# Quantum Programming via Linear Homotopy Types

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QUANTUM &
TOPOLOGICAL
SYSTEMS

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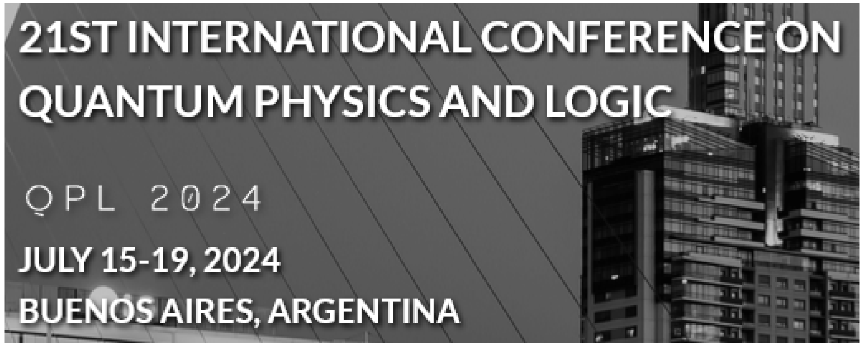
talk at:



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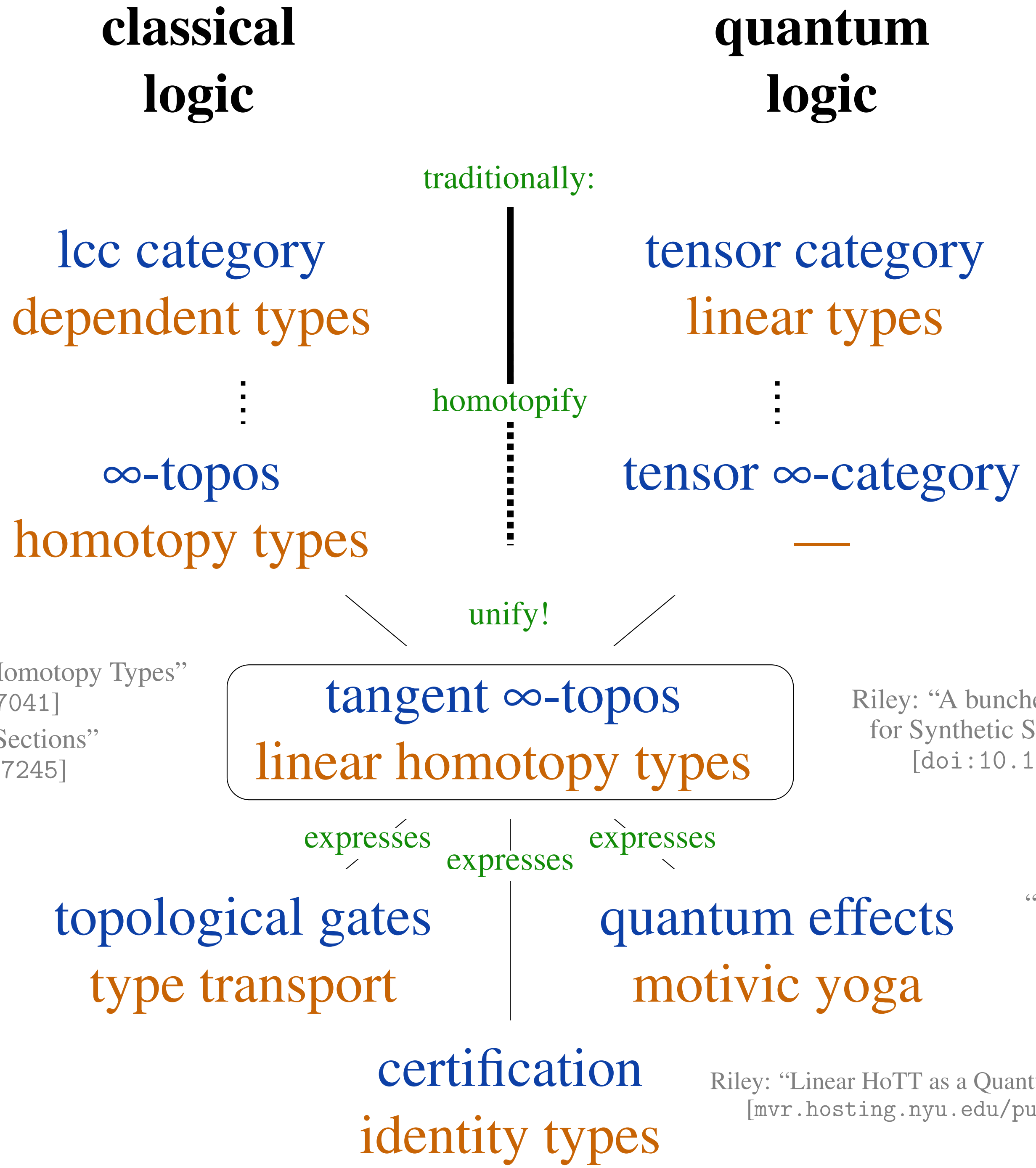
these slides available at: `ncatlab.org/schreiber/show/QPL2024`

**The Gist**



**classical logic**

**quantum logic**

traditionally:

lcc category
dependent types

tensor category
linear types

homotopify

∞-topos
homotopy types

tensor ∞-category
—

unify!

“Quantization via Linear Homotopy Types”
[arXiv:1402.7041]
“Entanglement of Sections”
[arXiv:2309.07245]

tangent ∞-topos
linear homotopy types

Riley: “A bunched Homotopy Type Theory
for Synthetic Stable Homotopy Theory”
[doi:10.14418/wes01.3.139]

expresses

expresses

expresses

“Topological Quantum Gates
in Homotopy Type Theory”
[arXiv:2303.02382]

topological gates
type transport

quantum effects
motivic yoga

“The Quantum Monadolgy”
[arXiv:2310.15735]
“Quantum and Reality”
[arXiv:2311.11035]

certification
identity types

Riley: “Linear HoTT as a Quantum Certification Language”
[mvr.hosting.nyu.edu/pubs/translation.pdf]

## Algebraic Topology — Quantum Physics — Linear Homotopy Types

Highlights

### Parameterized Quantum Systems

**local system** of state spaces
i.e., flat vector bundle over
classical parameter space

$$\begin{tikzcd}
\text{linear state space bundle}\ \mathcal{H} \arrow[r] \arrow[d, dashed, "\text{quantum state fibration}"'] & \widehat{\mathrm{LinType}} \arrow[d, dashed, "\text{univalent linear fibration}"] \\
\text{parameter space}\ X \arrow[r, "\vdash \mathcal{H}"] & \mathrm{LinType}\ \text{linear type universe}
\end{tikzcd}$$

**linear types dependent**
on homotopy type

### Topological Quantum Gates

**parallel** (adiabatic) **transport**
along path in parameter space

$$\begin{tikzcd}
\mathcal{H}_x \arrow[rr, bend left, "U_p", "\text{gate operator}"'] & & \mathcal{H}_y \\
& \updownarrow \text{lift} & \\
x \arrow[rr, squiggly, "p", "\text{path in parameter space}"'] & & y
\end{tikzcd}$$

**type transport** along
identification in base type

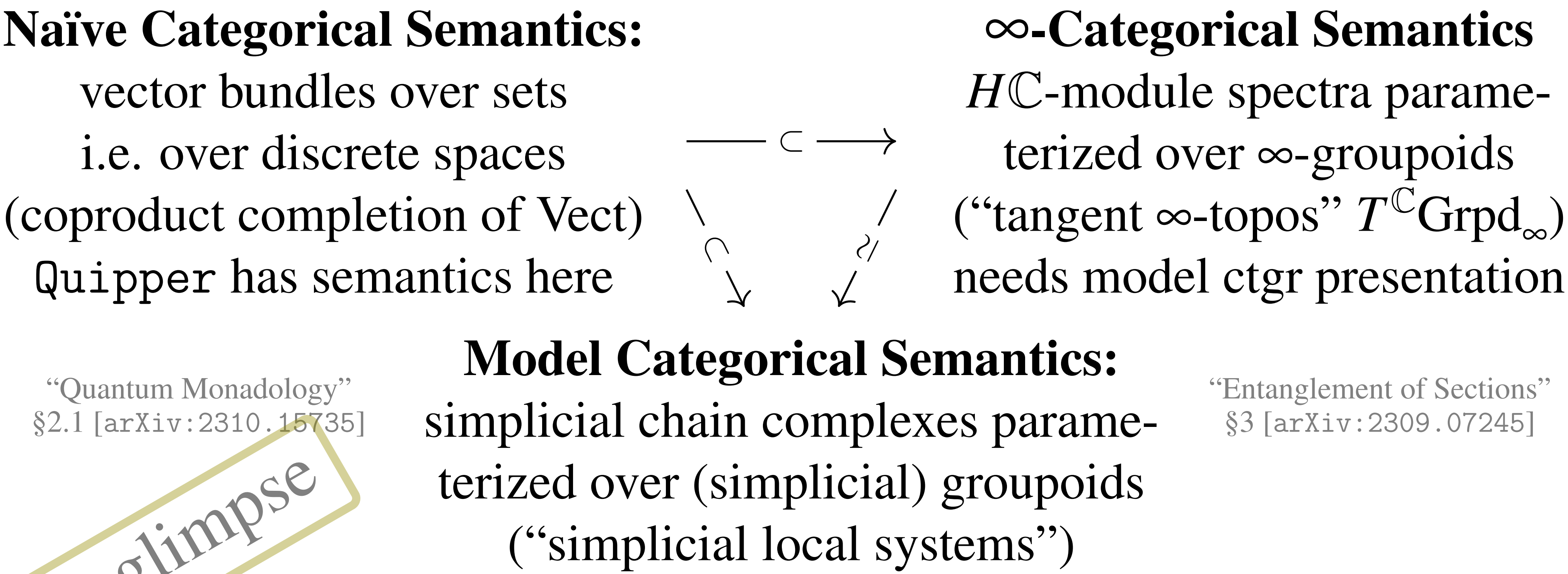
### Quantum Measurement

**count** of the
**descent comonad**
(“necessity” modality)

$$\begin{tikzcd}
\underset{\text{actual world}}{x' : X} \vdash \bigoplus_X \mathbb{C} \arrow[rr, "\sum_x q_x |x\rangle \mapsto q_{x'} |x'\rangle", "\text{state collapse}"'] & & \mathbb{C} \\
\Box \mathbb{1}_X \arrow[rr, "\eta^{\Box}", "\text{necessity counit}"'] \arrow[d] & & \mathbb{1}_X \arrow[d] \\
\text{possible worlds}\ X \arrow[rr, equal] & & X\ \text{measurement basis (finite)}
\end{tikzcd}$$

**eliminator** of
**dependent product**
(here: direct sum)

# Default Categorical Semantics



**Naïve Categorical Semantics:**

vector bundles over sets
i.e. over discrete spaces
(coproduct completion of Vect)
`Quipper` has semantics here

$\xhookrightarrow{\phantom{xxxx}}$

**∞-Categorical Semantics**

$H\mathbb{C}$-module spectra parameterized over ∞-groupoids
(“tangent ∞-topos” $T^{\mathbb{C}}\mathrm{Grpd}_\infty$)
needs model ctgr presentation

$\searrow$ (⊂) $\qquad \swarrow$ (≃)

**Model Categorical Semantics:**

simplicial chain complexes parameterized over (simplicial) groupoids
(“simplicial local systems”)

“Quantum Monadology”
§2.1 [`arXiv:2310.15735`]

“Entanglement of Sections”
§3 [`arXiv:2309.07245`]

Tech glimpse

In all cases: the corresponding Grothendieck construction of tensor categories of linear bundles $\mathcal{H}$ over base spaces $X$:

all types are...

...linear types over...

...classical base types.

$$\overbrace{\int^{X \in \mathrm{ClaType}} \mathrm{QuType}_X}^{\mathrm{Type}} = \left\{ \begin{tikzcd} \mathcal{H} \arrow[rr, "\text{fiberwise linear map}"] \arrow[d, "\text{quantum state fibration}"'] & & \mathcal{H}' \arrow[d, "\text{quantum state fibration}"] \\ {} \arrow[rr] & & {} \\ X \arrow[rr, "\text{map of base spaces}"'] & & X' \end{tikzcd} \right\}$$

these dependent linear types
are “infinitesimally cohesive”

“Diff. cohomology in cohesive ∞-topos”
Prop. 4.1.9 [`arXiv:1310.7930`]

**modality:**
“**classically**”

$$\begin{tikzcd}
\begin{bmatrix} 0_X \\ \downarrow \\ X \end{bmatrix} & X \arrow[l, mapsto] \\
\mathrm{Type} \arrow[loop left, "\natural"] \arrow[r, shift right=2] \arrow[r, phantom, "\bot\ \top"] & \mathrm{ClaType} \arrow[l, hook', shift right=2] \\
\begin{bmatrix} \mathcal{H} \\ \downarrow \\ X \end{bmatrix} \arrow[r, mapsto] & X
\end{tikzcd}$$

Linear HoTT is foremost HoTT
with infntsml cohesive modality ♮

Finster et al: “Synthetic Spectra” [`arXiv:2102.04099`]

Tech glimpse

this allows to *construct*
linear/quantum types

$$\mathrm{QuType} \;\equiv$$

$$(\mathcal{H} : \mathrm{Type}) \times (\natural\mathcal{H} = *)$$

**modality:**
“**quantumly**”

$$\begin{tikzcd}
\begin{bmatrix} \mathcal{H} \\ \downarrow \\ * \end{bmatrix} & \mathcal{H} \arrow[l, mapsto] \\
\mathrm{Type} \arrow[loop left, "\triangleright"] \arrow[r, shift right=2] \arrow[r, phantom, "\top"] & \mathrm{QuType} \arrow[l, hook', shift right=2] \\
\begin{bmatrix} \mathcal{H} \\ \downarrow \\ X \end{bmatrix} \arrow[r, mapsto] & \bigoplus\limits_{x:X} \mathcal{H}_x
\end{tikzcd}$$

and a Girard !-modality of linear logic
via a Bierman-Benton adjunction

“The Quantum Monadology”
Prop. 2.7 [`arXiv:2310.15735`]

$$\begin{tikzcd}
\mathrm{QuType} \arrow[loop left, "!"] \arrow[r, hook, shift right=2] \arrow[r, phantom, "\bot"] \arrow[rr, bend right=40, "\text{“classicized” } \mathrm{C} \equiv \Omega^\infty_+"'] & \mathrm{Type} \arrow[l, shift right=2, "\triangleright"'] \arrow[r, shift right=2, "\natural(\mathbb{1}\to(-))"'] \arrow[r, phantom, "\bot"] & \mathrm{ClaType} \arrow[l, <->, shift right=2, "\mathbb{1}\times(-)"'] \arrow[ll, bend right=40, "\text{“quantized” } \mathrm{Q} \equiv \Sigma^\infty_+"']
\end{tikzcd}$$

# Schreiber@QPL2024: “Quantum Programming via Linear Homotopy types”

| **Syntax** | **Semantics** |
|---|---|
| **♮-FORM** $\dfrac{\Gamma \vdash A : \mathrm{Type}}{\Gamma \vdash \natural A : \mathrm{Type}}$ | $$\begin{tikzcd} A \arrow[d, two heads, "p_A"] \\ \natural\Gamma \end{tikzcd}$$ <br> $$\begin{tikzcd} (5)\ \natural^{\mathrm{rel}}_{\Gamma} A \arrow[r] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & \natural(\eta^{\natural}_{\Gamma})^{*}A \arrow[r, "\natural q_A"] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & \natural A \arrow[d, two heads, "\natural p_A"] \\ \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, "\natural\eta^{\natural}_{\Gamma}"] & \natural\natural\Gamma \end{tikzcd}$$ |
| **♮-INTRO** $\dfrac{\Gamma \vdash a : A}{\Gamma \vdash a^{\natural} : \natural A}$ | $$\begin{tikzcd} \natural\Gamma \arrow[r, "\vdash a"] \arrow[d, equal] & A \arrow[d, two heads, "p_A"] \\ \natural\Gamma \arrow[r, equal] & \natural\Gamma \end{tikzcd}$$ <br> $$\begin{tikzcd} \Gamma \arrow[rrr, "\eta^{\natural}_{\Gamma}"] \arrow[dr, dashed] \arrow[ddrr, dashed, bend right, "\vdash a^{\natural}"'] \arrow[ddd, equal] & & & \natural\Gamma \arrow[r, "\vdash a"] \arrow[ddd, equal] & A \arrow[r, "\eta^{\natural}_{A}"] \arrow[ddd, two heads, "p_A"] & \natural A \arrow[ddd, two heads, "\natural p_A"] \\ & (\eta^{\natural}_{\Gamma})^{*}A \arrow[urrr, "q_A"] \arrow[dr, dashed, "\eta^{\natural^{\mathrm{rel}}_{\Gamma}}_{A}"] \arrow[dd] & (8) & & & \\ & & \natural^{\mathrm{rel}}_{\Gamma} A \arrow[uurrr] \arrow[d] & & & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, equal] & \natural\Gamma \arrow[r, "\eta^{\natural}_{\natural\Gamma}"] & \natural\natural\Gamma \end{tikzcd}$$ |
| **♮-ELIM** $\dfrac{\Gamma \vdash b : \natural A}{\Gamma \vdash b_{\natural} : A}$ | $$\begin{tikzcd} \Gamma \arrow[r, "\vdash b"] \arrow[d, equal] & \natural^{\mathrm{rel}}_{\Gamma} A \arrow[r] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & \natural\big((\eta^{\natural}_{\Gamma})^{*}A\big) \arrow[r, "\natural q_A"] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & \natural A \arrow[d, two heads, "\natural p_A"] \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, "\natural\eta^{\natural}_{\Gamma}"] & \natural\natural\Gamma \end{tikzcd}$$ <br> $$\begin{tikzcd} \Gamma \arrow[rr, "\vdash b"] \arrow[dr, dashed, "b_{\natural}"'] \arrow[dd, equal] & & \natural^{\mathrm{rel}}_{\Gamma} A \arrow[r] \arrow[dd, two heads] \arrow[ddr, phantom, "\text{(pb)}"] & \natural\big((\eta^{\natural}_{\Gamma})^{*}A\big) \arrow[r] \arrow[dd, two heads] \arrow[ddr, phantom, "\text{(pb)}"] & \natural A \arrow[r, "\varepsilon^{\natural}_{A}"] \arrow[dd, two heads, "\natural p_A"] & A \arrow[dd, two heads, "p_A"] \\ & (\eta^{\natural}_{\Gamma})^{*}A \arrow[urrrr] \arrow[d] & & & & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, "\natural\eta^{\natural}_{\Gamma}"] \arrow[rr, bend right, "\mathrm{id}"', "(16)"] & \natural\natural\Gamma \arrow[r, "\varepsilon^{\natural}_{\natural\Gamma}"] & \natural\Gamma \end{tikzcd}$$ |
| $\begin{array}{l} \Gamma \vdash a : A \\ \hline (24)\ \Gamma \vdash a : \underline{A} \\ \hline (30)\ \Gamma \vdash a^{\natural} : \natural\underline{A} \end{array}$ | $$\begin{tikzcd} \Gamma \arrow[rrr, "\eta^{\natural}_{\Gamma}"] \arrow[rrrrrrr, bend left, "\eta^{\natural}_{A}\circ(\vdash a)", "(10)"'] \arrow[dr, dashed, "\vdash a"] \arrow[ddrr, dashed, bend right, "\vdash a^{\natural}"'] \arrow[ddd, equal] & & & \natural\Gamma \arrow[rr, "\varepsilon^{\natural}_{\Gamma}"] \arrow[dr, dashed, "\vdash a"] \arrow[ddd, equal] & & \Gamma \arrow[r, "\vdash a"] \arrow[ddd, equal] & A \arrow[r, "\eta^{\natural}_{A}"] \arrow[ddd, two heads, "p_A"] & \natural A \arrow[r, "(\natural\nu_A)^{-1}", "\sim"'] \arrow[ddd, "\natural p_A"] & \natural\underline{A} \arrow[ddd, "\natural p_{\underline{A}}"] \\ & (\eta^{\natural}_{\Gamma})^{*}\underline{A} \arrow[rrr] \arrow[dr, dashed, "\eta^{\natural^{\mathrm{rel}}_{\Gamma}}_{\underline{A}}"] \arrow[dd] & & & \underline{A} \arrow[urr, "\nu_A"] \arrow[urrrr, "\eta^{\natural}_{\underline{A}}"] \arrow[dd, two heads, "p_{\underline{A}}"] & & (12) & (17) & \\ & & \natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[uurrrrrr] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & & & & & & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"'] & \natural\Gamma \arrow[r, equal] & \natural\Gamma \arrow[r, "\varepsilon^{\natural}_{\Gamma}"] \arrow[rrr, bend right, "\mathrm{id}"', "(9)"] & \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, "\sim", "\eta^{\natural}_{\natural\Gamma}"'] & \natural\natural\Gamma \end{tikzcd}$$ |

## some inference rules for the *classically* modality

| | |
|---|---|
| $\begin{array}{rcl} \Gamma & \vdash & A : \mathrm{Type} \\ \hline (19)\quad \Gamma,\ a{:}A & \vdash & a : A \\ \hline (34)\quad \Gamma,\ a{:}A & \vdash & \underline{a}^{\natural} : \underline{A} \\ \hline \Gamma & \vdash & (a \mapsto \underline{a}^{\natural}) : A \to \natural\underline{A} \end{array}$ | $$\begin{tikzcd} A \arrow[rr, "\eta^{\natural}_{A}"] \arrow[dr, dashed, "a\mapsto \underline{a}^{\natural}"] \arrow[dd, two heads, "p_A"] & & \natural A \arrow[dd, two heads, "\natural p_A"] \\ & \natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[ur] \arrow[d, two heads] \arrow[dr, phantom, "\text{(pb)}"] & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \end{tikzcd}$$ |

## some inference rules for linear types:

| **Syntax** | **Semantics** |
|---|---|
| $\dfrac{\Gamma \vdash A{:}\mathrm{Type} \qquad \Gamma \vdash \underline{x}{:}\natural\underline{A}}{\Gamma \vdash A_{\underline{x}} :\equiv (a{:}A)\times \mathrm{Id}(\underline{a}^{\natural}, \underline{x})}$ <br> linear fiber | $$\begin{tikzcd} A_{\underline{x}} \arrow[rr, "\text{(pb)}"'] \arrow[dr, "\text{(pb)}"'] \arrow[dd, two heads] \arrow[dddd, two heads, bend right, "p_{A_{\underline{x}}}"'] & & \natural\Gamma \arrow[r, "\vdash\underline{x}", "\text{(pb)}"'] \arrow[dd, "(\vdash\underline{x},\mathrm{id})"] & \natural A \arrow[dd, two heads, "\Delta_A"] \\ & \natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[urr, "\text{(pb)}"'] \arrow[dd, two heads, "\Delta_{\natural^{\mathrm{rel}}_{\Gamma}\underline{A}}"] & & \\ A \arrow[rr, "(\eta^{\natural}_{A},\ \eta^{\natural}_{\Gamma}\circ p_A)"'] \arrow[dr, "\vdash((-)^{\natural},\underline{x})", "(35)"'] \arrow[dd, two heads, "p_A"] & & \natural A\times\natural\Gamma \arrow[r, "\mathrm{id}\times\vdash\underline{x}"] & \natural A\times\natural A \arrow[dd, two heads, "\natural p_A\times\natural p_A"] \\ & \natural^{\mathrm{rel}}_{\Gamma}\underline{A}\times_{\Gamma}\natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[urr, "\text{(pb)}"'] \arrow[d, two heads] & & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \arrow[r, "\Delta_{\natural\Gamma}"] & \natural\Gamma\times\natural\Gamma \end{tikzcd}$$ |
| $\dfrac{\Gamma \vdash A{:}\mathrm{Type} \qquad \Gamma \vdash \underline{x}{:}\natural\underline{A}}{\Gamma \vdash \natural(A_{\underline{x}}) \simeq *}$ <br> linear fibers are indeed linear | $$\begin{tikzcd} \natural A_{\underline{x}} \arrow[r, equal, "\sim", "(37)"'] & \natural\Gamma \arrow[r, "\natural\eta^{\natural}_{\Gamma}", "(14)"'] \arrow[d, "\vdash\underline{x}"] \arrow[dr, phantom, "\text{(pb)}"] & \natural\natural\Gamma \arrow[d, two heads, "\natural(\vdash\underline{x},\mathrm{id})"] \\ & \natural A \arrow[r, "\natural\left(\eta^{\natural}_{A},\ \eta^{\natural}_{A}\circ p_A\right)"] & \natural\left(\natural A\times\natural\Gamma\right) \end{tikzcd}$$ |
| $\dfrac{\Gamma \vdash A : \mathrm{Type}}{\Gamma \vdash A \simeq \sum\limits_{x:\natural A} A_{\underline{x}}}$ <br> types are sums of their linear fibers | $$\begin{tikzcd} A \arrow[rr, "\eta^{\natural}_{A}"] \arrow[dr, "\text{(pb)}", "\text{(pb)}"'] \arrow[dd, two heads, "(\mathrm{id}_A,\ \eta^{\natural}_{A})"] \arrow[dddd, two heads, bend right, "p_A"'] & & \natural A \arrow[dd, two heads, "\Delta_{\natural A}"] \\ & \natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[ur, "\text{(pb)}"'] \arrow[dd, two heads] & \\ A\times_{\Gamma}\natural A \arrow[rr, "(\eta^{\natural}_{A},\ \mathrm{id}_{\natural A})"] \arrow[dr] \arrow[dd, two heads] & & \natural A\times_{\Gamma}\natural A \arrow[dd, two heads] \\ & \natural^{\mathrm{rel}}_{\Gamma}\underline{A}\times_{\Gamma}\natural^{\mathrm{rel}}_{\Gamma}\underline{A} \arrow[ur, "\text{(pb)}"'] \arrow[d, two heads] & \\ \Gamma \arrow[r, equal] & \Gamma \arrow[r, "\eta^{\natural}_{\Gamma}"] & \natural\Gamma \end{tikzcd}$$ |

to give an impression

miracle
occurs

## key: **quantum measurement** effect handling captured by dependent linear type formation

recall: given a context extension

classical type

$$\begin{tikzcd} X \arrow[rr, two heads] & & * \end{tikzcd}$$

then dependent sum and dependent product form the base change adjoint triple, where...

**dependent sum**

*X*-dependent...

...classical types

$$\begin{tikzcd}[column sep=huge] \mathrm{ClaType}_X \arrow[r, shift left=4, "\Sigma_X"] \arrow[r, phantom, shift left=2, "\perp"] \arrow[r, shift right=4, "\Pi_X"'] \arrow[r, phantom, shift right=2, "\perp"] & \mathrm{ClaType} \arrow[l, "*_X \times (\text{-})"'] \end{tikzcd}$$

**dependent products**

...the induced (co-)modalities exhibit S5 modal logic

---

now for dependent *linear* types over finite bases this is the direct sum...

**direct sum**

*X*-dependent...

...linear types

$$\begin{tikzcd}[column sep=huge] \mathrm{QuType}_X \arrow[loop above, "\Diamond_X"] \arrow[loop below, "\Box_X"'] \arrow[r, shift left=4, "\oplus_X"] \arrow[r, phantom, shift left=2, "\perp"] \arrow[r, shift right=4, "\oplus_X"'] \arrow[r, phantom, shift right=2, "\perp"] & \mathrm{QuType} \arrow[l, "\mathbb{1}_X \otimes (\text{-})"'] \arrow[loop above, "\star_X"] \arrow[loop below, "\bigcirc_X"'] \end{tikzcd}$$

$\Diamond_X \simeq \Box_X$ $\qquad$ $\star_X \simeq \bigcirc_X$

$X$ finite:

**direct sum**

... which yields pair of Frobenius monads

⇒ **quantum necessity monad**
models “dynamic lifting” of quantum measurement;
Kleisli equivalence proves deferred msrmnt principle

“Quantum Monadology”
§2 [`arXiv:2310.15735`]

**quantum indefiniteness monad**
$\simeq$ Coecke’s measurement monad
as now used in `xz-calculus`

this way, quantum-logic & -effects are incarnated in homotopy type theory by a **system of modalities/monads**

axiomatized:

Riley: “A bunched HoTT” [doi:10.14418/wes01.3.139]

- $\natural$ classically
- $\otimes$ entangled

induced:

“Quantum Monadology” §2 [arXiv:2310.15735]

- $\triangleright$ quantumly
- $!$ $2^{nd}$ quantized
- $\square$ necessarily
- $\bigcirc$ indefinitely

(necessarily, indefinitely:) quantum measurement effects

via do-notation for monadic effects this permits a domain specific **quantum language** to be embedded into LHoTT

e.g. snippet from embedded error-correction peudocode:

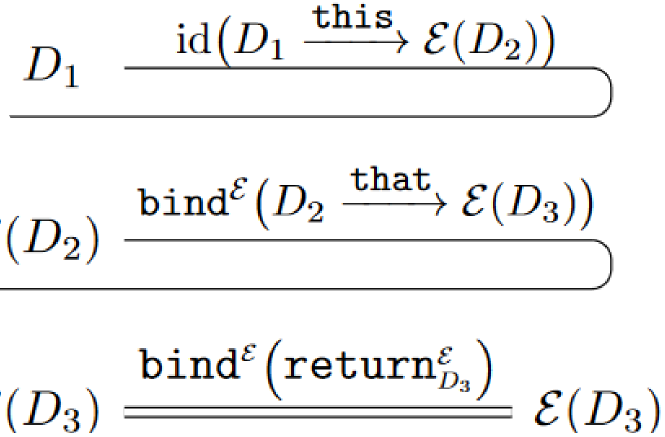
```
$\mathrm{correct\_error} \;:\; \mathrm{LgclQBit} \multimap \bigcirc_{\mathrm{Syndrome}} \mathrm{LgclQBit}$

$\mathrm{correct\_error} \;\equiv$
    for $|b_1, b_2, b_3\rangle$
    do
        for $|\psi\rangle$ in $\mathrm{measure\_syndrome}(|b_1, b_2, b_3\rangle)$
        do
            if measured $(s_1, s_2)$
            then $\mathrm{BitFlip}_{(s_1, s_2)}|\psi\rangle$
```

$1^{st}$ punchline

“Quantum Monadology” §3 [arXiv:2310.15735]

Kleisli morphism for monadic effect $\mathscr{E}$



$$\begin{tikzcd}
D_1 \arrow[r, "\mathrm{id}\left(D_1 \xrightarrow{\texttt{this}} \mathcal{E}(D_2)\right)"] & \mathcal{E}(D_2) \arrow[r, "\texttt{bind}^{\mathcal{E}}\left(D_2 \xrightarrow{\texttt{that}} \mathcal{E}(D_3)\right)"] & \mathcal{E}(D_3) \arrow[r, equal, "\texttt{bind}^{\mathcal{E}}\left(\texttt{return}^{\mathcal{E}}_{D_3}\right)"] & \mathcal{E}(D_3)
\end{tikzcd}$$

“Quantum Monadology” Lit 1.19 [arXiv:2310.15735]

corresponding do-notation

```
$\mathrm{prog} \;:\; D_1 \to \mathcal{E}(D_3)$

$\mathrm{prog} \;\equiv$
    for d2 in this d1
    do
        for d3 in that d2
        do return d3
```

beyond offering certification for traditional quantum languages (e.g. `Quipper`)

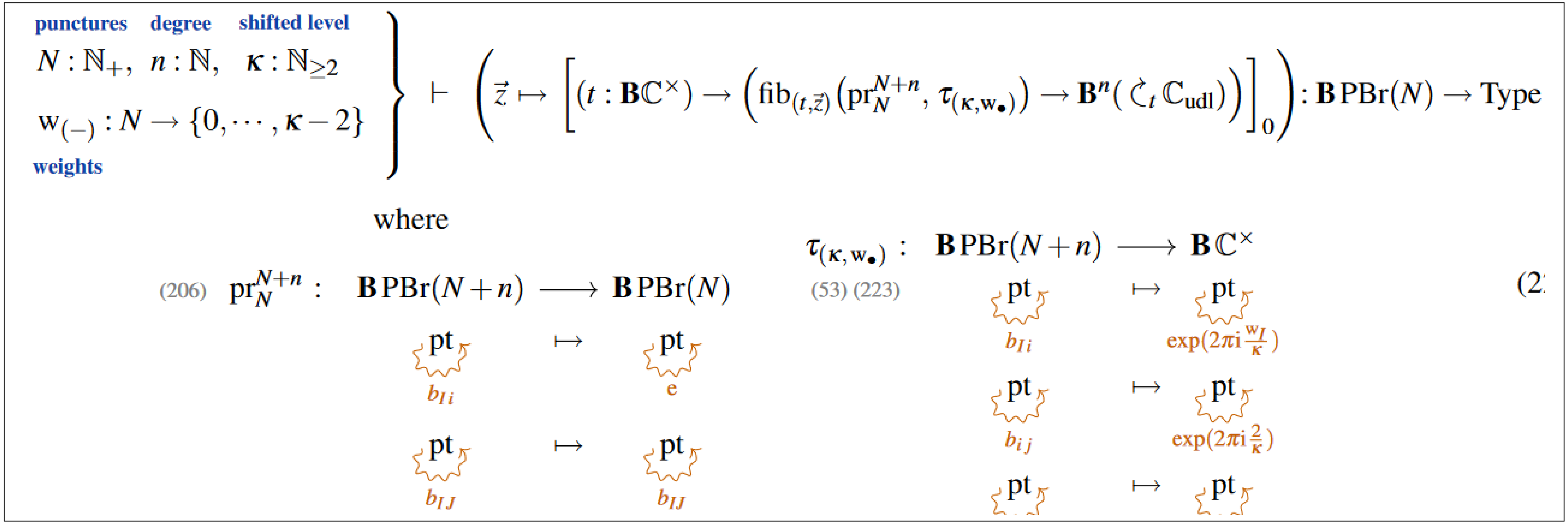
Riley: “Linear HoTT as a Quantum Certification Language”
[`mvr.hosting.nyu.edu/pubs/translation.pdf`]

the homotopy-typing reflects
**topological quantum gates**

2nd punchline

**Theorem:** *Type transport in the following dependent type family expresses, in the above simplicial categorical semantics, topologial braid quantum gates of* $\mathfrak{su}(2)$*-anyons (Majorana, Fibonacci, ...)*

“Topological Quantum Gates
in Homotopy Type Theory”
Thm. 6.8 [`arXiv:2303.02382`]



$$\left.\begin{array}{l} \overset{\text{punctures}}{N : \mathbb{N}_+}, \; \overset{\text{degree}}{n : \mathbb{N}}, \; \overset{\text{shifted level}}{\kappa : \mathbb{N}_{\geq 2}} \\ \underset{\text{weights}}{\mathrm{w}_{(-)} : N \to \{0, \cdots, \kappa - 2\}} \end{array}\right\} \vdash \left(\vec{z} \mapsto \Big[(t : \mathbf{B}\mathbb{C}^\times) \to \Big(\mathrm{fib}_{(t,\vec{z})}\big(\mathrm{pr}^{N+n}_N, \tau_{(\kappa,\mathrm{w}_\bullet)}\big) \to \mathbf{B}^n(\circlearrowleft_t \mathbb{C}_{\mathrm{udl}})\Big)\Big]_0\right) : \mathbf{B}\,\mathrm{PBr}(N) \to \mathrm{Type}$$

where

(206) $\mathrm{pr}^{N+n}_N : \mathbf{B}\,\mathrm{PBr}(N+n) \longrightarrow \mathbf{B}\,\mathrm{PBr}(N)$

$\underset{b_{Ii}}{\mathrm{pt}} \mapsto \underset{\mathrm{e}}{\mathrm{pt}}$

$\underset{b_{IJ}}{\mathrm{pt}} \mapsto \underset{b_{IJ}}{\mathrm{pt}}$

(53) (223) $\tau_{(\kappa,\mathrm{w}_\bullet)} : \mathbf{B}\,\mathrm{PBr}(N+n) \longrightarrow \mathbf{B}\mathbb{C}^\times$

$\underset{b_{Ii}}{\mathrm{pt}} \mapsto \underset{\exp(2\pi\mathrm{i}\frac{\mathrm{w}_I}{\kappa})}{\mathrm{pt}}$

$\underset{b_{ij}}{\mathrm{pt}} \mapsto \underset{\exp(2\pi\mathrm{i}\frac{2}{\kappa})}{\mathrm{pt}}$

$\mathrm{pt} \mapsto \mathrm{pt}$

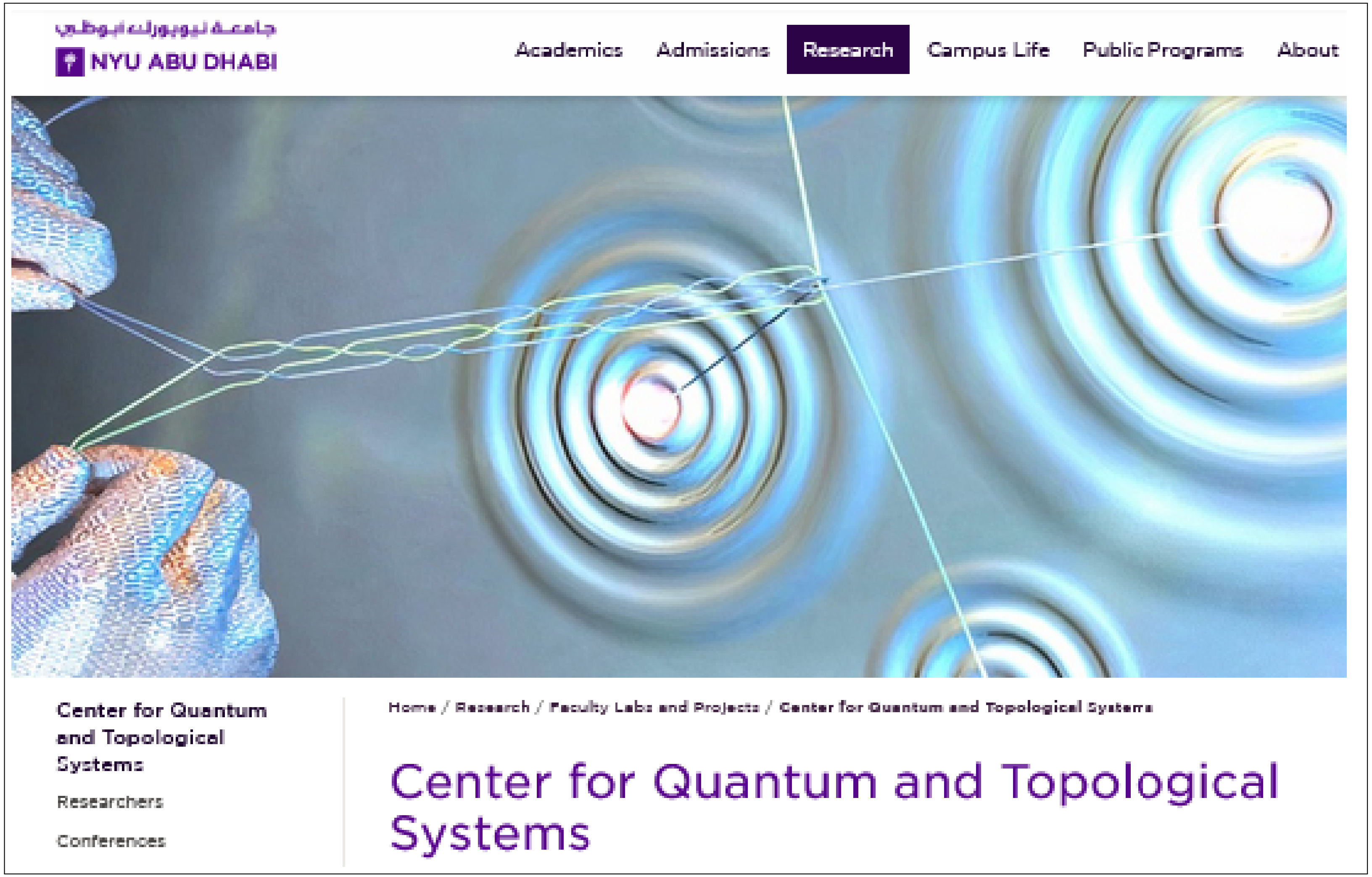
(2

in combination $\Rightarrow$ major potential use-case:
*certifying topological quantum compilation*
(arguably inevitable for industry-scale TQC)

the reported results are theoretical ($\exists$ on paper)

If you are a young energetic type theorist
interested in coding implementations $\Rightarrow$
contact us: `nyuad.cqts.info@nyu.edu` !

Thanks!



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Conferences