Differential generalized cohomology and Fundamental physics in Cohesive homotopy type theory

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Abstract

These talk notes survey the problem of characterizing and constructing twisted differential generalized cohomology theories; and its solution by cohesive homotopy type theory; and some implications for practical foundations of modern fundamental physics.

[Schreiber-Shulman 12] [Schreiber 13a]
[Bunke-Nikolaus-Völkl 13]
[Schreiber 13c, Schreiber 14]

Contents

1 Introduction

The proverbial practicing mathematician usually says "this follows formally" to mean "without hard analysis of point-set models".

But the sea keeps rising [Grothendieck 85]:

1. Over time, much hard analysis ends up following formally as the theory building improves.
2. For fully formalized mathematics,
   with unsuitable axiomatics (ZFC point-set models)
   rich parts remain unfeasible.

Therefore we need
pertinent axioms that capture the *relevant* structure.

The approach aiming for this is called "synthetic":

- [Lawvere 67, 86, 97]: "synthetic differential geometry"
  [Lawvere 91, 94, 07]: "axiomatic cohesion"
- [Quillen 67, HoTT 13]: "synthetic homotopy theory".

I will now be talking about combining these to

cohesive homotopy theory

and how this gives synthetic

twisted differential generalized cohomology theory.

2 Cohomology

I'll be speaking in terms of ∞-*topos theory* [Brown 73, Toën-Vezzosi 02, Rezk 10, Lurie 09] thought of [Awodey 10] as the categorical semantics for *homotopy type theory* [HoTT 13]:

Claim 2.1 ([Shulman 12, 13, Shulman-Lumsdaine 12], [Lumsdaine-Warren 14])**.**

1. *Locally cartesian closed presentable ∞-categories*[1] *interpret homotopy type theory without HITs and without univalent universes.*

2. *∞-Toposes over elegant Reedy sites interpret homotopy type theory with HITs and univalent type universes à la Russell.*

3. *∞-Toposes $\mathrm{Sh}_\infty(S)$ over general ∞-sites S interpret homotopy type theory with HITs and univalent type universes à la Tarski.*

See homotopytypetheory/show/model+of+type+theory+in+an+(infinity,1)-topos

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I'll talk semantically as traditional in mathematics, but phrased such that it should lend itself to a fully syntactic formulation in homotopy type theory, following [Schreiber-Shulman 12].

[1] Presented by a suitable Cisinski model structure on simplicial presheaves.

Definition 2.2. *For* $A \in \mathrm{Ab}(\mathbf{H}_{\leq 0})$ *an abelian group, and* $n \in \mathbb{N}$, *write*

$$\mathbf{B}^n A \in \mathbf{H}$$

for the corresponding Eilenberg-MacLane type, characterised by

$$\pi_k(\mathbf{B}^n A) = \begin{cases} A & \text{if } k = n \\ * & \text{otherwise} \end{cases}$$

[Licata-Finster 14]

Example 2.3. *Write*

$$\mathbf{H} = \infty\mathrm{Grpd} \simeq \mathrm{Sh}_\infty(*)$$

for the model of homotopy type theory in simplicial sets [Kapulkin-Lumsdaine-Voevodsky 12]. Here an EM type is a traditional Eilenberg-MacLane space

$$\mathbf{B}^n A \simeq K(A, n)\,.$$

The 0-truncation of function types into it is ordinary cohomology*:*

$$H^n(X, A) \simeq \pi_0(X \to \mathbf{B}^n A)$$

More generally, "generalized cohomology", has coefficients in stable homotopy types (spectra):

Definition 2.4. *1. Write* $(\Omega \dashv \Sigma) : \mathbf{H} \underset{\Sigma}{\overset{\Omega}{\rightleftarrows}} \mathbf{H}^{*/}$ *for suspension* $* \underset{(-)}{\sqcup} *$ *and looping* $* \underset{(-)}{\times} *$.

2. *A homotopy type* $X \in \mathbf{H}$ *is called* stable *if* $\eta : X \to \Omega\Sigma X$ *is an equivalence.*

3. *A spectrum object* E *in* $\mathbf{H}$ *is a collection of objects* $(E_n \in \mathbf{H}^{*/})_{n\in\mathbb{Z}}$ *equipped with equivalences* $(E_n \xrightarrow{\simeq} \Omega E_{n+1})$.

4. *The obvious maps of diagrams between spectrum objects in* $\mathbf{H}_{/X}$ *as* $X \in \mathbf{H}$ *ranges form the* tangent ∞-category $T\mathbf{H}$ *of* $\mathbf{H}$. *[Joyal 08]*

Proposition 2.5. $T\mathbf{H}$ *is itself an* ∞*-topos. The types in* $T_X\mathbf{H}$ *are stable homotopy types in* $\mathbf{H}_{/X}$. *[Joyal 08]*

Example 2.6. *For* $\mathbf{H} = \infty\mathrm{Grpd}$ *then a homotopy type in* $T_*\mathbf{H}$ *is equivalently an ordinary spectrum and for any type* $X \in \infty\mathrm{Grpd} \hookrightarrow T\infty\mathrm{Grpd}$ *then*

$$E^\bullet(X) \simeq (X \to E)$$

is the E*-cohomology spectrum of* X. *For* $\tau \in T_X\mathbf{H}$ *a bundle of spectra whose fibers are equivalent to* E, *then*

$$E^{\bullet+\tau}(X) \simeq (X \to \tau)$$

is the τ*-twisted* E*-cohomology spectrum of* X *[ABGHR 14].*

Notice that even if you build $\mathbf{H} = \infty\mathrm{Grpd}$ from topological spaces, as a homotopy theory it knows nothing about continuity. Or even smoothness. Nothing about *geometry*. A homotopy type in $\mathbf{H} = \infty\mathrm{Grpd}$ is a *geometrically discrete homotopy type*.
To fix this:

Proposition 2.7. *For S a site let $\mathbf{H} = \mathrm{Sh}_\infty(S)$. The stable Dold-Kan correspondence turns a sheaf of chain complexes $A_\bullet \in \mathrm{Ch}_\bullet(\mathrm{Sh(S)})$ into an S-geometric stable homotopy type $HA \in T_*\mathbf{H} \hookrightarrow T\mathbf{H}$. Then the function type*

$$HA^\bullet(X) \simeq (X \to HA)$$

is the abelian sheaf hypercohomology *of X with coefficients in A.*

[Brown 73]

Example 2.8.

$S := \mathrm{SmthMfd} := \{$*smooth manifolds + open cover topology*$\}$

$\mathbf{H} = \mathrm{Smooth}\infty\mathrm{Grpd} := \mathrm{Sh}_\infty(\mathrm{SmthMfd})$. *Contains*

- *de Rham complex* $\Omega^{\bullet \geq 1} \in T_*\mathbf{H}$*;*
- *Lie groups such as* $U(1) \in \mathrm{Grp}(\mathbf{H})$.

The latter canonically acts on the former by gauge transformation $A \mapsto A + d\mathrm{log} g$.

Proposition 2.9 ([Schreiber 13a]). *In* Smooth∞Grpd

$$\theta : \mathbf{B}^n U(1) \longrightarrow \mathrm{Type}$$
$$\theta(*) := \mathbf{B}^{n-1}\Omega^{1\leq \bullet \leq n}$$

then $\sum_{\mathbf{B}^n U(1)} \theta$ *is the Deligne complex, whose sheaf hypercohomology is degree-n* Deligne cohomology

$$\hat{H}^{n+1}(X, \mathbb{Z}) \simeq \pi_0(X \to \mathbf{B}^n U(1)_{\mathrm{conn}})$$

hence equivalently

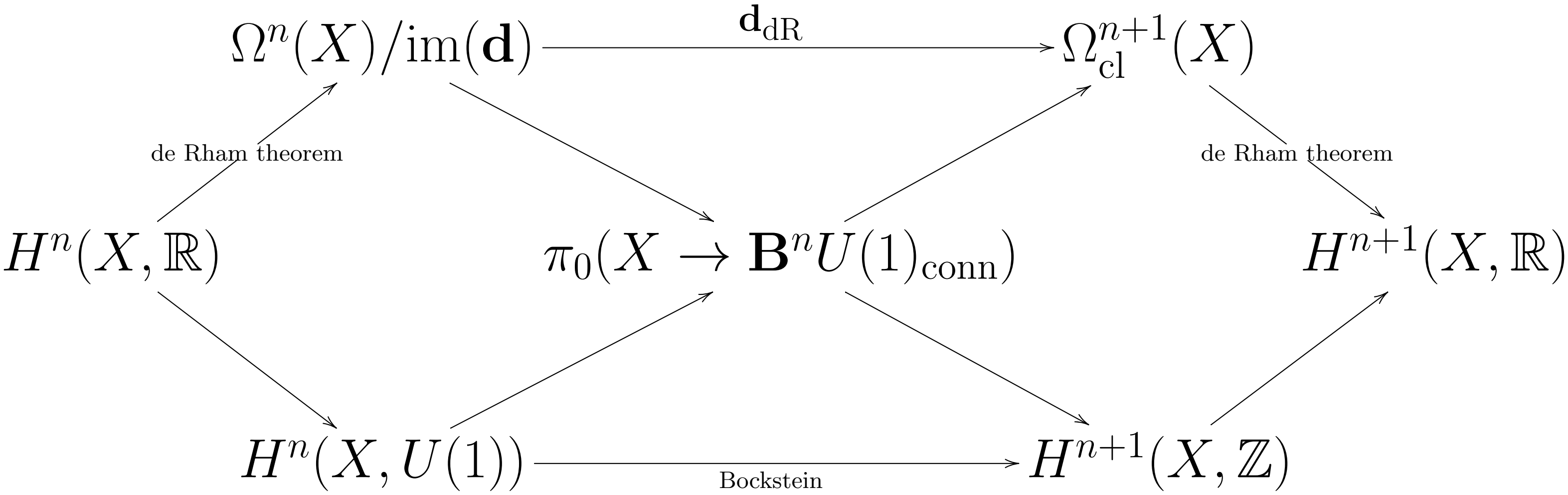
- *degree-$n+1$* ordinary differential cohomology *of X;*
- *circle n-bundles with connection ("higher bundle gerbes");*

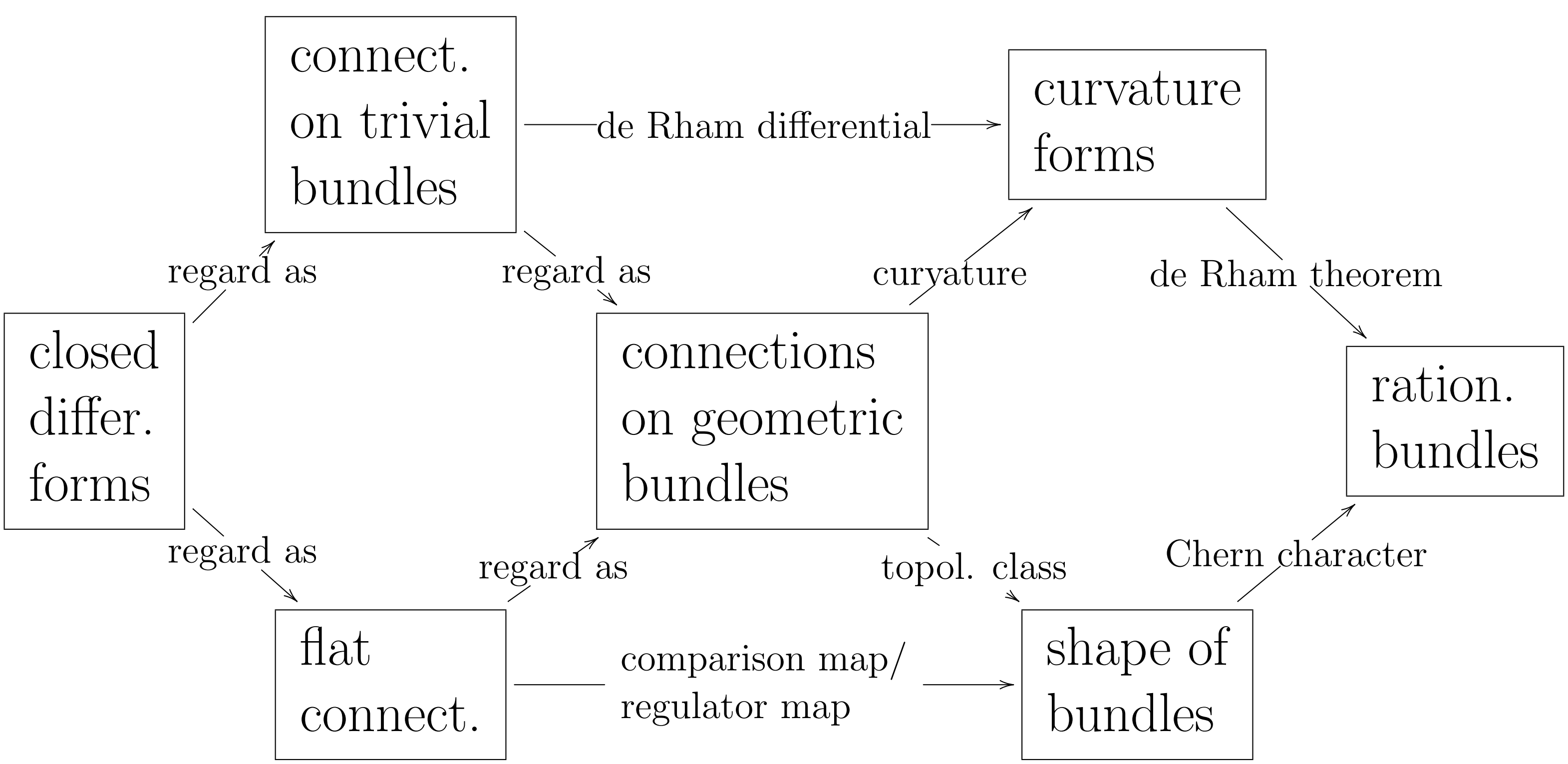
This governs fundamental reality: If X models (compactified) spacetime, then for example

- terms of type $\pi_0(X \to \mathbf{B}U(1)_{\mathrm{conn}})$ are configurations of the electromagnetic fields;
- terms of type $\pi_0(X \to \mathbf{B}^2 U(1)_{\mathrm{conn}})$ model (hypothetical) magnetic currents; as well as configurations of the (hypothetical) "Kalb-Ramond B-field";
- terms of type $\pi_0(X \to \mathbf{B}^3 U(1)_{\mathrm{conn}})$ model instanton currents of the electroweak field; as well as configurations of the (hypothetical) "supergravity C-field";
- terms of type $\pi_0(X \to \mathbf{B}^7 U(1)_{\mathrm{conn}})$ model currents of (hypothetical) 5-brane charge.

3 Simons-Sullivan's Question

[Simons-Sullivan 08] observed that $\pi_0(X \to \mathbf{B}^n U(1)_{\mathrm{conn}})$ is uniquely characterized by this hexagon of exact sequences:





The first "generalized" cohomology theory beyond "ordinary" cohomology is complex K-theory

$$\mathrm{KU} \in T_* \infty\mathrm{Grpd}$$

This has a differential refinement [Bunke-Gepner 13]

$$\mathrm{KU}_{\mathrm{conn}} \in T_* \mathrm{Smooth}\infty\mathrm{Grpd}$$

Was found via string theory
[Freed-Hopkins 00, Freed 00, Hopkins-Singer 02]:

- terms of type $\tau_0(X \to \mathrm{KU}_{\mathrm{conn}})$ are D-brane currents in type II string theory

Plenty of pure math descends from type II string theory:

- homological mirror symmetry,
- elliptic cohomology/genera,
- topological T-duality,
- generalized complex geometry, ...

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only that this is not quite true:
generally the D-brane currents should be in
twisted differential KU-theory.
Indeed, the fine structure of type II superstring theory should to large degree be that of twisted differential equivariant KU [Distler-Freed-Moore 09]...
but the nature of this twisted theory wasn't clear...

[Simons-Sullivan 08] asked the evident question:

Problem 3.1.
Is every generalized *differential cohomology theory characterized by/constructed via an exact hexagon?*

And in view of the above discussion we add:

Problem 3.2.
What is differential twisted generalized cohomology?

The classical representability theorem [Brown 62]
in view of the above discussion
suggests that the answer lies in completing this analogy:

generalized cohomology theory	$\simeq$ type in $T_*\infty\mathrm{Grpd}$
twisted generalized cohomology theory	$\simeq$ type in $T\infty\mathrm{Grpd}$
differential twisted generalized cohomology theory	$\simeq$ type in ???

4 Cohesion

Definition 4.1 ([Schreiber-Shulman 12])**.** *Cohesion on the homotopy type theory* $\mathbf{H}$ *is an adjoint triple of higher modalities [Shulman 12]*

$$(\Pi \dashv \flat \dashv \sharp) : \mathbf{H} \to \mathbf{H}$$

that encodes a three-fold (co-)reflection of the form

$$\mathbf{H} \;\begin{array}{c} \times\!\!\longrightarrow \\ \overset{\perp}{\hookleftarrow} \\ \overset{\perp}{\longrightarrow} \\ \overset{\perp}{\hookleftarrow} \end{array}\; \mathbf{B}$$

such that Π *preserves finite products.*

Example 4.2.

1. **higher differential geometry**.
$\mathrm{Sh}_\infty(S, \mathbf{B})$ *over sites* S *of manifolds (topological, smooth, formal, super) is cohesive over* $\mathbf{B}$ *[Schreiber 13a]. Here* Π *is* $\mathbb{R}^1$*-homotopy localization = fat geometric realization = classifying space functor.*

2. **equivariant homotopy theory**.
$\mathrm{PSh}_\infty(\mathrm{GlobOrbitCategory})$ *is cohesive over* $\infty\mathrm{Grpd}$ *[Rezk 14]. Here* Π *forms fixed point homotopy types.*

3. **parameterized stable homotopy theory**.
If $\mathbf{H}$ *is cohesive over* $\mathbf{B}$ *then* $T\mathbf{H}$ *is cohesive over* $T\mathbf{B}$*. (see section 4.1 of [Schreiber 13a])*

Definition 4.3 (secondary cohesive modalities).

1. $\mathbf{B}G \in \mathbf{H}$ *for a pointed connected cohesive homotopy type;*
2. $G := \Omega \mathbf{B}G \in \mathrm{Grp}(\mathbf{H})$ *for its loop type;*
3. $G \xrightarrow{\theta_G} \flat_{\mathrm{dR}} G \longrightarrow \flat \mathbf{B}G \longrightarrow \mathbf{B}G$ *for the long fiber sequence of the* $\flat$*-counit;*
4. $\mathbf{B}G \to \Pi \mathbf{B}G \to \Pi_{\mathrm{dR}} G$ *for the cofiber of the* Π*-unit.*

Example 4.4 (higher Cartan differential geometry). *For* $H = \mathrm{Smooth}\infty\mathrm{Grpd}$ *and* $G \in \mathrm{Grp}(\mathbf{H})$ *a Lie group with Lie algebra* $\mathfrak{g}$*, then:*

1. $\flat_{\mathrm{dR}} G = \{$*sheaf of flat* $\mathfrak{g}$*-valued diff. forms*$\}$*;*
2. $\theta_{\mathbf{BG}}$ *is the Maurer-Cartan form;*
3. $(X \to \mathbf{B}G)$ *is the moduli stack of* G*-bundles;*
4. $\sharp(X \to \flat \mathbf{B}G) \underset{\sharp(X \to \mathbf{B}G)}{\times} (X \to \mathbf{B}G)$ *is the moduli stack of flat connections;*

   [Schreiber 13a, Fiorenza-Rogers-Schreiber 13b]

5. $$\begin{array}{ccccc} & & & & \mathbf{B}^2 U(1)_{\mathrm{conn}} \\ & \overset{\mathrm{WZW}}{\nearrow} & & & \downarrow{\scriptstyle F} \\ G & \xrightarrow[\theta_G]{} & \flat_{\mathrm{dR}} G & \xrightarrow[\mu]{} & \Omega^3_{\mathrm{cl}} \end{array}$$ is the WZW gerbe.

[Fiorenza-Sati-Schreiber 13]

Example 4.5. *(higher differential moduli stacks)*
There are stages of differential refinement:

$$
\begin{array}{ccc}
\mathbf{B}^n U(1)_{\mathrm{conn}} & \xrightarrow{\theta_{\mathbf{B}^n U(1)\mathrm{conn}}} & \Omega^{n+1}_{\mathrm{cl}} \\
\downarrow & & \downarrow \\
\vdots & & \vdots \\
\downarrow & & \downarrow \\
\mathbf{B}^n U(1)_{\mathrm{conn}_{n-1}} & \xrightarrow{\theta_{\cdots}} & \mathbf{B}^{n-1}\Omega^{\bullet \geq 2} \\
\downarrow & & \downarrow \\
\mathbf{B}^n U(1) & \xrightarrow{\theta_{\mathbf{B}^n U(1)}} & \mathbf{B}^n \Omega^{\bullet \geq 1} \\
\downarrow & & \downarrow \\
\mathbf{B}^{n+1}\mathbb{R} & \xrightarrow{\mathrm{ch}} & \mathbf{B}^{n+1}\mathbb{R}
\end{array}
$$

Set

$$
\mathbf{B}^{n-1}U(1)\mathrm{Conn}(X) = \sharp_1[X, \mathbf{B}^n U(1)_{\mathrm{conn}}] \underset{\sharp_1[X, \mathbf{B}^n U(1)_{\mathrm{conn}_{n-1}}]}{\times} \sharp_2[X, \mathbf{B}^n U(1)_{\mathrm{conn}_{n-1}}] \underset{\cdots}{\times} \cdots \underset{\cdots}{\times} [X, \mathbf{B}^n U(1)]
$$

where

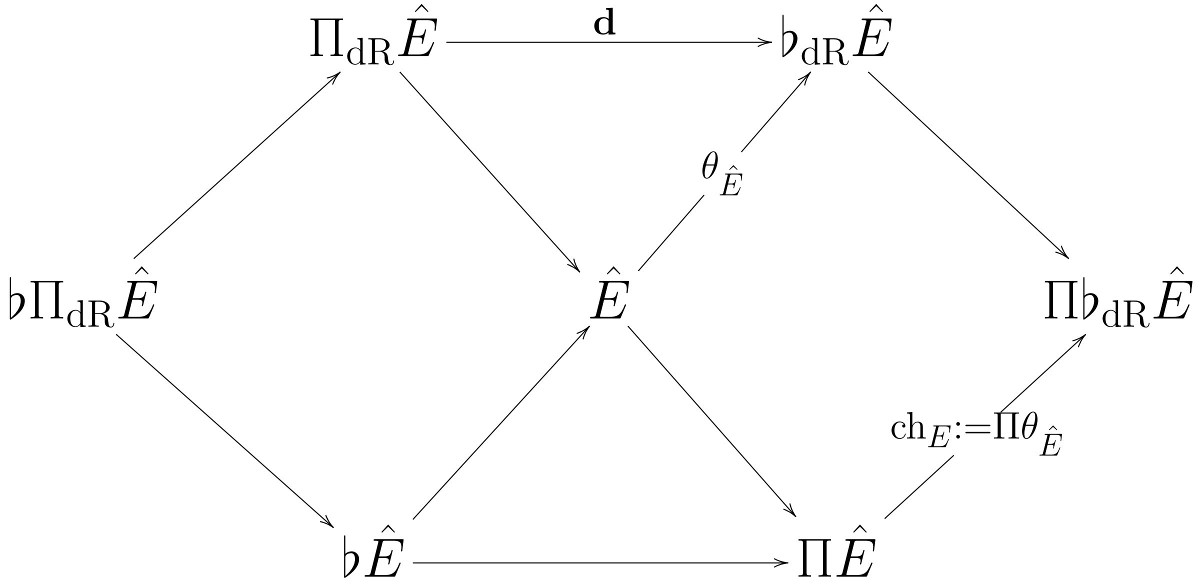
$$
\sharp_k := \mathrm{im}_k(\mathrm{id} \to \sharp)
$$

is the k-image ([Rijke-Spitters 13])
of the unit of the $\sharp$-modality.
This is the moduli n-stack *of circle n-connections.*

[Fiorenza-Rogers-Schreiber 13b] sect. 2.3.4,
[Schreiber 13a] sect. 3.9.6.4

5 The Answer

Theorem 5.1 ([Bunke-Nikolaus-Völkl 13]). *For $\hat{E}$ a stable cohesive homotopy type, then its canonical* $(\Pi \dashv \flat)$*-fracture hexagon*



is homotopy exact
(diagonals and boundary are homotopy fiber sequences, the squares are homotopy cartesian).

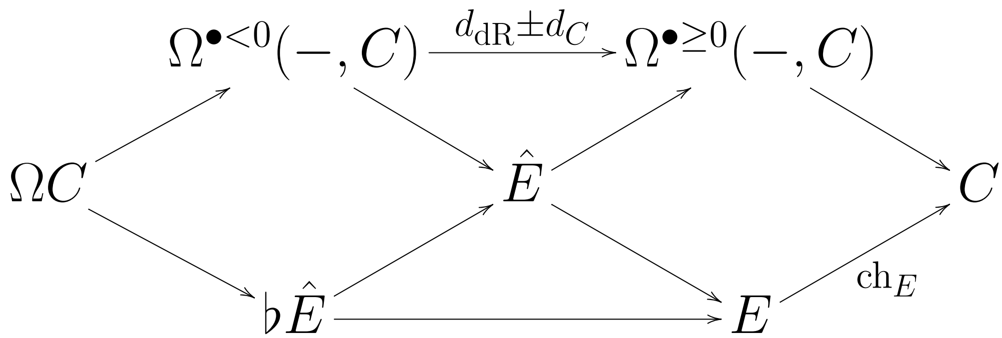
Proof. In the fiber-wise characterzation of stable homotopy pullbacks use the natural equivalence $\flat \xrightarrow{\simeq} \Pi\flat$ of cohesion, and dually. Then use the pasting law. ☐

Differential generalized cohomology theory

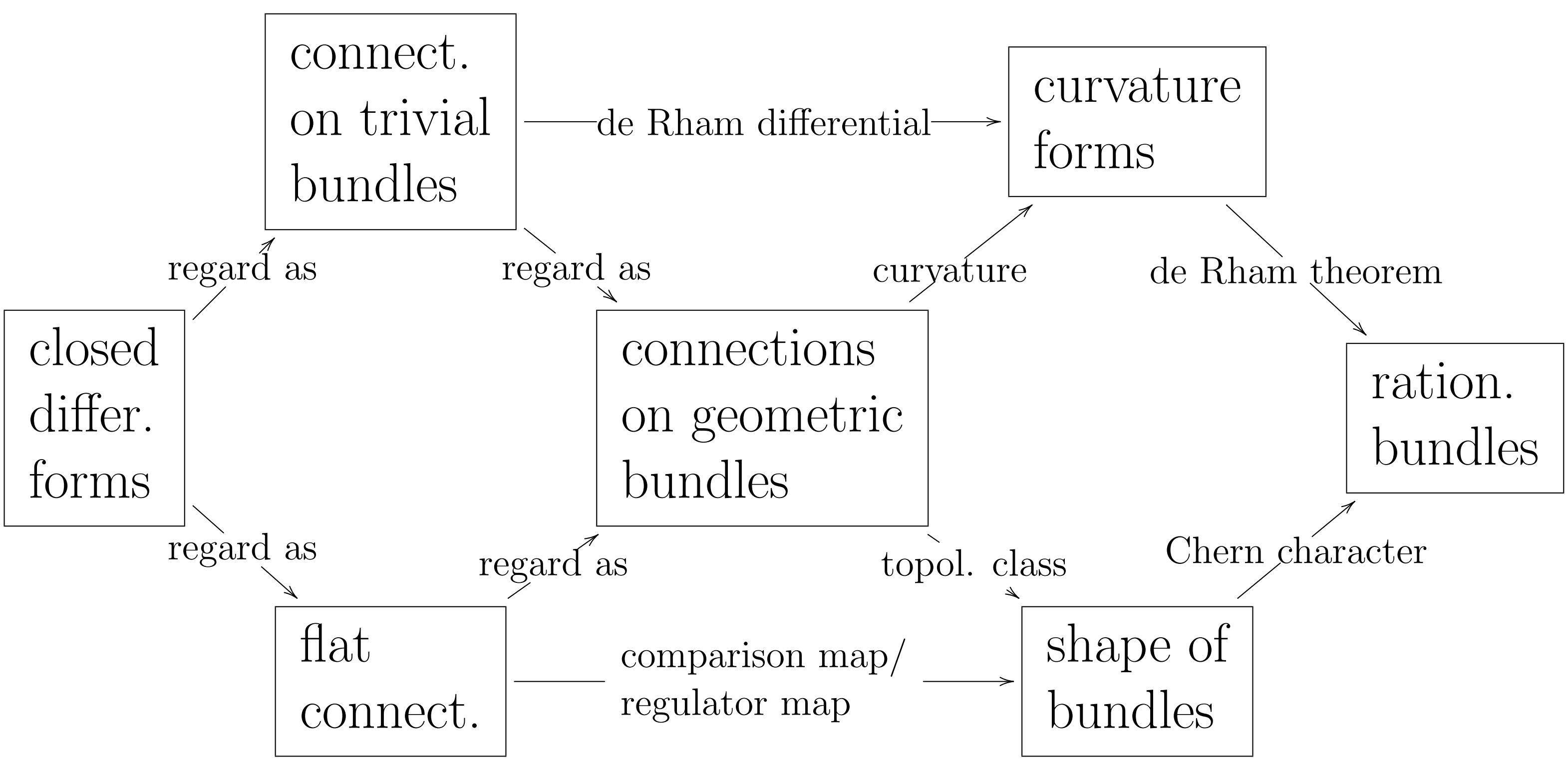
is

stable cohesive homotopy type theory.

Example 5.2 ([Bunke-Gepner 13, Bunke-Nikolaus-Völkl 13]). *For $E \in T_*\infty\mathrm{Grpd}$ a spectrum, $C \in \mathrm{Ch}_\bullet \stackrel{\mathrm{DK}}{\to} T_*\infty\mathrm{Grpd}$ the connected part of a chain complex model for its rationalization, then $\hat{E} \in T_*\mathrm{Smooth}\infty\mathrm{Grpd}$ in*



is the differential E-cohomology theory of [Hopkins-Singer 02].



So:

	differential generalized cohomology: maps in $T_*\mathbf{H}$
$\Rightarrow$	twisted differential generalized cohomology: maps in $T\,\mathbf{H}$

Definition 5.3. *For* $E \in \mathrm{CMon}_\infty(T_*\mathbf{H})$ *an* E_∞*-ring and* $\mathrm{Pic}(E) \in \mathbf{H}$ *its moduli of* E*-lines, write*

$$\mathcal{E} \in T_{\mathrm{Pic}(E)}\mathbf{H}$$

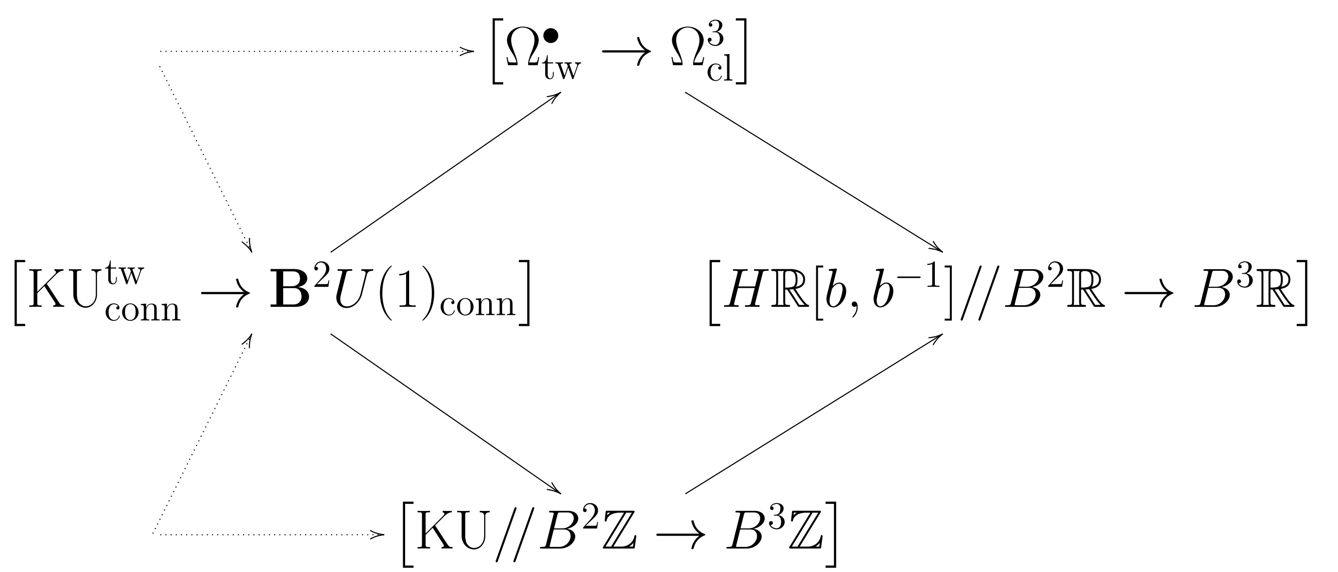
for the univeral E*-line.*

Proposition 5.4. *For* $X \in \mathbf{H} \hookrightarrow T\mathbf{H}$ *the function type*

$$(X \to \mathcal{E}) \in T_{(X \to \mathrm{Pic}(E))}\mathbf{H}$$

is over a given twist $\tau : X \to \mathrm{Pic}(E)$ *the* τ*-twisted* E*-cohomology spectrum* $E^{\bullet+\tau}(X)$*.*

Example 5.5 (twisted differential KU)**.**



6 Lagrangian Field Theory

An application:

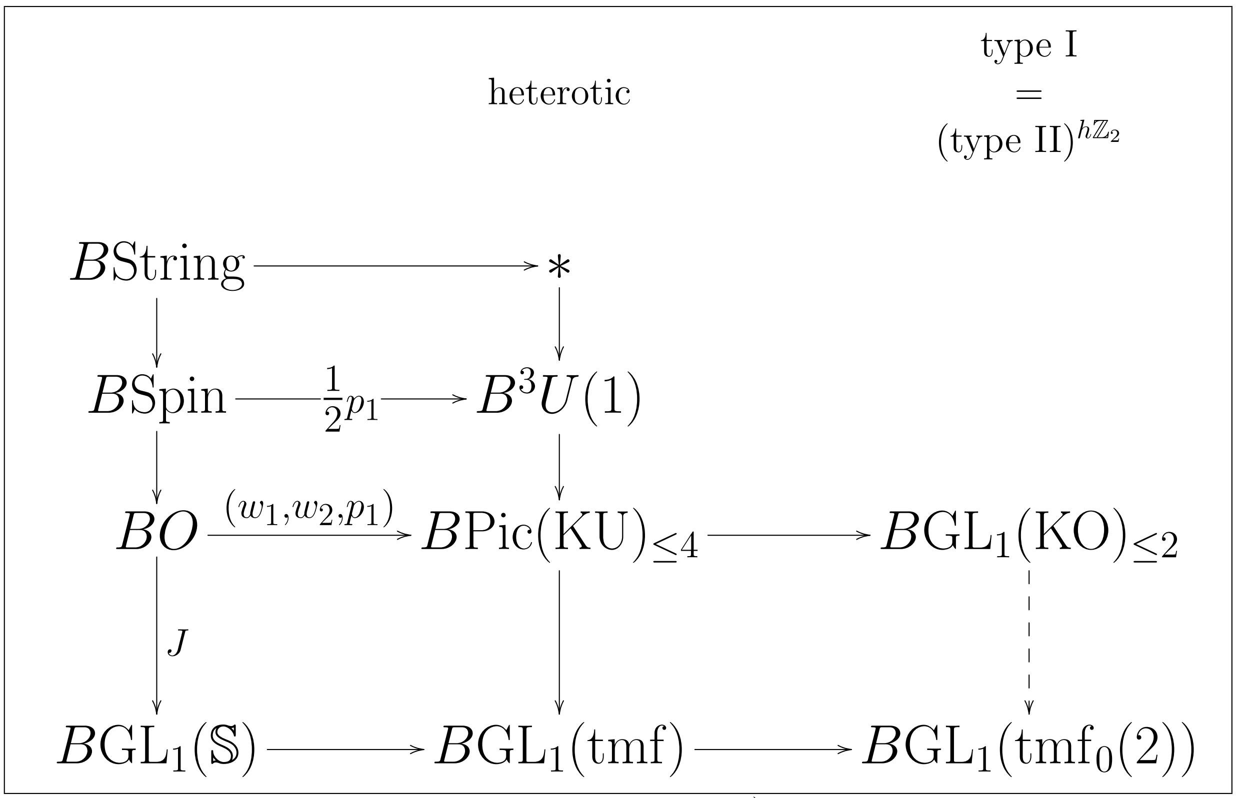
Problem 6.1. *There is an oracle which produces mathematical problems whose eventual solution produces Fields medals:*

- *(monster) vertex algebras; Goddard-Thorn theorem;*
- *mirror symmetry via A/B-model topological string;*
- *Gromov-Witten theory;*
- *formality theorem and Poisson deformation via holographic PSM string;*
- *generalized elliptic genera and regidity theorem via superstring partition functions;*
- *knot invariants via WZW-string/Chern-Simons theory;*
- *Ricci flow for string σ-model in dilaton background*
- *...*

An open problem is to understand the oracle as such: what is string theory?

String theoretic aside for the cognoscenti:

The fine structure of perturbative string theory (i.e.. anomaly cancellation, brane currents, BPS states,...) is known[2] to be captured by the differential refinement of a network of twists in generalized cohomology:



(for the dashed arrow the E_∞-structure is unclear to date)

and the non-perturbative refinement (F/M) is conjectured [Kriz-Sati 05] to involve a lift to G-equivariant tmf for G something like $\mathrm{GL}_2(\mathbb{Z})$ as in [Hill-Lawson 13].

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[2] See ... for a review.

Problem 6.2. *Which axiomatics produces physics from such data?*

Short answer [Schreiber 13b, Schreiber 14]:

Twisted duality/ambidexterity[3] in
linear homotopy type theory dependent[4]
on these differential coefficient types,
as in $T\mathbf{H}$.

Time is long up. But if you are still reading, the following pages have a few more pointers on this statement:

1. Classical field theory
   [Schreiber 13c]
2. Quantum mechanics
   [Bongers 14, Nuiten 13]
3. Path integral quantization
   [Nuiten 13, Schreiber 14]

[3] Strict ambidexterity is studied in [Hopkins-Lurie 14].

[4] Syntax for dependent linear type theory has been considered in [Pfenning 96, 03, Vákár 14].

1. Classical field theory

Synthetic Hamilton-Lagrange-Jacobi mechanics [Arnold 89]:

Theorem 6.3 ([Schreiber 13c])**.**
In $\mathbf{H} = \mathrm{Smooth}\infty\mathrm{Grpd}_{/\mathbf{B}U(1)_{\mathrm{conn}}}$

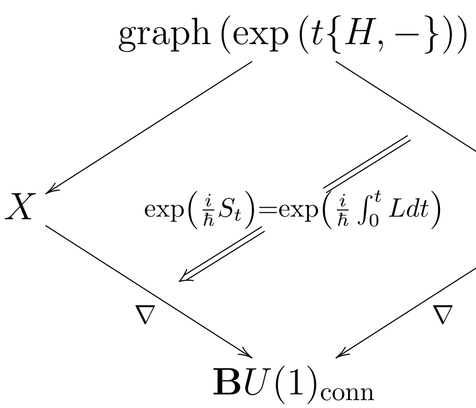
1. *a type* ∇ *is a* prequantized phase space*;*

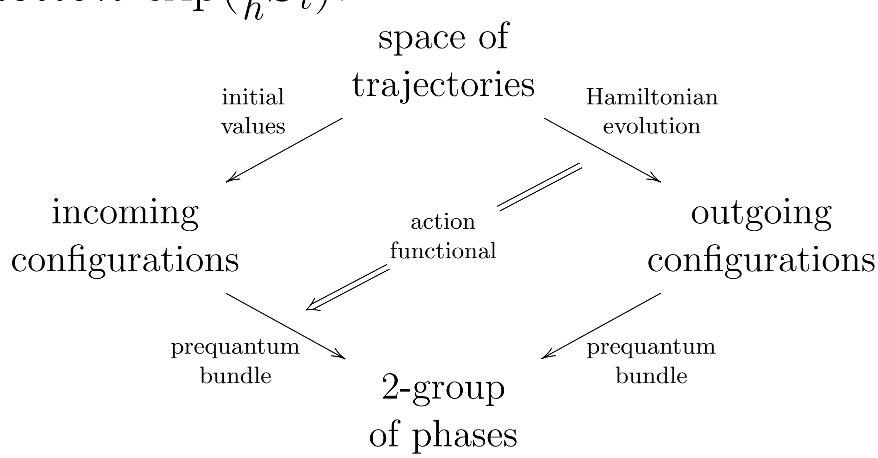
2. *an equivalence is a* Hamiltonian symplectomorphism*.*

A $\sharp$*-concrete function term*

$$H : \mathbf{B}\mathbb{R} \longrightarrow \prod_{\mathbf{B}U(1)_{\mathrm{conn}}} \mathbf{B}(\nabla \overset{\simeq}{\to} \nabla)$$

is a choice of Hamiltonian*. Sends parameter* $t : \mathbb{R}$ *to Hamiltonian evolution* $\exp(t\{H, -\})$
with Hamilton-Jacobi action $\exp(\frac{i}{\hbar} S_t)$*.*





Remark 6.4 ([Schreiber 13a], section 1.2.11)**.** *Replacing here* $\mathbf{B}U(1)_{\mathrm{conn}}$ *with* $\mathbf{B}^n U(1)_{\mathrm{conn}}$ *produces n-dimensional classical field theory in covariant Hamilton-de Donder-Weyl formulation on dual jet spaces of the field bundle (see e.g. [Román-Roy 05]).*

2. Quantum mechanics

Theorem 6.5. *For* (X, π) *a Poisson manifold, then its non-perturbative 2d Poisson-Chern-Simons theory [Fiorenza-Rogers-Schreiber 12, Fiorenza-Rogers-Schreiber 13b has as local action functional the prequantization of its symplectic groupoid*

$$\exp(\tfrac{i}{\hbar} S_{PCS}) : \mathrm{SymplGrpd} \longrightarrow \mathbf{B}(\mathbf{B}U(1)_{\mathrm{conn}}) \,,$$

[Bongers 14]

Theorem 6.6. *For* X *compact, boundary quantization of* $\exp(\frac{i}{\hbar} S_{PCS})$ *in* KU*-linear homotopy type theory reproduces traditional Kostant-Soriau geometric quantization...*

[Nuiten 13]

Remark 6.7. *... and thus generalizes it to Poisson manifolds, capturing for instance the universal orbit method of [Freed-Hopkins-Teleman 05].*

3. Path integral quantization

Theorem 6.8. *Dependent duality of functions in E-linear homotopy type theory reproduces Umkehr/Gysin/pushforward maps in twisted generalized E-cohomology as in [Ando-Blumberg-Gepner 10].*

And applied to local action functionals

$$\exp(\tfrac{i}{\hbar}S) : \mathbf{Fields} \to B\mathrm{GL}_1(E)$$

as above this produces cohomological path integral quantization for TQFTs.

[Nuiten 13, Schreiber 14]

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