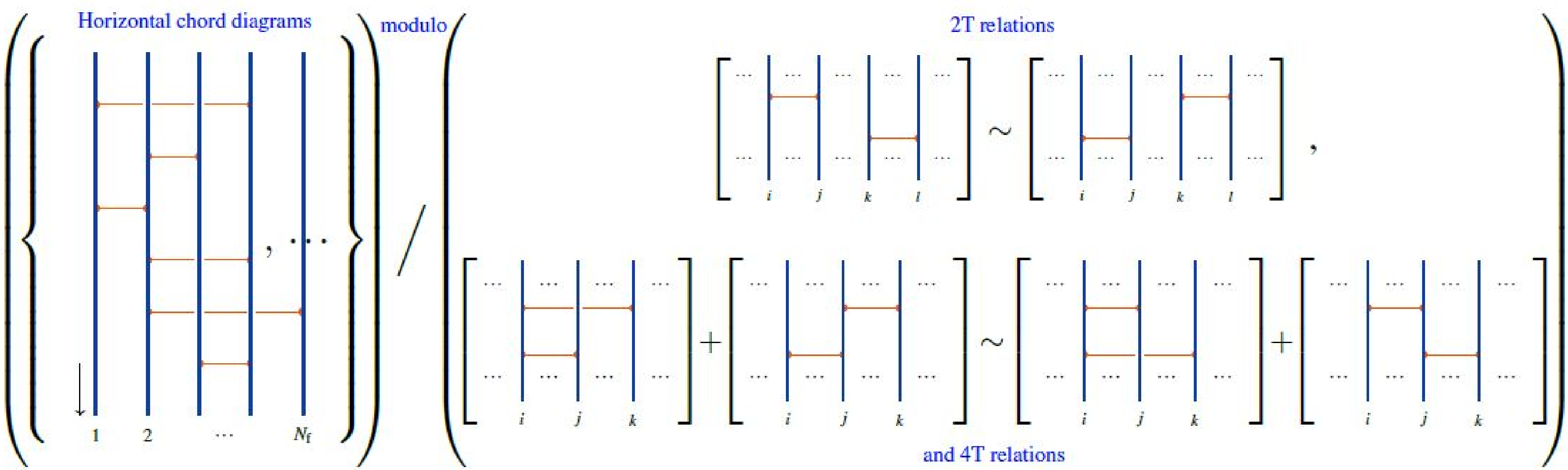
$$\simeq \bigoplus_{N_{\mathrm{f}} \in \mathbb{N}} \mathcal{A}^{\mathrm{pb}}_{N_{\mathrm{f}}}$$

Fadell-Husseini theorem

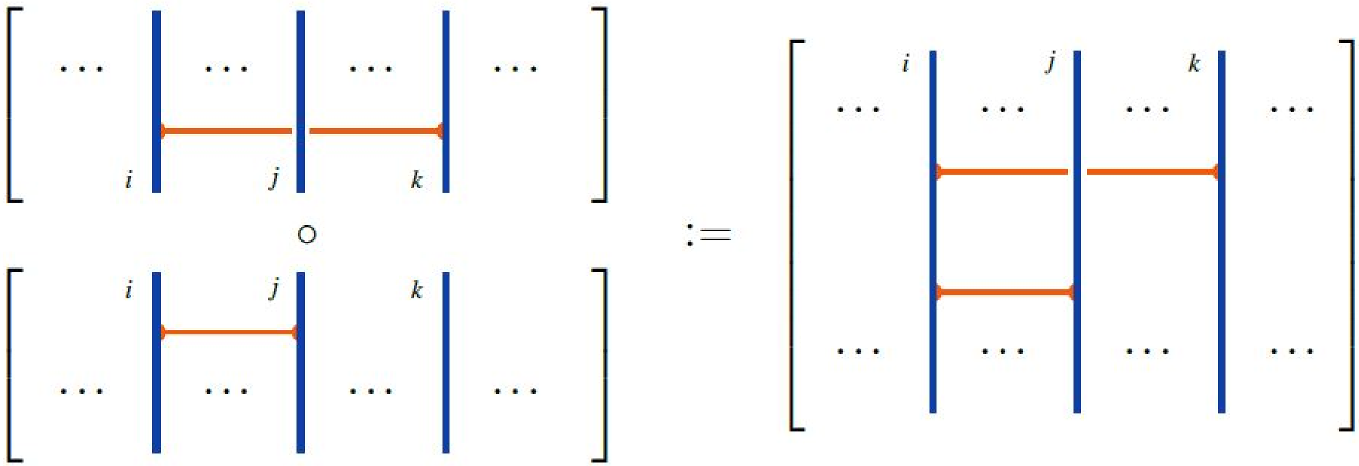
are algebra of **horizontal chord diagrams**:



$\mathscr{A}^{\mathrm{pb}}_{N_{\mathrm{f}}} :=$ Span



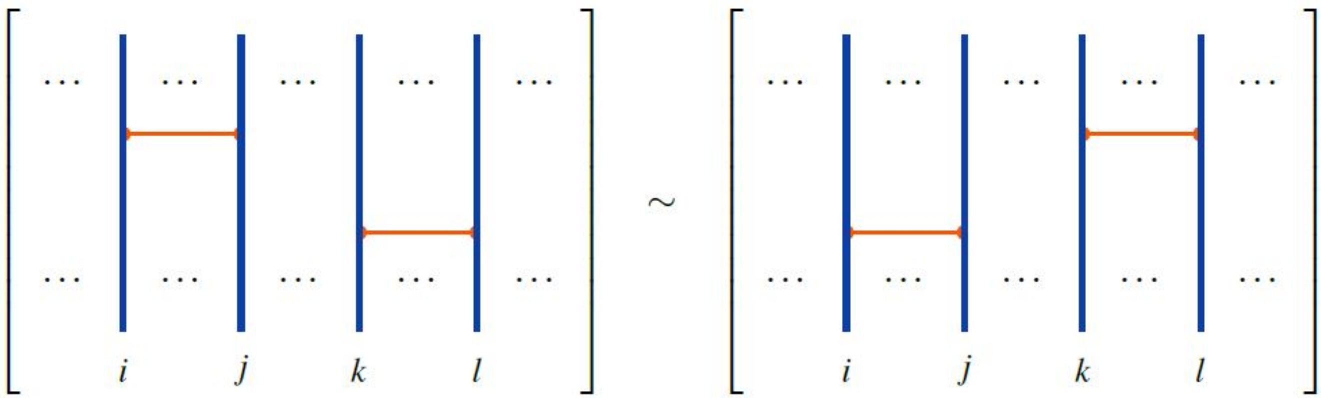
Horizontal chord diagrams form **algebra under concatenation of strands**.



$$t_{ik} \circ t_{ij} \;=\; t_{ik} t_{ij}$$

This is universal enveloping algebra of the infinitesimal braid Lie algebra (Kohno):

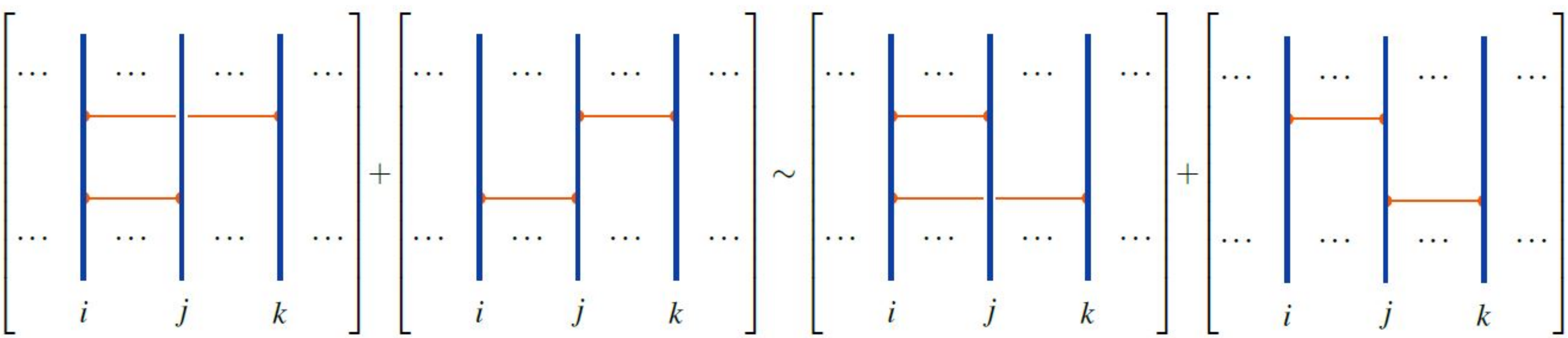
(i) the *2T relations*:



$$[t_{ij}, t_{kl}] = 0$$

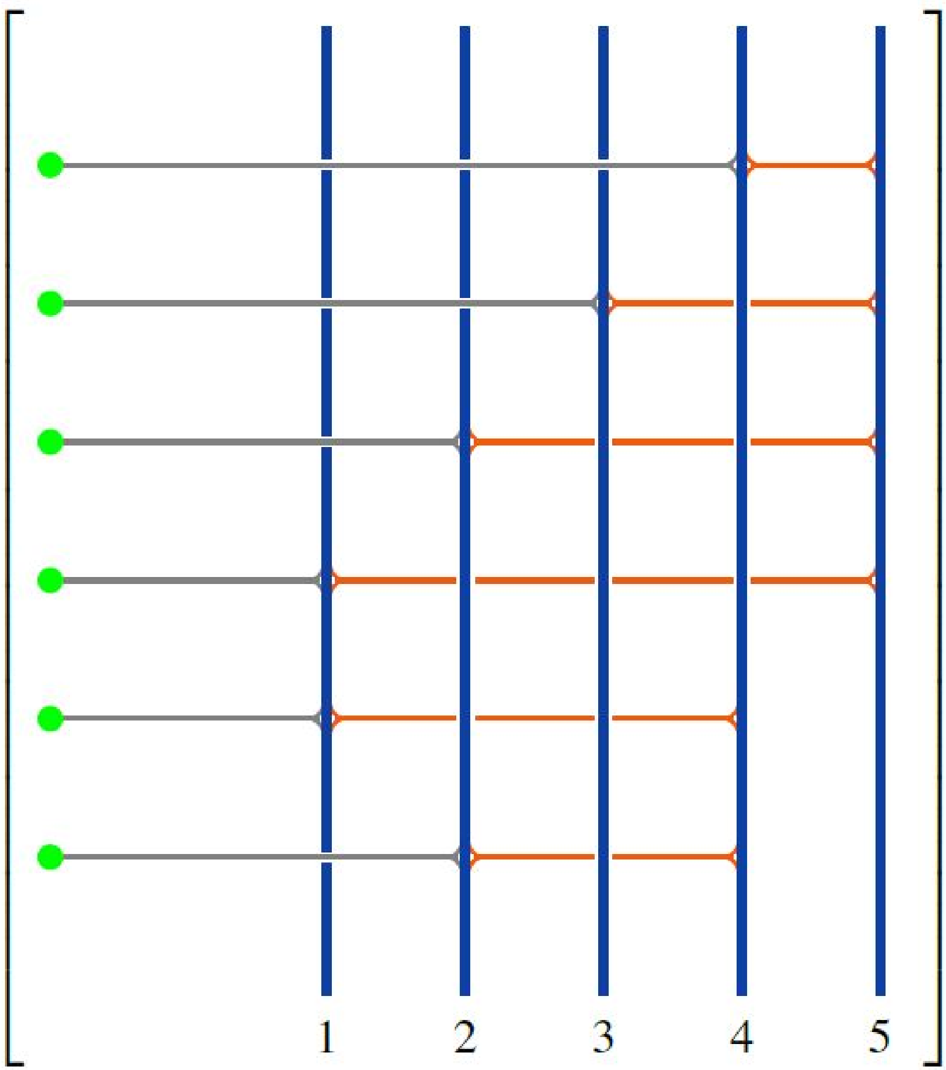
(ii) the *4T relations*

$$[t_{ij}, t_{ik} + t_{jk}] = 0$$



Consider the subspace of skew-symmetric co-observables,

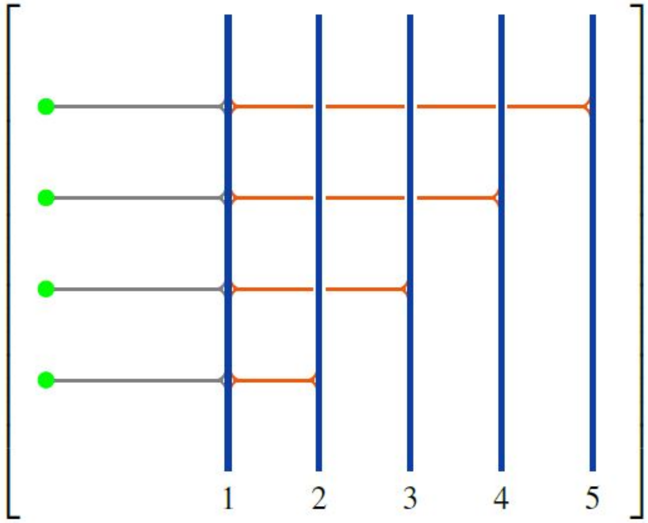
denote elements as follows:



$$= t_{45} \wedge t_{35} \wedge t_{25} \wedge t_{15} \wedge t_{14} \wedge t_{24}$$

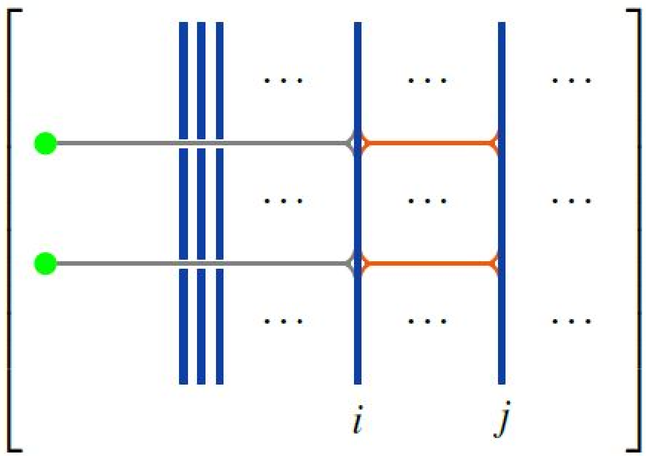
In the subspace of skew-symmetric co-observables we find:

the 2T relations become the *ordering constraint*



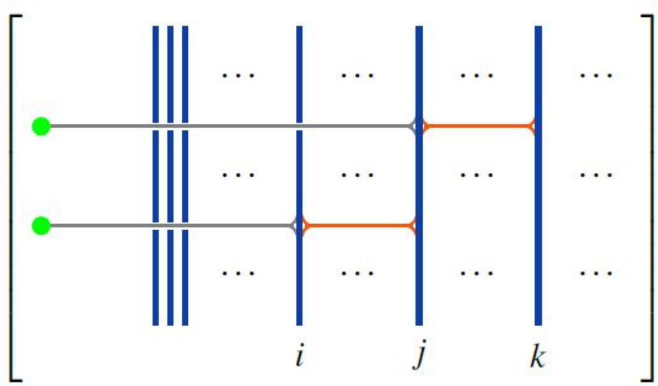
= form of any non-vanishing element

skew-symmetry becomes the *s-rule*

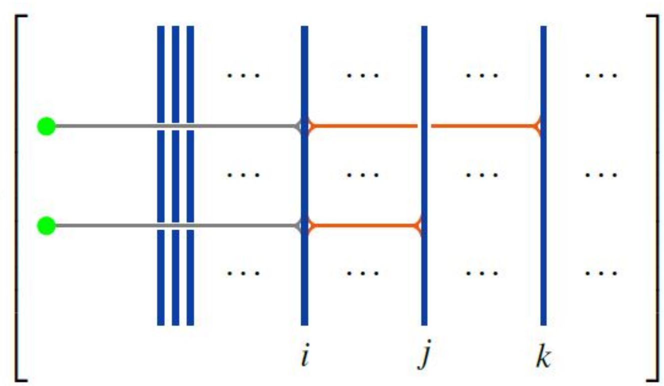


= 0

the 4T relations become the *breaking rule*



= −



these are the rules of Hanany-Witten theory
for NS5 $\perp$ Dp $\perp$ D$(p+2)$-brane intersections

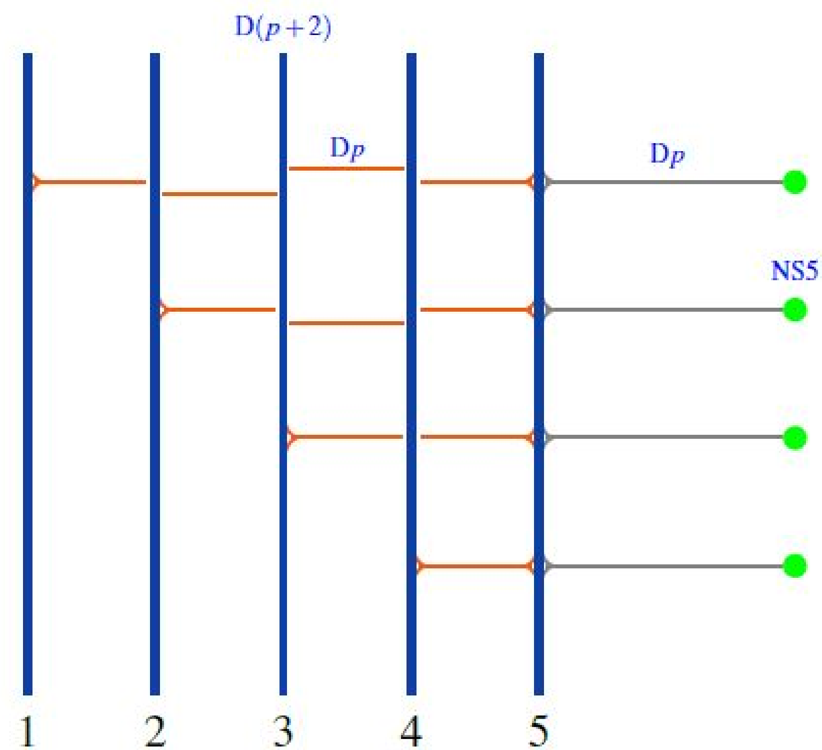
if we identify horizontal chord diagrams as follows:

(i) strands as D$(p+2)$-branes;

(ii) chords as Dp-branes,
stretching between D$(p+2)$s;

(iii) green dots as NS5-branes;

(iv) gray lines as Dp-branes,
stretching from NS5 to D$(p+2)$.



(5)

Chan-Paton data

implied by

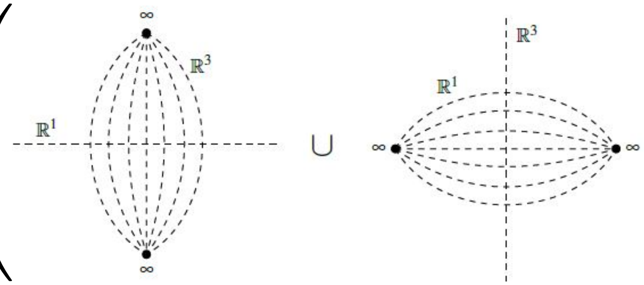
Hypothesis H with Bar-Natan' theorem

Sati-Schreiber 19c [arXiv:1912.10425]

back to top

higher co-states on D6 ⊥ D8 -intersections

$$H^\bullet\left(\underset{\text{phase space}}{\overset{\text{topological}}{\underset{[c]}{\sqcup}\,\Omega_c\,\boldsymbol{\pi}^4_{\mathrm{diff}}}} \left(\cdots \right)\right)$$



$$\simeq H^\bullet\left(\underset{N_{\mathrm{f}}\in\mathbb{N}}{\sqcup} \underset{\{1,\cdots,N_{\mathrm{f}}\}}{\mathrm{Conf}}(\mathbb{R}^3)\right) \quad \text{(by the above)}$$

$$\simeq \underset{N_{\mathrm{f}}\in\mathbb{N}}{\bigoplus} \mathcal{W}^{\mathrm{pb}}_{N_{\mathrm{f}}}$$

Kohno & Cohen-Gitler theorem

are **horizontal weight systems**:

Cohomology — weight systems

Higher observables $H^\bullet\Big(\underbrace{\underset{N_{\mathrm{f}}\in\mathbb{N}}{\sqcup}\ \Omega \underset{\{1,\cdots,N_{\mathrm{f}}\}}{\mathrm{Conf}}(\mathbb{R}^3)}_{\text{phase space}}\Big) \simeq \mathscr{W} = (\mathscr{A})^*$ Higher co-states

dualization

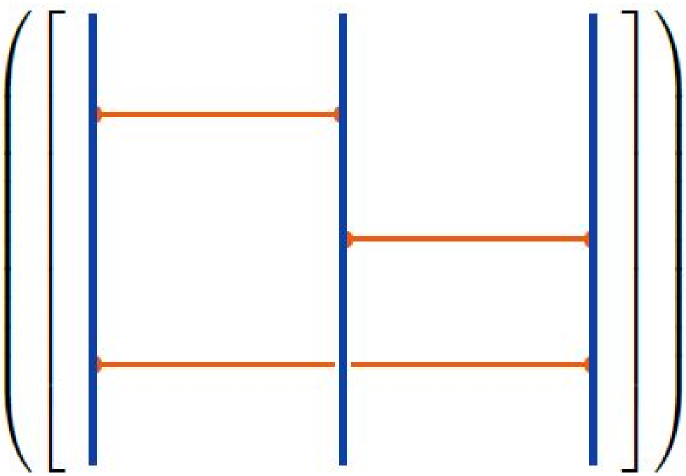
Higher co-observables $H_\bullet\Big(\overbrace{\underset{N_{\mathrm{f}}\in\mathbb{N}}{\sqcup}\ \Omega \underset{\{1,\cdots,N_{\mathrm{f}}\}}{\mathrm{Conf}}(\mathbb{R}^3)}\Big) \simeq \mathscr{A} = (\mathscr{W})^*$ Higher states

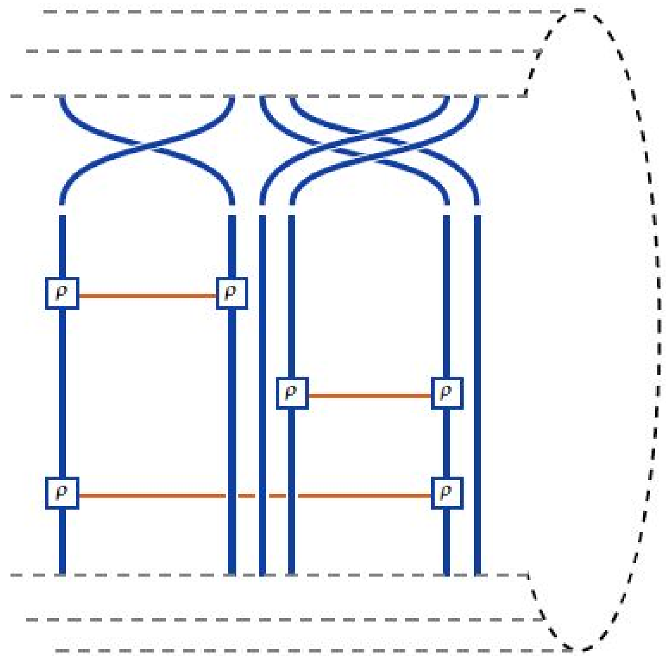
Homology — chord diagrams

All **horizontal weight systems** $w : \mathcal{A}^{\text{pb}} \to \mathbb{C}$ **come from Chan-Paton data**:

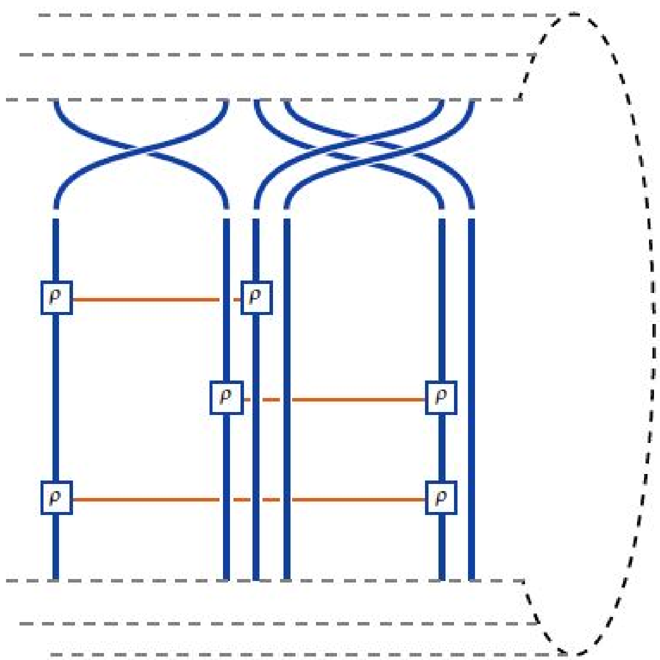
1) metric Lie representations ρ	2) stacks of coincident strands	3) winding monodromies:

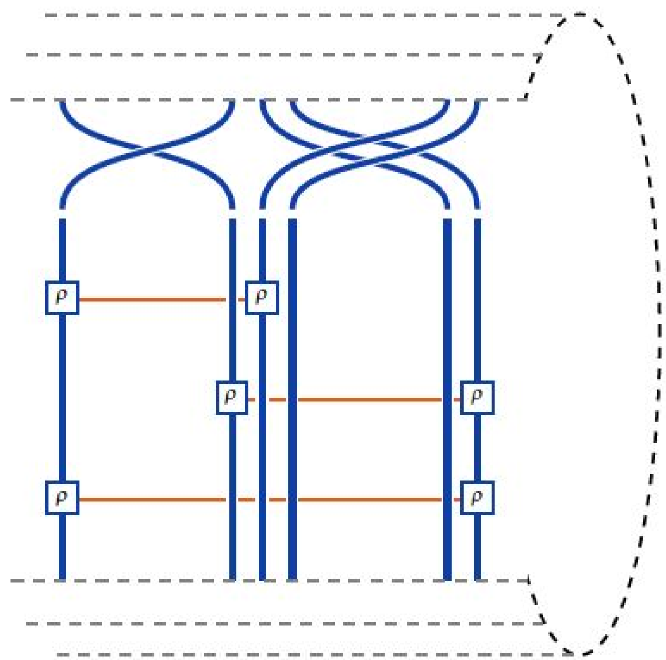
Bar-Natan theorem

w  $=$

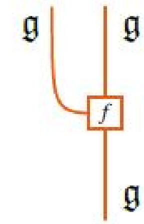
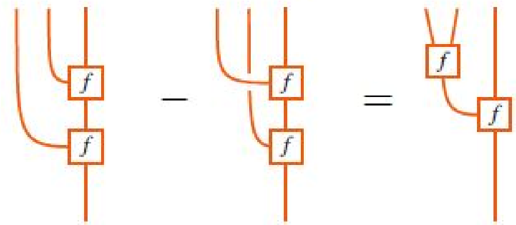
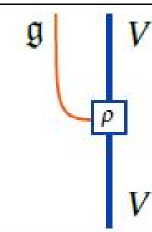
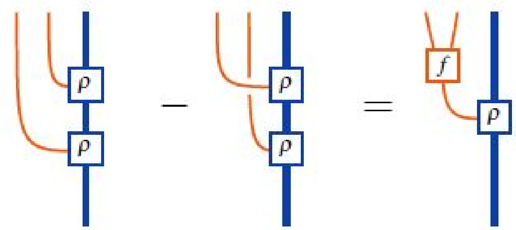
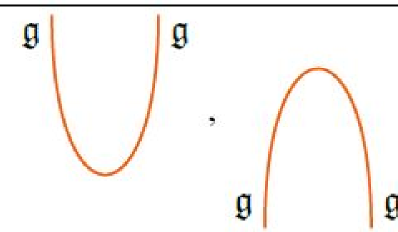
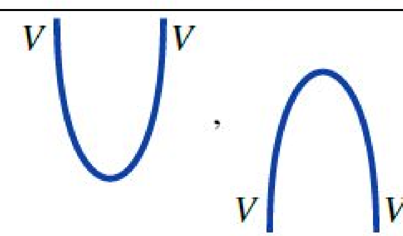


$+$

$+$ 

$+$ 

$+ \cdots$

Data of metric Lie representation	Category notation	Penrose notation	Index notation
Lie bracket	$\begin{array}{c} \mathfrak{g}\otimes\mathfrak{g} \\ \downarrow f \\ \mathfrak{g} \end{array}$		$f_{ab}{}^{c}$
Jacobi identity	$\begin{array}{ccc} \mathfrak{g}\otimes\mathfrak{g}\otimes\mathfrak{g} & \xrightarrow{\mathrm{id}\otimes f - f\otimes\mathrm{id}} & \mathfrak{g}\otimes\mathfrak{g} \\ \downarrow{\scriptstyle \sigma_{213}\circ(\mathrm{id}\otimes f)} & & \downarrow f \\ \mathfrak{g}\otimes\mathfrak{g} & \xrightarrow[f]{} & \mathfrak{g} \end{array}$		$f_{ae}{}^{d} f_{bc}{}^{e} - f_{be}{}^{d} f_{ac}{}^{e} = f_{ec}{}^{d} f_{ab}{}^{e}$
Lie action	$\begin{array}{c} \mathfrak{g}\otimes V \\ \downarrow \rho \\ V \end{array}$		$\rho_{a}{}^{i}{}_{j}$
Lie action property	$\begin{array}{ccc} \mathfrak{g}\otimes\mathfrak{g}\otimes V & \xrightarrow{\mathrm{id}\otimes\rho - f\otimes\mathrm{id}} & \mathfrak{g}\otimes V \\ \downarrow{\scriptstyle \sigma_{213}\circ(\mathrm{id}\otimes\rho)} & & \downarrow \rho \\ \mathfrak{g}\otimes V & \xrightarrow[\rho]{} & V \end{array}$		$\rho_{a}{}^{j}{}_{l}\,\rho_{b}{}^{l}{}_{i} - \rho_{b}{}^{j}{}_{l}\,\rho_{a}{}^{l}{}_{i} = f_{ab}{}^{c}\,\rho_{c}{}^{j}{}_{i}$
Metric	$\begin{array}{c} \mathfrak{g}\otimes\mathfrak{g} \\ \downarrow g \\ \mathbf{1} \end{array}$, $\begin{array}{c} \mathbf{1} \\ \downarrow g^{-1} \\ \mathfrak{g}\otimes\mathfrak{g} \end{array}$		g_{ab} , g^{ab}
	$\begin{array}{c} V\otimes V \\ \downarrow k \\ \mathbf{1} \end{array}$, $\begin{array}{c} \mathbf{1} \\ \downarrow k^{-1} \\ V\otimes V \end{array}$		k_{ij} , k^{ij}

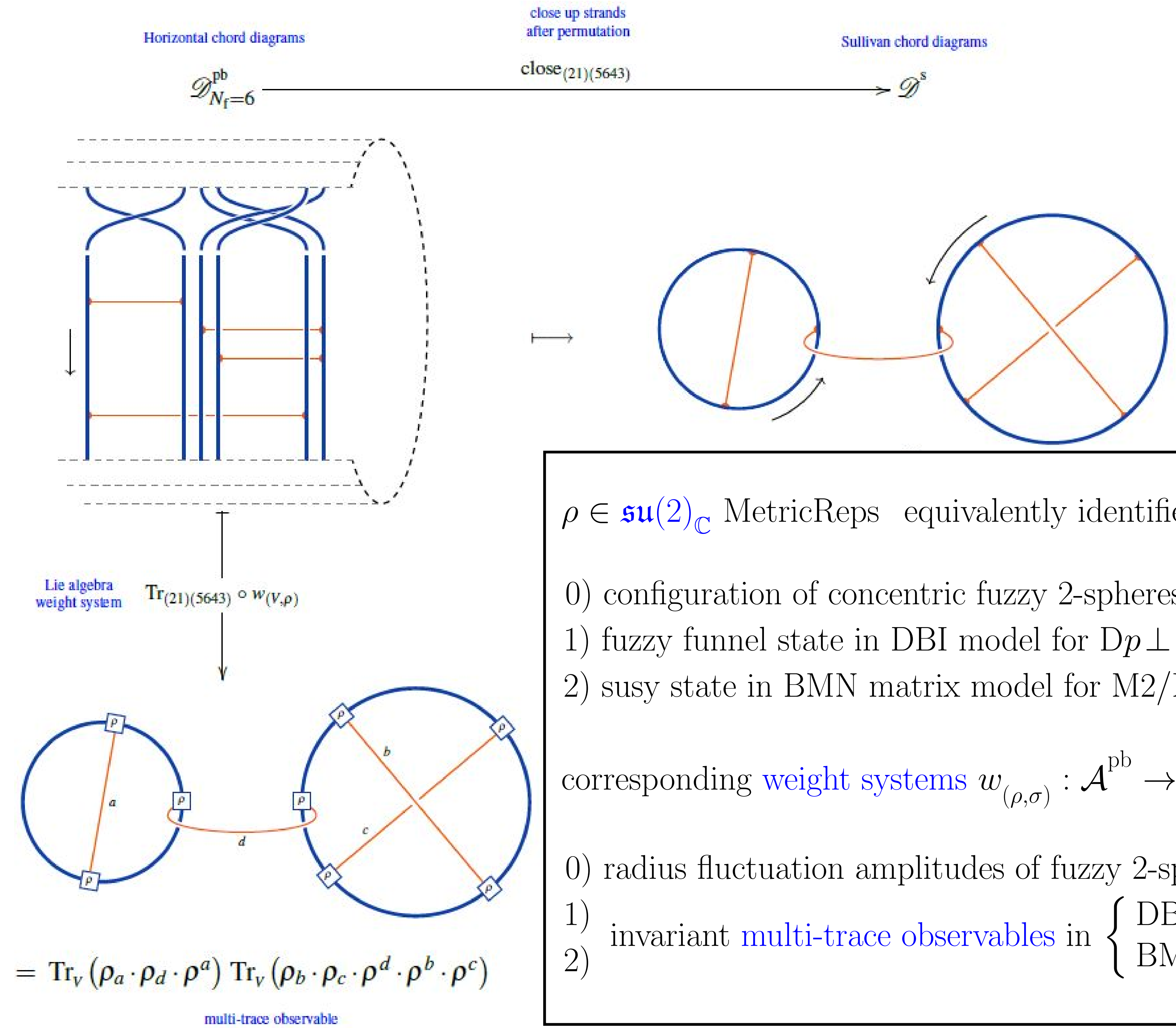
(6)

BMN Matrix Model States

implied by

Hypothesis H

back to top



$\rho \in \mathfrak{su}(2)_{\mathbb{C}}$ MetricReps equivalently identified with:

0) configuration of concentric fuzzy 2-spheres
1) fuzzy funnel state in DBI model for $\mathrm{D}p \perp \mathrm{D}(p+2)$
2) susy state in BMN matrix model for M2/M5

corresponding weight systems $w_{(\rho,\sigma)} : \mathcal{A}^{\mathrm{pb}} \to \mathbb{C}$ are:

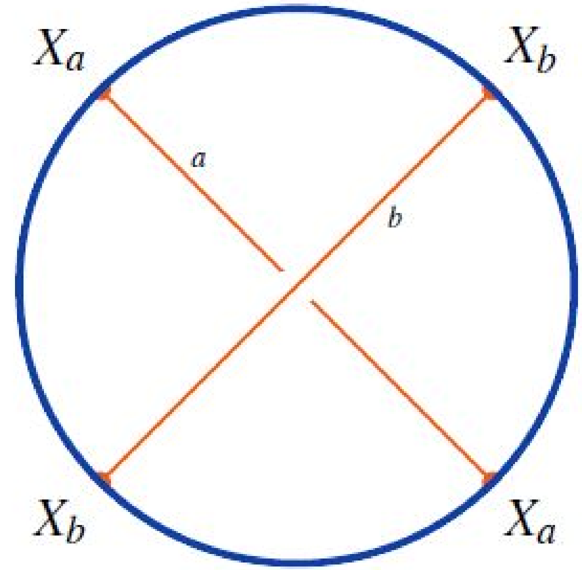
0) radius fluctuation amplitudes of fuzzy 2-spheres
1) 2) invariant multi-trace observables in $\begin{cases} \text{DBI model} \\ \text{BMN model} \end{cases}$

,

0) Radius fluctuation observables on N-bit fuzzy 2-spheres S^2_N are $\mathbf{N} \in \mathfrak{su}(2)_{\mathbb{C}}$MetricReps weight systems on chord diagrams:

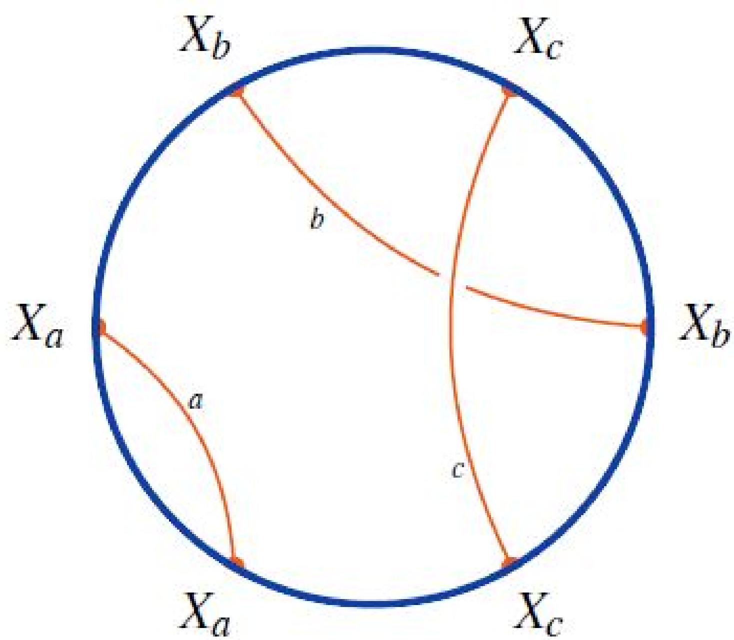
$$\int_{S^2_N} (R^2)^2$$ 

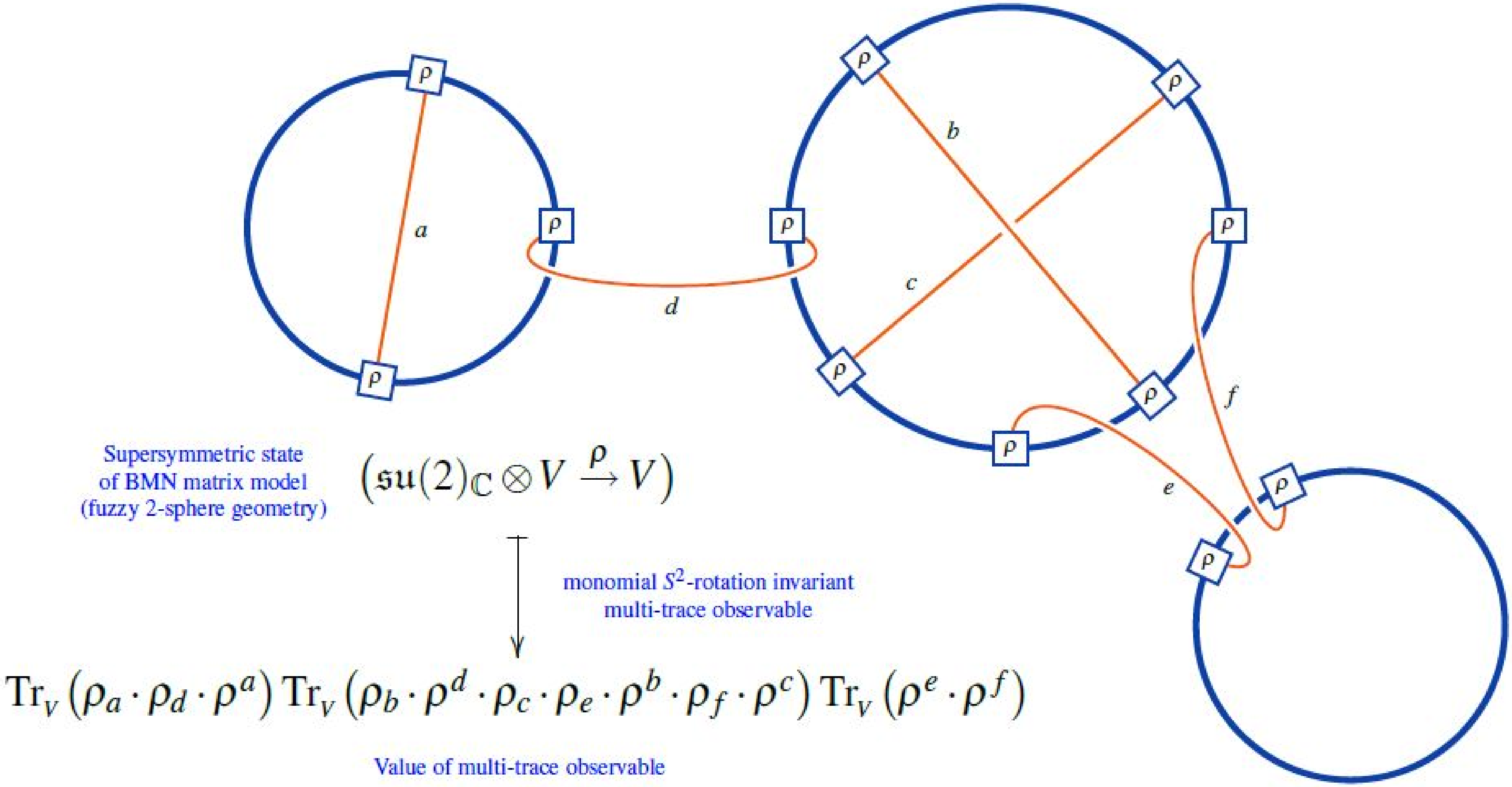
$$= \frac{4\pi}{\sqrt{N^2-1}} \mathrm{Tr}\big(X_a \cdot X_b \cdot X^a \cdot X^b\big)$$



$$\int_{S^2_N} (R^2)^3$$ 

$$= \frac{4\pi}{\sqrt{N^2-1}} \mathrm{Tr}\big(X_a \cdot X^a \cdot X_b \cdot X_c \cdot X^b \cdot X^c\big)$$





1,2) weight system w_ρ is the observable aspect of matrix model state ρ:

linear combinations of
finite-dim $\mathfrak{su}_{\mathbb{C}}$-representations

weight systems on
horizontal chord diagrams

$$\mathrm{Span}\left(\mathfrak{su}(2)_{\mathbb{C}}\mathrm{MetricReps}\right) \xrightarrow{\rho \mapsto w_\rho} \mathcal{W}^{\mathrm{pb}}$$

naive funnel- / susy-states of
DBI model / BMN matrix model

states of DBI model / BMN matrix mode
as observed by invariant multi-trace observables

(7)

M2/M5 Brane Bound States

implied by

Hypothesis H

back to top

Given a *sequence* of susy states in the BMN matrix model

$$\overbrace{(V,\rho)}^{\substack{\text{M2/M5-brane state} \\ (\text{finite-dim } \mathfrak{su}(2)_{\mathbb{C}}\text{-rep})}} \;:=\; \underbrace{\bigoplus_i}_{\substack{\text{stacks of coincident branes} \\ (\text{direct sum over irreps})}} \overbrace{\left(N_i^{(\mathrm{M2})} \cdot \mathbf{N}_i^{(\mathrm{M5})}\right)}^{\substack{\text{M2/M5-brane charge in } i\text{th stack} \\ (i\text{th irrep with multiplicity})}} \;\in\; \mathfrak{su}(2)_{\mathbb{C}}\mathrm{MetricReps}_{/\sim}$$

---

this is argued to converge to macroscopic M2- *or* M5-branes depending on how the sequence behaves in the large N limit:

Stacks of macroscopic...

	M2-branes	*M5-branes*	
If for all i	$N_i^{(\mathrm{M5})} \to \infty$	$N_i^{(\mathrm{M2})} \to \infty$	(the relevant large N limit)
with fixed	$N_i^{(\mathrm{M2})}$	$N_i^{(\mathrm{M5})}$	(the number of coincident branes in the ith stack)
and fixed	$N_i^{(\mathrm{M2})}/N$	$N_i^{(\mathrm{M5})}/N$	(the charge/light-cone momentum carried by the ith stack)

Given a *sequence* of susy states in the BMN matrix model

$$\overbrace{(V,\rho)}^{\substack{\text{M2/M5-brane state} \\ \text{(finite-dim } \mathfrak{su}(2)_{\mathbb{C}}\text{-rep)}}} \;:=\; \underbrace{\bigoplus_i}_{\substack{\text{stacks of coincident branes} \\ \text{(direct sum over irreps)}}} \overbrace{\Big(N_i^{(\mathrm{M2})} \cdot \mathbf{N}_i^{(\mathrm{M5})}\Big)}^{\substack{\text{M2/M5-brane charge in } i\text{th stack} \\ (i\text{th irrep with multiplicity})}} \;\in\; \mathfrak{su}(2)_{\mathbb{C}}\mathrm{MetricReps}_{/\sim}$$

---

the large N

limit does *not* exist

here:

but

does exist in weight systems

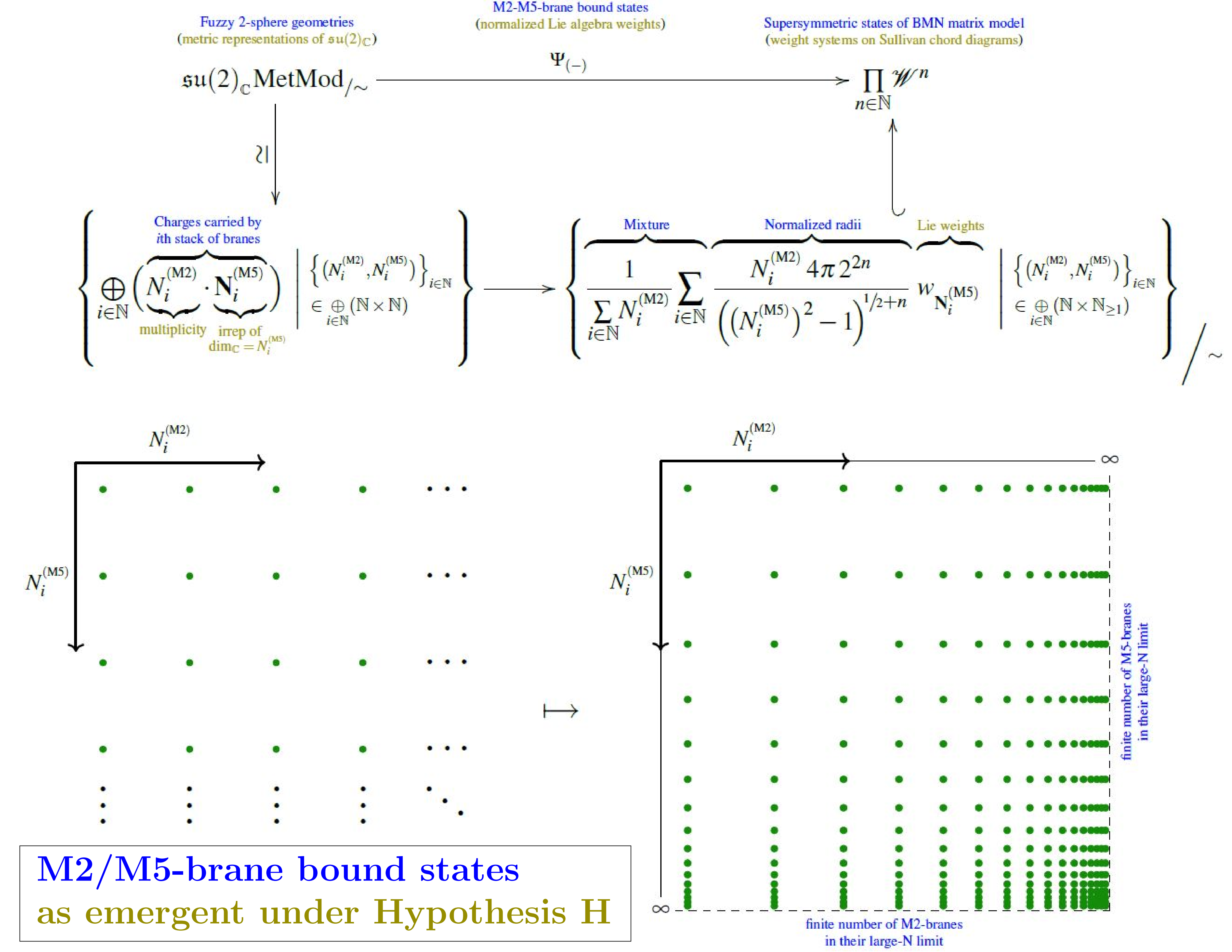
$$\mathrm{Span}\big(\mathfrak{su}(2)_{\mathbb{C}}\mathrm{MetricReps}\big) \xrightarrow{\;\rho \mapsto w_\rho\;} \mathcal{W}^{\mathrm{pb}}$$

if we normalize by the scale of the fuzzy 2-sphere geometry:

$$\underbrace{\frac{4\pi\, 2^{2n}}{\Big(\big(N^{(\mathrm{M5})}\big)^2 - 1\Big)^{1/2+n}}\, w_{\mathbf{N}^{(\mathrm{M5})}}}_{\substack{\text{Single M2-brane state in BMN model} \\ \text{(multiple of } \mathfrak{su}_{\mathbb{C}}\text{-weight system)}}} \qquad \in \qquad \mathcal{W}^{\mathrm{pb}}$$

states as seen by multi-trace observables

(weight systems on chord diagrams)


Fuzzy 2-sphere geometries
(metric representations of $\mathfrak{su}(2)_{\mathbb{C}}$)
M2-M5-brane bound states
(normalized Lie algebra weights)
Supersymmetric states of BMN matrix model
(weight systems on Sullivan chord diagrams)
$\Psi_{(-)}$
$\mathfrak{su}(2)_{\mathbb{C}}\mathrm{MetMod}_{/\sim}$
$\prod_{n\in\mathbb{N}} \mathscr{W}^n$
Charges carried by
ith stack of branes
multiplicity
irrep of
$\dim_{\mathbb{C}} = N_i^{(M5)}$
$\left\{ \bigoplus_{i\in\mathbb{N}} \left(N_i^{(M2)} \cdot \mathbf{N}_i^{(M5)} \right) \;\middle|\; \left\{ (N_i^{(M2)}, N_i^{(M5)}) \right\}_{i\in\mathbb{N}} \in \bigoplus_{i\in\mathbb{N}} (\mathbb{N}\times\mathbb{N}) \right\}$
Mixture
Normalized radii
Lie weights
$\left\{ \frac{1}{\sum_{i\in\mathbb{N}} N_i^{(M2)}} \sum_{i\in\mathbb{N}} \frac{N_i^{(M2)}\, 4\pi\, 2^{2n}}{\left((N_i^{(M5)})^2 - 1 \right)^{1/2+n}} w_{\mathbf{N}_i^{(M5)}} \;\middle|\; \left\{ (N_i^{(M2)}, N_i^{(M5)}) \right\}_{i\in\mathbb{N}} \in \bigoplus_{i\in\mathbb{N}} (\mathbb{N}\times\mathbb{N}_{\geq 1}) \right\} \Big/ \sim$
$N_i^{(M2)}$
$N_i^{(M5)}$
$\mapsto$
$N_i^{(M2)}$
$N_i^{(M5)}$
∞
∞
finite number of M5-branes
in their large-N limit
finite number of M2-branes
in their large-N limit
M2/M5-brane bound states
as emergent under Hypothesis H

End.

back to top