Urs Schreiber on joint work with Hisham Sati:

invitation to our preprint: [`arXiv:2507.00138`]

Identifying Anyonic Topological Order in FQAH Systems

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(September 2025) find these slides at: [`ncatlab.org/schreiber/show/QMATH16`]

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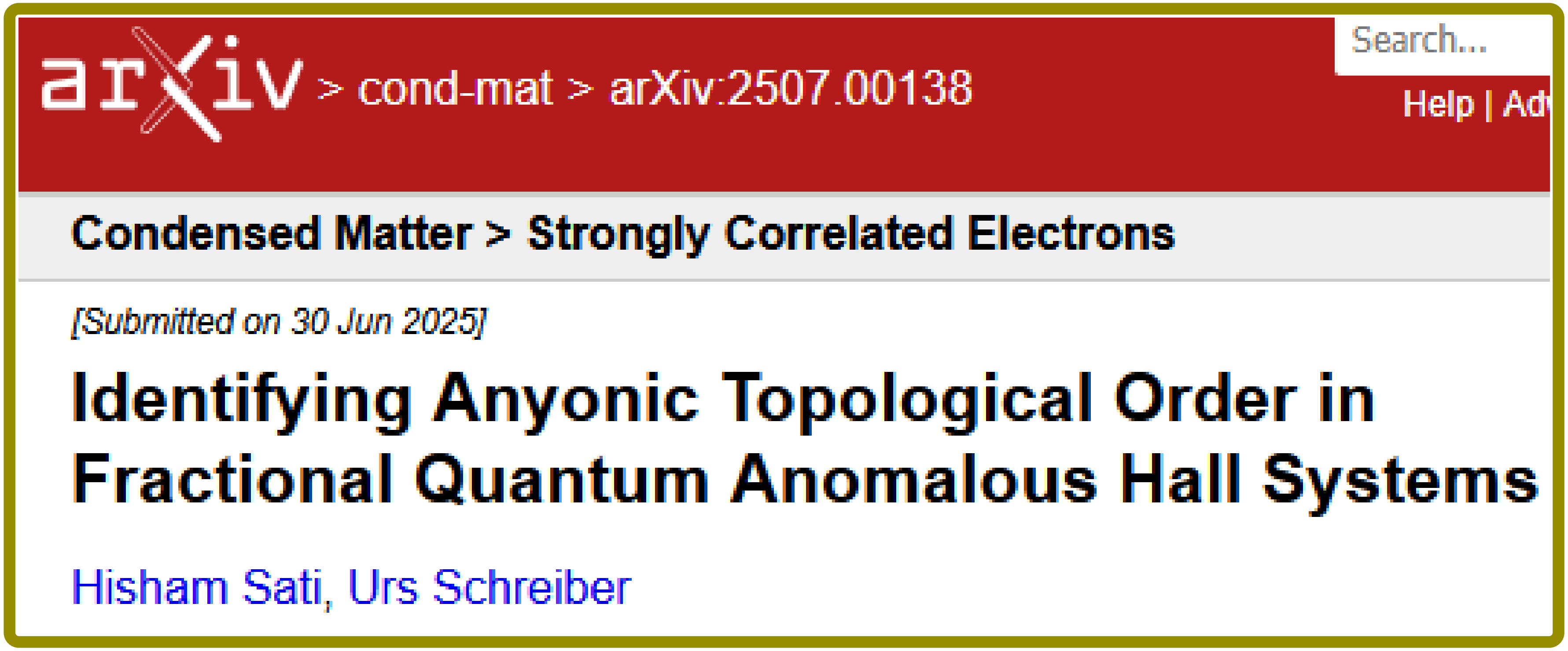

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[Submitted on 30 Jun 2025]

Identifying Anyonic Topological Order in Fractional Quantum Anomalous Hall Systems

Hisham Sati, Urs Schreiber

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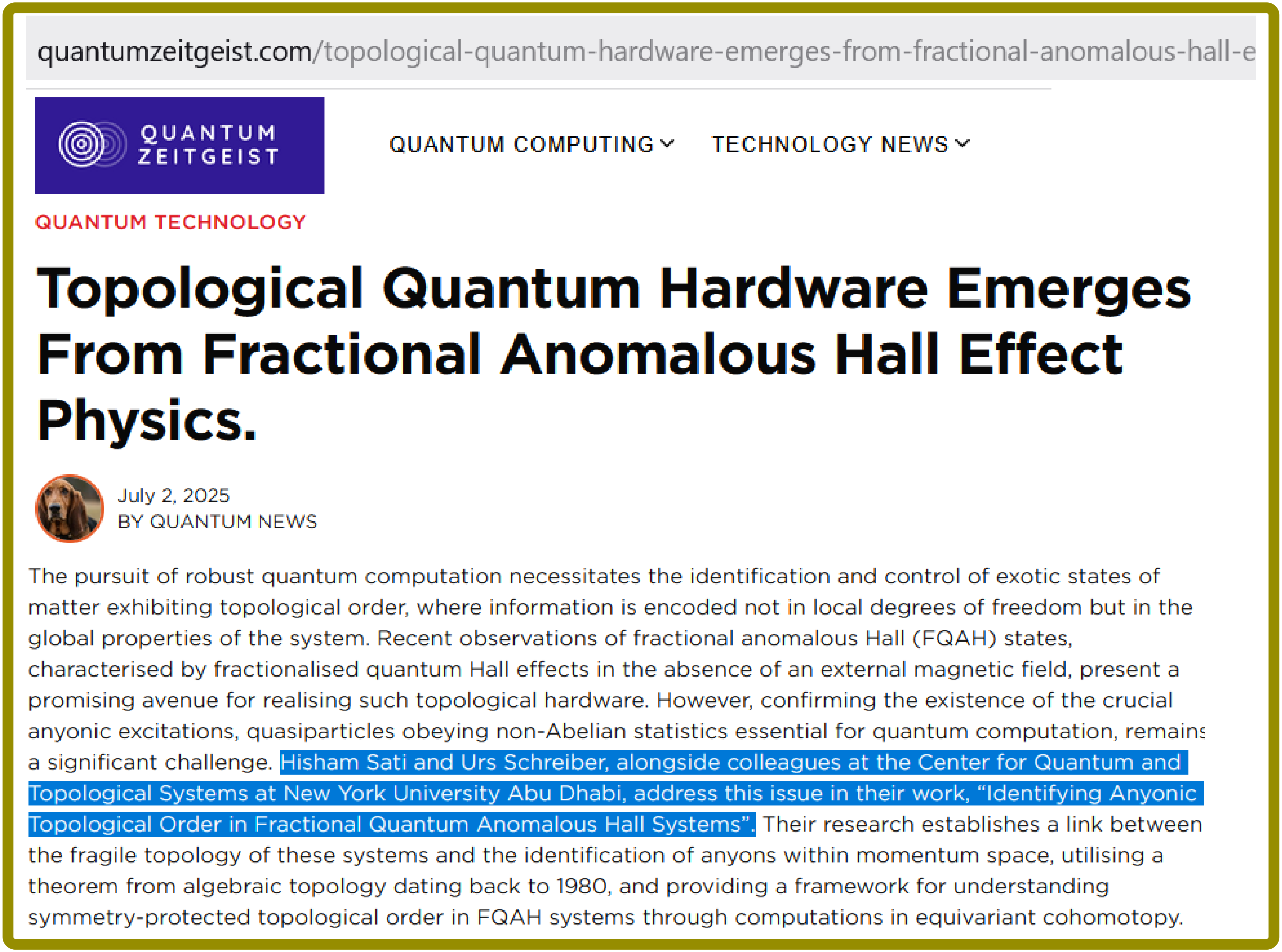
Featured at *Quantum Zeitgeist.*



QUANTUM TECHNOLOGY

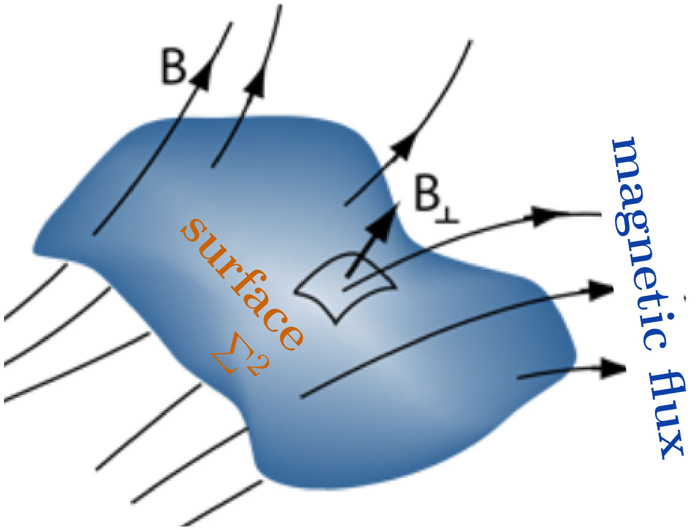
Topological Quantum Hardware Emerges From Fractional Anomalous Hall Effect Physics.

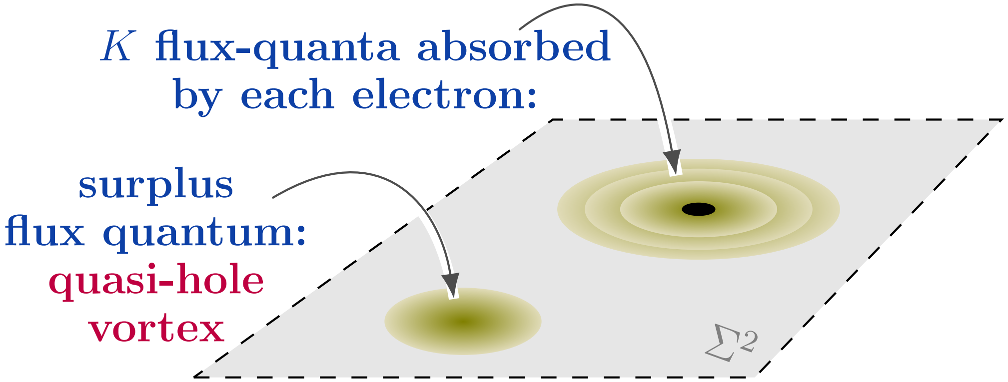
July 2, 2025
BY QUANTUM NEWS

The pursuit of robust quantum computation necessitates the identification and control of exotic states of matter exhibiting topological order, where information is encoded not in local degrees of freedom but in the global properties of the system. Recent observations of fractional anomalous Hall (FQAH) states, characterised by fractionalised quantum Hall effects in the absence of an external magnetic field, present a promising avenue for realising such topological hardware. However, confirming the existence of the crucial anyonic excitations, quasiparticles obeying non-Abelian statistics essential for quantum computation, remains a significant challenge. Hisham Sati and Urs Schreiber, alongside colleagues at the Center for Quantum and Topological Systems at New York University Abu Dhabi, address this issue in their work, "Identifying Anyonic Topological Order in Fractional Quantum Anomalous Hall Systems". Their research establishes a link between the fragile topology of these systems and the identification of anyons within momentum space, utilising a theorem from algebraic topology dating back to 1980, and providing a framework for understanding symmetry-protected topological order in FQAH systems through computations in equivariant cohomotopy.

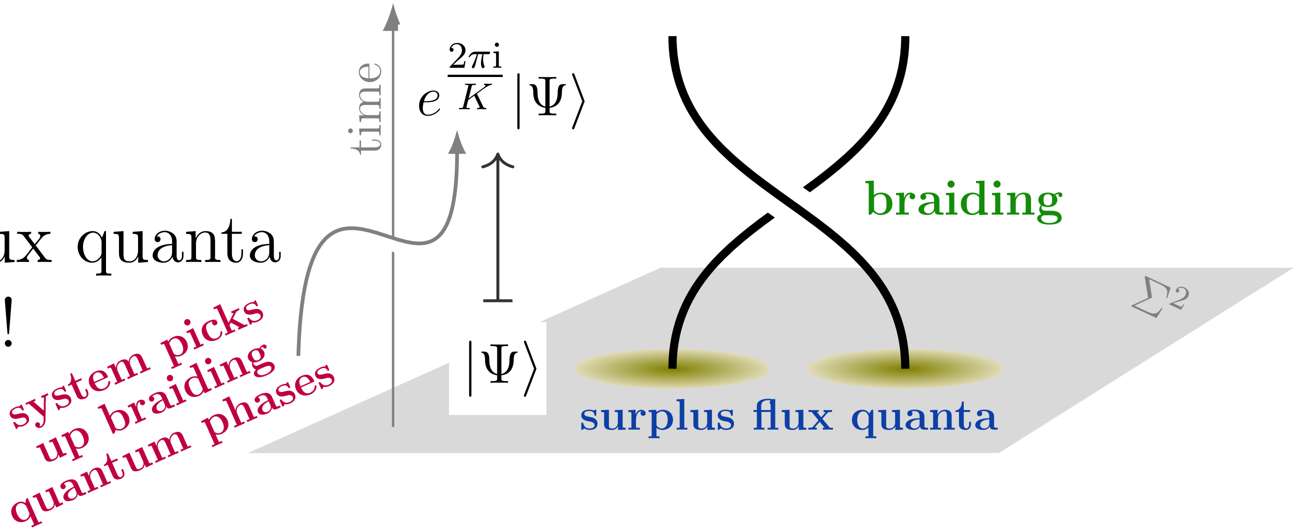
Fractional Quantum Hall systems.

electron gas in 2D semiconductor Σ^2
subject to transverse magnetic field
at *rational filling fraction* of
electrons per magnetic flux quanta





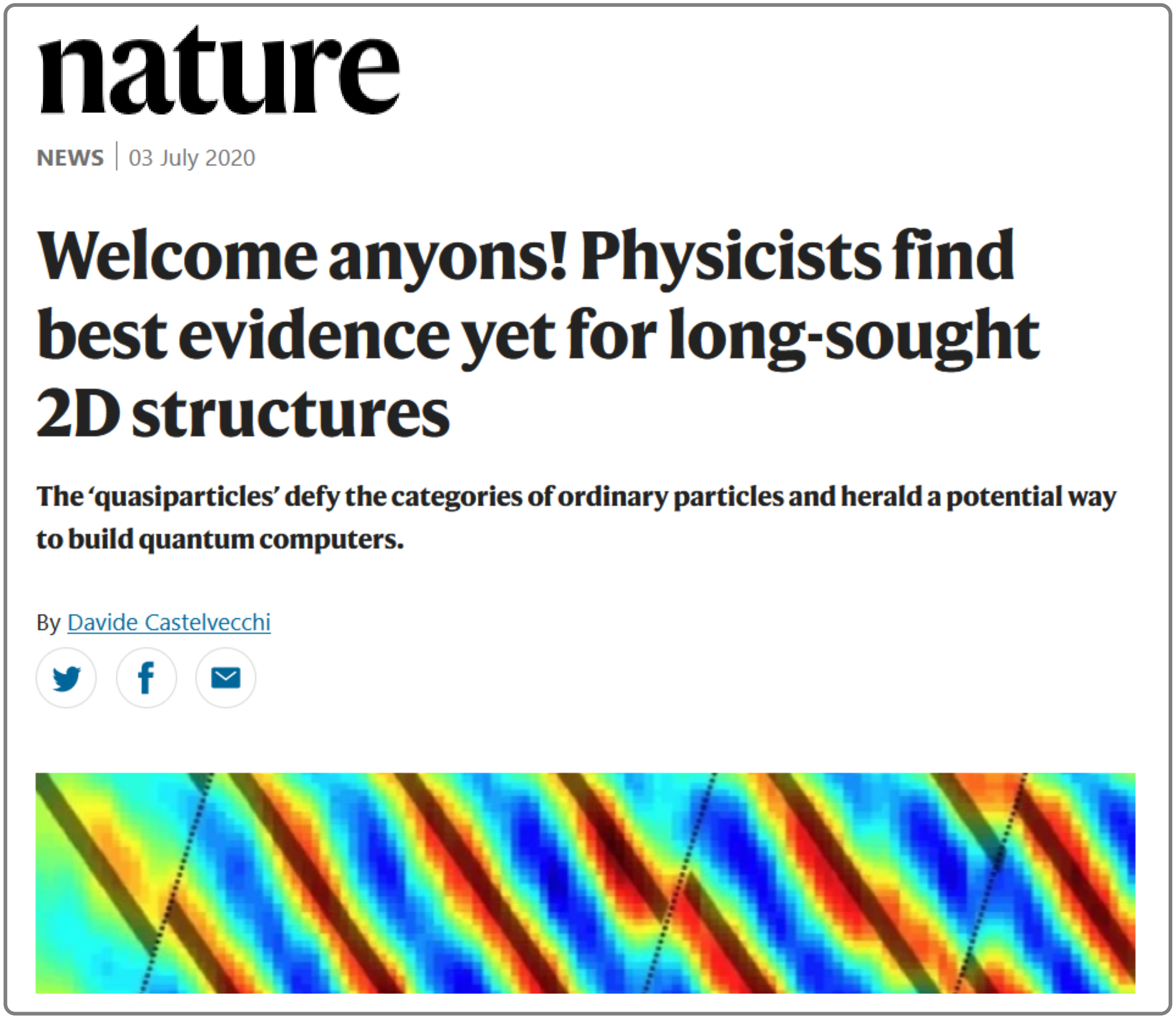
remarkably:
surplus magnetic flux quanta
are solitonic anyons!



Fractional Quantum Hall systems.

in recent years, these FQH anyons
are experimentally observed

[Nakamura et al. 2020]
[Nakamura et al. 2023]
[Ruelle et al. 2023]
[Glidic et al. 2023]
[Kundu et al. 2023]
[Veillon et al. 2024]
[Ghosh et al. 2025]



nature

NEWS | 03 July 2020

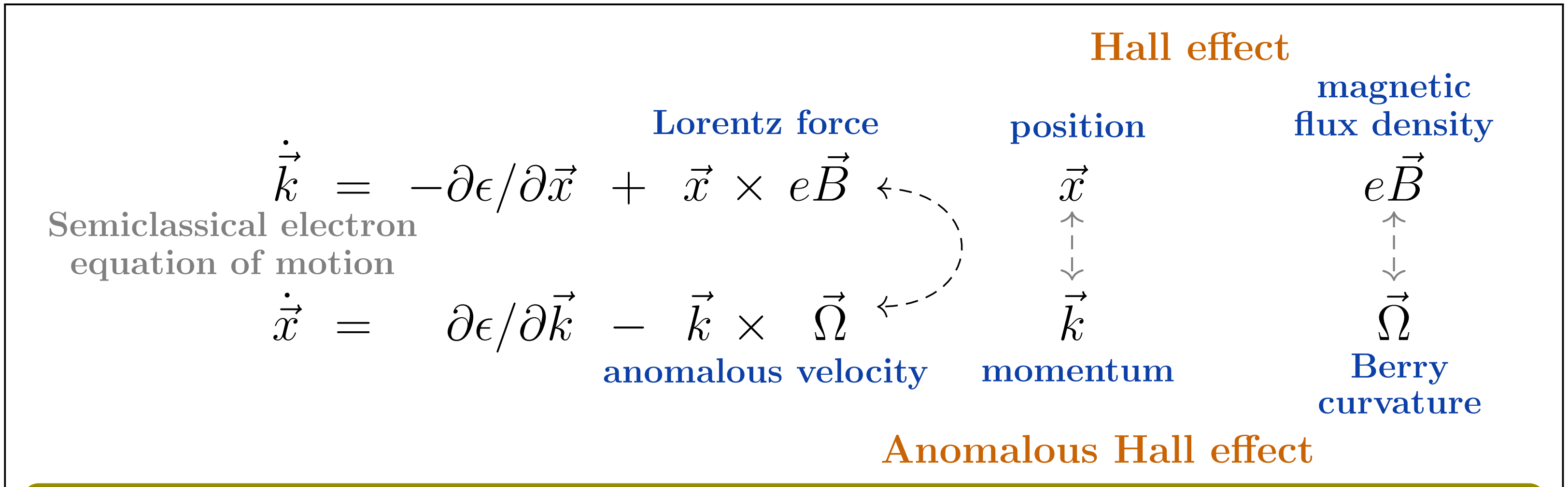
Welcome anyons! Physicists find best evidence yet for long-sought 2D structures

The 'quasiparticles' defy the categories of ordinary particles and herald a potential way to build quantum computers.

By Davide Castelvecchi

Fractional Quantum Anomalous Hall systems.

Very recently also observed:
2D crystals, exhibiting “dual” FQH, where
Berry curvature over momentum space $\widehat{T}^2$
plays the role of flux in position space



Hall effect

Lorentz force

position

magnetic flux density

$$\dot{\vec{k}} = -\partial\epsilon/\partial\vec{x} + \vec{x} \times e\vec{B}$$

Semiclassical electron equation of motion

$$\dot{\vec{x}} = \partial\epsilon/\partial\vec{k} - \vec{k} \times \vec{\Omega}$$

anomalous velocity

$\vec{x}$ ↕ $\vec{k}$

$e\vec{B}$ ↕ $\vec{\Omega}$

momentum

Berry curvature

Anomalous Hall effect

nature

Article | Published: 17 August 2023

Observation of fractionally quantized anomalous Hall effect

Heonjoon Park, Jiaqi Cai, Eric Anderson, Yinong Zhang, Jiayi Zhu, Xiaoyu Liu, Chong Wang, William Holtzmann, Chaowei Hu, Zhaoyu Liu, Takashi Taniguchi, Kenji Watanabe, Jiun-Haw Chu, Ting Cao, Liang Fu, Wang Yao, Cui-Zu Chang, David Cobden, Di Xiao & Xiaodong Xu ✉

Nature **622**, 74–79 (2023) | Cite this article

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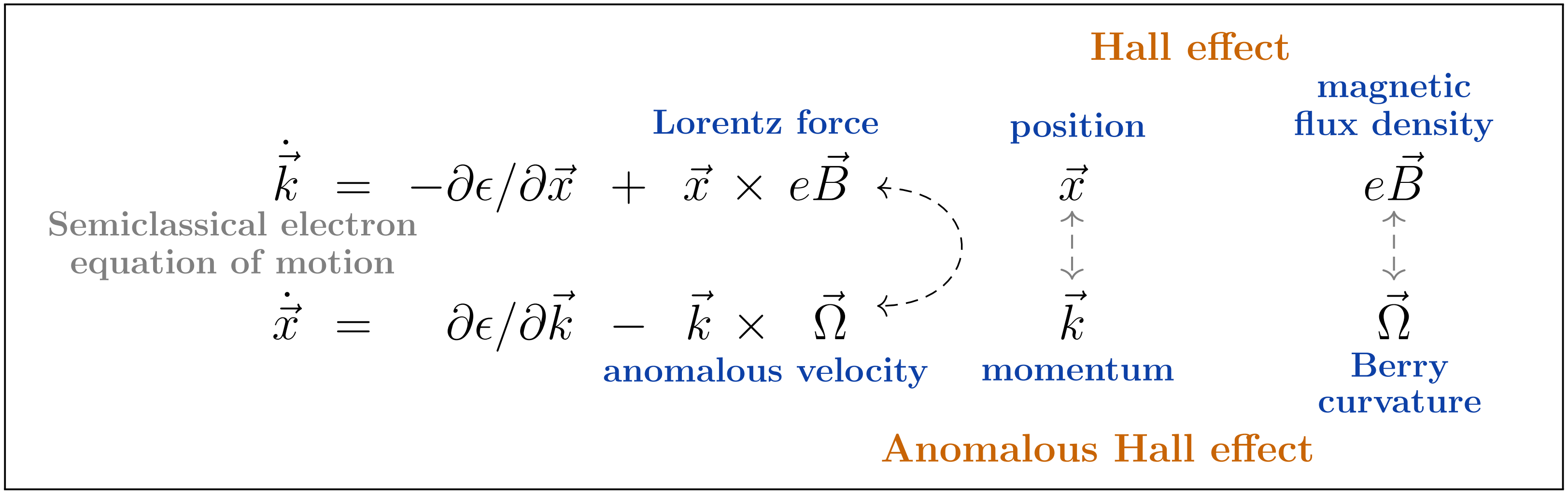


Hall effect

Semiclassical electron equation of motion

$$\dot{\vec{k}} = -\partial\epsilon/\partial\vec{x} + \vec{x} \times e\vec{B}$$

$$\dot{\vec{x}} = \partial\epsilon/\partial\vec{k} - \vec{k} \times \vec{\Omega}$$

Lorentz force

anomalous velocity

position $\vec{x}$ ↔ $\vec{k}$ **momentum**

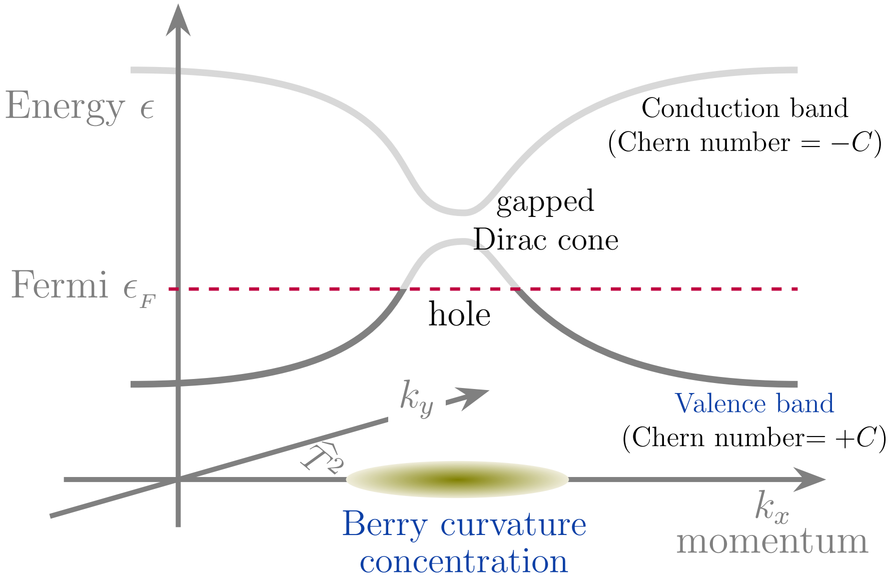
magnetic flux density $e\vec{B}$ ↔ $\vec{\Omega}$ **Berry curvature**

Anomalous Hall effect

suggests FQAH systems as viable topological quantum hardware!

but **open question**:

Where & how are FQAH anyons?



Answer – in general:

Given:

crystal symmetry group G

Bloch Hamiltonian space $\mathcal{A}$

Then:

connected components

mapping space

equivariant maps

equivariant extraordinary nonabelian cohomology

topological phase: $C \in \pi_0\Big(\mathrm{Map}\big(\widehat{T}^2, \mathcal{A}\big)^G\Big) \equiv H^1_G\big(\widehat{T}^2; \Omega\mathcal{A}\big)$

central claim

topological order: $\mathcal{H} \in \pi_1\Big(\mathrm{Map}\big(\widehat{T}^2, \mathcal{A}\big)^G_C\Big)\text{-IrrRep}$

fundamental group

unitary irreps

by topological adiabatic theorem:

gapped ground state Hilbert spaces

form local system/flat bundle

over external parameter space

⇔ rep of fundamental group

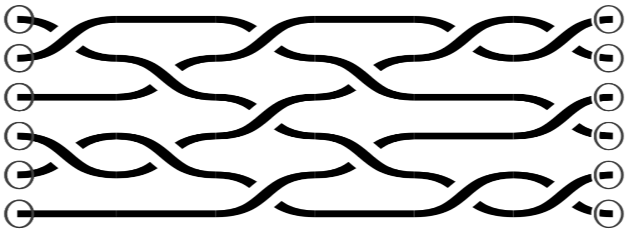
$$\begin{tikzcd}
 & \mathcal{H}_{p_2} \arrow[dr, "U_{\gamma_{23}}"] & \\
\mathcal{H}_{p_1} \arrow[ur, "U_{\gamma_{12}}"] \arrow[rr, "U_{\gamma_{13}}"] & & \mathcal{H}_{p_3}
\end{tikzcd}$$

$$\begin{tikzcd}
 & p_2 \arrow[dr, squiggly, "\gamma_{23}"] & \\
p_1 \arrow[ur, squiggly, "\gamma_{12}"] \arrow[rr, squiggly, "\gamma_{13}"] & & p_3
\end{tikzcd}$$

Familiar example – FQH systems:

parameters = configurations of anyons in plane
monodromy = braiding of their worldlines
irrep = braid representation = anyon statistics

$$\mathcal{H} \xrightarrow{\quad U \quad} \mathcal{H}$$



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topological order: $\mathcal{H} \in \pi_1\Big(\mathrm{Map}\big(\widehat{T}^2, \mathcal{A}\big)^G_C\Big)$-IrrRep

more precisely, the ground state TQFT is *generally covariant*

⇒ band topologies differing by diffeos are equivalent to the TQFT:

topological phase: $C = \pi_0\Big(\underbrace{\mathrm{Map}\big(\widehat{T}^2, \mathcal{A}\big)^G}_{\text{band topology}} /\!/ \underbrace{\mathrm{Diff}^+\big(\widehat{T}^2\big)^G}_{\text{diffeos}} \Big)$

as seen by

topological order: $\mathcal{H} \in \pi_1\Big(\overbrace{\mathrm{Map}\big(\widehat{T}^2, \mathcal{A}\big)^G} \underset{\text{homotopy quotient}}{/\!/} \overbrace{\mathrm{Diff}^+\big(\widehat{T}^2\big)^G}, C \Big)$-IrrRep

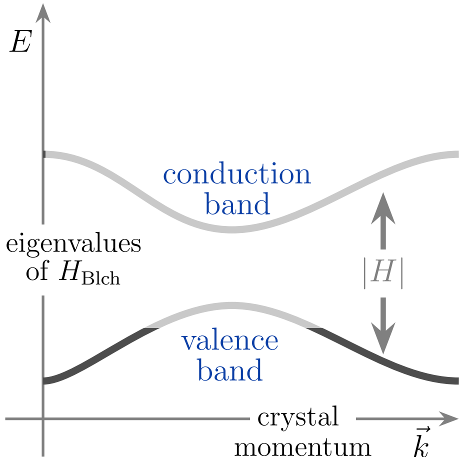
Answer – for FQAH: 1.) Bloch Hamiltonian space

2-band systems

$\Rightarrow$ Bloch Hamiltonian expanded in Pauli σ

$$\begin{tikzcd} \widehat{T}^2 \arrow[r, "H_{\mathrm{Blch}}"] & \mathrm{Mat}_{2\times 2}(\mathbb{C}) \\ \vec{k} \arrow[r, mapsto] & H_{\mathrm{Blch}}(\vec{k}) \end{tikzcd}$$

$$\equiv h_0(\vec{k}) + \overbrace{\textstyle\sum_{i=1}^{3} h_i(\vec{k})\,\sigma_i}^{\substack{\text{relative}\\ \text{Bloch Hamiltonian}\\ H(\vec{k})}}$$



which are gapped

$$\Rightarrow \quad \big|H(\vec{k}\,)\big| := \sqrt{\textstyle\sum_{i=1}^{3}\big(h_i(\vec{k}\,)\big)^2} \;>\; 0\,,$$

$\Rightarrow$ valence bundle is $\mathrm{Eig}_{-1}\big(H/|H|\big)$

$\Rightarrow$ Hamiltonian space is $\dfrac{\mathrm{U}(2)}{\mathrm{U}(1)\times\mathrm{U}(1)} \simeq S^2 \equiv \mathcal{A}$

$$\begin{tikzcd} \widehat{T}^2 \arrow[r, "H/|H|"] & S^2 \subset \mathbb{R}^3 \\ \vec{k} \arrow[r, mapsto] & \vec{h}(\vec{k})/|H|\,. \end{tikzcd}$$

Answer – for FQAH: 2.) topological phases

$\Rightarrow$ valence bundle is pullback of tautological/universal line bundle

$$\begin{tikzcd}
\text{valence bundle} & \text{tautological complex line bundle} & & \text{universal complex line bundle} \\
\mathcal{V} \arrow[r] \arrow[d] \arrow[dr, phantom, "\text{(pb)}"] & \mathcal{L} \arrow[rr] \arrow[d] \arrow[drr, phantom, "\text{(pb)}"] & & E\mathrm{U}(1) \underset{\mathrm{U}(1)}{\times} \mathbb{C} \arrow[d] \\
\widehat{T}^2 \arrow[r, "H/|H|"] & S^2 = \frac{\mathrm{U}(1)}{\mathrm{U}(1)\times\mathrm{U}(1)} \arrow[r, hook] & \bigcup\limits_{c\in\mathbb{N}} \frac{\mathrm{U}(1+c)}{\mathrm{U}(1)\times\mathrm{U}(c)} \arrow[r, "\sim"] & B\mathrm{U}(1)\,, \\
 & \text{fragile band topology} & \text{stable band topology} &
\end{tikzcd}$$

$\Rightarrow$ topological phases characterized by an integer

$$C := \big[H/|H|\big] \;\in\; \pi_0 \, \mathrm{Map}\big(\widehat{T}^2,\, S^2\big) \;\simeq\; \pi_0 \, \mathrm{Map}\big(\widehat{T}^2,\, B\mathrm{U}(1)\big) \;\simeq\; \mathbb{Z}$$

winding number

Chern number

$\Rightarrow$ fragile = stable, for topological phases

but not so for topological order...

Answer – for FQAH: 3.) topological order

the stable band monodromy is

$$\pi_1\Big(\mathrm{Map}\big(\widehat{T}^2,\, B\mathrm{U}(1)\big)_C\Big) \;\simeq\; \Big\langle A, B\Big\rangle \Big/ \Big(AB = BA\Big) \;\simeq\; \mathbb{Z}^2$$

Maxwell Wilson loop observables

have 1-dim irreps ⇒ no topological order

but

the *fragile* band monodromy is

[Sati & S. 2025, using
Larmore & Thomas 1980,
Kallel 2001,
going back to Hansen 1974]

$$\pi_1\Big(\mathrm{Map}\big(\widehat{T}^2,\, S^2\big)_C\Big) \;\simeq\; \Big\langle A, B, \underset{\text{central}}{\zeta}\Big\rangle \Big/ \Big(AB = \zeta^2 BA\Big)$$

Chern-Simons Wilson loop observables!

irreps are the anyonic

topological order on the torus!

⇒ solitonic anyons localized in momentum space

so far this assumed *all* symmetries broken

Answer – for FQAH: 4.) crystal symmetry

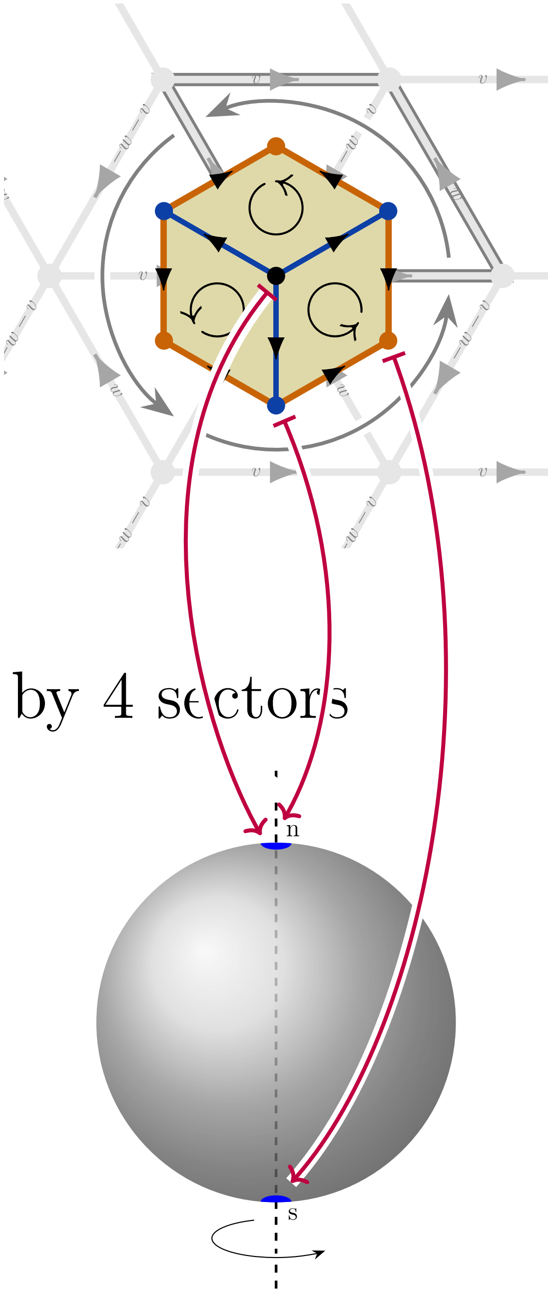
fragile crystalline Chern phases

(apparently not computed before)

Example: crystalline p3 symmetry $\mathbb{Z}_3$

topological phase: $(C, c) \in \boxed{3\mathbb{Z} \times [4]}$

Chern number is multiple of 3 & adjoined by 4 sectors

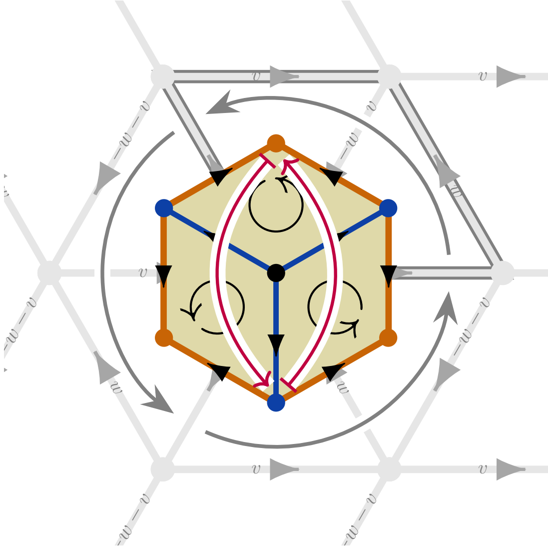


labeled by how
the 3 fixed/high symmetry points
map to either fixed pole on the sphere

Answer – for FQAH: 4.) crystal symmetry

fragile crystalline Chern phases

(apparently not computed before)



Example: crystalline p3 symmetry $\mathbb{Z}_3$

topological phase: $(C, c) \in 3\mathbb{Z} \times [4]$

Chern number is multiple of 3 & adjoined by 4 sectors

topological order: $\mathcal{H} \in \Big(\mathbb{Z}^3 \rtimes \mathrm{Sym}(3) \Big)$-IrrRep

braid the three high-symmetry points

nonabelian!

high symmetry points as defect para-anyons!

standard Sym(3) irrep implements Z-and rotation-gate $R_y(2\pi/3)$

[Sati & S 2025, Prop. 3.61]

Conclusion.

FQAH topological order theoretically seen in

monodromy of fragile band topology

$\Rightarrow$ clear predictions via equivariant (co)homotopy theory

band topology is maps to Bloch Hamiltonian space $\mathcal{A}$

topological phases is components π_0

topological order is monodromy π_1

For FQAH Chern insulators: $\mathcal{A} = S^2$ (2-Cohomotopy)

all symmetry broken $\Rightarrow$ solitonic anyons in momentum space!

p3 symmetry $\mathbb{Z}_3$ $\Rightarrow$ defect para-anyons in momentum space!

$\Rightarrow$ tantalizing targets for experiment

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